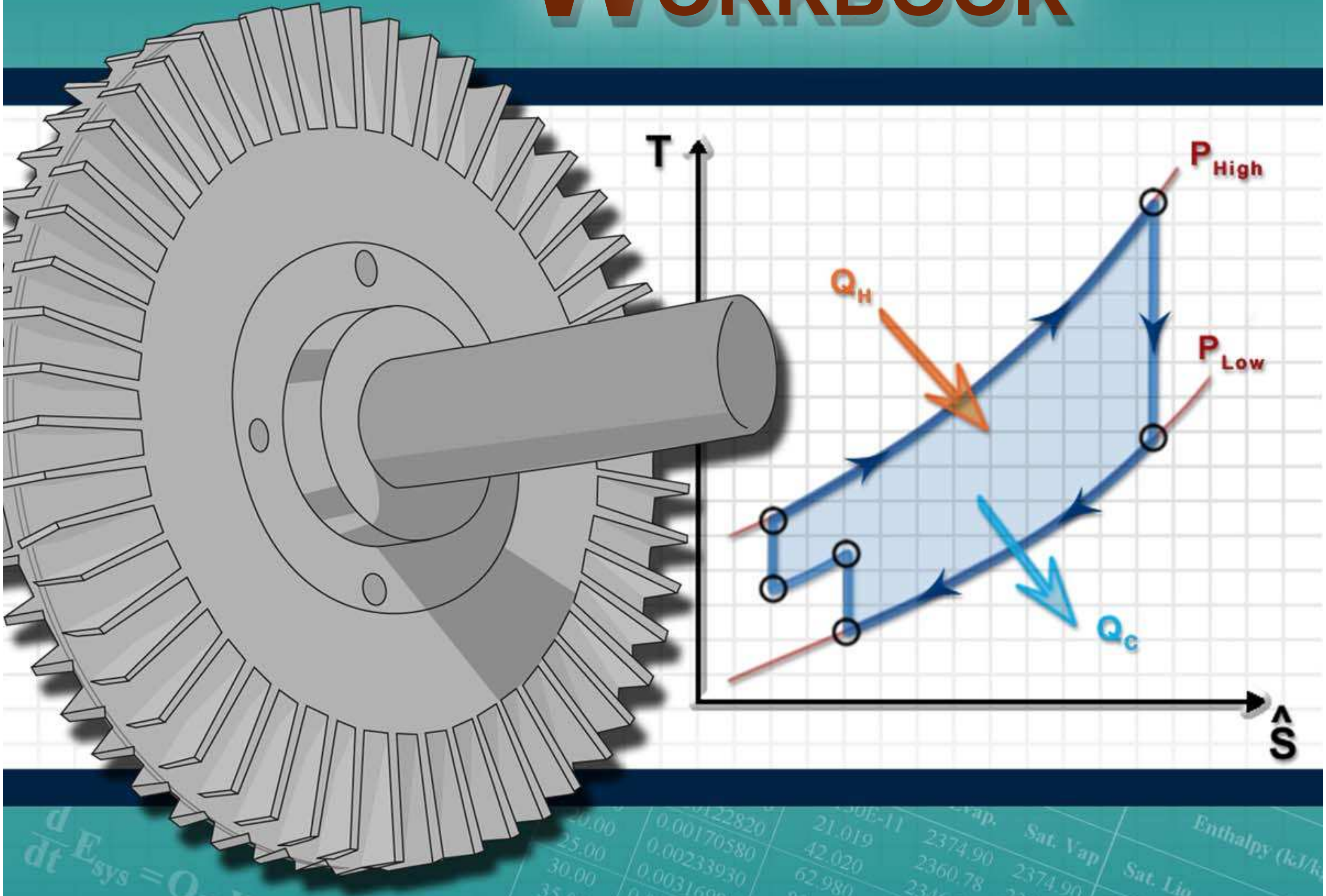
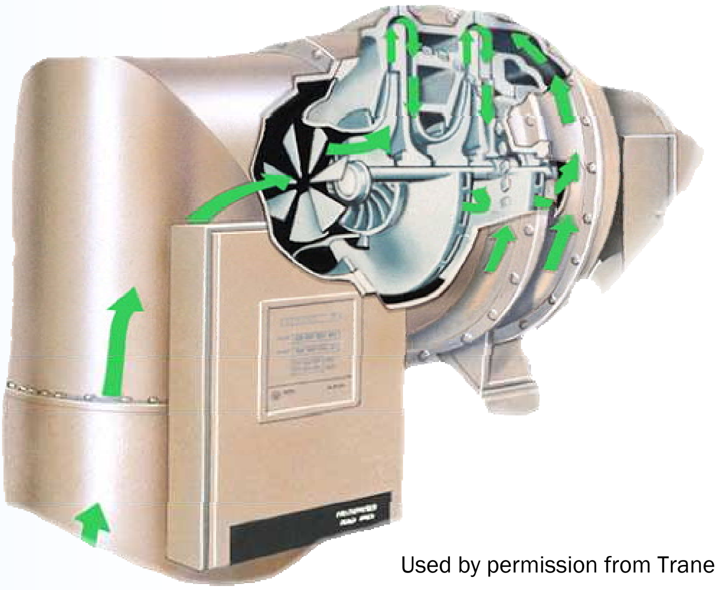
LEARN THERMODYNAMICS WORKBOOK



by

William B. Baratuci

LearnThermo



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Introduction to Engineering Thermodynamics

4th Edition

2014

By

William B. Baratuci

B - Cubed

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Foreword

This **Workbook** is an integral part of the LearnThermo learning package for the Introduction to Thermodynamics Course. This **Workbook** is intended to provide five benefits to the student of thermodynamics.

1- The **Workbook** provides an off-line reference for the LearnThermo website.
2- It provides a structured environment for taking **notes** both in class and while interacting with the LearnThermo website.
3- It provides all of the thermodynamic data to solve the homework problems, test problems and many real-world problems.
4- The **Workbook** contains brief summaries of the chapters that make up the LearnThermo website. These include all of the key equations and concepts from each of the chapters.
5- The **Workbook** also contains the **151 example problems** from the LearnThermo website. These problems are worked out in great detail and the solutions follow the problem solving procedure developed on the LearnThermo website.

LearnThermo.com is an interactive website designed to help students learn thermodynamics without excessive reading. Learning still takes time and effort, but the mini-lecture format of each screen in the LearnThermo website allows students to see and read equations and graphs as well as to hear an explanation of their meaning, use and function.

How to Use This Workbook

Although the LearnThermo website is the centerpiece of this learning package, the key to success is the effective combination of this **Workbook** with the LearnThermo website. This **Workbook** is filled with hyperlinks to specific pages on the LearnThermo website. The header and footer on almost every page are linked to the website. Almost every image in the chapters of the **Workbook** is linked to the related information on the website. Some of the links in Chapter 1 are highlighted to help you learn to use the links.

A good strategy is to take **notes** while studying the LearnThermo website. It might be best to take those **notes** in this **Workbook** using the highlighting and sticky-note capabilities of **Acrobat Reader 11** (or newer). Then, you can bring this **Workbook** to class. You can ask **questions** in class based on the **notes** derived from the LearnThermo website and supplement your **notes** with information from the lecture. This process of repeated exposure to the material, linked by this **Workbook**, helps many students learn thermodynamics faster, more easily and more thoroughly. I hope it works well for you.

Learning thermodynamics is challenging. I hope that the LearnThermo learning package makes it easier. Enjoy the learning process as you might enjoy practicing and training for a sporting event.

William Baratuci
March 27, 2014

TABLE OF CONTENTS

LEARNTHERMO CHAPTER SUMMARIES

LEARNTHERMO NOTEBOOK

APPENDICES - THERMODYNAMIC DATA TABLES

Appendix A – STEAM TABLES

Appendix B – AMMONIA TABLES

Appendix C – R-134A TABLES

Appendix D – IDEAL GAS PROPERTY TABLES

Appendix E - IDEAL GAS, CONSTANT PRESSURE HEAT CAPACITY

Thermodynamics
Comes from the Greek words therme (heat) and dynamis (power).

Classical Thermodynamics
Characterizes the behavior of large groups of molecules based on properties of the entire group of molecules, such as **T** & **P**.

Pure Component Thermodynamics
Characterize the behavior of systems that contain a pure component.

Phase Equilibrium Thermodynamics
Characterizes the behavior of multiple phases that exist in equilibrium with each other.

1st Law of Thermo
Energy can neither be created nor destroyed; it can only change forms.

2nd Law of Thermo
Energy in the form of heat only flows spontaneously from regions of higher **T** to regions of lower **T**.

Internal Energy
Energy associated with the structure and motion of molecules within the system.

Potential Energy
Energy associated with the position of the system within a potential field.

$$E_P = m\frac{g}{g_c}z$$

Kinetic Energy
Energy associated with the net linear or angular velocity of the system.

$$E_K = \frac{1}{2}\frac{m\,v^2}{g_c}$$

Fundamental Dimensions
Mass, Length, Time, Temperature, moles and sometimes Force.

Derived Dimensions
Can be calculated or derived by multiplying or dividing fundamental dimensions. Examples: area, velocity, density, and volume.

Dimensions
Multiplication/Division is always possible with any dimensions. Addition/Subtraction is only allowed when both quantities have the same dimensions. (If an eqn follows this rule the eqn is said to be dimensionally homogeneous.)

Closed System
A fixed amount of mass. No mass can cross the boundary of the system.

Open System
A fixed region of space, a device, or a collection of devices in which mass crosses the boundary during a process.

State
The condition of a piece of matter or system as determined by its intensive properties.

Properties
Characteristics of a substance that do not depend on the events that brought the substance to its current condition. Examples include: **P**, **T**, **m**, **V**, & **U**.

Intensive Properties
Do not depend on the size of the system during a process. Examples include: **P** & **T**.

Extensive Properties
Do depend on the size of the system. Examples include: **m**, **V**, & **U**.

Molar Properties
The ratio of any extensive property to the number of moles in the system. (intensive properties) Examples include: molar volume, and molar internal energy.

Specific Properties
The ratio of any extensive property to the mass of the system. (Intensive property) Examples include: specific volume and specific internal energy.

Process
When the value of a property of the system changes the system is in a different state.

Process Path
The series of states that a system passes through as it moves from an initial state to a final state.

Isobaric- Constant Pressure
Isothermal- Constant Temperature
Isochoric- Constant Volume

Cycle
A process in which the initial and final states are the same.

Thermodynamic Cycle
When 2 or more processes occur and the system returns to its initial states.

Quasi-Equilibrium
A process during which the system only deviates from equilibrium by an infinitesimal amount.

Equilibrium
A system is at equilibrium when no unbalanced potentials or driving forces exists within the system boundary.

Thermal Equilibrium
No **T** driving force

Chemical Equilibrium
No chemical driving force

Phase Equilibrium
No mass transfer driving force

Mechanical Equilibrium
No unbalanced forces

Pure Substance
A substance comprised of a single chemical element or compound.

Gas Phase
Molecules move randomly with more vibrational, rotational & translational energies than the liquid & solid phases & are separated by large distances, & travel a long way between collisions.

Miscible
Multiple liquid components that completely dissolve in each other. (Ex. Water and Vinegar)

Homogeneous Mixture
A uniform mixture that can often be treated as a pure substance.

Liquid Phase
Molecules move randomly with more vibrational, rotational & translational energies than the solid phase but less than the gas phase but are much closer together than in the gas phase & cannot travel far between collisions.

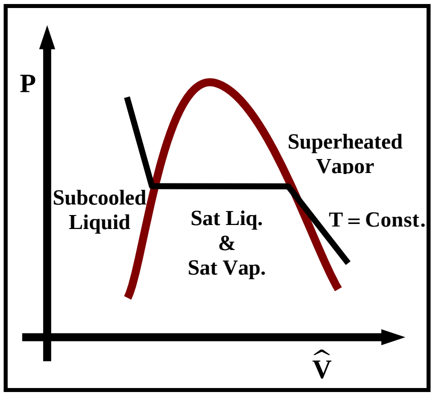
Immiscible
Multiple liquid components that do not dissolve in each other. (Ex. Oil and Water)

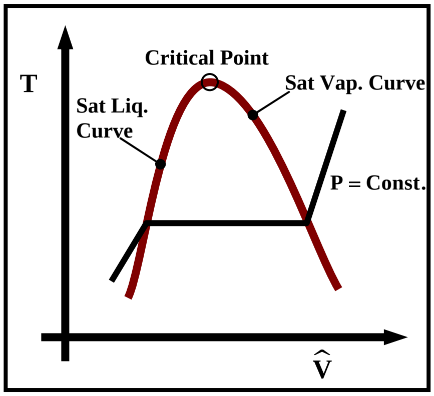
Heterogeneous Mixture
A non-uniform mixture that cannot be treated as a pure substance.

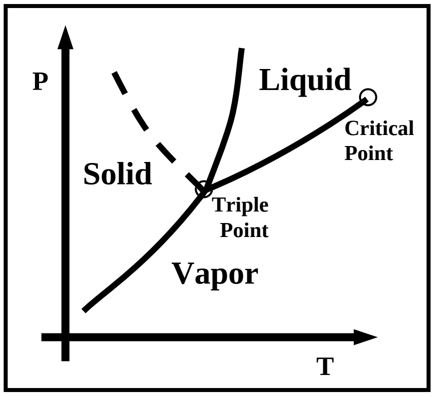
Solid Phase
Molecules move with less vibrational, rotational & translational energies than the gas & liquid phases & are very close together & cannot travel far at all before they collide.

Molar Volume $\tilde{V} = \dfrac{V}{n}$

Specific Volume $\hat{V} = \dfrac{V}{m}$







Subcooled Liquid
A liquid at a T below its b.p. ($T < T_{sat}$) for the existing **P**.

Saturated Liquid
A liquid at exactly T_{sat} at which it would boil at the existing **P** ($P = P^*$).

Saturated Mixture
A mixture of sat. liquid & sat. vapor in equilibrium at T_{sat} and P^*.

Saturated Vapor
A vapor at exactly Tsat at which it would condense at the existing **P** ($P = P^*$).

Superheated Vapor
A vapor at a **T** above its b.p. ($T > T_{sat}$) for the existing **P**.

Saturated Liquid Curve
The curve where only sat liq exists.

Saturated Vapor Curve
The curve where only sat vap exists.

Critical Point
pt where the sat. liq phase = sat. vap phase

Quality

$$x = \frac{m_{vap}}{m} = \frac{m_{vap}}{m_{vap} + m_{liq}}$$

$$\hat{V} = \hat{V}_{sat\,liq} + x\left(\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}\right)$$

Vaporization or Boiling
Process where a liq changes into a vap (reverse process is condensation)

Melting
Process where a solid changes into a liq (reverse process is freezing or fusion)

Sublimation
Process where a solid evaporates directly into a vap without melting first. (reverse process is desublimation)

Triple Point
pt where solid, liq & vap can all exist in equilibrium.

Linear Interpolation

$$y_? = y_1 + \left(x_? - x_1\right)\left[\frac{y_2 - y_1}{x_2 - x_1}\right]$$

Partial Pressure
The fraction of the total **P** in a gas phase due to the presence of 1 particular substance.

Pure Substance $P_i = (y_i)(P_{total})$

At equilibrium: $P_i = P_{total} = P^*$

Single Condensible Species

$$P_i = (y_i)(P_{total})$$
$$P_j = (y_j)(P_{total})$$
$$y_i + y_j = 1$$
$$P_i + P_j = P_{tot}$$

At equilibrium: $P = P_i = P_i^*(T)$

Humidity
Refers to an air-water system.

Saturation
Refers to any gas-vapor system.

Relative Humidity/Saturation

$$h_r = s_r = \frac{P_i}{P_i^*(T)} \times 100\%$$

Absolute Humidity/Saturation

$$h_a = s_a = \frac{P_i\,Mw_i}{P_j\,Mw_j}$$

IDEAL GAS PHASE ONLY

Evaporation
Occurs at liq-vap interface
$P_{total} > P^*_{H_2O}(T_{liq}) > P_{H_2O}$

Boiling
Occurs at solid-liq interface
$P^*_{H_2O}(T_{liq}) \geq P_{total}$

Boiling Point (T_{bp})
Lowest T that a pure liq can boil at P.
$P^*_{H_2O}(T_{bp}) = P_{total}$

Normal Boiling Point (T_{nbp})
Lowest T that a pure liq can boil at 1 atm.
$P^*_{H_2O}(T_{nbp}) = 1\,atm$

Equations of State (EOS)

An equation that relates **P**, **T** & molar **V** of a substance.

Ideal Gas EOS

Good approximation at high **T** (above 0^oC) & low **P** (1 atm or less).

$$PV = nRT$$

$$P\tilde{V} = RT$$

Universal Gas Constant

$$
\begin{aligned}
& 8.3144621 \text{ kJ/kmol}\bullet\text{K} \\
& 8.3144621 \text{ kPa}\bullet\text{m}^3\text{/kmol}\bullet\text{K} \\
& 0.083144621 \text{ bar}\bullet\text{m}^3\text{/kmol}\bullet\text{K} \\
R = {} & 82.05746 \text{ L}\bullet\text{atm/kmol}\bullet\text{K} \\
& 1.985884 \text{ Btu/lbmol}\bullet\text{R} \\
& 1545.348 \text{ ft}\bullet\text{lb}_f\text{/lbmol}\bullet\text{R} \\
& 10.73159 \text{ psia}\bullet\text{ft}^3\text{ / lbmol}\bullet\text{R}
\end{aligned}
$$

Ideal Gas at Std. Cond.

0^oC, 1atm: 22.415 L / mol

Ideal Gas EOS Validity

$$\varepsilon = \frac{X_{ideal} - X_{true}}{X_{true}} \times 100$$

$|\varepsilon| < 1\%$ if :

$\tilde{V} = RT/P > 20\,L/mol$ for most gases

$\tilde{V} = RT/P > 5\,L/mol$ for diatomic & noble gases

Graphical EOS:

Compressibility Charts

Z accounts for the deviation from IG.

$$PV = nZRT$$

$$P\tilde{V} = ZRT$$

$$Z = \frac{PV}{nRT}$$

Critical Properties

Properties at the critical point such as the critical **T**, critical **P** & critical molar **V** (although the ideal critical molar volume is ofen used instead).

$$\tilde{V}_C^{ideal} = RT_C / P_C$$

Reduced Properties

Dimensionless properties determined from critical properties.

$$P_R = \frac{P}{P_C} \qquad T_R = \frac{T}{T_C}$$

$$\tilde{V}_R^{ideal} = \frac{\tilde{V}}{\tilde{V}_C^{ideal}} = \frac{\tilde{V}}{RT_C / P_C}$$

Virial EOS

Based on the IG EOS where **Z** is expanded as an infinite power series.

$$Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B(T)}{\tilde{V}} + \frac{C(T)}{\tilde{V}^2} + \ldots$$

Truncated Virial EOS

$$Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B(T)}{\tilde{V}}$$

Species		**ω**
Ammonia		0.250
Argon	-	0.004
Carbon Dioxide		0.225
Carbon Monoxide		0.049
Chlorine		0.073
Ethane	0.	098
Hydrogen Sulfide		0.100
Methane		0.008
Methanol		0.559
Nitrogen		0.040
Oxygen	0.	021
Propane	0.	152
Sulfur Dioxide		0.251
Water	0.	344

Estimating the first virial coefficient:

$$B = \frac{RT_C}{P_C}\left(B_0 + \omega B_1\right)$$

$$B_0 = 0.083 - \frac{0.422}{T_R^{1.6}}$$

$$B_1 = 0.139 - \frac{0.172}{T_R^{4.2}}$$

Van der Waal EOS

Constant b accounts for the volume the molecules occupy.

$a/\tilde{V}_2$ accounts for attractive forces between molecules (Van der Waal's forces).

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}^2}$$

$$a = \frac{27 R^2 T_C^2}{64 P_C} [=] \frac{P \bullet V^2}{mol^2}$$

$$b = \frac{RT_C}{8P_C} [=] \frac{V}{mol}$$

Redlich-Kwong

Modification of the Van der Waal EOS which includes a correction for the **T** dependence of the Van der Waal attraction term.

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}\left(\tilde{V} + b\right)T^{1/2}}$$

$$a = 0.42748 \frac{R^2 T_C^{5/2}}{P_C}$$

$$b = 0.08664 \frac{R T_C}{P_C}$$

Soave-Redlich-Kwong

$$P = \frac{RT}{\tilde{V} - b} - \frac{\alpha a}{\tilde{V}\left(\tilde{V} + b\right)}$$

$$a = 0.42748 \frac{R^2 T_C^2}{P_C}$$

$$b = 0.08664 \frac{R T_C}{P_C}$$

$$m = 0.48508 + 1.55171\omega - 0.1561\omega^2$$

$$\alpha = \left[1 + m\left(1 - \sqrt{T_R}\right)\right]^2$$

Real Gases
U is a strong function of T & a weak function of P.
H is a function of both T & P.

Enthalpy $H = U + PV$

Ideal Gases
U is a function of T only.
H is a function of T only.

Incompressible Liquid & Solid
U is a function of T only.
H is a function of both T & P.

NIST WebBook
http://webbook.nist.gov

NIST Thermophysical Properties of Fluid Systems
http://webbook.nist.gov/chemistry/fluid/

Reference State
A state (**T**, **P**, phase) where 1 property is arbitrarily assigned a value of zero & from this all other properties are determined.

Molar Heat Capacity
Energy required to raise the **T** of a mole of a substance by 1 degree.

Constant V Heat Capacity

$$\tilde{C}_V = \left(\frac{\partial \tilde{U}}{\partial T}\right)_{\tilde{V}}$$

Constant P Heat Capacity

$$\tilde{C}_P = \left(\frac{\partial \tilde{H}}{\partial T}\right)_{P}$$

Specific Heat Capacity
Energy required to raise the **T** of a unit mass of a substance by 1 degree.

Constant V Specific Heat

$$\hat{C}_V = \left(\frac{\partial \hat{U}}{\partial T}\right)_{\hat{V}}$$

Constant P Specific Heat

$$\hat{C}_P = \left(\frac{\partial \hat{H}}{\partial T}\right)_{P}$$

Heat Capacity Ratio

$$\gamma = \frac{\tilde{C}_P}{\tilde{C}_V} = \frac{\hat{C}_P}{\hat{C}_V}$$

IG Heat Capacities
U & **H** are functions of **T** only.

Indicates IG

$$\tilde{C}_P^{o} = \left(\frac{\partial \tilde{H}}{\partial T}\right) \qquad \tilde{C}_V^{o} = \left(\frac{\partial \tilde{U}}{\partial T}\right)$$

Gibbs Phase Rule
Lets you determine how many intensive variables you can choose arbitrarily for a system.

$$°F = C - P + 2$$

Change in Internal Energy
As a function of IG const **V** heat capacity.

$$\Delta\tilde{U} = \int_{T_1}^{T_2} \tilde{C}_V^{o}\, dT$$

Change in Enthalpy
As a function of IG const **P** heat capacity.

$$\Delta\tilde{H} = \int_{T_1}^{T_2} \tilde{C}_P^{o}\, dT$$

IG Heat Capacity Relationships

$$\tilde{C}_P^{o} = \tilde{C}_V^{o} + R$$

$$\hat{C}_P^{o} = \hat{C}_V^{o} + \frac{R}{MW}$$

IG Heat Capacity Approx
For near room temperature

$\tilde{C}_P^{o} = (5/2)R$ Monatomic Gases:

$\tilde{C}_P^{o} = (7/2)R$ Diatomic Gases:

For Liq & Solids
(over a moderate **T** & **P** range)

$$\hat{C}_P = \hat{C}_V$$

$$\tilde{C}_P = \tilde{C}_V$$

NIST IG Heat Capacity Polynomial
Approx.in terms of **T** where **a**, **b**, **c**, **d** & **e** are constants for a given substance.

$$\tilde{C}_P^{o} = A + Bt + Ct^2 + Dt^3 + E/t^2$$

$$\tilde{C}_P^{o}\,[=]\,J/mol\cdot K \qquad t = T(K)/1000$$

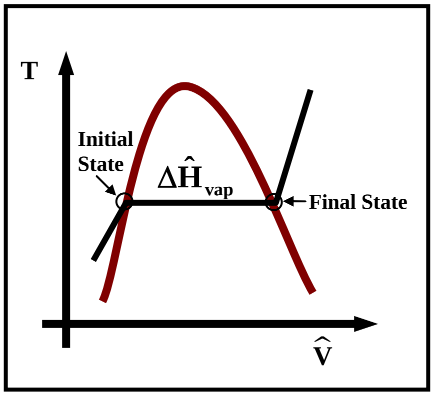
Process Path
The actual series of states that the system passes through as it moves from the initial state to the final state during a process.

State Variables
They are not path dependent.

Hypothetical Process Path
A path that connects the initial & final states and allows you to solve the problem easier. Construct a path in which each step involves a change in just one key intensive property (usually **P**, **T**, or **V**).

IG HPP Simplification
The gas must be considered an IG at both the initial & final states.

$$\Delta\tilde{H} = \int_{T_1}^{T_2} \tilde{C}_P^{o}\, dT$$



Enthalpy of Vaporization
Energy needed to convert 1 mol of sat liq into 1 mol sat vap at T_{sat} & **P***.

$$\Delta\hat{H}_{vap} = \hat{H}_{sat\,vap} - \hat{H}_{sat\,liq}$$

Clapeyron Equation
Relationship of how **P*** depends on **T**.

$$\frac{dP^*}{dT} = \frac{\Delta\tilde{H}_{vap}}{T\left(\tilde{V}_{sat\,vap} - \tilde{V}_{sat\,liq}\right)}$$

Clausius-Clapeyron Equation

1- Plot **Ln P*** vs. **1/T(K)** and use the slope to determine **ΔHvap**.
2- Linearly interpolate on **Ln P*** vs. **1/T(K)** to determine **P*** at an intermediate T.
3- Correlate **P***, **T** data by fitting it to an eqn in terms of **Ln P*** vs. **1/T(K)**.

$$Ln\,P^* = -\frac{\Delta\tilde{H}_{vap}}{R}\frac{1}{T} + C$$

Antoine Equation

$$Log_{10}P^* = A - \frac{B}{T + C}$$

Work

Is energy transfer associated with a force acting through a distance.

$$W = \int_1^2 F\,dx$$

Work Rate (Power)

$$\dot{W} = \frac{\delta W}{dt}$$

Watt (W): $1\,W = 1\,J/s$

Accelerational Work

W associated w/ change in velocity of sys.

$$W_a = \frac{m}{2g_c}\left(v_2^2 - v_1^2\right)$$

Gravitational Work

W done by or against a gravitational field.

$$W_g = m\left(g/g_c\right)\left(z_2 - z_1\right)$$

Electrical Work

W done on the sys whenever electrons cross the boundary in response to an externally supplied electrical potential.

Heat

A form of energy that is transferred between two systems driven by ΔT.

$$Q = \int_{t_1}^{t_2} \dot{Q}\,dt \qquad Q = \dot{Q}\,\Delta t$$

Heat Flux (W/m^2)

Heat transfer rate per unit area (area of heat transfer normal/perpendicular to the direction of heat flow).

$$\dot{Q} = \int_A \dot{q}\,dA$$

Conduction

A mode of heat transfer where more energetic atoms, molecules or electrons interact/collide w/less energetic ones.

Fourier's Law of Conduction

Heat conduction flux is proportional to the T gradient driving force. The proportionality constant, **k**, is the thermal conductivity.

$$q = \frac{\dot{Q}_{cond}}{A} = -k\frac{dT}{dx}$$

Conduction	k (W/m K) Order of magnitude
Metals	100
Nonmetallic Solids	1 - 10
Liquids	0.1 -10
Insulation Materials	0.1 – 0.01
Gases	0.1 – 0.01

Exact Differential

The differentials of state variables (**V**, **T**, **P**, **U** & **H**) are exact (they depend only on the state and not the details of the process) & are represented by the prefix "**d**".

Inexact Differential

The differentials of path variable (**W** & **Q**) are inexact (we need to know the process path). & are represented by the prefix "δ".

Mechanical Work

W associated with the movement of the boundary of a sys or with the movement of the entire sys as a whole. (includes shaft, boundary & spring work)

Spring Work

W that results in a change in the length or displacement of a spring.

$$W_{sp} = 1/2\,k_{sp}\left(x_2^2 - x_1^2\right)$$

Thermo-CD Sign Convention

W > 0	work done on the surr (positive)
W = 0	no work done
W < 0	work done on the sys (negative)
Q > 0	heat transferred to the sys (positive)
Q = 0	no heat transferred (adiabatic)
Q < 0	heat transferred from the sys (negative)

Adiabatic Process

A process where no heat transfer across the sys boundary occurs.

$$Q = 0$$

Convection

A mode of heat transfer usually between a solid surface at one **T** & an adjacent moving fluid at another **T**. **Forced**- fluid motion caused by external forces. **Free**- fluid motion caused by density differences & buoyancy forces within the fluid phase.

Newton's Law of Cooling

Convection heat flux at a solid-fluid interface is proportional to **ΔT** between the bulk fluid & the solid surface. The proportionality constant, **h**, is the convective heat transfer coefficient.

$$q = \frac{\dot{Q}_{conv}}{A} = h\left(T_s - T_f\right)$$

Condvection	h (W/m^2 K)
Free Conv. of Gases	2 - 25
Free Conv. of Liquids	50 - 1000
Forced Conv. of Gases	25 - 250
Forced Conv. of Liquids	50 -20000
Boiling Phase Change	2500 - 100000

Quasi-Equilibrium Process

A process in which the sys remains in equilibrium at all times.

$$W_b = nRT\,Ln\frac{V_2}{V_1}$$

IG Isothermal

$$W_b = \frac{P_2V_2 - P_1V_1}{1-n}$$

Polytropic
V & **P** are related by: **PV^n=C**

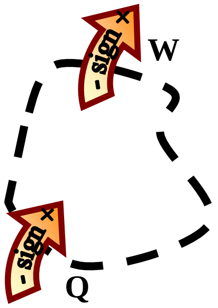
$$W_b = \frac{nR(T_2 - T_1)}{1-n}$$

IG Polytropic

Shaft Work

W associated w/energy transmission what is either driven by a rotating shaft (pump/compressor) or that causes a shaft to rotate (turbine).

$$W_{sh} = 2\pi\tau N \qquad \dot{W}_{sh} = 2\pi\tau\dot{N}$$



Radiation

A mode of heat transfer that is caused by the emission of photons by the molecules giving up energy & the absorption of photons by the molecules gaining energy.

Stefan-Boltzmann Law

Allows you to calculate the radiation heat transfer rate between 2 objects based on **T** & surface properties of the emitter & absorber.

$$\dot{Q}_{e,max} = \sigma A T_s^4$$

Max

$$\dot{Q}_e = \varepsilon\sigma A T_s^4$$

Real Surfaces

$$\dot{Q}_{net} = \varepsilon\sigma A\left(T_s^4 - T_{surr}^4\right)$$

Net

Emissivity

Quantifies the ability of a surface to radiate. Blackbody ε = **1** & Real substance **0** < ε < **1**.

Absorptivity

Fraction of the incident radiation energy that is absorbed by a surface. Blackbody α = **1** & Real substances **0** < α < **1**.

First Law of Thermodynamics

Relationship between **Q**, **W** & **E**$_{total}$.
Energy cannot be created or destroyed; it can only change forms.

$$\Delta E = \Delta E_K + \Delta E_P + \Delta U$$

$$\Delta E = E_2 - E_1$$

$$\frac{dE}{dt} = \frac{dE_P}{dt} + \frac{dE_K}{dt} + \frac{dU}{dt} = \dot{Q} - \dot{W}$$

First Law for Adiabatic, Closed Systems

The <u>only</u> form of energy which crosses the sys boundary is **W**. For this process the total work depends <u>only</u> on the initial & final states (not path dependent).

$$\Delta E = \Delta E_K + \Delta E_P + \Delta U = -W_{ad}$$

$$\Delta E = E_2 + E_1 = -\left(W_b + W_{sh} + W_e\right)$$

First Law for Non-Adiabatic, Closed Systems

Energy can cross the sys boundary as <u>both</u> **Q** & **W**. For this process the total work <u>depends</u> on the process path.

$$\Delta E = \Delta E_K + \Delta E_P + \Delta U = Q - W$$

$$\Delta E = Q - \left(W_b + W_{sh} + W_e\right)$$

$$dE = \delta Q - \delta W$$

Problem Solving Procedure

1 – Read Carefully
2 – Draw a Diagram
3 – List Given Information
4 – List All Assumptions
5 – Write Equations and Lookup Data
6 – Solve Equations
7 – Verify Assumptions
8 - Answers

First Law for a Thermodynamic Cycle

A sys completes a thermo cycle when it undergoes two or more processes & the system returns to its initial state.

$$Q_{cycle} = W_{cycle}$$

First Law for an Isobaric, Quasi-Equilibrium Process in a Closed System

$$\Delta U = Q - W_{other} - P\left(V_2 - V_1\right)$$

$Q = \Delta H$ if process involves only W_b

Reservoirs

Systems that are so large that their intensive properties (**T**, **P**) always remain constant.

First Law for an Isochoric, Quasi-Equilibrium Process in a Closed System

$$\Delta U = Q - W_{other} - W_b$$

$Q = \Delta U$ if process involves only W_b

First Law for Power Cycles

Purpose: to transfer a net work to the surr.

$$W_{HE} = Q_H - Q_C$$

Thermal Efficiency for Power Cycles

$$\eta = \frac{W_{HE}}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\eta = 1 - \frac{Q_C}{Q_H}$$

First Law for Refrigeration Cycles

Purpose: to maintain the **T** within the sys to be <u>below</u> the **T** of the surr.

$$W_R = Q_H - Q_C$$

COP for Refrigeration Cycles

$$COP_R = \frac{Q_C}{W_{ref}} = \frac{Q_C}{Q_H - Q_C}$$

$$COP_R = \frac{1}{Q_H / Q_C - 1}$$

First Law for Heat Pump Cycles

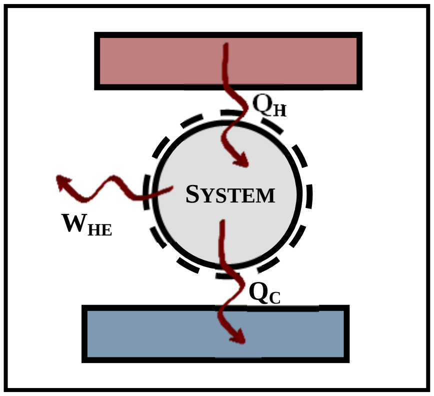
Purpose: to maintain the **T** within the system to be above the **T** of the surr.

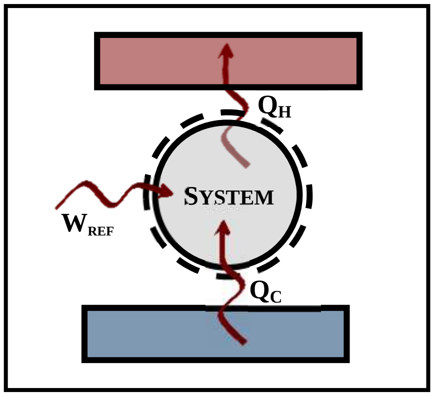
$$W_{HP} = Q_H - Q_C$$

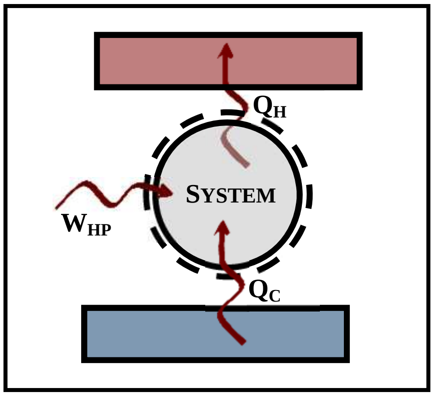
COP for Heat Pump Cycles

$$COP_{HP} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_C}$$

$$COP_{HP} = \frac{1}{1 - Q_C / Q_H}$$







Mass Balance

Mass cannot be created or destroyed.

Integral Mass Balance

$$\Delta m_{sys} = \sum_i m_{in,i} - \sum_j m_{out,j}$$

$$\Delta m_{sys} = (m_{sys})_{initial} - (m_{sys})_{final} = m_{in} - m_{out}$$

Integral Mass Balance

$$m = \rho \langle v \rangle A = \frac{\langle v \rangle A}{\hat{V}}$$

Differential Mass Balance

$$\frac{d}{dt} m_{sys} = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{d}{dt} m_{sys} = \sum_i \dot{m}_{in} - \sum_j \dot{m}_{out}$$

Integral Energy Balance

$$\Delta E_{sys} = (E_{sys})_{initial} - (E_{sys})_{final} = E_{in} - E_{out}$$

Flow Work

Work done when a volume of fluid flows into or out of the system opposed by a pressure.

$$\dot{W}_{flow} = P_{out}\left(\dot{m}_{out}\,\hat{V}_{out}\right) - P_{in}\left(\dot{m}_{in}\,\hat{V}_{in}\right)$$

Differential Energy Balance

$$\frac{d}{dt} E_{sys} = \dot{E}_{in} - \dot{E}_{out}$$

$$\frac{d}{dt} E_{sys} = \dot{Q} - \dot{W} + \dot{m}_{in}\left[\hat{U}_{in} + \frac{\langle v_{in} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{in}\right] - \dot{m}_{out}\left[\hat{U}_{out} + \frac{\langle v_{out} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{out}\right]$$

Enthalpy Form of the Differential Energy Balance

$$\frac{d}{dt} E_{sys} = \dot{Q} - \dot{W}_{sh} + \dot{m}_{in}\left[\hat{H}_{in} + \frac{\langle v_{in} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{in}\right] - \dot{m}_{out}\left[\hat{H}_{out} + \frac{\langle v_{out} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{out}\right]$$

$$\frac{d}{dt} E_{sys} = \dot{Q} - \dot{W}_{sh} + \sum_{i}^{in} \dot{m}_{in}\left[\hat{H}_{in} + \frac{\langle v_{in} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{in}\right] - \sum_{j}^{out} \dot{m}_{out}\left[\hat{H}_{out} + \frac{\langle v_{out} \rangle^2}{2 g_c} + \frac{g}{g_c} z_{out}\right]$$

SS Mass & Energy Balance

$$\frac{d}{dt} m_{sys} = 0 \qquad \dot{m}_{in} = \dot{m}_{out}$$

$$\frac{d}{dt} E_{sys} = 0$$

SISO, SS with no electrical or boundary work interactions

$$\dot{Q} - \dot{W}_{sh} = \dot{m}\left[\Delta \hat{H} + \frac{\Delta v^2}{2 g_c} + \frac{g}{g_c} z\right]$$

1st Law for SISO, SS Nozzles & Diffusers	$\Delta \hat{H} + \frac{\Delta v^2}{2 g_c} = 0$	NOZZLE- increases the velocity of a fluid by decreasing P. DIFFUSER- increases the **P** of a fluid by decreasing its velocity.
1st Law for SISO, SS Turbines & Compressors	$\dot{W}_{sh} = -\dot{m}\,\Delta \hat{H}$	TURBINE- **W** is produced by decreasing the enthalpy. COMPRESSOR- **W** is input to increase the **H** & **P** of fluid.
1st Law for SISO, SS Throttling Devices	$\Delta \hat{H} = 0$	THROTTLING DEVICES- Decreases **P** no **W** & little heat transfer.
1st Law for SISO, SS Heat Exchanger	$\dot{Q} = \dot{m}\,\Delta \hat{H}$ $m_C\,\Delta \hat{H}_C = -m_H\,\Delta \hat{H}_H$	HEAT EXCHANGER- 2 streams exchange heat without mixing.
1st Law for MIMO, SS Mixing Chamber	$\sum_{j}^{out} \dot{m}_j \hat{H}_j - \sum_{i}^{in} \dot{m}_i \hat{H}_i = 0$	MIXING CHAMBER- 2 or more feed streams mix & form 1 effluent.
Bernoulli Equation	$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2 g_c} + \frac{g}{g_c} \Delta z = 0$	PIPES OR DUCTS- fluid flow follows Bernoulli Equation

Transient Process (or Unsteady Process)
A process in which 1 or more properties or process variables change as a function of time.

Uniform Flow
The properties and flow rates of all inlet and outlet streams are constant over the cross-sectional area for flow and are also constant with respect to time.

Uniform State
At all times, the properties of the outlet stream are exactly the same as the properties of the system at that point in time. The properties do change with time, but they are always the same as the properties of the outlet stream.

SISO Transient Mass Balance

$$\Delta m_{sys} = m_{sys}(t_2) - m_{sys}(t_1) = \dot{m}_{in}(t_2 - t_1) - \dot{m}_{out}(t_2 - t_1) = m_{in} - m_{out}$$

MIMO Transient Mass Balance

$$\Delta m_{sys} = m_{sys}(t_2) - m_{sys}(t_1) = (t_2 - t_1)\sum_{i}^{in} \dot{m}_i - (t_2 - t_1)\sum_{j}^{out} \dot{m}_j$$

SISO Transient Energy Balance

$$\Delta E_{sys} = Q - W_{sh} + m_{in}\left[\hat{H}_{in} + \frac{< v_{in} >^2}{2g_c} + \frac{g}{g_c} z_{in}\right] - m_{out}\left[\hat{H}_{out} + \frac{< v_{out} >^2}{2g_c} + \frac{g}{g_c} z_{out}\right]$$

MIMO Transient Energy Balance

$$\Delta E_{sys} = Q - W_{sh} + \sum_{i}^{in} m_{in}\left[\hat{H}_{in} + \frac{\langle v_{in} \rangle^2}{2g_c} + \frac{g}{g_c} z_{in}\right] - \sum_{j}^{out} m_{out}\left[\hat{H}_{out} + \frac{\langle v_{out} \rangle^2}{2g_c} + \frac{g}{g_c} z_{out}\right]$$

SISO, Uniform State Energy Balance with Negligible Kinetic and Potential Energies

$$m_{sys,2}\hat{U}_{sys,2} - m_{sys,1}\hat{U}_{sys,1} = Q - W_{sh} + m_{in}\hat{H}_{in} - m_{out}\hat{H}_{out}$$

Power Cycles

Purpose: to transfer a net work to the surroundings.

$$W_{HE} = Q_H - Q_C$$

$$\eta = \frac{W_{HE}}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\eta = 1 - \frac{Q_C}{Q_H}$$

Refrigeration Cycles

Purpose: to maintain the **T** within the sys to be below the **T** of the surr.

$$W_R = Q_H - Q_C$$

$$COP_R = \frac{Q_C}{W_{ref}} = \frac{Q_C}{Q_H - Q_C}$$

$$COP_R = \frac{1}{Q_H / Q_C - 1}$$

Heat Pump Cycles

Purpose: to maintain the **T** within the sys to be above the **T** of the surr.

$$W_{HP} = Q_H - Q_C$$

$$COP_{HP} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_C}$$

$$COP_{HP} = \frac{1}{1 - Q_C / Q_H}$$

Clausius Statement

A system operating in a thermodynamic cycle can not transfer energy (heat) from a cooler body to a hotter body and produce no other effect.

Kelvin-Planck Statement

A system operating on a thermodynamic cycle cannot deliver a net amount of work as a result of exchange of heat from a single thermal reservoir.

Perpetual-Motion Machines

1st Kind- creating mass, energy, or work from nothing. (violates 1st Law)

2nd Kind- extracting heat from a source and converting it completely into work. (violates 2nd Law)

3rd Kind- produces no work, has no friction, and runs indefinitely.

Reversible Process

Processes where both the system and its surroundings can return to their initial states after the process is completed.

$$W_{cycle} = 0$$

$$Q_{cycle} = 0$$

$$\Delta E_{cycle} = 0$$

Internally Reversible

Tthere are no irreversibilities in the system but there may be irreversibilities in the surroundings.

Externally Reversible

There are no irreversibilities in the surroundings but there may be irreversibilities in the system.

Irreversible Process

Processes where the system and its surroundings cannot return to their initial states after the process is completed

1st Carnot Principle

No cyclic process can operate with 100% efficiency, and the reversible process is the most efficient.

$$W_{net} = W_{irrev} - W_{rev} < 0$$

$$\eta_{rev} > \eta_{irrev}$$

$$COP_{R,rev} > COP_{R,irrev}$$

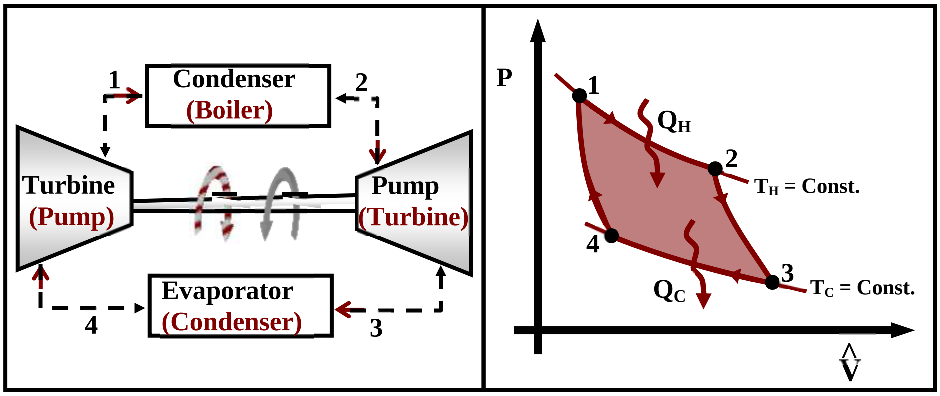
$$COP_{HP,rev} > COP_{HP,irrev}$$

2nd Carnot Principle

All reversible processes operating between the same two reservoirs will have the same efficiency.

Four Basic Processes of the Carnot Cycle

Process 1-2: Reversible Isothermal
Process 2-3: Reversible Adiabatic
Process 3-4: Reversible Isothermal
Process 4-1: Reversible Adiabatic



Kelvin Temperature Scale

Based in the triple pt of water (**273.15K**).

Celsius Temperature Scale

Based by assigning the normal melting pt of water a value of **0°C** & the normal boiling pt **100 °C**. (**273.15K = 0°C**)

Maximum Efficiencies

T in Kelvin or Rankine

$$\eta_{rev} = 1 - \frac{T_C}{T_H}$$

$$COP_{R,rev} = \frac{T_C}{T_H - T_C}$$

$$COP_{HP,rev} = \frac{T_H}{T_H - T_C}$$

Quality

100% **W** can be converted to **Q** but only fraction of **Q** can be converted to **W**.

Energy in the form of heat for HE

$\Delta T = T_H - T_C$

Large **ΔT**: thermal efficiency high

Small **ΔT**: thermal efficiency low.

Clausius Inequality

$$\oint \frac{\delta Q}{T} \le 0$$

$$\oint \frac{\delta Q}{T} = 0 \quad \text{Internally Reversible}$$

$$\oint \frac{\delta Q}{T} < 0 \quad \text{Irreversible}$$

Trends for HE

As the irreversibility of the cycle increases, the work output decreases, & the left-hand side of the Clausius Inequality becomes more negative.

Trends for Refrigerators & Heat Pumps

As the irreversibility of the cycle increases, the work input increases & the left-hand side of the Clausius Inequality becomes more negative.

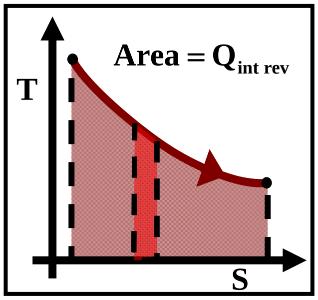
Entropy

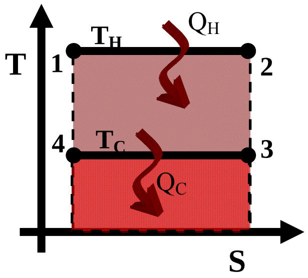
A property that allows us to apply the 2nd Law to processes & to quantitatively assess & compare their performance.

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

$$\Delta S = \left(S_2 - S_1\right)_{\text{int rev}} = \left(S_2 - S_1\right)_{\text{irrev}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

$$\Delta S = S_2 - S_1 = \frac{Q}{T_0} \quad \text{Internally Reversible, Isothermal Process}$$





Carnot Cycle

Step 1-2 $\quad Q_H = \int_1^2 T\,dS = T_H\left(S_2 - S_1\right)$

Step 2-3 $\quad S_2 = S_3$

Step 3-4 $\quad Q_C = -\int_3^4 T\,dS = T_C\left(S_3 - S_4\right)$

Step 4-1 $\quad S_4 = S_1$

1st & 2nd Gibbs Equations

$T\,dS = dU + P\,dV$ Easier to apply to closed systems.

$T\,dS = dH - V\,dP$ Easier to apply to open systems.

Incompressible Liquids & Solids

$$\Delta\tilde{S} = \int \tilde{C}(T)\frac{dT}{T}$$

$$\Delta\tilde{S} \cong \tilde{C}_{ave} \ln\frac{T_2}{T_1} \quad \text{Constant Heat Capacity}$$

Entropy Generated

Reversible Processes: $S_{gen} = 0$

Irreversible Processes: $S_{gen} > 0$

Impossible Processes: $S_{gen} < 0$

$$dS = \left(\frac{\delta Q}{T}\right) + \delta S_{gen}$$

$$\Delta S = S_2 - S_1 = \int \left(\frac{\delta Q}{T}\right) + S_{gen}$$

$$S_{gen} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} - \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{irrev}}$$

Principle of Increasing Entropy

Entropy of the universe is ALWAYS increasing.

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr} = S_{gen} \ge 0$$

Heat Transfer For an Irrev Path

$$Q = \text{Area} - \int_1^2 T\,\delta S_{gen}$$

Methods for Ideal Gases

METHOD 1: CONSTANT HEAT CAPACITY

$$\Delta\tilde{S} = \tilde{C}_V^o\ \text{Ln}\left(\frac{T_2}{T_1}\right) + R\ \text{Ln}\left(\frac{\tilde{V}_2}{\tilde{V}_1}\right)$$

$$\Delta\tilde{S} = \tilde{C}_P^o\ \text{Ln}\left(\frac{T_2}{T_1}\right) - R\ \text{Ln}\left(\frac{P_2}{P_1}\right)$$

METHOD 2: HEAT CAPACITY POLYNOMIAL

$$\tilde{C}_P^o = A + BT + CT^2 + DT^3 + E/T^2$$

METHOD 3: IDEAL GAS ENTROPY TABLES

$$\tilde{S}_T^o = \int_{Tref}^{T} \tilde{C}_P^o \frac{dT}{T}$$

$$\Delta\tilde{S} = \tilde{S}_{T2}^o - \tilde{S}_{T1}^o - R\,\text{Ln}\left(\frac{T_2}{T_1}\right) + R\,\text{Ln}\left(\frac{\tilde{V}_2}{\tilde{V}_1}\right)$$

$$\Delta\tilde{S} = \tilde{S}_{T2}^o - \tilde{S}_{T1}^o - R\,\text{Ln}\left(\frac{P_2}{P_1}\right)$$

Relative Molar Volume

$$\tilde{V}_r = \frac{\tilde{V}}{\tilde{V}_{ref}}$$

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \left(\frac{\tilde{V}_r(T_1)}{\tilde{V}_r(T_2)}\right)$$

$$\text{Ln}\,\tilde{V}_r = -\frac{\tilde{S}_T^o}{R} + \text{Ln}\left(\frac{T}{T_{ref}}\right)$$

Relative Pressure

$$P_r = \frac{P}{P_{ref}}$$

$$\frac{P_1}{P_2} = \left(\frac{P_r(T_1)}{P_r(T_2)}\right)$$

$$\text{Ln}\,P_r = \frac{\tilde{S}_T^o}{R}$$

Polytropic Processes

$$T_1\tilde{V}_1^{\delta-1} = T_2\tilde{V}_2^{\delta-1} = C_1$$

$$T_1P_1^{(1-\delta)/\delta} = T_2P_2^{(1-\delta)/\delta} = C_2$$

$$P_1\tilde{V}_1^{\delta} = P_2\tilde{V}_2^{\delta} = C_3$$

ISOBARIC PROCESS $\quad \delta = 0 \quad \tilde{W}_b = P\,\Delta\tilde{V}$

ISOTHERMAL PROCESS $\quad \delta = 1 \quad \tilde{W}_b = RT\,\text{Ln}\left(\frac{\tilde{V}_2}{\tilde{V}_1}\right)$

ISOCHORIC PROCESS $\quad \delta = \infty \quad \tilde{W}_b = 0$

ISENTROPIC PROCESS $\quad \delta = \gamma \quad \tilde{W}_b = -\tilde{C}_V^o\left(T_2 - T_1\right)$

Closed System Entropy Balance

Integral form $\Delta S = \int \left(\frac{\delta Q}{T}\right) + S_{gen}$

Differential form $dS = \left(\frac{\delta Q}{T}\right) + \delta S_{gen}$

Rate form $\frac{d}{dt}S = \frac{\delta \dot{Q}}{T} + \dot{S}_{gen}$

Reversible Processes

$\Delta S = \int \left(\frac{\delta Q}{T}\right)_{rev}$ **and** $S_{gen} = 0$

Irreversible Processes

$\Delta S > \int \left(\frac{\delta Q}{T}\right)_{irrev}$ **and** $S_{gen} > 0$

Open System Entropy Balance

$$\frac{d}{dt}S_{sys} = \sum_{in} \dot{m}_{in}\,\hat{S}_{in} - \sum_{out} \dot{m}_{out}\,\hat{S}_{out} + \sum_{sys} \int \frac{d\dot{Q}_{sys}}{T_{sys}} + \dot{S}_{gen}$$

SS $0 = \sum_{in} \dot{m}_{in}\,\hat{S}_{in} - \sum_{out} \dot{m}_{out}\,\hat{S}_{out} + \sum_{sys} \frac{\dot{Q}_{sys}}{T_{sys}} + \dot{S}_{gen}$

SS, SISO $\left(\hat{S}_{out} - \hat{S}_{in}\right) = \frac{1}{\dot{m}} \sum_{sys} \int \frac{d\dot{Q}_{sys}}{T_{sys}} + \frac{\dot{S}_{gen}}{\dot{m}}$

SS, SISO, Int. Rev., One Reservoir

$$\frac{\dot{Q}_{sys}}{\dot{m}} = \int_{in}^{out} T_{sys}\, d\hat{S}$$

SS, SISO, Int. Rev., One Reservoir, Isothermal

$$\frac{\dot{Q}_{sys}}{\dot{m}} = T_{sys}\left(\hat{S}_{out} - \hat{S}_{in}\right)$$

Mechanical Energy Balance (MEBE)

$$\frac{\dot{W}_{sh}}{\dot{m}} = -\int_{in}^{out} \hat{V}\,dP + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c}\Delta z$$

Bernoulli Eqn

$$-\int_{in}^{out} \hat{V}\,dP + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c}\Delta z = 0$$

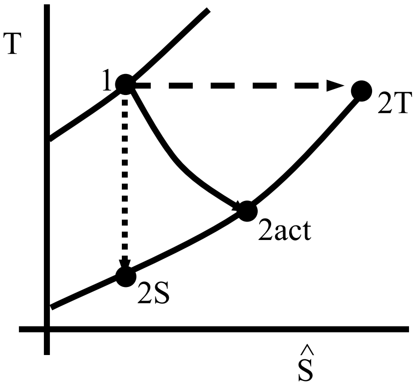
SISO, SS, Internally Reversible, Polytropic Processes where $\Delta E_P = \Delta E_K = 0$

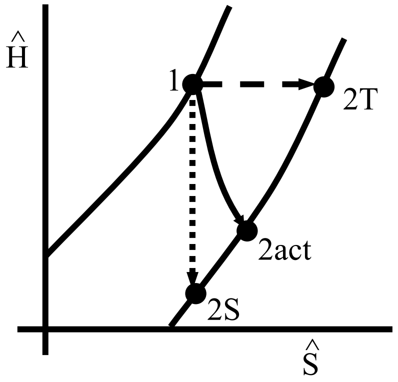
	Real Fluids	**Ideal Gas**
Isobaric $\delta = 0$	$\frac{\dot{W}_{sh}}{\dot{m}} = 0$	$\frac{\dot{W}_{sh}}{\dot{m}} = 0$
Isothermal $\delta = 1$	$\frac{\dot{W}_{sh}}{\dot{m}} = -P_1\hat{V}_1\,Ln(P_2/P_1)$ $= -P_2\hat{V}_2\,Ln(P_2/P_1)$	$\frac{\dot{W}_{sh}}{\dot{m}} = -T_1\left(\frac{R}{MW}\right)Ln(P_2/P_1)$ $= -T_2\left(\frac{R}{MW}\right)Ln(P_2/P_1)$
Polytropic $\delta \neq 1$	$\frac{\dot{W}_{sh}}{\dot{m}} = -\frac{\delta}{\delta - 1}(P_2\hat{V}_2 - P_1\hat{V}_1)$	$\frac{\dot{W}_{sh}}{\dot{m}} = -\frac{\delta}{\delta - 1}\left(\frac{R}{MW}\right)(T_2 - T_1)$
Isentropic **(γ = constant)** $\delta = \gamma$	$\frac{\dot{W}_{sh}}{\dot{m}} = -\frac{\gamma}{\gamma - 1}\left(P_2\hat{V}_2 - P_1\hat{V}_1\right)$	$\frac{\dot{W}_{sh}}{\dot{m}} = -\frac{\gamma}{\gamma - 1}\frac{R}{MW}\left(T_2 - T_1\right)$
Isochoric $\delta = \infty$	$\frac{\dot{W}_{sh}}{\dot{m}} = -V\left(P_2 - P_1\right)$	$\frac{\dot{W}_{sh}}{\dot{m}} = -\frac{R}{MW}\left(T_2 - T_1\right)$

Turbine Isentropic Efficiency

$$\eta_{S,turb} = \frac{\hat{H}_1 - \hat{H}_{2,act}}{\hat{H}_1 - \hat{H}_{2,S}}$$

Adiabatic Turbines:
Best Case = Isentropic (internally reversible
Worst Case = Isothermal (highly irreversible)





Nozzle Isentropic Efficiency

$$\eta_{S,noz} = \frac{v_{2a}^2 / 2g_c}{v_{2S}^2 / 2g_c}$$

If $A_1 >> A_2$ and the process is not highly irreversible:

$$\eta_{S,noz} = \frac{\hat{H}_1 - \hat{H}_{2,act}}{\hat{H}_1 - \hat{H}_{2,S}}$$ only exact when $v_1 = 0$

Adiabatic Compressor Isentropic Efficiency

$$\eta_{S,comp} = \frac{\hat{H}_{2S} - \hat{H}_1}{\hat{H}_{2,act} - \hat{H}_1}$$

Multistage Compressor Isentropic Efficiency

$$\eta_{T,multi} = \frac{\hat{H}_{2T} - \hat{H}_1}{\hat{H}_{2,act} - \hat{H}_1}$$

Optimal Intermediate Pressure for a 2-stage Compressor

$$P_x = \sqrt{P_1 P_2} \qquad \frac{P_1}{P_x} = \frac{P_x}{P_2}$$

Lost Work
Work that is wasted due to irreversibilities in a process.

$$\dot{W}_{Sh,lost} = \dot{W}_{Sh,rev} - \dot{W}_{Sh,act}$$

$$\dot{W}_{Sh,lost} = T_{surr}\,\dot{S}_{gen,total}$$

Processes that only exchange heat with surroundings

$$\dot{W}_{Sh,lost} = \dot{m}\,T_{surr}\Delta\hat{S} - \dot{Q}_{act}$$

$$\dot{S}_{gen,total} = \dot{m}\,\Delta\hat{S} - \frac{\dot{Q}_{act}}{T_{surr}}$$

Processes that exchange heat with a Thermal Reservoir

$$\dot{W}_{Sh,lost} = \dot{m}\,T_{surr}\Delta\hat{S} - \dot{Q}_{act}\frac{T_{surr}}{T_{res}}$$

$$\dot{S}_{gen,total} = \dot{m}\,\Delta\hat{S} - \frac{\dot{Q}_H}{T_{res}}$$

Compressor 2nd Law Efficiency

$$\eta_{II,comp} = \frac{\dot{W}_{Sh,rev}}{\dot{W}_{Sh,act}}$$

Turbine 2nd Law Efficiency

$$\eta_{II,turb} = \frac{\dot{W}_{Sh,act}}{\dot{W}_{Sh,rev}}$$

Lost Work Analysis for a Vapor Cycle

$$(\dot{S}_{gen})_{cycle} = \sum_{i}^{Processes} (\dot{S}_{gen})_i = \sum_{i}^{Processes} (-\dot{Q}_i / T_{i,ext})$$

$$\dot{W}_{Sh,lost} = T_{surr}(\dot{S}_{gen})_{cycle} = T_{surr}\sum_{i}^{Processes} (-\dot{Q}_i / T_{i,ext})$$

Rankine Cycle

Process 1-2: Boiler	Heat added at constant pressure.
Process 2-3: Turbine	Isentropic expansion.
Process 3-4: Condenser	Heat rejected at constant pressure.
Process 4-1: Pump	Isentropic compression.

Assumptions in applying the 1st Law

1- Int. Rev., SS, SISO Process

2- Negligible E_K & E_P

3- No shaft work in boiler or condenser

4- No heat transfer occurs in the turbine or pump

1st Law: Boiler

$$\frac{\dot{Q}_H}{\dot{m}} = \hat{H}_2 - \hat{H}_1$$

1st Law: Turbine

$$\frac{\dot{W}_{S,turb}}{\dot{m}} = \hat{H}_2 - \hat{H}_3$$

1st Law: Condenser

$$\frac{\dot{Q}_C}{\dot{m}} = \hat{H}_4 - \hat{H}_3$$

1st Law: Pump

$$\frac{\dot{W}_P}{\dot{m}} = \hat{H}_4 - \hat{H}_1$$

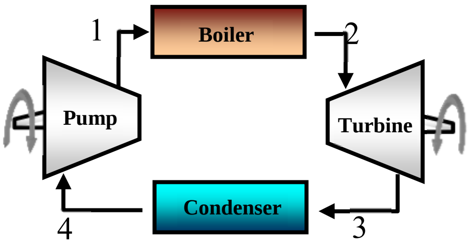
Thermal Efficiency of a Power Cycle

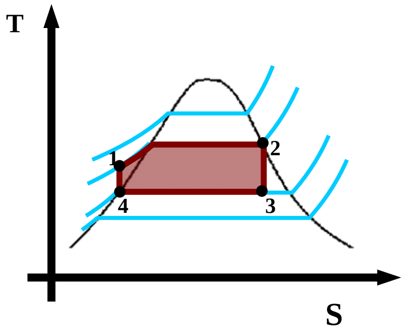
$$\eta = \frac{W_{cycle}}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

Back Work Ratio (BWR)

Ratio of pump work required and turbine work generated.

$$BWR = \frac{-\hat{W}_P}{\hat{W}_T} = \frac{\hat{H}_1 - \hat{H}_4}{\hat{H}_2 - \hat{H}_3}$$





Increasing Boiler Pressure

Q_H increases significantly and Q_C decreases slightly.

Therefore, η_t increases.

Decreasing Condenser Pressure

Q_C decreases significantly and Q_H increases slightly.

Therefore, η_t increases.

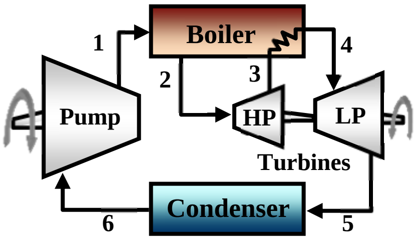
Superheat Rankine Cycle

Boiler produces superheated vapor.

Result: η_t increases and quality of the turbine effluent increases.

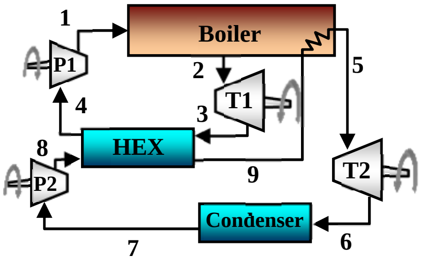
Reheat Rankine Cycle

A superheat Rankine Cycle that uses 2 turbines to allow the condenser to operate at a low P and still maintain a very high quality at the turbine effluent. The working fluid is reheated in the boiler before it enters the 2nd turbine.



Binary Vapor Rankine Cycle

Consists of 2 separate Rankine Vapor Power Cycles that use different working fluids.



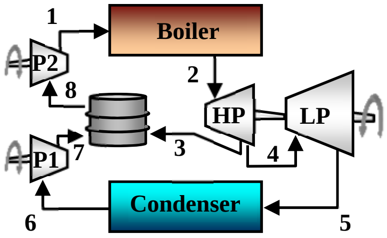
Supercritical Rankine Cycle

Boiler operating pressure is greater than the critical pressure of the working fluid.

Result: η_t increases (Q_H increases greatly while Q_C increases very little) and quality of the turbine effluent can increase.

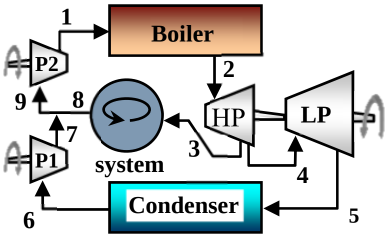
Rankine Cycle With Regeneration

The condenser effluent is pumped back up to the high pressure of the boiler in 2 stages. At the intermediate P between the 2 pumps, the working fluid is heated back to the sat. liquid state in the feedwater heater (open or closed).



Cogeneration Gas-Vapor Rankine Cycle

Uses heat from a high temperature reservoir to simultaneously produce work and heat, at a moderately high temperature and pressure, for use in an external system.



Main reasons real vapor power cycles are not as efficient as ideal Rankine Vapor Power Cycle

1. Heat Loses
2. Fluid Friction
3. Mechanical Losses
4. Condenser Subcooling

Lost Work

$$\dot{W}_{S,lost} = T_{surr}\left(\frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}\right) + \dot{Q}_{lost}$$

Air-Standard Assumptions

1- The working fluid is air and it behaves as an ideal gas.
2- The cycle is modeled as a closed cycle with the air cooled in HEX #2 and recirculated to the compressor.
3- The combustion reactor is replaced HEX #1.
4- All processes are internally reversible.

Assumptions in applying the 1st Law

5- Int. Rev., SS, SISO Process
6- Negligible E_K & E_P
7- No shaft work in HEX #1 or HEX #2
8- No heat transfer occurs in the turbine or comp.

1st Law: HEX #1

$$\frac{\dot{Q}_H}{\dot{m}} = \frac{\dot{Q}_{12}}{\dot{m}} = \hat{H}_2 - \hat{H}_1$$

1st Law: Turbine

$$\frac{\dot{W}_{S,turb}}{\dot{m}} = \hat{H}_2 - \hat{H}_3 > 0$$

1st Law: HEX #2

$$\frac{\dot{Q}_C}{\dot{m}} = \frac{\dot{Q}_{34}}{\dot{m}} = \hat{H}_4 - \hat{H}_3$$

1st Law: Compressor

$$\frac{\dot{W}_{S,comp}}{\dot{m}} = \hat{H}_4 - \hat{H}_1 < 0$$

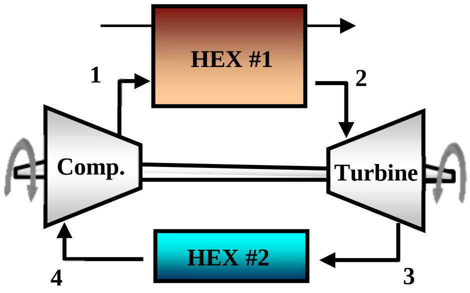
Back Work Ratio (BWR)

Fraction of work produced by the turbine that is consumed by the compressor.

$$BWR = \frac{-\hat{W}_C}{\hat{W}_T} = \frac{\hat{H}_1 - \hat{H}_4}{\hat{H}_2 - \hat{H}_3}$$

The Air-Standard Brayton Cycle

Process 1-2: HEX #1
Heat transferred to the working fluid at constant pressure from the external heat source.
Process 2-3: Turbine
Hot gases expand isentropically to produce shaft work.
Process 3-2 HEX #2
Heat is rejected to the low temperature reservoir at constant pressure.
Process 4-1: Compressor
Cool gas is compressed isentropically.



Cold Air-Standard Assumptions

Heat capacities of air are constant and have the values determined at **25°C**.

Cold Air-Standard Brayton Cycle Thermal Efficiency

$$\eta = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

Compressor Pressure Ratio

$$r_p = \frac{P_1}{P_4}$$

Regenerative Air-Standard Brayton Cycle

$$\eta_{Regeb} = 1 - \frac{T_1}{T_4}(r_p)^{(\gamma-1)/\gamma}$$

Simple Air-Standard Brayton Cycle

$$\eta = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

Refrigeration Cycle Maximum COP

The Carnot Refrigeration Cycle provides the upper limit of the **COP**.

$$COP_{R,max} = \frac{Q_C}{W_{cycle}} = \frac{1}{T_H / T_C - 1} = \frac{T_C}{T_H - T_C}$$

Impracticalities of the Carnot Vapor-Compression Refrigeration Cycle

Problem: Most Compressors do not work well on two-phase mixtures.

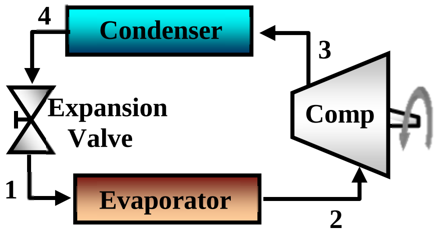
Solution: Completely vaporize the refrigerant in the evaporator.

Problem: Turbines are not designed to handle vapor-liquid mixtures.

Solution: Replace the turbine with an expansion valve.

Ideal Vapor-Compression Refrigeration Cycle

Process 1-2: Evaporator
Heat absorbed at constant pressure.
Effluent is a saturated vapor.
Process 2-3: Compressor
Isentropic compression.
Process 3-4: Condenser
Heat is rejected at constant pressure.
Process 4-1: Expansion Valve
Isenthalpic expansion.



COP of Ideal Vapor-Compression Refrigeration Cycles

$$COP_R = \frac{Q_C}{W_{cycle}} = \frac{\hat{H}_2 - \hat{H}_1}{\hat{H}_3 - \hat{H}_2}$$

Factors to Consider in Choosing a Refrigerant

- Cost, the P*-T relationship, enthalpy of vaporization, chemical stability, corrosiveness, toxicity, and flammability.

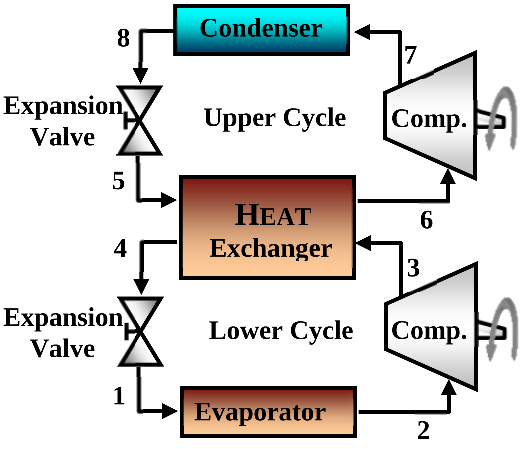
The P*-T Relationship

The saturation temperature of the working fluid at the operating pressure of the **condenser** must be greater than the temperature of the hot reservoir in order to reject heat.
The saturation pressure of the refrigerant should not be too high because a condenser that is capable of operating at high pressure is much more expensive and hazardous.

The saturation temperature of the working fluid at the operating pressure of the **evaporator** must be less than the temperature of the cold reservoir in order to absorb heat. The saturation pressure of the refrigerant should not be less than atmospheric pressure because air leaks into the system can severely decrease the COP.

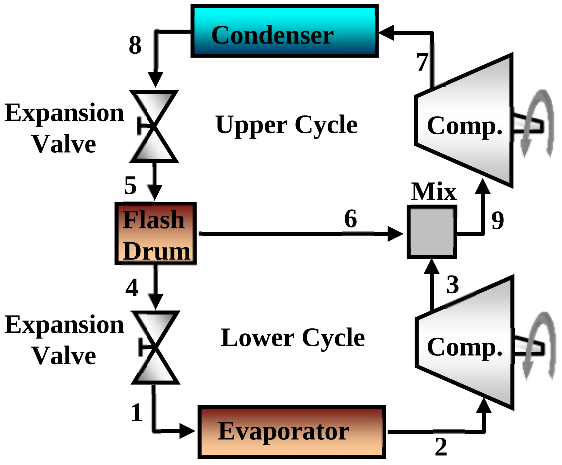
Cascade Vapor-Compression Refrigeration Cycle

- Two refrigeration cycles that use two different refrigerants are linked by a HEX.
- The lower cycle is colder and it absorbs heat from the refrigerated space.
- The lower cycle rejects heat into the upper cycle through the HEX.
- The upper cycle absorbs heat from the lower cycle through the HEX.
- The upper cycle is hotter and can reject heat to a very hot reservoir.
- Can use the same refrigerant in both cycles or use a refrigerant with a low vapor pressure in the upper cycle and one with a relatively high vapor pressure in the lower cycle.



Multistage Vapor-Compression Refrigeration Cycle

- Similar to the Cascade Cycle.
- Both cycles must use the same refrigerant.
- Instead of exchanging heat between two cycles, the flash tank separates the saturated liquid and the saturated vapor.
- The saturated liquid leaves the flash drum in stream 4 and proceeds to the lower expansion valve.
- The saturated vapor leaves the flash drum in stream 6 and flows to the mixer and passes through the upper cycle again.
- The saturated vapor, in stream 6, recycled back into the upper cycle has a high enthalpy. By diverting this high enthalpy stream from the lower cycle into the upper cycle, we have accomplished the same energy transfer that the heat exchanger does in a Cascade cycle.



Heat Pump Cycle Maximum COP

The Carnot Heat Pump Cycle provides the upper limit of the COP.

$$\mathrm{COP}_{\mathrm{HP,max}} = \frac{Q_H}{W_{\mathrm{cycle}}} = \frac{1}{1 - T_C / T_H}$$

COP of Heat Pump Cycles

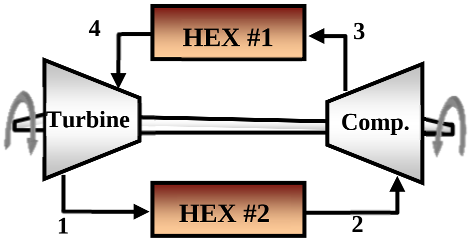
$$\mathrm{COP}_{\mathrm{HP}} = \frac{Q_H}{W_{\mathrm{cycle}}} = \frac{\hat{H}_3 - \hat{H}_4}{\hat{H}_3 - \hat{H}_2}$$

Gas Refrigeration Cycles

Disadvantage: tend to be less efficient than Vapor-Compression Cycles.
Advantages: relatively lightweight and are capable of achieving very low temperatures.

Air-Standard Assumptions

1. The working fluid is air and it behaves as an ideal gas.
2. The cycle is modeled as a closed cycle with the air cooled in HEX #2 and recirculated to the compressor.
3. The combustion reactor is replaced HEX #1.
4. All processes are internally reversible.



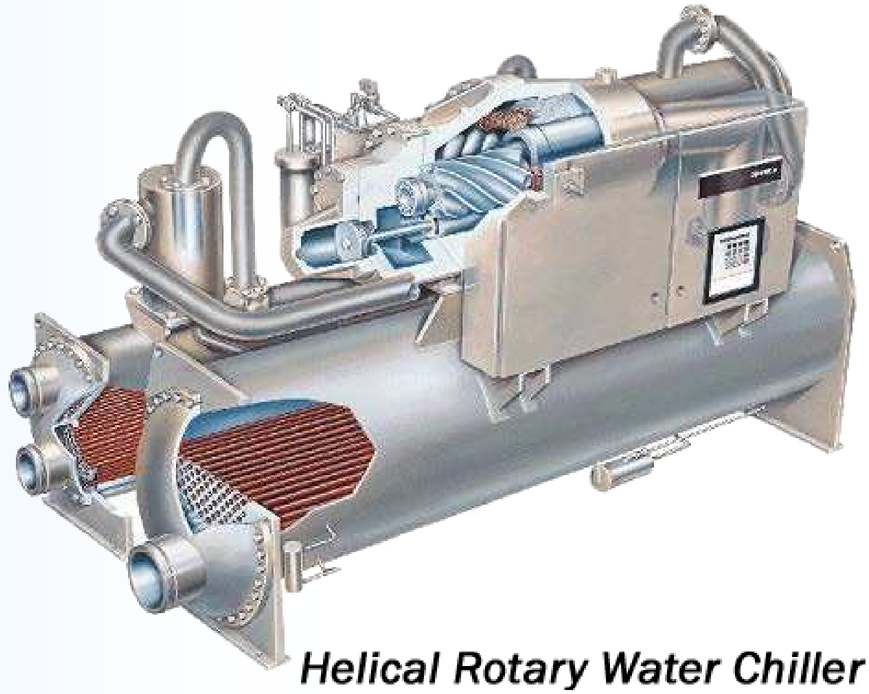
Cold Air-Standard Assumptions

Heat capacities of air are constant and have the values determined at **25°C**.

Regenerative Gas Refrigeration Cycles

Increases the coefficient of performance and it reduces the minimum temperature in the cycle.
Multistage regeneration allows the air-standard refrigeration cycle to achieve cryogenic temperatures.

Chapter 1



Helical Rotary Water Chiller

Used by permission from Trane

Introduction: Basic Concepts of Thermodynamics

In this chapter, you will discover the nature of thermodynamics and how it effects your life. The scope of this book is provided so you will know what you will and will not learn. Many of the key terms that are used to discuss thermodynamics are introduced and explained.

How Does Thermo Affect You ?

- Engines: Trains, Planes & Automobiles
- Heating Systems: Heat Pumps
- Cooling Systems:
  - ◊ Air Conditioning
  - ◊ Refrigeration
- Others:
  - ◊ Aspects of Thermo that are beyond the scope of this course make it a key aspect of chemical and biological systems

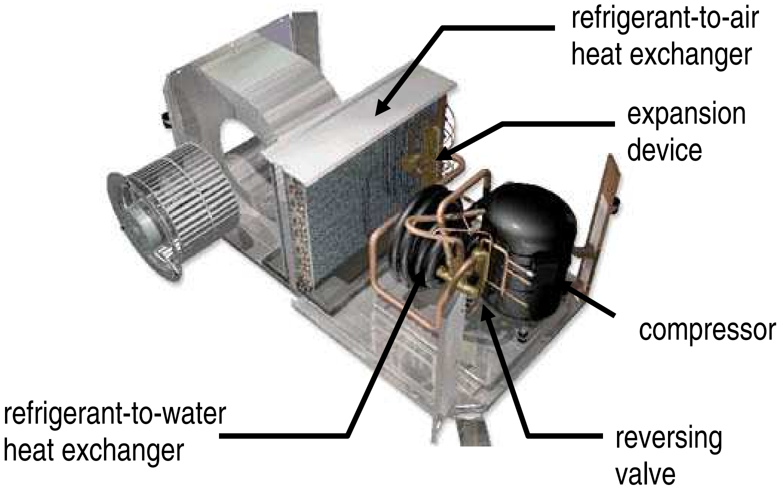
- Thermo touches the lives of most people as a means of transportation.
- Heating and cooling systems make many people's lives more comfortable and keep food from spoiling.
- Thermo is also the key to electrical power generation.

Water-Source Heat Pump

Used by permission from Trane

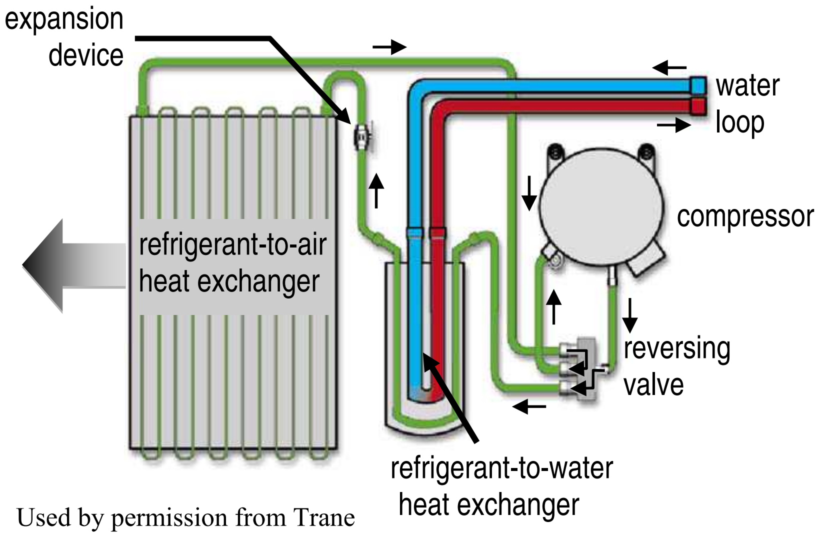
- If you live in a mild climate, such as Seattle's, you may have one of these behind your house.
- Ever wonder what is inside or how it works ?

Water-Source Heat Pump



- Looks kind of complicated.
- Reversible heat pumps, like this one, can also be used as air-conditioners
- So, what is the purpose of this device ?

Heat Pump in Cooling Mode



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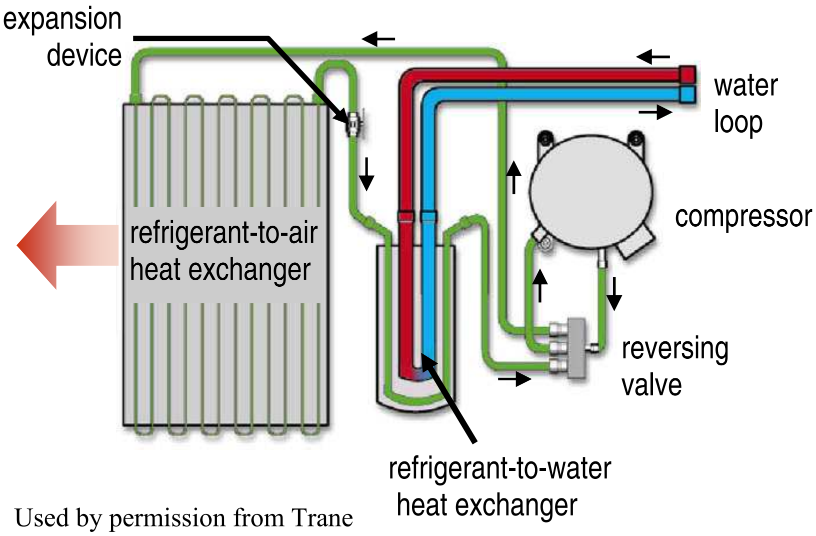
- In cooling mode, the heat pump uses electrical work to transfer heat from cool air (inside your home) into the warm air (outside your home).
- The cool part is that, for each Watt of electrical power used by the A/C unit, more than one Watt of energy is removed from the air inside your home.
- Not bad !
- What happens when the reversing valve is switched ?

Heat Pump in Heating Mode



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- In heating mode, the heat pump uses electrical work to transfer heat from cool air (outside your home) into the warm air (inside your home).
- The cool part is that, for each Watt of electrical power used by the heat pump unit, more than one Watt of energy is added to the air inside your home.
- So, a heat pump is more efficient than an electrical resistance heater ! Nice.
- A refrigerator works in much the same way that a heat pump does.
- In fact an A/C unit is technically a refrigerator because the purpose is to keep the cool space cool.

Classical Thermodynamics

- **Large Groups of Molecules – Continuum Scale**
- **The Laws of Thermodynamics**
  - 1^{st} Law: Energy can neither be created nor destroyed. It can only change form.
  - 2^{nd} Law: Energy in the form of heat only flows spontaneously from regions of higher temperature to regions of lower temperature.
- **Forms of Energy**

Gravitational Potential	$E_P = m\frac{g}{g_C}z$	$\hat{E}_P = \frac{g}{g_C}z$
Kinetic	$E_K = \frac{1}{2}m\frac{v^2}{g_C}$	$\hat{E}_K = \frac{1}{2}\frac{v^2}{g_C}$
Internal	U	$\hat{U}$
Heat	Q	$\hat{Q}$
Work	W	$\hat{W}$

- The Continuum Scale
  - A large group of molecules is larger than 100 mm across
  - Behavior of individual molecules is not studied
  - Only the properties of the large group of molecules are studied: think P, V and T
- 1st Law – Energy is a conserved quantity
  - This is the basis for much of this course.
- 2nd Law – A simple and sensible idea, right ?
  - The implications of the 2nd law are ENORMOUS
- Forms of Energy
  - There are many types of potential energy, but we will generally only consider gravitational potential energy
  - I suspect you already have a pretty good understanding of Kinetic Energy
  - You were introduced to Internal Energy in general chemistry, but in this class, you will learn a lot more about how to use this quantity to solve problems.

Dimensions & Units

- Dimensions: Mass, Length, Time
- Units: m, ft, kg, lb_m, J, Btu
- Force
    - IS a fundamental unit in the AE System
    - Is NOT a fundamental unit in the SI System
    - Newton's 2nd Law of Motion: $\mathbf{g_C\, F = m\, a}$
    - AE: $\mathbf{g_C = 32.174\ \dfrac{lb_m \cdot ft}{lb_f \cdot s^2}}$
    - SI: $\mathbf{g_C = 1\ \dfrac{kg \cdot m}{N \cdot s^2}}$
- Conversion Factors
    - Download from the course website
    - Online: "The Foot Rule" website
        - http://www.FootRule.com

- Dimensions are more fundamental than units
- Other dimensions include electrical charge and temperature
- Units are not very difficult except when we must convert between systems of units, such as AE and SI.
- The easiest way to keep this straight is to use g_c and Newton's 2nd Law of Motion.
- We will work a problem using this in class.
- Online, I like the FootRule website. Check it out !

Terminology or Nomenclature

- System: The material or volume that we are studying
    - Systems have boundaries
    - Closed Systems: Mass does not cross the boundary
    - Open Systems: Mass does cross the boundary
- Properties
    - Intensive vs. Extensive Properties
        - Extensive properties depend on the size of the system, intensive properties do not.
    - Molar Properties: per mole. Molar volume: $\tilde{\mathbf{V}}$
    - Specific Properties: per kg or per lb_m. Specific volume: $\hat{\mathbf{V}}$
- States
    - The condition of a piece of matter or system as determined by its intensive properties.
    - If ANY intensive property is different, then the system is in a different state.

- Systems
    - The entire universe is divided into two regions: the system and the surroundings.
    - The surface that separates the system from the surroundings is called the boundary of the system.
    - If mass flows across the system boundary, then the system is called OPEN.
    - If NO mass flows across the system boundary, then the system is called CLOSED.
- Properties
    - Intensive properties are more important in this course because they determine the STATE of the system.
    - Molar and specific properties are intensive variables. We will use them MUCH more than we will use extensive properties, such as volume.
- States
    - Consider a system that contains a pure substance in a single phase.
    - If we measure just TWO intensive properties of the system, then we don't need to measure any more properties.
    - They are all fixed and could be determined from the two values that we measured.
    - This cool part is a special case of the Gibbs Phase Rule. We'll learn more about this in Lesson 3C.

More Nomenclature

- Process
  - ◊ A change in the state of a system
- Process Path
  - The series of states that a system moves through on its way from the initial state to the final state.
- Special Types of Processes
  - ◊ Isobaric – constant pressure
  - ◊ Isothermal - constant temperature
  - ◊ Isochoric - constant volume
- Cycles
  - ◊ Special process paths in which the initial state is the same as the final state
  - ◊ Thermodynamic cycles are a key topic in this course

- Most of the problems we solve in this course will involve the analysis of processes
- One key to understanding the effects of any process on a system is to know which states the system passes through during the process. That is, we need to understand the process path
- Special types of processes are a little bit easier to analyze because one property does not change.
- These are the big three special processes, but we will add 3 or 4 more special types of processes later in the course.
- Cycles are special and cool types of processes.
- Engines, refrigerators and heat pumps all operate on Thermodynamic Cycles.
- Analysis of cycles is the ultimate goal of this course !

Equilibrium

- A system is in equilibrium when no unbalanced potentials or driving forces exist within the system boundary.
  - Thermal: no temperature driving forces
  - Chemical: no chemical driving forces
  - Phase: no mass transfer driving forces
  - Mechanical: no unbalanced mechanical forces
- Quasi-Equilibrium Processes
  - A process during which the system only deviates from equilibrium by an infinitessimal amount.
  - Every state along the process path is very nearly an equilibrium state.

- Equilibrium
- For a system to truly be in equilibrium, it must be in equilibrium thermally, chemically, mechanically and it must be in phase equilibrium as well.
- Quasi-Equilibrium Processes
  - ◊ All forces are balanced or nearly balanced throughout the process
  - ◊ Q-E processes must occur very slowly so the system only deviates slightly from equilibrium.
  - ◊ Although no real process is actually a Q-E process, we can use the Q-E process as a best case.
  - ◊ Then we can compare the performance of real processes to the performance of equivalent Q-E processes.

Pressure, Volume & Temperature

- Volume: $\tilde{V} = \frac{V}{n}$ $\hat{V} = \frac{V}{m}$ $\rho = \frac{m}{V} = \frac{1}{\hat{V}}$
  - ◊ SI: L, m^3, mL=cm^3
  - ◊ AE: ft^3
- Pressure: acts in all directions ⊥ to all surfaces
  - ◊ SI: Pa, kPa, MPa, bar, atm
  - ◊ AE: psia
  - ◊ Absolute, Gage and Vacuum Pressures

- Volume is pretty straightforward and the units should be very familiar.
- The units of pressure for SI may not all be familiar.
- My favorite unit for pressure is the kPa. 1 atm = 101.325 kPa = 14.696 psia
- Oddly enough, there are different types of pressures.
- Before today, you only worked with absolute pressure and that is what you want to continue to use.
- So, if you are given a vacuum pressure or gage pressure, the first thing you want to do is convert it to absolute pressure.
- $P_{gage} = P_{abs} - P_{atm}$
- $P_{vac} = P_{atm} - P_{abs}$
- Remember that absolute pressure cannot be negative, but gage pressure can.
- Technically, vacuum pressure can be negative, but it is not usually expressed as vacuum pressure in that case.

Manometers

- Barometer Eqn: $\mathbf{P_2 = P_1 + \rho_f \frac{g}{g_c} h}$
- Manometer Eqn: $\mathbf{P_{in} = P_{out} + \rho_f \frac{g}{g_c} h}$
- Differential Manometer Eqn: $\mathbf{P_{up} = P_{down} + (\rho_m - \rho_f) \frac{g}{g_c} h}$

- The barometer eqn helps you calculate the pressure the bottom of a tank, P2, given the pressure at the top of the tank, P1, and the depth and density of the fluid.
- Remember that the pressure at the bottom of the tank is always greater than the pressure at the surface of the liquid.
- A manometer lets you calculate the difference between the pressure in two different locations, such as the difference between the pressure inside a tank and the ambient pressure.
- All you need to know is the density of the manometer fluid and the difference in the height of the manometer fluid in the two legs of the manometer. (assuming the density of the fluid in the tank is negligible, usually this means it is a gas).
- The differential manometer eqn lets you calculate the difference in pressure between two locations, even if the fluid density is similar to the density of the manometer fluid.
- This is especially useful for measuring the pressure drop that occurs as a fluid flows through a pipe.

Temperature

- Thermometers and Thermocouples
- Temperature conversions are straightforward
- $\Delta T\ (^oC) = \Delta T(K)$ and $\Delta T\ (^oF) = \Delta T(^oR)$
- Ideal Gas Thermometry
  - ◊ Must be calibrated
  - ◊ Tedious, but extremely accurate
  - ◊ IG T-scale is identical to the Kelvin Scale !

- Engineers usually use thermocouples or other temperature measuring devices because they are well-suited to electronic data acquisition.
- Ideal Gas Thermometers provide some of the most accurate temperature measurements available.
- The downside is that several experiments must be run at decreasing pressure so the results can be extrapolated back to ZERO pressure.
- Why ? Because only at zero pressure is any real gas actually ideal.
- Once this calibration is complete, the IG Thermometer is ready to measure the temperature of a sample of the same gas that was used in the calibration.
- In order to measure the temperature of a different gas, the IG Thermometer must be recalibrated.
- The IG thermometer is very accurate, but not practical for most applications.

1A-1 Kinetic and Potential Energy of an Airplane in Flight 6 pts

Gravitational acceleration is less at higher altitudes than at sea level. Assume gravitational acceleration as a function of altitude is described by **$g(m/s^2) = 9.806 - 3.2 \times 10^{-6} * h$**, where **h** is the altitude (relative to sea level).
Consider an aircraft flying at **750 km/h** at an altitude of **12 km**. Before take-off, the aircraft weighed **50 kN** at sea level.
Determine its:
a.) kinetic energy
b.) potential energy relative to sea level.

Read : Since gravitational acceleration is LESS at higher altitude, the gravitational potential energy of the airplane will not be quite as great as you might ordinarily expect. We need the weight of the airplane at sea level in order to determine the mass of the airplane. We need ot know the mass in order to calculate both the kinetic and gravitation potential energies of the plane.

Given: **$g(m/s^2) = a - b * h$**

a	**9.8066**	m/s^2	**v**	**750**	km/h
b	**3.20E-06**	s^{-2}	**h**	**12000**	m
			$W_{sea\ level}$	**50**	kN
			g_c	**1**	$kg\text{-}m/N\text{-}s^2$

Find: a.) E_{kin} ??? MJ b.) E_{pot} ??? MJ

Assumptions: 1- Gravitational acceleration is a function of altitude only.

Equations / Data / Solve :

Let's begin by determining the mass of the airplane from the weight at sea level.

Newton's 2nd Law of Motion:

$$g_c F = m a \qquad \text{Eqn 1}$$

or

$$g_c W_{sea\,level} = m g \qquad \text{Eqn 2}$$

At sea level, according to the eqn given in the problem statement, the acceleration of gravity is :

g **9.8066** **m/s^2**

Now, we can solve **Eqn 1** for **m** and plug in values :

$$m = W_{sea\,level} \frac{g_c}{g} \qquad \text{Eqn 3}$$

m **5099** **kg**

Now, we are ready to solve the rest of the problem.

Part a.) The definition of kinetic energy is :

$$E_K = \frac{1}{2} m \frac{v^2}{g_C} \qquad \text{Eqn 4}$$

E_k **1.106E+08** **J**

E_k **111** **MJ**

Part b.) The definition of gravitational potential energy, relative to sea level, is :

$$E_P = m\frac{g}{g_C}h \qquad \textbf{Eqn 5}$$

The problem is what value of g do we use ? Do we simply use **g** at the altitude of the plane? Or do we use some sort of average value of **g** ?

Let's think about this part a bit more carefully.

$$E_P = m\frac{\bar{g}}{g_C}h \qquad \textbf{Eqn 6}$$

The differential increase in the potential energy of an object infinitessimally above sea level is:

$$dE_P = m\frac{g}{g_C}dx \qquad \textbf{Eqn 7}$$

So, the gravitational potential energy of an oject that is a distance h above sea level is :

$$E_P = \int_0^{E_P} dE_P = \int_0^h m\frac{g}{g_C}dx \qquad \textbf{Eqn 8}$$

Now, if **m**, **g** and $\mathbf{g_c}$ are all constants, Eqn 7 simply reduces to Eqn 5 because the integral of **dx** from **0** to **h** is just **h**. In our problem, however, **g** is NOT a constant. Therefore :

$$E_P = \frac{m}{g_C}\int_0^h (a - b\,x)\,dx = \frac{m}{g_C}\left[a\,x - b\frac{x^2}{2}\right]_0^h = \frac{m}{g_C}\left[a\,h - b\frac{h^2}{2}\right] = \frac{m}{g_C}\left[a - b\frac{h}{2}\right]h \qquad \textbf{Eqn 9}$$

E_{pot} 5.988E+08 J

E_{pot} 598.8 MJ

So, what "average" value of g should we have used in Eqn 5 ? Let's combine Eqns 5 and 8 and see what we get.

$$\cancel{m}\frac{\bar{g}}{\cancel{g_C}}\cancel{h} = \frac{\cancel{m}}{\cancel{g_C}}\left[a - b\frac{h}{2}\right]\cancel{h} \qquad \text{or :} \qquad \bar{g} = \left[a - b\frac{h}{2}\right] \qquad \textbf{Eqn 10}$$

So, the average effective value of the gravitational acceleration for determining the potential energy of the airplane in this problem is equal to the gravitational acceleration at HALF of the actual altitude of the airplane. Would this be true if $\mathbf{g = a - b\,x - c\,x^2}$?? Nope. What is special about the equation for **g** given in this problem that leads to the interesting result in Eqn 10 ?

Answers : **a.)** **E_k 111 MJ** **b.)** **E_{pot} 599 MJ**

1A-2 Conversion of Kinetic Energy into Spring Potential Energy 5 pts

It takes energy to compress a spring. This energy is stored as spring potential energy, which can be calculated using: $\mathbf{E_{spring} = 1/2\ K\ x^2}$, where **K** is the spring constant and **x** is the distance the spring is compressed.
At a dock, a boat with a **mass** of **50,000 kg** hits a bumper supported by **two** springs that **stop** the boat and absorb its kinetic energy.

Determine the **spring constant** of the springs that is required if the maximum compression is to be **60 cm** for a boat speed of **2.4 m/s**.

Read: It is important to note that two springs are used to stop the vehicle.
All of the initial kinetic energy of the vehicle must be absorbed by the springs and converted to spring potential energy.

Given:

$$E_{spring} = \frac{1}{2} K x^2 \qquad \textbf{Eqn 1}$$

K = spring constant

x = displacement of the spring, in this case compression.

m	**50000**	**kg**
v	**2.4**	**m/s**
x	**0.6**	**m**

g_c	**1**	$\mathbf{kg\text{-}m/N\text{-}s^2}$

Find: **K** **???** **N/m**

Assumptions:
1- The spring is a linear spring and therefore the given equation applies.
2- All of the kinetic energy of the boat is absorbed by the springs.

Equations / Data / Solve:

The key to solving this problem is to recognize that the final potential energy of the two springs must be equal to the initial kinetic energy of the vehicle. So, we should begin by calculating the initial kinetic energy of the boat.

$$E_{kin} = \frac{1}{2 g_c} m v^2 \qquad \textbf{Eqn 2}$$

Because there are two identical springs :

$$E_{spring} = \frac{E_{kin}}{2} \qquad \textbf{Eqn 3}$$

Plugging given values into **Eqn 2** and **Eqn 3** yields:

E_{kin}	144000	J
E_{spring}	72000	J

Next, we can solve **Eqn 1** algebraically for the spring constant, **K**.

$$K = \frac{2 E_{spring}}{x^2} \qquad \textbf{Eqn 4}$$

First, let's work on the units.

$$\mathbf{K} \; [=] \; \frac{\mathbf{J}}{\mathbf{m^2}} \; [=] \; \frac{\mathbf{N \cdot m}}{\mathbf{m^2}} \; [=] \; \frac{\mathbf{N}}{\mathbf{m}}$$

Eqn 5

Now, let's calculate the value of the spring constant.

K	**4.00E+05**	**N/m**

Answers:

K	**4.00E+05**	**N/m**

1B-1 Mass, Weight and Gravitational Acceleration 4 pts

A curious astronaut took her bathroom scale (a spring scale) and a beam scale (compares **masses**) to the **moon**. On **Earth**, she determined that her **mass** was **124 lb_m**. The gravitational acceleration on the moon is **$a = 5.3\ ft/s^2$**.
When she reaches the **moon**…
a.) What will she weigh on the spring scale?
b.) What will the beam scale indicate for her **mass**?

Read: The key here is that a spring scale actually measures weight (which is a force) and not mass.
Use gc in Newton's 1st Law of Motion to answer this question.

Given: m 124 lb_m a 5.3 ft/s^2

Find: a.) F_{wt} ??? lb_f b.) m_{beam} ??? lb_m

Assumptions:

Equations / Data / Solve:

Part a.) The key equation here is Newton's 1st Law of Motion :

$$g_C\, F_{wt} = m\, a \quad \text{Eqn 1}$$

Because a spring scale measures weight, which is the force exerted by the astronaut on the scale, we need to solve **Eqn 1** for F_{wt}.

$$F_{wt} = \frac{m\, a}{g_C} \quad \text{Eqn 2}$$

Now, we can plug values into **Eqn 2** :

g_C 32.174 $lb_m\text{-}ft/lb_f\text{-}s^2$

F_{wt} 20.43 lb_f

Part b.) A beam balance is the kind of scale used in most medical and sports facilities.

Known masses are moved along the length of the beam until the net torque around the fulcrum is zero. That is the astronaut's weight multiplied by a fixed distance along the beam to fulcrum is equal to the weight of the masses hanging from the beam multiplied by their distance from the fulcrum.

Weight is still involved in a beam balance,but the key is that the local gravitational acceleration applies to BOTH the astronaut AND the masses on the beam balance. This scale will read the same on the moon as it would in the astronaut's bathroom ! Therefore, it is actually measuring the astronaut's MASS (which is the same on the moon as in her or his bathroom.

m_{beam} 124 lb_m

Answers:

a.)	F_{wt}	20.4	lb_f
b.)	m_{beam}	124	lb_m

1B-2 Mass, Force and Acceleration 4 pts

A strong boy throws a rock straight up with a **force** of **100 N**. At what rate does the rock initially accelerate upwards, in **m/s^2**? Assume the **mass** of the rock is **2 kg** and the local gravitational acceleration is **9.74 m/s^2**.

Read: The key here is to recognize that two forces are acting on the rock: the 100 N and the weight of the rock (due to gravity). Then, the problem becomes an application of Newton's 1st Law of Motion.

Given:

m	2	kg	g	9.74	m/s^2
F_{throw}	100	N			

Find:

a	???	m/s^2

Assumptions: None.

Equations / Data / Solve:

The key equation here is Newton's 1st Law of Motion :

$$g_C\, F_{net} = m\, a \qquad \textbf{Eqn 1}$$

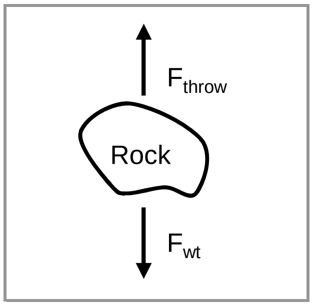
We can solve **Eqn 1** for the rate at which the rock accelerates :

$$a = \frac{g_C\, F_{net}}{m} \qquad \textbf{Eqn 2}$$

We know that : g_C 1 kg-m/N-s^2

So, all we need to is determine the net force acting on the stone.

A free-body diagram might be helpful.



The net force acting on the rock in the upward direction is :

$$F_{net} = F_{throw} - F_{wt} \qquad \textbf{Eqn 3}$$

We can apply Newton's 1st Law of Motion again to evaluate F_{wt}.

$$g_C\, F_{wt} = m\, g \qquad \textbf{Eqn 4}$$

We can solve **Eqn 4** for F_{wt}, as follows :

$$F_{wt} = m \frac{g}{g_C} \qquad \textbf{Eqn 5}$$

Plugging values into **Eqn 5**, then **Eqn 3** and, finally, **Eqn 2** yields :

F_{wt}	19.5	N
F_{net}	80.52	N
a	**40.26**	**m/s^2**

Answers:

a	**40.3**	**m/s^2**

1B-3 Units and Carbon Dioxide Emissions 4 pts

A new, energy-efficient home refrigerator consumes about **350 kW-h** of electricity **per year**. Assume the electricity is generated by burning **fossil fuels**. When **fossil fuels** like **coal**, **oil**, **natural gas** and **gasoline** are burned, most of the carbon in the fuel burns completely to form **carbon dioxide** (**CO_2**). **CO_2** is a greenhouse gas that contributes to global warming and that is undesirable. In a typical **natural gas** power plant, **0.5 kg** of **CO_2** is produced for each **kW-h** of electricity generated. Consider a city with **500,000** homes with **one** refrigerator in each home. How much **CO_2**, in **metric tons**, is produced by the refrigerators in the city in **one year**?

Read: This problem is all about unit conversions.
Keep careful track of units and it will not be difficult.
A **kW-h** is unit of energy. A **Watt** is energy per **time** and when you multiply by **hours** (**time**) you are left with units of **energy**. A **kW-h** is **kW** times **hours**. It is the amount of energy consumed in an **hour** by a device that uses **1 kW** of power (**1 kJ/s** for **3600 sec**).

Given:

m_{CO2}	0.5	kg CO_2 / kW-h	Power	350	kW-h/house/year
N	500000	houses			

Find: M_{CO2} ??? mton CO_2 / year

Assumptions: None.

Equations / Data / Solve:

The total mass of CO_2 produced per year is the product of the rate of CO_2 production per kW-h, the rate at which power is used by each house and the number of houses.

$$M_{CO2} = Power \cdot m_{CO2} \cdot N_{houses} \qquad \text{Eqn 1}$$

Check the units in this equation :

$$M_{CO2} = \left(\frac{kW \cdot h}{house \cdot year}\right)\left(\frac{kg\,CO_2}{kW \cdot h}\right)\cdot(house) = \left(\frac{kg\,CO_2}{year}\right) \qquad \text{Eqn 2}$$

Plug in the values : M_{CO2} 8.75E+07 kg CO_2 / year

Unit conversion factor : 1 metric ton = 1000 kg

Therefore : M_{CO2} 87500 mton CO_2 / year

Answers: M_{CO2} 87500 mton CO_2 / year

1B-4 Force Required to Accelerate a Rocket — 4 pts

NASA would like a rocket to accelerate upward at a rate of **125 ft/s^2**. The **mass** of the rocket is **35,000 lb$_m$**. Determine the upward thrust **force**, in **lb$_f$**, that the rocket engine must produce.

Read: This is a direct application of Newton's 2nd Law of Motion in the AE System of units.
The key to solving this problem is a clear understanding of **g$_c$**.

Given:

m	35000	lb$_m$	g_c	32.174	lb$_m$-ft/lb$_f$-s^2
a	125	ft/s^2			

Find:

F_{up}	???	lb$_f$

Assumptions: 1- Assume: g 32.174 ft/s^2

Equations / Data / Solve:

We begin with Newton's 2nd Law of Motion :

$$g_c F = m a \qquad \text{Eqn 1}$$

The force required to lift the rocket and accelerate it upward depends on both the weight of the rocket (and therefore the **g**) and the rate at which the rocket must be accelerated…**120 ft/s^2**. Therefore:

$$F_{up} = F_{weight} + F_{accelerate} \qquad \text{Eqn 2}$$

We can now substitute **Eqn 1** into **Eqn 2** to get :

$$F_{up} = \frac{m\,g}{g_c} + \frac{m\,a}{g_c} = m\frac{(g+a)}{g_c} = m\frac{a_{total}}{g_c} \qquad \text{Eqn 3}$$

Now, we can plug in the values :

a_{total}	157.174	ft/s^2
F_{wt}	35000	lb$_f$
F_{acc}	135979	lb$_f$
F_{total}	170979	lb$_f$

Note, in the absence of gravity, weightlessness, it would still require a force of **F_{acc} = 135,979 lb$_f$** to accelerate the rocket at a rate of **125 ft/s^2**.

Answers:

F_{up}	171000	lb$_f$

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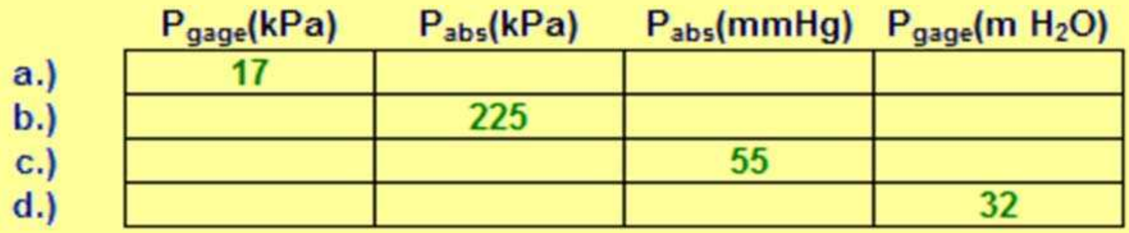
1B-5 Relationships between Different Types of Pressures 5 pts

Fill in the blank values in the table below. Assume P_{atm} = 100 kPa and the **density** of liquid **mercury** (**Hg**) is 13,600 kg/m^3.



	P_{gage}(kPa)	P_{abs}(kPa)	P_{abs}(mmHg)	P_{gage}(m H_2O)
a.)	17			
b.)		225		
c.)			55	
d.)				32

Read: This problem requires an understanding of the relationship between absolute and gage pressure. It will also require the effective use of unit conversions.

Given:

P_{atm}	100	kPa	ρ_{H2O}	1000	kg/m^3
g_c	1	kg-m/N-s^2	ρ_{Hg}	13600	kg/m^3

a.)	P_{gage}	17	kPa	c.)	P_{abs}	55	mmHg
b.)	P_{abs}	225	kPa	d.)	P_{gage}	32	m H_2O

Assumptions: 1- Assume: g 9.8066 m/s^2

Find:

P_{gauge}	???	kPa	P_{abs}	???	mmHg
P_{abs}	???	kPa	P_{gauge}	???	m H_2O

Equations / Data / Solve:

There are two key relationships in the solution to this problem.
The first is the relationship between absolute and gage pressure :

$$P_{gage} = P_{abs} - P_{atm} \qquad \textbf{Eqn 1}$$

The second relationship is required in order to make sense of the units for pressure in the last two columns of the table in the problem statement. The 2nd relationship is the Manometer Equation.

$$P_{in} = P_{out} + \rho_f \frac{g}{g_c} h \qquad \textbf{Eqn 2}$$

The reason we use the Manometer Equation is that when a pressure unit involves a length of a given fluid, as in the last two columns of the table given in this problem, it really means that this is the height that an open-ended manometer (for gage pressure) or a closed end manometer (for absolute pressure) would read if the given fluid were used as the manometer fluid.

Now, let's see how we use these 2 equations to complete the table.

Part a.) In order to fill in the 2nd column, we must solve Eqn 1 for the absolute pressure :

$$P_{abs} = P_{gage} + P_{atm} \qquad \textbf{Eqn 3}$$

Therefore : **P_{abs} 117 kPa**

To complete column 3, we must convert the units from our result for **P_{abs}** using Eqn 2.
In this case, **P_{in} = P_{abs}** and **P_{out} = 0** (because it is a closed-end manometer).
Actually, **P_{out}** should be equal to the vapor pressure of the manometer fluid, but that is a topic for chapter 2.

Therefore, **Eqn 2** becomes :

$$P_{abs} = \rho_{Hg} \frac{g}{g_c} h \qquad \textbf{Eqn 4}$$

In this case, the answer for column 3 is actually the value of **h**, so we need to solve **Eqn 4** for **h** :

$$h = \frac{P_{abs}}{\rho_{Hg}} \frac{g_c}{g} \qquad \textbf{Eqn 5}$$

Be sure to convert **kPa** to **Pa=N/m^2** when plugging values into **Eqn 5**.

P_{abs} = h = 0.877 m Hg **P_{abs} 877 mm Hg**

Column 4 requires a gage pressure, so the open-end form of the Manometer Equation is used :

In this case, **P_{in} = P_{abs}** and **P_{out} = P_{atm}** (because it is a closed-end manometer).

$$P_{abs} = P_{atm} + \rho_{H2O} \frac{g}{g_c} h \qquad \textbf{Eqn 6}$$

Next, we solve **Eqn 6** for h and make use of **Eqn 1** if we want to use the given value of the gage pressure.

$$h = \frac{\left(P_{abs} - P_{atm}\right)}{\rho_{H2O}} \frac{g_c}{g} = \frac{P_{gage}}{\rho_{H2O}} \frac{g_c}{g} \qquad \textbf{Eqn 7}$$

P_{gage} = h = 1.7335 m H_2O **P_{gage} 1.734 m H_2O**

Parts b-d) The solution of the remaining parts of this problem involve the algebraic manipulation of **Eqns 1**, **3**, **5** and **7**, but does not involve any additional concepts, techniques or data. The final answers to **parts b** through **d** are provided in the table below.

Answers:

	P_{gage} (kPa)	P_{abs} (kPa)	P_{abs} (mmHg)	P_{gage} (m H_2O)
a.)	17	117	877	1.73
b.)	125	225	1690	22.9
c.)	-92.7	7.34	55	-0.909
d.)	314	414	3100	32

Notice that I chose to use 3 significant figures in my answers.
This is somewhat arbitrary since the problem statement does not make it very clear how many significant figures exist in the given information. When in doubt, 3 significant figures is a reasonable choice.

1B-6 Force Required to Lift an Underwater Gate 5 pts

A steel plate rests on the horizontal bottom of a **water** tank that is **3 m** deep. What upward **force**, in **N**, must be applied to the steel plate to just barely lift it straight upward? The plate is **20 cm** in **diameter**.

Assume the **weight** of the plate is negligible.

Read: The key to this problem is to recognize that the TOTAL force required to just barely lift the man-hole cover is equal to the force exerted on the top surface of the man-hole cover by both the atmosphere and the water. This force is equal to the absolute pressure at the bottom of the tank times the area of the man-hole cover. The force a man or machine would need to exert in order to lift the man-hole cover is less because atmospheric pressure is also acting on the outer or bottom surface of the man-hole cover.

Given:

D	**0.2**	**m**	g_c	**1**	**kg-m/N-s^2**
h	**3**	**m**			

Find: F_{up} **???** **kN**

Assumptions: 1- Assume:

g	**9.8066**	**m/s^2**
P_{atm}	**100**	**kPa**
ρ_{H2O}	**1000**	**kg/m^3**

Equations / Data / Solve:

The total force required to just barely lift the man-hole cover is:

$$F_{bottom} = P_{bottom} \; A_{gate} \qquad \text{Eqn 1}$$

The gate is circular, so :

$$A_{gate} = \frac{\pi}{4} D^2 \qquad \text{Eqn 2}$$

Plug values into **Eqn 2** : A_{gate} 0.03142 m^2

Next, we can use the Barometer Equation to determine the pressure at the bottom of the tank.

$$P_{bottom} = P_{surface} + \rho_{H2O} \frac{g}{g_c} h = P_{atm} + \rho_{H2O} \frac{g}{g_c} h \qquad \text{Eqn 3}$$

P_{bottom} 129.4 kPa

Finally, we substitute values into **Eqn 1** to answer the question : F 4066 N

This is the total force required to lift the man-hole cover. But atmospheric pressure is also acting on the outside or bottom surface of the manhole cover as well. So, since we want to determine how much additional force must be applied in order to lift the man-hole cover, we must subtract the upward force attributable to atmospheric pressure below the cover.

$$\mathbf{F_{atm} = P_{atm}\ A_{gate}}$$ **Eqn 4**

F_{atm} 3142 N

The additional force that must be exerted to lift the man-hole cover is the difference between the total force required and the force exerted by the atmosphere.

$$\mathbf{F_{up} = F_{total} - F_{atm}}$$ **Eqn 5**

F_{up} 924.3 N

If the door had a hinge, the problem would be a bit more challenging because it would involve the computation of torques acting around the axis of the hinge. Torques and lever arms are not part of this thermodynamics course.

Answers: **F_{up} 924 N**

1B-7 Mass, Weight and Gravitational Acceleration: Keebos and Tweeks 5 pts

In the future, we may encounter a civilization from another planet. They will not use the SI or AE systems of units. Suppose we meet aliens who use the **Zerf** as a unit of **mass** and the **Spund** as a unit of **force**.

One **Spund** is the **weight** of a **mass** of one **Zerf** on the surface of **Mars**. The **gravitational acceleration** on **Mars** is **3.7 m/s^2**.

a.) What is g_c in the alien system of units? Be sure to include the numerical value and the units.

b.) How much would a **350 Zerf** object **weigh** on the surface of **Mars**, in **Spunds**?

c.) How much would a **350 Zerf** object **weigh** on the surface of the **Earth**, in **Spunds**?

d.) To which system of units is the **Zerf-Spund** system most similar, SI or AE?

Read: **1 Zerf** exerts **1 Spund** of force when **a = 3.7 m/s^2**.
The key here is that Newton's 2nd Law of Motion applies regardless of where the system is located or what system of units is used.

Given:

a_{moon}	3.7	m/s^2
m	350	Zerf

Find:

a.)	g_c	???	m-Zerf/Spund-s^2				
b.)	F_{wt}	???	Spund	for	m_{Mars}	350	Zerf
c.)	F_{wt}	???	Spund	for	m_{Earth}	350	Zerf
d.)	Is this **Z-S** system similer to the **SI** or **AE** system?						

Assumptions: 1- The acceleration of gravity at the surfae of the Earth is: a_{Earth} **9.8066 m/s^2**

Equations / Data / Solve:

Newton's 2nd Law :

$$g_c F = m a \qquad \text{Eqn 1}$$

Part a.) Solve Newton's 2nd Law for g_c :

$$g_c = \frac{m a}{F} \qquad \text{Eqn 2}$$

$$g_c = \frac{(1\,\text{Zerf})(3.7\,\text{m/s}^2)}{1\,\text{Spund}}$$

g_c	3.70	m-Zerf/Spund-s^2

Part b.) Solve Newton's 2nd Law for **F** :

$$F = \frac{m\,a}{g_c} \qquad \textbf{Eqn 3}$$

$$F = \frac{(350\ \text{Zerf})(3.7\,\text{m/s}^2)}{3.7\ \text{m}\cdot\text{Zerf/Spund}\cdot\text{s}^2}$$

F_{wt}	**350 Spund**

Part c.) Solve Newton's 2nd Law for **F** :

$$F = \frac{m\,a}{g_c} \qquad \textbf{Eqn 4}$$

$$F = \frac{(350\ \text{Zerf})(9.8066\,\text{m/s}^2)}{3.7\ \text{m}\cdot\text{Zerf/Spund}\cdot\text{s}^2}$$

F_{wt}	**928 Spund**

There are two important points to this problem:
1 - The value of g_c is the same everywhere in the universe.
2 - If a person weighs a certain amount more on the Earth than on Mars in one system of units, then he or she also weighs proportionally more on the Earth than on Mars in any other sytem of units as well !

Part d.) The Zerf-Spund system of units is analogous to the American Engineering System of units because g_c is not equal to 1 and 1 unit of mass exerts 1 unit of force (on the Mars).

Answers:			
a.)	g_c	**3.70**	**m-Zerf/Spund-s^2**
b.)	F_{wt}	**350**	**Spund**
c.)	F_{wt}	**930**	**Spund**
d.)	**AE**		

1B-8 Dimensionless Groups and Equations 5 pts

Consider the following equation. All three of the terms in parentheses are dimensionless groups. Because k_C is difficult to determine directly, the other variables are measured and k_C is calculated from the given equation.

$$\left(\frac{k_C\,D}{D_{AB}}\right) = 0.023\left(\frac{\mu}{\rho\,D_{AB}}\right)^{1/3}\left(\frac{d_p\,v\,\rho}{\mu}\right)^{0.8}$$

What is the estimated value of k_C ? What are the units of k_C ? Show your work.
The following values were measured:
$D = 8.0$ mm, $D_{AB} = 0.475$ cm^2/s, $\mu = 1.12 \times 10^{-3}$ N-s/m^2, $\rho = 1.00 \times 10^{-3}$ g/cm^3, $v = 18.3$ m/s.

Read: The key here is that the equation and the groups in parentheses are dimensionless.
This tells us that the constant **0.023** is also dimensionless.
So, we can plug numbers and units into the right-hand side of the equation to determine the value of the dimensionless group on the left-hand side. Then, we use the values and units of **D** and D_{AB} to determine the value and units of k_C.

It is probably wise to begin the problem by converting all of the given values to one consistent system of units. I chose the MKS system.

Given:

D	0.008	m	μ	1.12E-03	N-s/m^2
D_{AB}	4.75E-05	m^2/s	ρ	1.000	kg/m^3
			v	18.3	m/s

Find: k_C ??? ??? (units)

Assumptions: None.

Equations / Data / Solve:

Once all the given values are in a consistent set of units, we can evaluate each of the dimensionless groups in Eqn 1.

$$\left(\frac{D\,v\,\rho}{\mu}\right) = \quad 131 \qquad\qquad \left(\frac{\mu}{\rho\,D_{AB}}\right) = \quad 23.579$$

Let's double check the units on these groups using the SI units.

$$\left(\frac{D\,v\,\rho}{\mu}\right) [=] \frac{m(m/s)(kg/m^3)}{(kg\cdot m/s^2)\cdot s/m^2} [=] \frac{kg/m\cdot s}{kg/m\cdot s}$$

$$\left(\frac{\mu}{\rho D_{AB}}\right) [=] \frac{(kg\cdot m/s^2)\cdot s/m^2}{(kg/m^3)(m^2/s)} [=] \frac{kg/m\cdot s}{kg/m\cdot s}$$

All the units cancel, so we can conclude that both of these groups are indeed dimensionless.

$$\left(\frac{k_C\,D}{D_{AB}}\right) = 0.023\left(\frac{\mu}{\rho\,D_{AB}}\right)^{1/3}\left(\frac{D\,v\,\rho}{\mu}\right)^{0.8}$$ 3.253 (dimensionless)

First, let's determine the units for **k_g**.

$$k_C \;[=]\; \frac{D_{AB}}{D} = \frac{m^2\,/\,s}{m} \;[=]\; m\,/\,s$$

$$k_C = 3.253\,\frac{D_{AB}}{D} =$$ 0.0193

Answers:

k_C	**0.0193 m/s**
	1.93 cm/s

1C-1 Identifying Open and Closed Sysytems 4 pts

State whether each of the following systems is an **open** system or a **closed** system. **Explain** the reasoning behind your answer. **a.)** human being, **b.)** a bicycle tire, **c.)** your refrigerator, **d.)** the planet Earth.

Read: Not much to say here.

Given:

Find:
a.) Human being
b.) Bicycle tire
c.) A refrigerator
d.) Planet Earth

Assumptions: Assumptions are part of the **answers** in this problem.

Equations / Data / Solve: None.

Answers:

a.) **Human Being**

If we assume the person is alive, then, under most circumstances she or he should be treated as an open system because air and other gases move in and out of the person's lungs when they breathe in and out. For a short period of time, a person might hold their breathe and you could make a case that a closed system would be a reasonable choice. Over a longer period of time, perhaps days or years, a person would be best described as an open system due to the consumption of food and water and the elimination of waste.

b.) **Bicycle Tire**

If the tire has no leaks and is not being filled over the time period of interest, the tire can be reasonably modeled as a closed system. If the tire has a leak or the filling process is being studied, then air does cross the system boundary and the tire is an open system.

c.) **Refrigerator**

There are a few subtleties here.
Some refrigerators have ice makers and water chillers. Water flows into the system in this case. But water only flows into the system when ice is being made or the water tap is being used to obtain a glass of cold water. When water is flowing in and out of the system or ice is flowing out of the system, then the refrigerator can best be described as an open system.

When a person is putting groceries into the refrigerator or taking food out of the refrigerator, mass crosses the system boundary and the refrigerator is again best described as an open system.

Most of the hours in a given day, however, there is no mass crossing the system boundary and the refrigerator can be considered to be a closed system.

d.) **Earth**

This is a tricky one.
The general answer is that the Earth can be described as a closed system.
But this is not entirely true since meteors are routinely bombarding our atmosphere, if not the surface of the Earth. It is also true that molecules do leave our atmosphere and move off into space. Although these mass flow rates are very modest compared to the mass of the Earth and its atmosphere, under some circumstances, these factors are important and Earth must be considered to be an open system.

1C-2 Identifying Intensive and Extensive Properties 5 pts

State whether each of the following properties is an **intensive** property or an **extensive** property. **Explain** the reasoning behind your answer.

a.) **Density**
b.) **Internal energy**
c.) Average **molecular weight**
d.) **Molar volume**
e.) Number of **moles**.

Read: Not much to say here.

Given:

Find: **Intensive or Extensive ?**

a.) Density
b.) Internal energy
c.) Average molecular weight
d.) Molar volume
e.) Number of moles

Assumptions: None.

Equations / Data / Solve: None.

Answers:

a.) **Density** **Intensive**

Dividing in half a tank of water at a given T, P and density yields two smaller tanks containing water, but the density of the water in each new, smaller tank is the same as it was intially in the one big tank.

Density is the ratio of mass to volume. The ratio of two extensive properties is an intensive property.

b.) **Internal Energy** **Extensive**

Dividing in half a tank of water at a given T, Pwith a given total internal energy yields two smaller tanks containing water at the same T & P, but the internal energy of the water in each new, smaller tank is half as much as it was intially in the one big tank.

c.) **Average Molcular Weight** **Intensive**

Dividing in half a tank of air at a given T, P and average molecular weight yields two smaller tanks containing air, but the average molecular weight of the air in each new, smaller tank is the same as it was intially in the one big tank.

Average molecular weight is the ratio of mass to moles. The ratio of two extensive properties is an intensive property.

d.) **Molar Volume** **Intensive**

Dividing in half a tank of air at a given T, P and molar volume yields two smaller tanks containing air, but the molar volume of the air in each new, smaller tank is the same as it was intially in the one big tank.

Molar volume is the ratio of volume to moles. The ratio of two extensive properties is an intensive property.

e.) **Number of moles** **Extensive**

Dividing in half a tank of water at a given T, P with a givennumber of moles inside yields two smaller tanks containing water at the same T & P, but thenumber of moles of water in each new, smaller tank is half as much as it was intially in the one big tank.

1C-3 **Intensive Properties and the State of a System** **2 pts**

Can pure liquid **water** exist at **99°C** and **1 atm** with a **density** of **0.980 g/cm^3**? Explain your reasoning.

Read: Not much to say here.

Given:

T	**99**	**°C**	ρ	**0.98**	**g/cm^3**
P	**1**	**atm**			

Find: **Can pure liquid water exist under these conditions?**

Assumptions: None.

Equations / Data / Solve:

Answers: **No.**

The state of a pure substance that exists in a single phase is uniquely determined when **2** intensive properties are known. Here, the values of **3** intensive properties are specified. Any one of the three could be considered too much, extraneous or redundant information. For example, at the given **T** & **P**, the density of liquid water is **0.959 g/cm^3**, not **0.980 g/cm^3**. Similarly, liquid water at the given **T** & **density** exists at a pressure of about **0.471 atm**, not **1 atm**.

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1D-1 Bomb Calorimeter 2 pts

Propane and **oxygen** undergo a combustion reaction in a strong, rigid, stainless steel canister called a **bomb calorimeter**. Is this process an isothermal, isobaric or isochoric process? **Explain** your reasoning.

Read: Not much to say here.

Given:

Find:

Assumptions:

Equations / Data / Solve:

Answers: **Isochoric**

The combustion of propane is exothermic so the temperature will increase. The reaction results in an increase in the number of moles as **1** molecule of propane combines with **5** molecules of **oxygen** to produce **3** molecules of **CO_2** and 4 molecules of water. As a result of the increase in **T** and the number of molecules in the system, the pressure will increase. So, the process is neither **isothermal** nor **isobaric**. The volume of the rigid bomb calorimeter remains constant throughout this process. Therefore, this process is an **isochoric** process.

1D-2 Thermodynamic Cycles in Normal Life 3 pts

List **three** common devices that make use of thermodynamic cycles and **explain** how you know they operate using a thermodynamic cycle.

Read: Not much to say here.

Given:

Find:

Assumptions:

Equations / Data / Solve:

Answers:

1- **Refrigerator**

Work and **heat** cross the system boundary, but **mass** does not. The **refrigerant** undergoes a cyclic process involving changes in both **T** & **P**, but the **refrigerant** always returns to the same **initial state** to begin the cycle again.

2- **Air conditioner**

Work and **heat** cross the system boundary, but **mass** does not. The **refrigerant** undergoes a cyclic process involving changes in both **T** & **P**, but the **refrigerant** always returns to the same **initial state** to begin the cycle again.

3- **Automobile**

This one is a bit trickier. **Work** and **heat** cross the system boundary. **Heat** enters the working fluid (air) from the heat source (burning gasoline) and **work** leaves the system in the form of a rotating drive shaft or gear train. It is not as clear that the air undergoes a cyclic process. It may help to consider all the air in the world as the working fluid for the cycle. The engine draws in air at ambient temperature. It undergoes changes in **T** & **P** inside the engine and is exhausted back into the environment at a higher temperature. Eventually, the exhausted air cools back down and returns to its initial state to complete the cycle.

1D-3 Identifying Types of Equilibria 4 pts

State whether each of the following systems are in thermal equilibrium, chemical equilibrium, phase equilibrium or mechanical equilibrium.

Give **all** the correct answers for each system. **a.)** snow resting on the roof of my house on a very cold day, **b.)** a cup of coffee sitting on my desk, **c.)** a bicycle rolling along a level road, **d.)** a river flowing past your city.

Read: Not much to say here.

Given:

a.)	**Snow**	c.)	**Bike**
b.)	**Coffee**	d.)	**River**

Find: Is each system in **thermal equilibrium**, **chemical equilibrium**, **phase equilibrium** or **mechanical equilibrium**?

Assumptions:

Equations / Data / Solve:

Answers:

		Thermal	Chemical	Phase	Mechanical	
a.)	**Snow**	No	Yes	No	Yes	The snow is melting due to heat transfer from my roof and may also be sublimating.
b.)	**Coffee**	No	Yes	No	Yes	Water and oils are evaporating while heat is being transferred to the surroundings.
c.)	**Bike**	Yes	Yes	Yes	No	Unbalanced forces cause the bike to roll.
d.)	**River**	No	Yes	No	No	The water is absorbing heat and evaporating.

1D-4 Problem Title 4 pts

Which of the following processes could be reasonably described as a quasi-equilibrium process?

a.) a gas escaping from a high-pressure tank through a tiny pin hole into the surrounding **air**

b.) a boy jumping from a cliff into the ocean

c.) a woman blowing up a balloon

d.) **water** flowing from the bottom of a large tank through a hose and a shower-head.

Read: Not much to say here.

Given:

a.) **Gas escaping from a tank**
b.) **Boy jumping from a cliff**
c.) **Woman blowing up a balloon**
d.) **Water flowing from a tank**

Find: Can each process be considered to be a **quasi-equilibrium** process?

Assumptions:

Equations / Data / Solve:

Answers:

a.) **No.** The gas escapes from the high pressure tank very rapidly. The outward pressure force of the gas is not balanced by the pressure force of the surrounding atmosphere. This system deviates substantially from mechanical equilibrium.

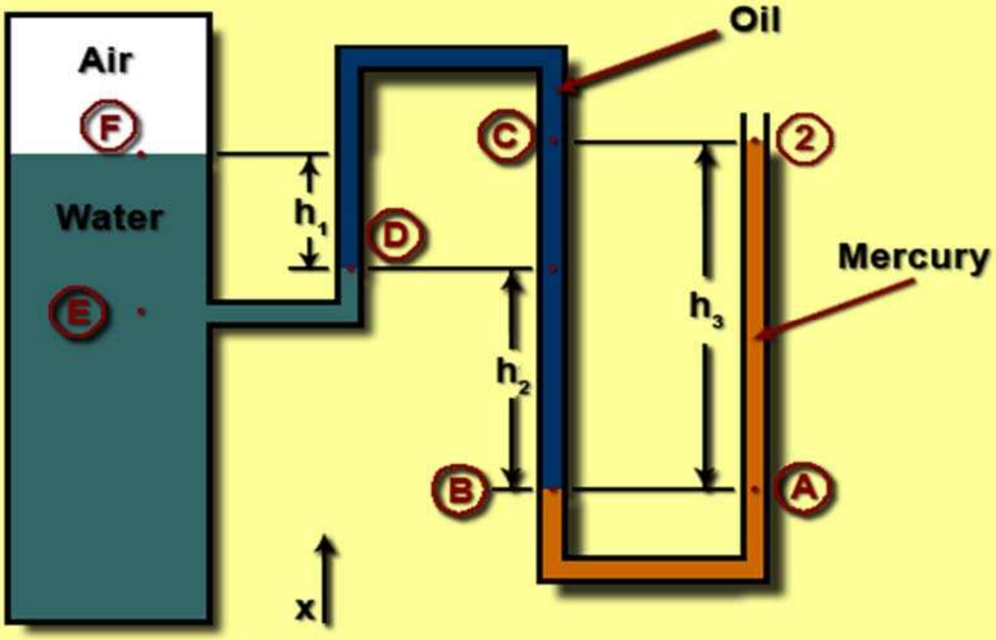
b.) **No.** The boy accelerates as he falls because the force of gravity acting on him is not balanced. This system deviates substantially from mechanical equilibrium.

c.) **Yes.** The air from the woman's lungs enters the balloon slowly and steadily in each breath. Acceleration is very modest. Under most circumstances, it would be reasonable to describe this as a quasi-steady-state process.

d.) **Yes.** The tank drains slowly. The water flows slowly and nearly steadily. There are unbalanced forces causing the water to flow from the tank through the tube and the shower head, but the imbalance is modest and the acceleration of the water is very gradual. The pressure at the bottom of the tank decreases as the water level drops. But this occurs slowly. Under most circumstances, it would be reasonable to describe this as a quasi-steady-state process.

1E-1 Pressure Measurement Using a Multi-Fluid Manometer 6 pts

A pressurized vessel contains **water** with some **air** above it, as shown below. A multi-fluid manometer system is used to determine the **pressure** at the **air-water** interface, point **F**. Determine the **gage pressure** at point **F** in **kPa gage**.



Data:
h_1 = 0.24 m, **h_2 = 0.35 m** and **h_3 = 0.52 m**
Assume the fluid **densities** are **water**: **1000 kg/m^3**, **oil**: **790 kg/m^3** and **mercury**(**Hg**): **13,600 kg/m^3**.

Read: Use the barometer equation to work your way through the different fluids from point 1 to point 2.
Remember that gage pressure is the difference between the absolute pressure and atmospheric pressure.

Given:	**h_1**	**0.24**	**m**	ρ_w	**1000**	**kg/m^3**
	h_2	**0.35**	**m**	ρ_{oil}	**790**	**kg/m^3**
	h_3	**0.52**	**m**	ρ_{Hg}	**13600**	**kg/m^3**
	P_2	101.325	kPa			

Find: **$P_{1,gauge}$** **???** **kPa gauge**

Assumptions:
1- The fluids in the system are completely static.
2- The densities of the liquids are uniform and constant.
3- The acceleration of gravity is:

g	**9.8066**	**m/s^2**
g_C	**1**	**kg-m/N-s^2**

Equations / Data / Solve:

Gage pressure is defined by :

$$P_{gage} = P_{abs} - P_{atm}$$ **Eqn 1**

If we assume that P_2 is atmospheric pressure, then **Eqn 1** becomes :

$$P_{1,gage} = P_1 - P_2$$ **Eqn 2**

The key equation is the Barometer Equation :

$$P_{bottom} = P_{top} + \rho \left(\frac{g}{g_C} \right) h$$ **Eqn 3**

Now, apply **Eqn 1** repeatedly to work our way from point 1 to point 2.

$$P_A = P_B$$ **Eqn 4**

Some key observations are:

$$P_C = P_D = P_E$$ **Eqn 5**

These are true because the points are connected by open tubing, the fluid is not flowing in this system and no change in the composition of the fluid occurs between **A** & **B** or **C** & **D** or **D** & **E**.

P_A > P_2, therefore :

$$P_A = P_2 + \rho_{Hg} \left(\frac{g}{g_C} \right) h_3$$ **Eqn 6**

P_E > P_1, therefore :

$$P_E = P_1 + \rho_w \left(\frac{g}{g_C} \right) h_1$$ **Eqn 7**

P_B > P_C, therefore :

$$P_B = P_C + \rho_{oil} \left(\frac{g}{g_C} \right) h_2$$ **Eqn 8**

Combine **Eqns 2, 5 & 6** to get :

$$P_2 + \rho_{Hg} \left(\frac{g}{g_C} \right) h_3 = P_C + \rho_{oil} \left(\frac{g}{g_C} \right) h_2$$ **Eqn 9**

Use **Eqns 3 & 5** to eliminate P_C from **Eqn 7** :

$$P_2 + \rho_{Hg} \left(\frac{g}{g_C} \right) h_3 = P_1 + \rho_w \left(\frac{g}{g_C} \right) h_1 + \rho_{oil} \left(\frac{g}{g_C} \right) h_2$$ **Eqn 10**

Now, solve for **P_1 - P_2** :

$$P_1 - P_2 = \rho_{Hg} \left(\frac{g}{g_C} \right) h_3 - \rho_w \left(\frac{g}{g_C} \right) h_1 - \rho_{oil} \left(\frac{g}{g_C} \right) h_2$$ **Eqn 11**

Combining **Eqns 10 & 2** yields :

$$P_{1,gage} = \left[\rho_{Hg}\ h_3 - \rho_w\ h_1 - \rho_{oil}\ h_2 \right] \left(\frac{g}{g_C} \right)$$ **Eqn 12**

Plugging values into **Eqn 12** yields :

$P_{1,gage}$	64287	Pa gage
$P_{1,gage}$	64.29	kPa gage

Answers: **$P_{1,gage}$ 64.3 kPa gage**

If you are curious :

P_1	165.61	kPa	$P_A = P_B$	170.68	kPa
P_2	101.325	kPa	$P_C = P_D = P_E$	167.97	kPa

1E-2 Pressure Gage and Manometer Readings 6 pts

A cylindrical tank contains pressurized **nitrogen** gas. The gage on the tank reads **88 kPa**. A manometer is attached to the tank to verify its **pressure**. Determine the manometer reading in **cm** if the manometer fluid is …

a.) $\rho_{mercury}$ = 13,600 kg/m^3, b.) ρ_{water} = 1000 kg/m^3

Read: Remember that gage pressure is the difference between the absolute pressure and atmospheric pressure. Apply the Barometer Equation with a known gage pressure to determine the manometer reading, h.

Given:

P_{gage}	88	kPa
ρ_w	1000	kg/m^3
ρ_{Hg}	13600	kg/m^3

Find: h ??? m

Assumptions:

1- The **nitrogen** and **water** in the system are completely static.
2- The densities of the **nitrogen** and **water** are uniform and constant.
3- The acceleration of gravity is:

g	**9.8066**	**m/s^2**
g_C	**1**	**kg-m/N-s^2**

Equations / Data / Solve:

Gage pressure is defined by :

$$P_{gage} = P_{abs} - P_{atm} \qquad \textbf{Eqn 1}$$

The **Manometer Equation** tells us that :

$$P_{abs} = P_{atm} + \rho\left(\frac{g}{g_C}\right)h \qquad \textbf{Eqn 2}$$

Combining **Eqns 1 & 2** gives us :

$$P_{gage} = \rho\left(\frac{g}{g_C}\right)h \qquad \textbf{Eqn 3}$$

Now, we can solve **Eqn 3** for **h** :

$$h = \frac{P_{gage}}{\rho\left(\frac{g}{g_C}\right)} \qquad \textbf{Eqn 4}$$

Now, we can plug values into **Eqn 4** :

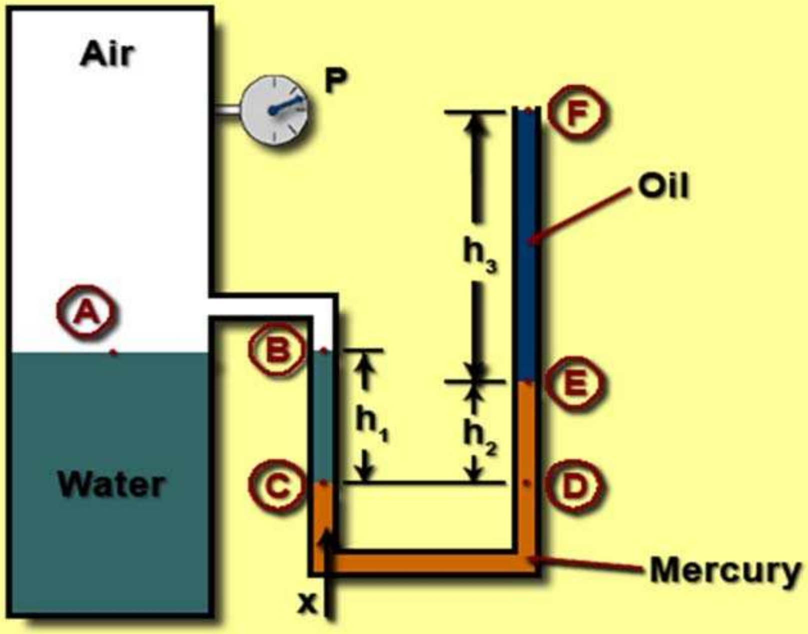
a.)	h	0.660	m
b.)	h	8.974	m

Answers:

a.)	**h**	**0.66**	**m**
b.)	**h**	**8.97**	**m**

1E-3 Pressure in a Tank Using a Complex Manometer 5 pts

The pressure gauge on the air in the tank shown below reads **87 kPa**. Determine the manometer reading, $\mathbf{h_2}$, in **cm**.



Data:
h_1 = 25 cm and h_3 = 65 cm
$SG_{mercury} = 13.6$, $SG_{oil} = 0.75$, $\rho_{water} = 1000\ kg/m^3$

Read: The density of the air is so much lover than the density of the liquids in this problem that the weight of the air can be considered negligible in the force balances we will write in this problem.

Given:

P_{gauge}	87	kPa		
h_{oil}	0.65	m	SG_{oil}	0.7
h_{water}	0.25	m	SG_m	13.6

Find: h_2 ??? cm

Assumptions:

1- The fluids in the system are completely static.
2- The densities of the liquids are uniform and constant.
3- The reference density of water used to determine specific gravity is: ρ_{water} 1000 kg/m^3
4- The acceleration of gravity is: g 9.8066 m/s^2
g_c 1 $kg\text{-}m/N\text{-}s^2$

Equations / Data / Solve:

Begin by writing the **Manometer Equation** for each interval between points **A** and **F** on the diagram.

$$P_e = P_f + \rho_{oil} \frac{g}{g_c} h_{oil}$$ **Eqn 1**

$$P_d = P_e + \rho_m \frac{g}{g_c} h$$ **Eqn 2**

$$P_c = P_d$$ **Eqn 3**

$$P_b = P_c - \rho_w \frac{g}{g_c} h_{water}$$ **Eqn 4**

$$P_a = P_b$$ **Eqn 5**

If we add all 5 of these equations together we obtain :

$$P_a = P_f + \rho_{oil} \frac{g}{g_c} h_{oil} + \rho_m \frac{g}{g_c} h - \rho_w \frac{g}{g_c} h_{water}$$ **Eqn 6**

The only unknown in this equation is **h**. So, the next step is to solve the equation for **h**.

$$h = \frac{\frac{g_c}{g}\left(P_a - P_f\right) - \rho_{oil}\, h_{oil} + \rho_w\, h_{water}}{\rho_m}$$ **Eqn 7**

Also, because $P_f = P_{atm}$ and the definition of gauge pressure we can use:

$$P_{gauge} = P_a - P_{atm} = P_a - P_f$$ **Eqn 8**

P_a-P_f = 87000 N/m^2

All we need to do is convert specific gravity into density and we are ready to plug values into **Eqn 7**.

The definition of specific gravity is :

$$SG = \frac{\rho}{\rho_{ref}} = \frac{\rho}{\rho_w}$$ **Eqn 9**

This helps us simplify **Eqn 7** to :

$$h = \frac{P_{gauge}}{\rho_m \frac{g}{g_c}} - \frac{SG_{oil}}{SG_m} h_{oil} + \frac{h_{water}}{SG_m}$$ **Eqn 10**

$$h = \frac{P_{gauge}}{\rho_m \frac{g}{g_c}} + \frac{h_{water} - SG_{oil} h_{oil}}{SG_m}$$ **Eqn 11**

From **Eqn 9** :

$$\rho = SG\ \rho_w$$ **Eqn 12**

ρ_m 13600 kg/m^3

Plugging values into **Eqn 11** yields :

h 0.652 m
65.2 cm

Answers: **h 65.2 cm**

1E-4 Temperature Unit Conversions 2 pts

The **temperature** on the surface of the radiator of an automobile is **134°C**. What is the **temperature** in **°R**, **K**, and **°F**?

Read: Not much to read here.

Given: T 134 °C

Find:

T	???	°R	T	???	°F
T	???	K			

Assumptions: None.

Equations / Data / Solve:

The relationship between Kelvin and Celsius is simplest, so let's begin there.

$$T(K) = T(^\circ C) + 273.15 \qquad \text{Eqn 1}$$

Therefore : **T(K)** **407.15** **K**

The relationship between Celsius and Fahrenheit is also fairly simple, so let's work on that next.

$$T(^\circ F) = 1.8\,T(^\circ C) + 32 \qquad \text{Eqn 2}$$

Therefore : **T(°F)** **273.20** **°F**

There are two straightforward ways to determine **T(°R)** at this point.
We can either convert **T(K)** to **T(°R)** or we can convert **T(°F)** to **T(°R)**.
Let's try both methods, just to be thorough.

$$T(^\circ R) = 1.8\,T(K) \qquad \text{Eqn 3}$$

T(°R) **732.87** **°R**

Alternatively :

$$T(^\circ R) = T(^\circ F) + 459.67 \qquad \text{Eqn 4}$$

T(°R) **732.87** **°R**

Answers:

T(°R)	733	°R
T(K)	407	K
T(°F)	273	°F

1E-5 Temperature Unit Conversions 2 pts

The **temperature** of a glass of **water** is **27°C**. What is the **temperature** of the **water** in **°R**, **K**, and **°F**?

Read: This one is a straightforward temperature conversion.

Given:

T	**27**	**°C**

Find:

T	**???**	**°R**
T	**???**	**K**
T	**???**	**°F**

Assumptions: None.

Equations / Data / Solve:

The key equations are those that relate **T** in **°C** to the desired temperature units.

$$\mathbf{T(K) = T(^{o}C) + 273.15} \qquad \text{Eqn 1}$$

T 300.15 K

$$\mathbf{T(^{o}F) = T(^{o}C) \cdot 1.8 + 32} \qquad \text{Eqn 2}$$

T 80.6 °F

$$\mathbf{T(^{o}R) = T(^{o}F) + 459.67} \qquad \text{Eqn 3}$$

T 540.27 °R

Alternatively, we could use :

$$\mathbf{T(^{o}R) = 1.8 \cdot T(K)} \qquad \text{Eqn 4}$$

T 540.27 °R

Answers:

T	**540.27**	**°R**
T	**300.15**	**K**
T	**80.6**	**°F**

1E-6 Temperature Change & Unit Conversions 2 pts

The **temperature** of a cup of **water** drops by **44°F** when it is placed in a refrigerator for **45 minutes**. How much did the **temperature** of the **water** change in **K**, **°R** and in **°C**.

Read: The key to this problem is that we are considering a **CHANGE** in temperature and **NOT** a temperature of 27°C.

Given:

ΔT	**-44**	**°F**

Find:

ΔT	**???**	**K**	**ΔT**	**???**	**°C**
ΔT	**???**	**°R**			

Assumptions: None.

Equations / Data / Solve:

Start with Rankine. The temperature increments for the Fahrenheit and Rankine scales are the same.

ΔT	**-44**	**°R**

Next, let's work on getting ΔT in the Kelvin scale.
This is straightforward because a temperature change of 1 K is equal to a temperature change of 1.8 °R.

$$\Delta T(K) = \frac{\Delta T(^{\circ}R)}{1.8} \qquad \textbf{Eqn 1}$$

ΔT	**-24.444**	**K**

Finally, the temperature increments for the Celsius and Kelvin scales are the same.

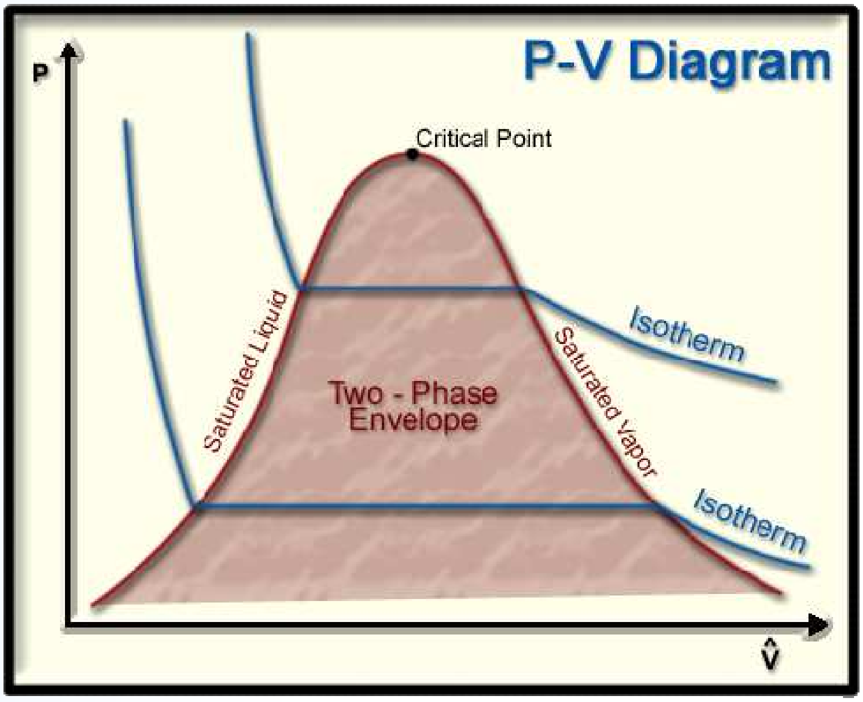
Therefore :

ΔT	**-24.444**	**°C**

Answers:

ΔT	**-24.4**	**K**
ΔT	**-44**	**°R**
ΔT	**-24.4**	**°C**

Chapter 2



Properties of Pure Substances

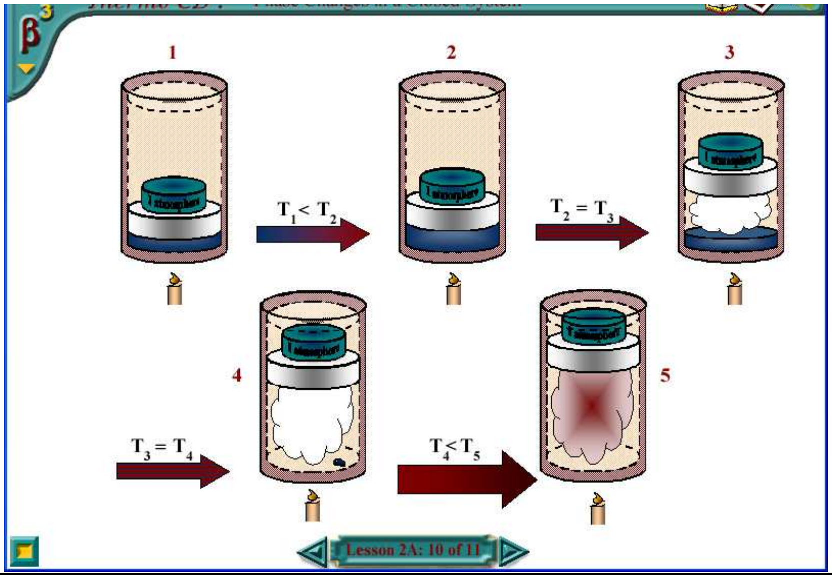
In this chapter, you will learn to recognize a pure substance. This book focuses on the thermodynamics of pure substances. We will only consider the special case of systems that contain a mixture that consists of a single condensable species, such as the air-water system. The topic of vapor-liquid equilibrium is the central topic of this chapter and it is crucial that you understand this topic well. You will learn to obtain thermodynamic properties from the tables in the Appendices of this book. The chapter concludes with a discussion of equations of state. An equation of state is a relationship between pressure, volume and temperature. You should already be familiar with the Ideal Gas Equation of State. Here you will learn about its limitations and more accurate alternatives.

Nomenclature

- **Pure Substance**
  - ◊ Uniform chemical composition throughout the system
- **Phases**
  - ◊ Liquids: Multiple liquid phase
  - ◊ Solids: Multiple solid phases
  - ◊ Gases: Only ONE gas phase can exist
- **Phase Changes**
  - ◊ Liquid » Gas: Boiling or Evaporating / Condensing
  - ◊ Liquid » Solid: Melting / Freezing
  - ◊ Gas » Solid: Sublimating / Desublimating
- **Liquid » Gas : An Isobaric Process Path**
  - ◊ Consider the isobaric process on the next slide in which energy is added to a closed system that initially contains liquid water at a $T < T_{sat}$

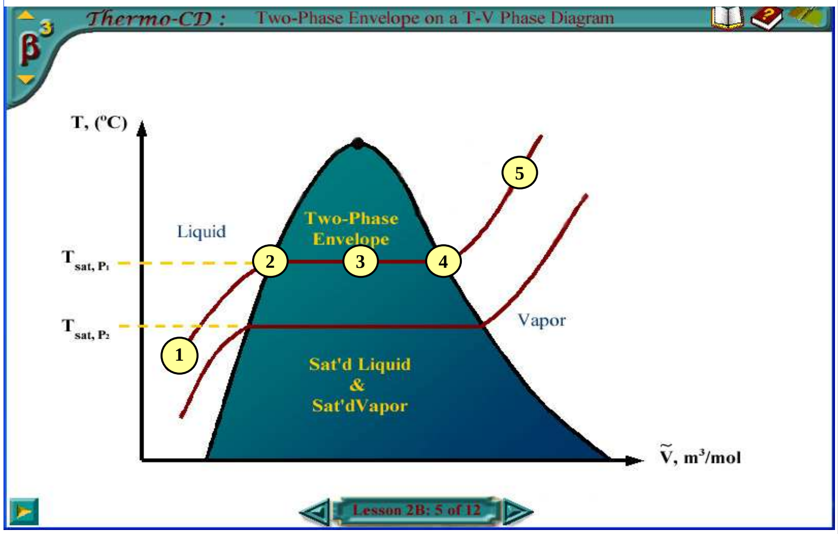
- Pure Substance
  - ◊ Dry air is a pure substance
  - ◊ Humid air can be considered to be a pure substance
  - ◊ A tank containing liquid water with humid air above it cannot be considered to be a pure substance !
- Phases
  - ◊ Multiple liquid phases can exist in equilibrium: oil and water
  - ◊ Multiple solid phases can exist in equilibrium: diamond and carbon, different types of ice crystals, different types of steels.
  - ◊ Only ONE gas phase can exist at equilibrium. All the molecules always mix.
- Phase Changes
  - ◊ What is the difference between boiling and evaporating ?
  - ◊ Sublimation: Did you ever notice that old ice cubes in your home freezer have shrunk ? Do you think they melted ? No. They sublimated !
- Phase Diagrams
  - ◊ An isobaric process path is a smooth way to introduce Phase Diagrams
  - ◊ Phase Diagrams are our FRIENDS…they make any process a little easier to understand.

Isobaric Heating of a Pure Substance



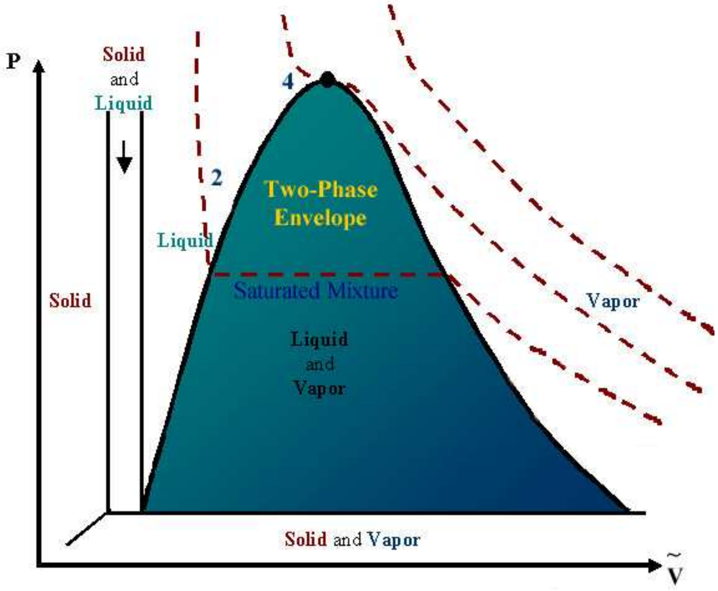
- Quality: x = fraction of the mass in the system that exists in the gas or vapor phase.
- 1 – Subcooled Liquid
  $T < T_{sat}$ and $P^*(T) < P$ and x is undefined
- 2 – Saturated Liquid
  $T = T_{sat}$ and $P^*(T) = P$ and $x = 0$
- 3 – Saturated Mixture
  $T = T_{sat}$ and $P^*(T) = P$ and $0 < x < 1$
- 4 – Saturated Vapor
  $T = T_{sat}$ and $P^*(T) = P$ and $x = 1$
- 5 – Superheated Vapor
  $T > T_{sat}$ and $P^*(T) > P$ and x is undefined

Isobaric Heating Process Path on a TV Diagram



- Elements of the Vapor-Liquid region of a phase diagram
  - ◊ Sat'd Liquid Curve
  - ◊ Sat'd Vapor curve
  - ◊ Critical Point
  - ◊ Two Phase Envelope where vapor and liquid both exist within the system at equilibrium
  - ◊ Subcooled liquid region
  - ◊ Superheated vapor region
  - ◊ Supercritical fluid region
- Isobaric Heating Process
  - ◊ Slides up and to the left along an isobar

Isothermal Heating Process Path on a PV Diagram



- This PV diagram extends down into the solid region
  - ◊ We will focus on the vapor-liquid region in this course.
- Same elements in this diagram as in the TV Diagram, but they are located in slightly different positions.
- Isobaric Heating Process
  - ◊ Slides to the left along a horizontal isobar

Nomenclature

- **Latent Heat vs. Sensible Heat**
  - ◊ Liquids: Multiple liquid phase
  - ◊ Liquids: Multiple liquid phase
  - ◊ Latent Heat: As heat is added, T does not change
  - ◊ Sensible Heat: As heat is added, T does change
- **Critical Point**
  - ◊ Above P_c or T_c two phases cannot exist at equilibrium
  - ◊ Only one phase exists: a supercritical fluid
- **Triple Point**
  - ◊ **Liquid ↔ Gas ↔ Solid :** All exist in the system at equilibrium
  - ◊ **Water:** 0.01°C and 0.612 kPa

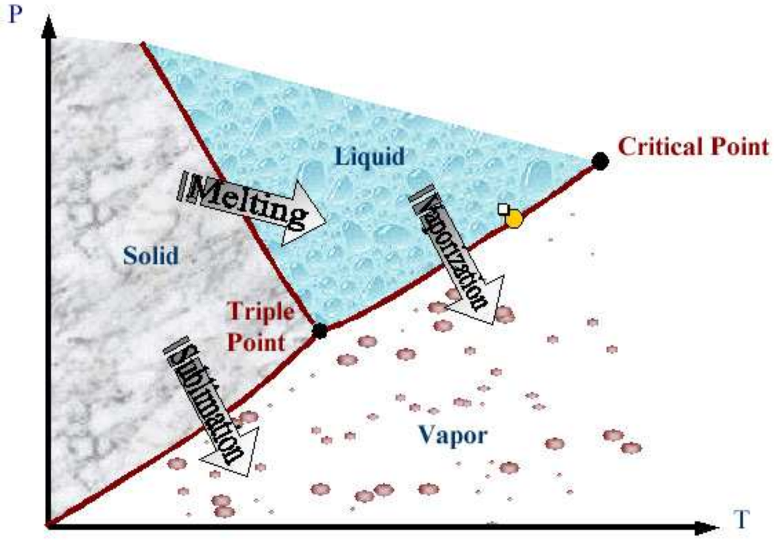
- Latent Heat
  - ◊ Heats associated with phase changes
- Sensible Heat
  - ◊ Heat flow which causes a change in temperature
- Critical Point
  - ◊ P_C, T_C and critical molar volume
  - ◊ Above P_C the substance is a supercritical fluid
  - ◊ Above T_C the substance is a supercritical fluid

PT Diagrams

- **Two Types for the Two Types of Substances**
  - ◊ Substances that contract when they freeze
    - Most substances fall into this category
  - ◊ Substances that expand when they freeze
    - There are very few of these.
    - Most important one is WATER

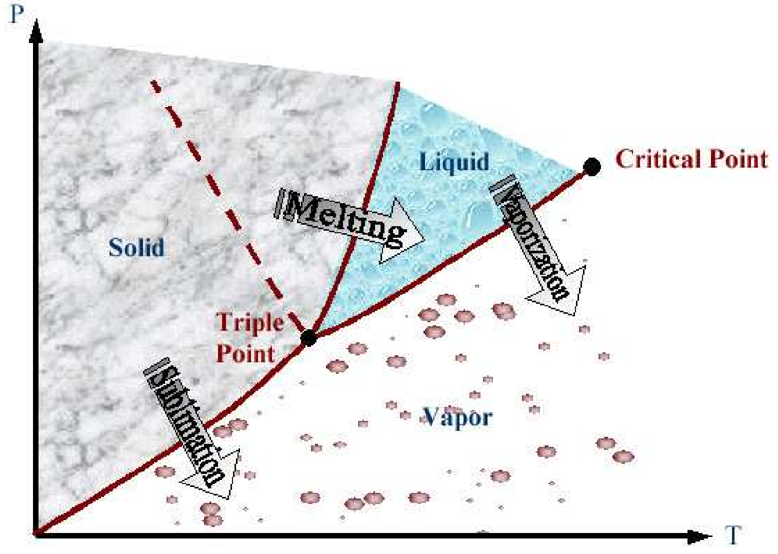
- Ice floats
- Most solids sink because they are more dense than the saturated liquid with which they exist at equilibrium.

Substances That Expand on Freezing



- Red lines are saturation curves
- Consider an increase in P at constant T for a system on a Saturation Curve
  - VLE: Vapor condenses into a liquid
    - Conclusion: liquid is more dense than vapor
  - SVE: Vapor desublimates into a solid
    - Conclusion: solid is more dense than vapor
  - SLE: Solid melts into a liquid
    - Conclusion: liquid is more dense than solid
    - The solid is less dense than the liquid because this substance expands on freezing
    - Example: Water

Substances That Contract on Freezing



- SLE: Liquid fuses into a solid
  - Conclusion: Solid is more dense than solid
  - The solid is more dense than the liquid because this substance contracts on freezing
  - Example: Almost everything <u>except</u> water.

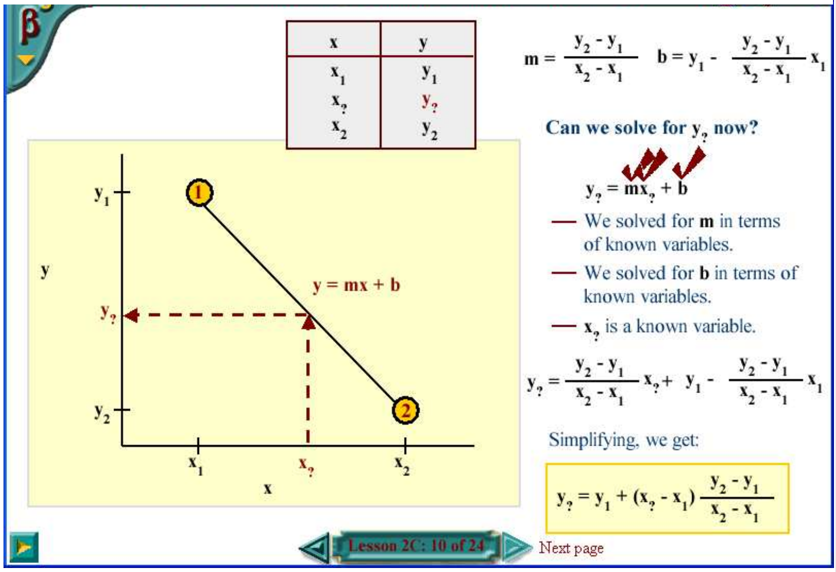
Thermodynamic Data Tables

- **Four Tables in a Complete Thermodynamic Data Table**
  - ◊ Saturation Temperature Table
  - ◊ Saturation Pressure Table
  - ◊ Subcooled Liquid Table
  - ◊ Superheated Vapor Table

- **Appendix**
  - ◊ Steam Tables, SI & AE
    - ◊ Appendix A
  - ◊ Ammonia Tables, SI & AE
    - ◊ Appendix B
  - ◊ R-134a Tables, SI & AE
    - ◊ Appendix C

- Look at Appendices A, B and C.
- You will become very good at using these data tables to solve problems.

Single Interpolation



x	y
x_1	y_1
$x_?$	$y_?$
x_2	y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad b = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

Can we solve for $y_?$ now?

✓✓✓

$$y_? = m x_? + b$$

— We solved for **m** in terms of known variables.

— We solved for **b** in terms of known variables.

— $x_?$ is a known variable.

$$y_? = \frac{y_2 - y_1}{x_2 - x_1} x_? + y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1$$

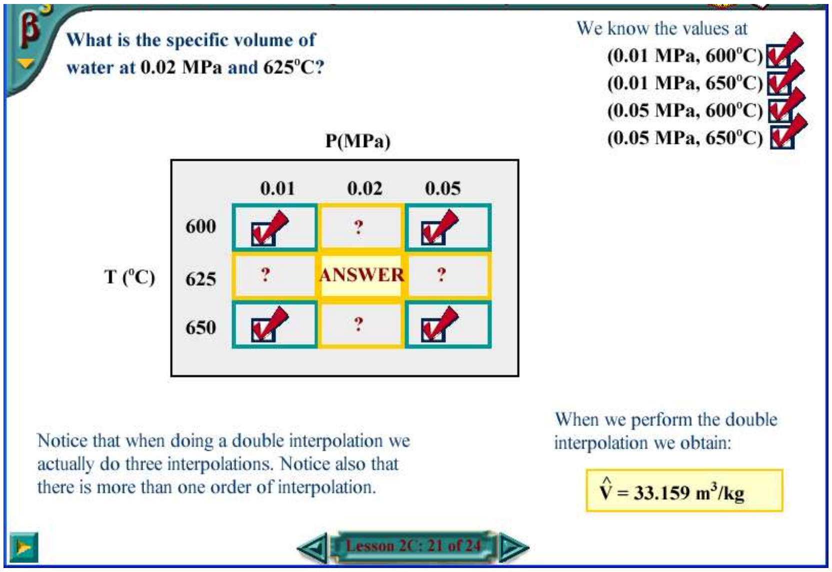
Simplifying, we get:

$$\boxed{y_? = y_1 + (x_? - x_1) \frac{y_2 - y_1}{x_2 - x_1}}$$

Lesson 2C: 10 of 24 Next page

- Interpolation is about drawing a line between known two points.
- Then, determine the Y value for a given X value that falls between the two know points.
- Y = m X + b
- Use the 2 known points to determine m and b
- Use m and b to estimate Y at some new value of X.

Double Interpolation



What is the specific volume of water at 0.02 MPa and 625°C?

We know the values at
(0.01 MPa, 600°C) ☑
(0.01 MPa, 650°C) ☑
(0.05 MPa, 600°C) ☑
(0.05 MPa, 650°C) ☑

P(MPa)

T (°C)		0.01	0.02	0.05
	600	☑	?	☑
	625	?	ANSWER	?
	650	☑	?	☑

Notice that when doing a double interpolation we actually do three interpolations. Notice also that there is more than one order of interpolation.

When we perform the double interpolation we obtain:

$\hat{V} = 33.159\ m^3/kg$

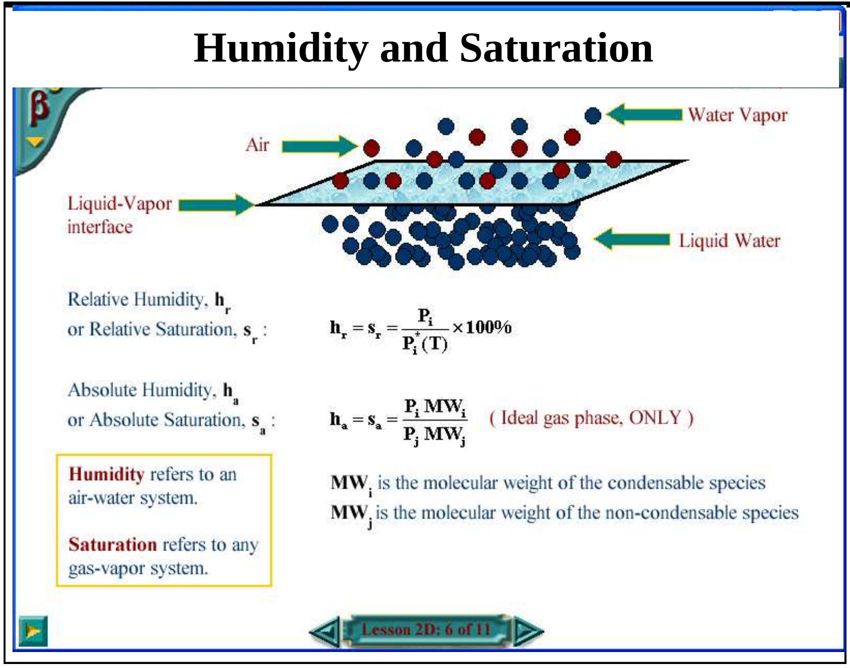
Lesson 2C: 21 of 24

- Double interpolation requires THREE single, linear interpolations.
- The only way to learn how to do double interpolation is to do a few of them.
- Unfortunately, they are pretty tedious.

VLE Nomenclature

- **Mole Fraction:** y_i
- **Partial Pressure:** P_i
  - ◊ Ideal Gases: $P_i = y_i P$
- **Vapor Pressure:** P_i^*
- **Condensable and non-Condensable Species**
  - ◊ Water / Air
- **Evaporation:** $P_{total} > P_i^*\left(T_{liquid}\right) > P_i$
- **Boiling:** $P_i^*\left(T_{liquid}\right) \geq P_{total}$

- Mole fraction is the fraction of all the molecules in a system that are chemical species "i"
- Partial pressure of species "i" is the pressure that WOULD exist in the system if all of the other chemical species were removed.
- Vapor pressure
  - ◊ Property of a pure species depends ONLY T
  - ◊ Same as saturation pressure: pressure at which a pure substance would boil at any fixed T
  - ◊ Pressure that molecules in the liquid phase can overcome in order to enter the vapor phase by boiling.
- Condensable Species
  - ◊ Could exist in the liquid phase at the system pressure if the temperature dropped enough
  - ◊ This means they are not supercritical fluids.
  - ◊ Water in the air is a condensable species
- Non-Condensable Species
  - ◊ Cannot condense at the system pressure at any T_{sys} because they are supercritical fluids.
  - ◊ The O_2 and N_2 in air are non-condensable.
- Evaporation
  - ◊ Molecules move from the liquid phase to vapor phase without bubble formation (boiling)
  - ◊ Vapor pressure is greater than the partial pressure of the condensable species in the gas phase, but less than the total pressure
  - ◊ Evaporation occurs because the water molecules in the liquid phase are pushing harder to get into the gas phase (P_i*) than the water molecules in the gas phase are pushing to get into the liquid phase (P_i).
- Boiling
  - ◊ Vapor bubbles of the condensable species form in the liquid phase
  - ◊ Condensable species is pushing so hard to get into the gas phase, that it overcomes the total pressure and suddenly bursts into a vapor bubble.
  - ◊ Boiling Point: $P_i^*(T_{BP}) = P$
  - ◊ Normal Boiling Point: $P_i^*(T_{NBP}) = 1$ atm



Humidity and Saturation

Relative Humidity, h_r or Relative Saturation, s_r :

$$h_r = s_r = \frac{P_i}{P_i^*(T)} \times 100\%$$

Absolute Humidity, h_a or Absolute Saturation, s_a :

$$h_a = s_a = \frac{P_i\, MW_i}{P_j\, MW_j}$$

(Ideal gas phase, ONLY)

Humidity refers to an air-water system.

Saturation refers to any gas-vapor system.

MW_i is the molecular weight of the condensable species

MW_j is the molecular weight of the non-condensable species

- Saturation is a general term that describes a relationship between the condensable and non-condensable species in a gas system.
- Humidity is the same as saturation except humidity only applies to a system that contains just air and water.
- Relative Humidity or Saturation
  - ◊ Presented in the daily weather report
  - ◊ Dimensionless because P_i and P_i* have the same units !
  - ◊ When h_r = 100%, the gas is "full" or saturated. No more water can evaporate.
- Absolute Humidity
  - ◊ Definition here only applies when the gas is an ideal gas
  - ◊ At atmospheric conditions, humid air usually behaves as an ideal gas because the molar volume > 20 L/mole.
  - ◊ We will learn more about this criterion for ideal gas behavior in the next class.
  - ◊ Units = [kg condensable / kg non-condensable] = [kg Water / kg Air]
  - ◊ MW = molecular weight [g/mole or lb_m/lbmole]

Equations of State

- **Relationship between** $P, \tilde{V}$ and T: $P = fxn\left(\tilde{V}, T\right)$
- **Ideal Gas EOS**: $P\tilde{V} = RT$
- **Universal Gas Constant: R**
  - ◊ R = 8.314 J/mol-K
    R = 0.082054 L-atm/mol-K
    R = 1.987 Btu/lbmole-^{o}R
- **When does the IG EOS apply ?**
  - ◊ When molecules interact very little with each other
  - ◊ At high T and low P
  - ◊ Generally: $\tilde{V} = \frac{RT}{P} > 20$ L / mole
  - ◊ Diatomic gases are especially unlikely to interact $\tilde{V}_{diatomic} = \frac{RT}{P} > 5$ L / mole

- The ideal gas equation is not a "Law" because it is generally NOT accurate !
- The ideal gas equation is an equation of state, a relationship between P, V and T
- We like to write all EOS's in terms of molar volume instead of volume and number of moles in the system.
  - ◊ In this way, our EOS only contains INTENSIVE variables and is therefore applicable to systems without consideration of their SIZE.
  - ◊ Remember that the values of intensive variables do not depend on the size of the system.
- Be careful, in some texts…
  - ◊ They use a different value of R for each substance and use R-bar for the universal gas constant
  - ◊ Their nomenclature is, R = R-bar / MW [=] J/g-K or kJ/kg-K
  - ◊ We will NOT follow this convention.

Advanced Equations of State

- **Compressibility Factor EOS (graphical)**
- **Virial EOS**
- **Van der Waals EOS**
- **Redlich-Kwong EOS**
- **Soave-Redlich-Kwong EOS**

- There are MANY EOS's.
- In fact, the "data" in our thermodynamic tables are not really data at all !
- The numbers in the thermodynamic tables are the output of very sophisticated EOS's that we will not study.
- We will just consider the values in thermodynamic tables to be data.
- We will only gain a passing familiarity with the compressibility EOS because it is an old style graphical EOS.
  - ◊ That is not to say it is not accurate.
  - ◊ It just is not compatible with our modern computing environment
- Van der Waals was the 1st cubic EOS
  - ◊ Very powerful
  - ◊ Can predict properties of liquids, as well as gases
- RK EOS is a popular EOS that can predict liquid properties reasonably well.
- SRK is an improved version of RK.
- None of these cubic EOS's work well for water or other highly polar molecules or ionic solutions
- Specialized EOS's have been developed for water.
- Ionic solutions are more complicated and are modeled by specialized EOS's.

Compressibility Factor EOS

- **Compressibility Factor:** $\mathbf{Z = \frac{P\tilde{V}}{RT} = \frac{\tilde{V}}{RT/P} = \frac{\tilde{V}}{\tilde{V}_{IG}}}$
- **Principle of Corresponding States**
- **Reduced Properties:**

$$\mathbf{T_R = \frac{T}{T_C} \qquad P_R = \frac{P}{P_C} \qquad \tilde{V}_R^{ideal} = \frac{\tilde{V}}{\tilde{V}_C^{ideal}} = \frac{\tilde{V}}{RT_C/P_C}}$$

- **Compressibility Charts**
  - ◊ **Z** vs $\mathbf{P_R}$
  - ◊ Curves of constant $\mathbf{T_R}$
  - ◊ Curves of constant $\mathbf{\tilde{V}_R^{ideal}}$

- For an IG, Z = 1
- The further Z is from 1, the more non-ideal the behavior of the gas.
- Many properties of gases, including Z, depend primarily on how close the state of a system is to the critical state of the gas.
- Reduced properties are the ratio of the real system properties to the properties of the system at the critical point.
- Reduced ideal molar volume is more convenient because you can calculate it without knowing the critical molar volume.
- This is important, because the critical molar volume is not as frequently tabulated as the critical T and P.

Virial EOS

- **Uses a power series expansion to describe deviations of Z from 1, the IG value**

$$Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B}{\tilde{V}} + \frac{C}{\tilde{V}^2} + \frac{D}{\tilde{V}^3} + \frac{E}{\tilde{V}^4} + \dots$$

- **B, C, D, etc are the Virial "constants"**
  - ◊ Functions of T, only
  - ◊ Determined experimentally
- **Truncated Virial EOS:** $Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B}{\tilde{V}}$
  - ◊ Estimating B: $B = \frac{RT_C}{P_C}\left(B_0 + \omega B_1\right)$

$$B_0 = 0.083 - \frac{0.422}{T_R^{1.6}} \qquad B_1 = 0.139 - \frac{0.172}{T_R^{4.2}}$$

- The Virial EOS can be very very accurate if you keep enough terms in the series.
- The problem is that it is difficult to determine the values of D, E etc.
- It is relatively easy to measure or look up values of the 1st 2 Virial constants, B and C.
- If you cannot find values for B & C, you can use the truncated Virial EOS and APPROXIMATE the value of B using the equations given here.

Van der Waals EOS

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}^2}$$

- **First cubic EOS**
- **Constants have physical interpretation**

$$a = \frac{27}{64}\frac{R^2 T_C^2}{P_C} \qquad b = \frac{1}{8}\frac{RT_C}{P_C}$$

- This EOS was a big breakthrough in modeling PVT behavior.
- It is not used much anymore.
- The constant a represents a correction for inter-molecular forces called Van der Waals Forces.
- a > 0
- B is the molar volume of the molecules themselves, with the absolute minimum amount of space between molecules
- This is the molar volume at T = 0 K

RK & SRK EOS's

- **Redlich-** $$P = \frac{R\,T}{\tilde{V} - b} - \frac{a}{\tilde{V}\left(\tilde{V} + b\right)T^{1/2}}$$

$$a = 0.42748\,\frac{R^2\,T_C^{5/2}}{P_C} \qquad b = 0.08664\,\frac{R\,T_C}{P_C}$$

- **Soave-Redlich-Kwong :** $$P = \frac{R\,T}{\tilde{V} - b} - \frac{\alpha\,a}{\tilde{V}\left(\tilde{V} + b\right)}$$

$$a = 0.42748\,\frac{R^2\,T_C^2}{P_C} \qquad b = 0.08664\,\frac{R\,T_C}{P_C}$$

$$\alpha = \left[1 + m\left(1 - \sqrt{T_R}\right)\right]^2$$

$$m = 0.48508 + 1.55171\,\omega - 0.1561\,\omega^2$$

- These two EOS's are very popular choices, especially for hydrocarbon molecules.
- SRK is preferred to RK
- SRK modification tries to take polarity and molecular geometry into account
- It still doesn't work well for water or other highly polar molecules

Application of EOS's

- **Given any 2 of the 3 variables,** $P, \tilde{V}$ **and** T**, determine the value of the unknown**
- **Cubic EOS's and other even more sophisticated EOS's can be used to…**
  - ◊ predict properties of liquids
  - ◊ Estimate molar internal energies, enthalpies and entropies of gases and liquids
  - ◊ In this way, sophisticated EOS's are used to generate the Thermodynamic "Data" Tables that we use

2A-1 Think About Tea 2 pts

Tea is a mixture of one or more tasty **oils** dissolved in hot **water**. Is this a heterogeneous or homogeneous mixture? Are there any conditions under which this **tea** can be considered to be a pure substance? **Explain**.

If the tea is well-mixed, perhaps by stirring, then the mixture can be adequately described as homogeneous.

As long as the composition remains constant and uniform throughout, a homogeneous mixture can be treated as a pure substance. We call this a pseudo-component.

2A-2 Automobile Fuel 2 pts

Automobile **fuel**, known as **gasoline** or **petrol**, is a mixture of many types of hydrocarbon molecules. Is the **fuel** a heterogeneous or homogeneous mixture?

Are there any conditions under which this **fuel** can be considered to be a pure substance? **Explain**.

If the fuel is well-mixed, then the mixture can be adequately described as homogeneous. The different types of hydrocarbon molecules in gasoline are **miscible**, so the fuel tends toward uniform composition because the molecules mix by a random, wandering motion called **diffusion**.

As long as the composition remains constant and uniform throughout, a homogeneous mixture can be treated as a pure substance. We call this a pseudo-component.

2A-3 Think About Fog 2 pts

Fog is very large number of liquid **water** droplets suspended in **air**. Can fog be considered to be a pure substance? Do you think fog is heterogeneous or homogeneous? **Explain** your answers.

As long as the composition remains constant and uniform throughout, a homogeneous mixture can be treated as a pure substance. We call this a pseudo-component.

This is a tricky question. If you look very closely or only consider a small sample of fog, say **1 mm^3**, then you could see the water doplets and the air surrounding them. You would conclude that the fog sample is not homogeneous but heterogeneous because the composition is **not** everywhere the same.

If you consider a medium-sized sample of fog, perhaps **1 m^3**, and you did **not** look at it with a **microscope**, you would conclude that the composition **is** the **same** at all positions and the fog was homogeneous. This would depend on how big your sample was from diferent positions within the **1 m^3** you were considering.

To make matters even more complex, if you considered a **cubic kilometer** of fog above your neighborhood, you might notice patches of dense fog with volumes of relatively clear air with few water droplets. In this case, you would conclude that fog is definitely heterogeneous.

From some perspective, the fog could be accurately described as homogeneous, but for **most** purposes, fog is heterogeneous.

2B-1 Condensing Water Vapor by Increasing the Pressure 3 pts

A piston-and-cylinder device contains **water** vapor at **135°C** and **1 atm** in **state 1**. The cylinder contains no **air** and no liquid **water**. The cylinder is surrounded by a water bath at **135°C** that keeps the temperature of the contents of the cylinder **constant**. The piston is pushed slowly downward causing the pressure to **increase** steadily. Eventually, a small droplet of liquid water forms in **state 2**.

a.) Describe the state of the contents of the cylinder in **state 1** and in **state 2**.

b.) Sketch the path of this process on a **PV Diagram**. Be sure to label **all** the regions on the diagram and include the two-phase envelope and all relevant isotherms.

Read : The keys to this problem are…

1- The process is isothermal because the constant temperature bath keeps the system at **135°C** and

2- The final state is saturated vapor because the 1st micro-droplet of lquid water condenses.

Given:

T_1	135	°C	T_2	135	°C
P_1	1	atm	$P_2 > P_1$		
			x_2	1	kg vap/kg

Find:

a.) Describe states **1** & **2**

b.) Sketch process path on a **PV Diagram**

Assumptions: 1- The constant temperature bath is prefectly effective in keeping the contents of the cylinder at a **constant** and **uniform temperature** of **135°C**.

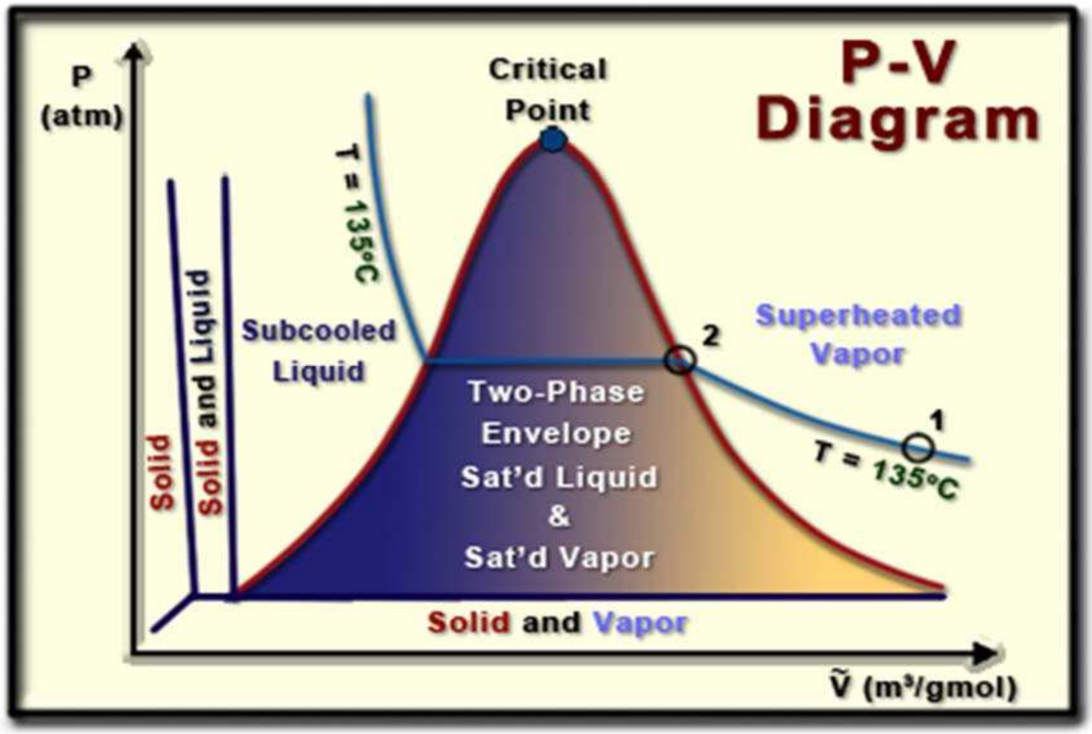
Equations / Data / Solve:

Part a.)

- In **state 1**, the cylinder contains all water vapor.
- The pressure must be increased on this vapor in order to cause any liquid water droplets to condense.
- We can conclude that **P_1 = 1 atm** is lower than the vapor pressure (or saturation pressure) of water at **T_1 = 135°C**.
- Therefore the water vapor in **state 1** is a **superheated vapor**.

- In **state 1**, the cylinder contains all water vapor.
- The pressure on this vapor is just exactly high enough to cause a micro-droplet of liquid water to condense.
- We can conclude that **P_2 = P*(135°C),** the vapor pressure (or saturation pressure) of water at **T_1 = 135°C**.
- Therefore the water vapor in **state 1** is a **saturated vapor**.

Part b.)



Verify: This assumption cannot be veriefied without experimentation.

Answers : See above.

2B-2 Problem Title 4 pts

Ammonia exists as a saturated mixture at **240.21 kPa** and **-14.6°C** in a rigid vessel with a volume of **1.0 m^3**. The specific volume of the saturated liquid and saturated vapor are **1.5195 L/kg** and **0.50063 m^3/kg**, respectively.
The **quality** of the **ammonia** is **0.275 kg vap/kg**. What is the total mass of **Ammonia** inside the vessel in **kg**?

Read : There are two keys to this problem. The first is the relationship between the total mass, the total volume and the overall, average or mixture specific volume. The other key is how to use quality and specific properties of saturated liquid and saturated vapor to determine a specific property of a saturated mixture.

Given:	**P**	**240.21**	**kPa**	$V_{sat\ liq}$	**1.5195**	**L**
	T	**-14.6**	**°C**	$V_{sat\ vap}$	**0.50063**	**m^3/kg**
	V_{total}	**1**	**m^3**	**x**	**0.275**	**kg vap/kg**
Find:	M_{total}	**???**	**kg**			

Assumptions: None.

Equations / Data / Solve:

Let's begin with the relationship between mass, volume and specific volume for the entire system.

$$M_{total} = \frac{V_{total}}{\hat{V}_{mix}} \qquad \text{Eqn 1}$$

We want to determine M_{total} and we know V_{total}, so all we need to do is determine V_{mix} and we will be able to use **Eqn 1** to solve this problem.

The specific volume of the two-phase mixture is related to the **quality** and the specific volumes of the saturated liquid and saturated vapor by the following equation.

$$\hat{V}_{mix} = x\,\hat{V}_{sat\,vap} + (1-x)\,\hat{V}_{sat\,liq} \qquad \text{Eqn 2}$$

We know the values of all of the variables on the right-hand side of **Eqn 2**, so we can plug-in values to determine V_{mix}.

V_{mix} **1.1373** **m^3/kg**

Be careful with the units in **Eqn 2**. You must convert **L** to **m^3** in $V_{sat\ liq}$ to make all of the units work out properly.

Now, we can plug values into **Eqn 1** to complete this problem.

M_{total} **0.8793** **kg**

Verify: There are no assumptions to verify in this problem.

Answers : M_{total} **0.879** **kg**

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2C-1 Specific Volume of Saturated Mixtures 4 pts

Calculate the **specific volume** for the following situations:

a.) Water at **300°C** and **72%** **quality**

b.) R-134a at **-50°C** and **83%** **quality**

c.) Ammonia at **750 kPa** and **48%** **quality**

Read : This is an exercise designed to drive home the meaning and use of the new concept of the quality of a saturated mixture. It is crucial to remember that if a quality is given, then the system contains a saturated mixture and you probably need to look up properties of both the saturated liquid and the saturated vapor.

Given:								
	a.)	**Water**	**T**	**300**	**°C**	**x**	**0.72**	**kg vap/kg tot**
	b.)	**R134a**	**T**	**-50**	**°C**	**x**	**0.83**	**kg vap/kg tot**
	c.)	**Ammonia**	**P**	**750**	**kPa**	**x**	**0.48**	**kg vap/kg tot**

Find: **V** **???** **m^3/kg** for each of the three parts of this problem.

Assumptions: None.

Equations / Data / Solve:

Part a.) Data from the Saturation Temperature Table of the Steam Tables at **300°C**:

$V_{sat\ liq}$	**0.0014042**	**m^3/kg**
$V_{sat\ vap}$	**0.02166**	**m^3/kg**
P*	**8587.9**	**kPa**

The key equation for this problem is the relationship between the properties of a saturated mixture and the properties of saturated liquid and vapor and the **quality**.

$$\hat{V} = x\,\hat{V}_{sat\,vap} + (1 - x)\,\hat{V}_{sat\,liq} \qquad \textbf{Eqn 1}$$

Now, we can plug numbers into **Eqn 1** to answer this part of the question.

V	**0.01599**	**m^3/kg**

Notice that I kept 4 significan figures in this answer istead of the usual 2 or 3 because there are 4 significant digits in the sat'd liquid and sat'd vapor values. Perhaps I should have only retained 2 significant digits because there only appear to be 2 significant digits in the quality. I have assumed that there are more than 2, really 4 or more, digits in the quality. This may not be a good assumption.

Part b.) Data from the Saturation Temperature Table of the R-134a Tables at **-50°C**:

$V_{sat\ liq}$ 0.00069142 m^3/kg
$V_{sat\ vap}$ 0.60620 m^3/kg
P* 29.451 kPa

Now, we can plug numbers into **Eqn 1** to answer this part of the question.

V	0.50326	m^3/kg

Significant figures are a bit tricky here.

Part c.) Data from the Saturation Pressure Table of the Ammonia Tables at **400 kPa**:

$V_{sat\ liq}$ 0.0016228 m^3/kg
$V_{sat\ vap}$ 0.16980 m^3/kg
T_{sat} 15.88 °C

Now, we can plug numbers into **Eqn 1** to answer this part of the question.

V	7.7095	m^3/kg

Significant figures are a bit tricky here as well.

Verify: No assumptions to verify.

Answers :

a.)	V	0.015988	m^3/kg	b.)	V	0.50326	m^3/kg
c.)	V	7.7095	m^3/kg				

2C-2 State of a System at a Given Temperature and Pressure 4 pts

Ammonia exists in a sealed tank at each of the following **temperatures** and **pressures**. In each case, what phase or phases **could** exist in the system? **Show your work** and **explain** your **reasoning**.

a.) 25°C, 114.7 kPa
b.) -31.59°C, 50 kPa
c.) -18.85°C, 200 kPa
d.) -37.41°C, 100 kPa

Read : At these temperatures and pressures, it is safe to assume that no solid water, or ice, exists in the system. So, the question becomes whether the system contains superheated vapor or subcooled liquid or whether the system is in VLE. If the system is in VLE, then **P = P*(T)**. Another way to look at this is that **$T = T_{sat}(P)$**. At saturation, it is not possible to determine the quality of the system without knowing the value of another intensive variable, such as v, u or h. So, we are not able to state whether both vapor and liquid exist, but we can say that both could exist under these conditions. Fortunately, that is exactly what we are asked to determine in this problem.

Given:

Part a.)	T	25	°C	P	114.7	kPa
Part b.)	T	-31.59	°C	P	50	kPa
Part c.)	T	-18.85	°C	P	200	kPa
Part d.)	T	-37.41	°C	P	100	kPa

Find: **What phase or phases could exist in the system? Explain.**

Assumptions: 1- No solid water, or ice, exists in the system.

Equations / Data / Solve:

In each part of this problem, we need to determine the saturation temperature associated with the system

If : $T_{sys} > T_{sat}$ Then : The system contains a superheated vapor.

If : $T_{sys} < T_{sat}$ Then : The system contains a subcooled liquid.
If : $T_{sys} = T_{sat}$ Then : The system could contain an equilibrium mixture of saturated

This is not always the easiest approach and for this part of the problem, there is an easier approach.

Another way to solve this problem is to determine the vapor pressure of water at the actual system temperature.

If : $P_{sys} < P^*(T_{sys})$ Then : The system contains a superheated vapor.
$P_{sys} > P^*(T_{sys})$
If : Then : The system contains a subcooled liquid.
If : $P_{sys} = P^*(T_{sys})$ Then : The system could contain an equilibrium mixture of saturated

Part a.) The second method is easier for **part (a)** because $\mathbf{T_{sys}} = \mathbf{25^oC}$ is listed in the Saturated Temperature Table while $\mathbf{P_{sys}}$ = **114.7 kPa** is not listed in the Saturation Pressure Table. So, let's use the 2nd method !

From the Saturation Temperature Table of the Ammonia Tables : **$P^*(25^oC)$** **1003.2** **kPa**

Since $\mathbf{P_{sys} < P^*(T_{sys})}$, we conclude : **The system contains superheated ammonia vapor.**

Part b.) In this part of the problem, it is easier to use method 1, described in **part (a)**, because $\mathbf{P_{sys}}$ appears in the

From the Saturation Pressure Table of the Ammonia Tables : **T_{sat}(50 kPa)** **-46.52** **^oC**

Since $\mathbf{T_{sys} > T_{sat}(P_{sys})}$, we conclude : **The system contains superheated ammonia vapor.**

Part c.) In this part of the problem, it is easier to use method 1, described in **part (a)**, because $\mathbf{P_{sys}}$ appears in the saturation tables, while $\mathbf{T_{sys}}$ does not.

From the Saturation Pressure Table of the Steam Tables : **T_{sat}(200 kPa)** **-18.85** **^oC**

Since $\mathbf{T_{sys} = T_{sat}(P_{sys})}$, we conclude : **The system could contain an equilibrium mixture of saturated liquid and saturated vapor.**

Part d.) In this part of the problem, it is easier to use method 1, described in part a, because $\mathbf{P_{sys}}$ appears in the saturation tables, while $\mathbf{T_{sys}}$ does not.

From the Saturation Pressure Table of the Ammonia Tables : **T_{sat}(100 kPa)** **-33.59** **^oC**

Since $\mathbf{T_{sys} < T_{sat}(P_{sys})}$, we conclude : **The system contains subcooled liquid ammonia.**

Verify: Since all of the temperature and pressure combination in this problm appear in the ammonia tables for vapor-liquid equilibrium, no solid ammonia, or ammonia ice, is present in any of the four systems considered.

Answers :				
	Part a.)	**Superheated vapor.**	**Part c.)**	**Saturated mixture.**
	Part b.)	**Superheated vapor.**	**Part d.)**	**Subcooled liquid.**

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2C-3 Water Boils at a Higher Temperature in a Covered Pot 4 pts

A large pot has a **diameter** of 14 cm. It is filled with water and covered with a **heavy lid** that weighs 3.7 kg. At what **temperature** does the water begin to boil if ambient **pressure** is 101.325 kPa?

Read : The key to this problem is to recognize that the total pressure at the surface of the liquid water must be greater than **101 kPa** before the water can boil because of the weight of the lid. This is true whether there is an air space between the liquid water and the lid or not. As the temperature of the contents of the pot rises, the pressure will increase. When the 1st bubble of water vapor forms, it will displace some air. The displaced air will escape by lifting the lid.

Given:

D	0.14	m	P_{atm}	101.325	kPa
m_{lid}	3.7	kg			

Find:

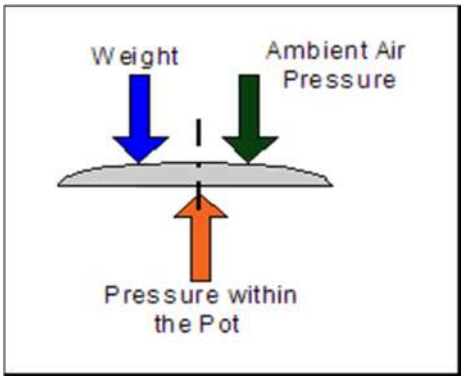
T_{boil}	???	^oC

Assumptions: None.

Equations / Data / Solve:

The liquid water will boil when it reaches the temperature at which the vapor pressure of the water is equal to the pressure required to lift the lid and let some air escape.

Let's begin by determining the pressure within the pot required to lift the lid. This can be accomplished by writing a force balance on the lid. See the diagram.



The lid will lift slightly and let some air escape when the upward force exerted by the gas inside the pot just balances the sum of the weight of the lid and the downward force due to atmospheric pressure on the outside of the lid.

$$F_{P,in} = F_{P,out} + F_{wt}$$ **Eqn 1**

The fact that the lid is not flat on top does not affect the solution of this problem, as long as the lid is axially symmetric about its centerline.

All of the horizontal components of the forces acting on the lid cancel each other out (vector sum is zero). The downward force is the same regardless of the shape of the top of the lid. Remember that pressure always acts in the direction perpendicular or normal to a surface. So as the lid surface curves, the downward component of the pressure force decreases. But the total surface area of the pot increases. These two factors are equal and opposite. The result is that the force exerted by the outside atmosphere on the pot lid is the same as if the lid were flat. The area of an equivalent flat surface is called the projected area (I use the symbol Aproj).

$$F_{P,out} = P_{atm}\ A_{proj}$$ **Eqn 2**

Following the same logic, the upward force exerted by the air in the pot on the lid can be determined using :

$$F_{P,in} = P_{in}\ A_{proj} \qquad \textbf{Eqn 3}$$

The only term left is the weight of the pot lid. This is an application of Newton's 2nd Law.

$$F_{wt} = m_{lid}\ \frac{g}{g_c} \qquad \textbf{Eqn 4}$$

Now, we can substitute Eqns 2, 3 & 4 into Eqn 1 :

$$P_{in}\ A_{proj} = P_{atm}\ A_{proj} + m_{lid}\ \frac{g}{g_c} \qquad \textbf{Eqn 5}$$

The goal is to determine the pressure inside the pot when the lid lifts and the water boils, so let's solve Eqn 5 for the unknown **P_{in}**.

$$P_{in} = P_{atm} + \frac{m_{lid}}{A_{proj}}\ \frac{g}{g_c} \qquad \textbf{Eqn 6}$$

The only unknown quantity on the right-hand side of Eqn 6 is the projected area. We can calculate its value using :

$$A_{proj} = \frac{\pi}{4} D^2 \qquad \textbf{Eqn 7}$$

A_{proj} **0.015394** **m^2**

At last, we can plug numbers into Eqn 6 and evaluate the pressure in the pot when the water boils.
Just be sure to use the unit conversion :

1 kPa **1000** **N/m^2**

g	**9.8066**	**m/s^2**
g_c	**1**	**$kg\text{-}m/N\text{-}s^2$**

P_{in} **103.68** **kPa**

Finally, we can go to the Saturation Pressure Table in the Steam Tables to determine the saturation pressure at **P_{in}**. This is the temperature at which the water in the pot will boil.

Because **102.25 kPa** is not an entry in the Saturation Pressure Table, an interpolation is required.

T_{sat} (^oC)	P_{sat} (kPa)
100.00	101.42
T_{boil}	103.68
105.00	120.90

Interpolation yields : **T_{boil}** **100.5810** **^oC**

Verify: None.

Answers : **T_{boil}** **100.6** **^oC**

2D-1 Isothermal Vaporization of Water 4 pts

A piston-and-cylinder device contains **25 lb_m** of **water** at **195°F**. The cylinder rests in a constant temperature bath that keeps the **temperature** of the **water** in the cylinder at **195°F**.
Weights are removed one at a time from the back of the piston causing the **pressure** inside the cylinder to drop from a very high value until the **water** inside begins to boil.

Eventually, the last drop of **water** in the cylinder **vaporizes**. Determine the **total volume** of the **steam** in the cylinder at this point, in **ft^3**.

Read : The initial state of the water is probably a subcooled liquid (or even a supercritical fluid), since the pressure is "very high". The final state of the water is a saturated vapor because the vaporization of the water is just barely complete. The temperature of the final saturated vapor is the same as the initial temperature: 205°F. This is an isothermal process !

Given: m	**25**	**lb_m**	**T**	**195**	**°F**
Find: V_{final}	**???**	**ft^3**			

Assumptions:

1- The initial and final states are equilibrium states.
2- In the initial state, the system contains subcooled liquid water.
3- In the final state, the system contains saturated water vapor.

x_{final} **1** **lb_m vap/lb_m**

Equations / Data / Solve:

We need to determine the volume of the system and we are given the mass of water in the system.

We need to determine the specific volume of the system because :

$$V = m\left(lb_m\right)\ \hat{V}\left(ft^3 / lb_m\right) \qquad \textbf{Eqn 1}$$

Because we know that the water in the final state is a saturated vapor, we can look up its specific volume in the Saturated Temperature Table of the Steam Tables at **195°F**.

The problem is that a temperature of **195°F** is not listed in the Saturation Temperature Table.

So, we must interpolate to determine the value :

T_{sat} (°F)	$V_{sat\ vap}$ (ft^3/lb_m)
190	40.916
195	???
200	33.609

$$V(195) = \text{slope} \cdot (195 - 190) + 40.916$$ **Eqn 2**

$$\text{slope} = \frac{33.609 - 40.916}{200 - 190}$$ **Eqn 3**

slope	**-0.73071**	**$(ft^3/lb_m)/^oF$**	**$V_{sat\ vap}$**	**37.263**	**ft^3/lb_m**

Now that we know the value of the specific volume of the saturated vapor, and the system contains ALL saturated vapor (x = 1), we can plug values into Eqn 1 and answer the question.

V_{final} **931.6** **ft^3**

Verify: None of these assumptions can be verified.

Answers : **V_{final}** **932** **ft^3**

2D-2 Dew Point Calculations for R-134a 4 pts

Ammonia is contained in sealed test tube at **25°C**. The test tube is slowly cooled until liquid **ammonia** droplets condense on the inside of the test tube. At this point, the **temperature** of the **ammonia** is **-20°C**.
Determine the initial **pressure** in the test tube, **before** the cooling process began.

Read : We know the initial temperature of the ammonia. If we knew the intial temperature, we could look up the specific volume in the Superheated Vapor Tables for ammonia. There is only one pressure that yields this value of the specific volume when the system is at **25°C**. Therefore, we could also look at the problem in the following way. We know the intitial temperature of the ammonia and IF we also knew the specific volume of the ammonia, we could use the Superheated Vapor Tables to work backwards and determine the initial pressure! That is what we are going to need to do in this problem.

When the 1st droplet of liquid appears on the wall of the glass vessel, the vapor inside the vessel is a satuated vapor. We can look up the properties of this saturated vapor in the Ammonia Tables. Since the vapor is saturated at **-20°C**, it must be superheated at **25°C**. But at both the initial and final state the specific volume must be the same because neither the mass nor the volume of the system changed! This is the key to the problem. Because we know the values of 2 intensive variables at the initial state, specific volume and temperature, and the initial state is a pure substance in a single phase, we can determine the values of ALL other properties! In this case we need to determine the pressure.

Given:

T_1	25	°C
T_2	-20	°C
x_2	1	kg vap/kg

Find: P_1 ??? kPA

Assumptions: 1- The system contains saturated vapor in the final state.

Equations / Data / Solve:

The specific volume of the system is equal to the specific volume of saturated ammonia vapor at T_2.

We can look up this value in the Saturated Temperature table of the Ammonia Tables at **-20°C** :

$V_{sat\ vap}$ **0.62373** m^3/kg

Next, we scan the Superheated R-134a Tables to determine the 2 pressures between which this value of specific volume falls, at the given temperature of **25°C**.

Here are the data at **25°C**:

P (kPa)	V (m^3/kg)
100	1.1381
200	0.59465
400	0.30941
500	0.25032
600	0.21035
700	0.18145

We need to interpolate between **100 kPa** and **200 kPa** to determine the system pressure that corresponds to our value of specific volume at a temperature of **25^oC**.

$$P(0.62373) = \text{slope} \cdot (0.62373 - 1.1381) + 100$$ **Eqn 1**

$$\text{slope} = \frac{200 - 100}{0.59465 - 1.1381}$$ **Eqn 2**

slope	**-184.0134**	**$kPa/(m^3/kg)$**	**P**	**194.6**	**kPa**

Verify: The assumption cannot be verified.

Answers : **P_1** **195** **kPa**

2D-3 Volume Occupied by 10 kg of Water at Various Temperatures 6 pts

Determine the **volume** occupied by **25 kg** of **R-134a** at a **pressure** of **800 kPa** and the following **temperatures**:
a.) -12°C, **b.) -40°C**, **c.) 70°C**, **d.) 160°C**, **e.) 325°C**

Read : This problem is an exercise in how to read and interpolate values from the Steam Tables.

It covers the use of the Subcooled Liquid Tables and the Superheated Vapor Tables, but does not involve double interpolation because the pressure value, **800 kPa** does appear explicitly in both the Subcooled Liquid and Superheated Vapor Tables.

Given:

m	25	kg	P	800	kPa
T_a	-12	°C	T_d	160	°C
T_b	-40	°C	T_e	325	°C
T_c	70	°C			

Find: V ??? m^3

Assumptions: None.

Equations / Data / Solve:

We need to determine the volume of the system and we are given the mass of water in the system.

We need to determine the specific volume of the system because :

$$V = m(kg)\ \hat{V}(m^3/kg) \qquad \text{Eqn 1}$$

So, for each part of this problem, we must evaluate the specific volume and plug this into **Eqn 1** to determine the total volume of the system.

The first step in determining the specific volume is to determine the phase or phases present in the system. From the R-134a Tables, we can obtain the saturation temperature associated with **800 kPa**.

T_{sat} **31.33** °C

This makes it easy to determine the phase or phases in the system for each part of the problem.

If :	$T_{sys} > T_{sat}$	Then :	The system contains a superheated vapor.
If :	$T_{sys} < T_{sat}$	Then :	The system contains a subcooled liquid.
If :	$T_{sys} = T_{sat}$	Then :	The system could contain an equilibrium mixture of saturated liquid and saturated vapor.

Part a.) The system contains a subcooled liquid. Here are the key data values from the Subcooled Liquid Table of the R-134a Tables :

T (^oC)	V (m^3/kg)
-10	0.00075236
-12	V_a
-20	0.00073517

$$V(5) = \text{slope} \cdot (5 - 0) + 0.0009952$$

$$\text{slope} = \frac{0.00099731 - 0.0009952}{5 - 0}$$ **Eqn 3**

slope	1.7191E-06	$(m^3/kg)/^oC$			
V	0.00074892	m^3/kg	V	0.018723	m^3
				18.723	L

Part b.) The system contains a subcooled liquid. Here are the key data values from the Subcooled Liquid Table of the R-134a Tables :

T_b	-40	^oC	No interpolation required !	V	0.00070451	m^3/kg
				V	0.017613	m^3
					17.613	L

Part c.) The system contains a superheated vapor. Here are the key data values from the Superheated Vapor Table of the R-134a Tables :

T_c	70	^oC	No interpolation required !	V	0.031340	m^3/kg
				V	0.78350	m^3
					783.50	L

Part d.) The system contains a superheated vapor. Here are the key data values from the Superheated Vapor Table of the R-134a Tables :

T_d	160	^oC	No interpolation required !	V	0.042291	m^3/kg
				V	1.05726	m^3
					1057.26	L

Part e.) T_e 325 ^oC

This temperature is too high for our steam tables !
At very high temperatures, most gases behave as Ideal Gases.
The criterion by which we know whether it is reasonable to approximate real gases as ideal gases is :

$$\tilde{V} > 20\,L\,/\,mole \qquad \text{Eqn 4}$$

The Ideal Gas EOS is :

$$P\,\tilde{V} = R\,T \qquad \text{Eqn 5}$$

or :

$$\tilde{V} = \frac{R\,T}{P} \qquad \text{Eqn 6}$$

R	8.314	**J/mole-K**	**V**	**0.006216**	**m^3/mole**
			V	**6.216**	**L/mole**

The Ideal Gas EOS does NOT apply because **V << 20 L/mole** !

Our only choice is to EXTRAPOLATE from the data in the steam tables.

That is not very safe and I do not want to encourage you to do this, so I will not do it here.

The best course of action is to find another data source.
The **NIST Webbook** yields:

V	0.060298	**m^3/kg**
V	**1.50744**	**m^3**
	1507.4	**L**

Verify: No assumptions to verify.

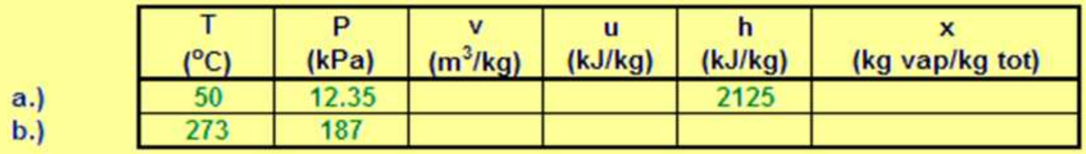
Answers :

V_a	**18.7**	**L**
V_b	**17.6**	**L**
V_c	**783**	**L**

V_d	**1060**	**L**
V_e	**1510**	**L**

2D-4 Determine Properties Using Thermodynamic Tables 8 pts

Complete the following table by determining the values of all the blank entries. The system contains only H_2O.



	T (°C)	P (kPa)	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	x (kg vap/kg tot)
a.)	50	12.35			2125	
b.)	273	187				

Read : The key to this problem is to recognize that all of the variables in the table are state variables (except maybe **x**), or properties, and that they are all intensive properties. It is also important to assume that either one or two phases exist. The triple point of water does not appear in this table. Also, since we have no data availabe about solid water, we can assume that we have either a subcooled liquid, a superheated vapor or an equilibrium mixture of saturated vapor and saturated liquid in the system. Gibbs Phase Rule tells us that for a pure substance in a single phase there are 2 degrees of freedom. If two phases are present, then there is just 1 degree of freedom. In either case, the two values of intensive properties given in each part of this problem will be sufficient to completely determine the values of all of the other intensive properties of the system. So, we are in good shape to move forward on solving this problem.

Given:

	T (°C)	P (kPa)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	x (kg vap/kg tot)
a.)	50	12.35			2125	
b.)	273	187				

Find:

a.)	V	???	m^3/kg	b.)	V	???	m^3/kg
	U	???	kJ/kg		U	???	kJ/kg
	H	???	kJ/kg		H	???	kJ/kg
					x	???	kg vap/kg

Assumptions: 1- No ice exists in the system in either part of the problem.

Equations / Data / Solve:

Part a.) **Given :**

T 50 °C
P 12.35 kPa

The first step in solving each part of this problem is to determine the state of the system. Is it subcooled liquid, superheated vapor or a two-phase VLE mixture.

We could do this by determining the boiling point or saturation temperature at the system pressure. But, since **12.35 kPa** does not appear in the Saturation Pressure Table for water, this would require an interpolation. It is easier to determine the saturation pressure or vapor pressure based on the system temperature because **50°C** does appear in the Saturation Temperature Table and therefore does not require an interpolation.

P*(kPa)	T_{sat} (°C)	$V_{sat\ liq}$ (m^3/kg)	$V_{sat\ vap}$ (m^3/kg)	$U_{sat\ liq}$ (kJ/kg)	$U_{sat\ vap}$ (kJ/kg)	$H_{sat\ liq}$ (kJ/kg)	$H_{sat\ vap}$ (kJ/kg)
12.352	50	0.0010121	12.027	209.33	2442.7	209.34	2591.3

P*(50°C) 12.352 **kPa**

Since the actual system pressure is EQUAL TO the vapor pressure, the system is at saturation.

In order to determine the values of the other properties of the system using the following equation, we will need to know the quality, **x**.

$$\mathbf{M}_{\mathbf{VLE}} = \mathbf{M}_{\mathbf{sat\,liq}} + \mathbf{x}\left(\mathbf{M}_{\mathbf{sat\,vap}} - \mathbf{M}_{\mathbf{sat\,liq}}\right) \qquad \textbf{Eqn 1}$$

We can determine x from the saturation data and the known value of u for the system using :

$$\mathbf{x} = \frac{\mathbf{V} - \mathbf{V}_{\mathbf{sat\,liq}}}{\mathbf{V}_{\mathbf{sat\,vap}} - \mathbf{V}_{\mathbf{sat\,liq}}} \qquad \textbf{Eqn 2}$$

x **0.804** **kg vap/kg**

Now, we can plug **x** back into **Eqn 1** and apply it to the unknown properties, **V** and **U**.

V **9.673** **m^3/kg** **U** **2005.5** **kJ/kg**

Part b.) **Given :** **T** **273** **^{o}C** **P** **187** **kPa**

We again begin by determining the state of the system. Unfortunately the system temperature is not listed in the Saturation Temperature Table and the system pressure is is not listed in the Saturation Pressure Table. Either way we go, interpolation is required.

P*(kPa)	T_{sat} (^{o}C)	$V_{sat\ liq}$ (m^3/kg)	$V_{sat\ vap}$ (m^3/kg)	$U_{sat\ liq}$ (kJ/kg)	$U_{sat\ vap}$ (kJ/kg)	$H_{sat\ liq}$ (kJ/kg)	$H_{sat\ vap}$ (kJ/kg)
5503.0	270	0.0013030	0.035621	1178.1	2593.7	1185.3	2789.7
5946.4	275	0.0013175	0.032766	1203.1	2590.3	1210.9	2785.2
175	116.0	0.0010568	1.0036	486.82	2524.5	239.42	2700.1
200	120.2	0.0010605	0.88568	504.49	2529.1	251.38	2706.2

We could interpolate to determine the saturation properties at **187 kPa**, but there isn't much point! Since the system temperature is higher than the saturation temperature at EITHER **175 kPa** or **200 kPa**, the system temperature must also be higher than the interpolated value of **T_{sat}(187 kPa)**.

Since the system temperature is greater than the saturation temperature at the system pressure, the system contains a superheated vapor. Therefore, we must use data from the Superheated Vapor Table to determine the unknown properties of the system.

x **N/A - Superheated**

The Superheated Vapor Table includes tables for pressure of **100** and **200 kPa**, but not **187 kPa**. These two tables include rows for **250^{o}C** and **300^{o}C**, but not for **273^{o}C**. Consequently a double interpolation is required for each unknown system propert, **V**, **U** and **H**.

The double interpolation can be done with the aid of tables like the ones developed in Lesson 2C on page 18. The data required for the double interpolation tables are :

P*(kPa)	T (^{o}C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)
100	250	2.4062	2733.9	2974.5
100	300	2.6388	2810.6	3074.5
200	250	1.1989	2731.4	2971.2
200	300	1.3162	2808.8	3072.1

Here is the double interpolation table for **V** :

	Pressure (kPa)		
T(^{o}C)	**100**	**187**	**200**
250	**2.4062**	**1.3558**	**1.1989**
273	**2.5132**	**1.4167**	**1.2529**
300	**2.6388**	**1.4882**	**1.3162**

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

V **1.417** **m^3/kg**

Here is the double interpolation table for **U** :

	Pressure (kPa)		
T(^{o}C)	**100**	**187**	**200**
250	**2733.9**	**2731.8**	**2731.4**
273	**2769.2**	**2767.3**	**2767.0**
300	**2810.6**	**2809.1**	**2808.8**

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

U **2767.3** **kJ/kg**

Here is the double interpolation table for **H** :

	Pressure (kPa)		
T(^{o}C)	**100**	**187**	**200**
250	**2974.5**	**2971.6**	**2971.2**
273	**3020.5**	**3018.0**	**3017.6**
300	**3074.5**	**3072.4**	**3072.1**

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

H **3018.0** **kJ/kg**

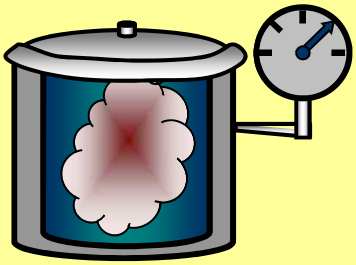
Verify: The assumption that there is no ice in the is confirmed by the fact that the the states in **part (a)** and **part (b)** are both located in the Steam Tables. The Steam Tables only consider states in which no ice can exist at equilibrium.

Answers :

	T (^{o}C)	**P (kPa)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**x (kg vap/kg tot)**
a.)	**50**	**12.35**	**9.673**	**2005.5**	**2125**	**0.804**
b.)	**273**	**187**	**1.417**	**2767.3**	**3018.0**	**N/A - Superheated**

2D-5 Relative and Absolute Humidity of Air 4 pts

A **pressure** gauge on a rigid steel tank reads **50 kPa**. The tank holds **2.1 kg** of **air** and **0.250 kg** of **water** vapor at **70°C**. Calculate the **relative humidity** of the **air** in the tank.



Read : The keys here are the definition of relative humidity and the relationships between mass, moles, molecular weight, mole fraction and partial pressure.

We need to assume the humid air behaves as an ideal gas in order to determine the partial pressures from the given $\mathbf{m_{water}}$ and $\mathbf{m_{BDA}}$.

Given:	T	70	°C	m_{tot}	2.100	kg wet air
	P_{tot}	50	kPa	m_{H2O}	0.250	kg H_2O

Find: h_r = ??? %

Assumptions:

1- Air is a non-condensable gas.
2- Humid air behaves as an ideal gas.

Equations / Data / Solve:

Let's begin with the definition of relative humidity:

$$h_r = \frac{P_{H20}}{P^*_{H20}} \quad \text{Eqn 1}$$

The vapor pressure is equal to the saturation pressure at the system temperature.
We can find this in the saturation temperature section of the steam tables:

P* (80°C) = 47.39 kPa

For an ideal gas:

$$P_{H2O} = y_{H2O}\ P_{tot} \quad \text{Eqn 2}$$

Test if ideal: Ideal Gas EOS :

$$P\,\tilde{V} = R\,T \quad \text{Eqn 3}$$

Solve for the molar volume :

$$\tilde{V} = \frac{R\,T}{P} \quad \text{Eqn 4}$$

V 57.06 L/mole

Therefore, since **V > 20 L/mole**, we can treat the wet gas as an ideal gas.

For all gases, mole fraction is defined as :

$$y_{H2O} = \frac{n_{H2O}}{n_{wet\,gas}} = \frac{n_{H2O}}{(n_{H2O} + n_{BDA})} \qquad \textbf{Eqn 5}$$

Where : $n_i = m_i / MW_i$ **Eqn 6**

Data: $MW_{H20} =$ 18.016 **g H_2O / mol H_2O**

$MW_{BDA} =$ 28.96 **g bone-dry air / mol bone-dry air**

Since we know the mass of water and bone-dry air in the tank, as well as their molecular weights, we can calculate the number of moles of water and BDA in the tank using **Eqn 6**.

Then, we can calculate the mole fraction of water in the gas in the tank, using **Eqn 5**. Next, we can use the given total pressure to calculate the partial pressure of water in the gas using **Eqn 2**.

Here are the numerical results:

n_{H2O}	13.88	mol H_2O
m_{BDA}	1.850	kg BDA
n_{BDA}	63.88	mol BDA
y_{H2O}	0.178	mol H_2O / mol wet gas
P_{H2O}	8.92	kPa

Finally, we can calculate the relative humidity using **Eqn 1**: $h_r =$ 18.8% %

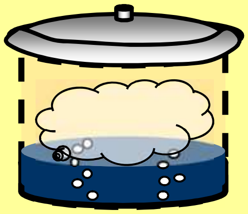
Verify: 1 - The assumption is sound since the normal boiling points of oxygen and nitrogen are **68 K** and **70 K** and our system is at **343 K**.

2 - The ideal gas assumption was verified above because **V = 57 L/mol** which is greater than **20 L/mole**. The limit for non-diatomic gases applies because of the water in the air.

Answers : $h_r =$ **19% %**

2D-6 Humidity and Partial Pressure in a Humid Ideal Gas 6 pts

The rigid tank shown below contains **5 kg** of a non-condensable gas with a **molecular weight** of **44.1 g/mol**. The tank also contains **water** vapor. The gas in the tank is at **140 kPa** and **80^{o}C** and the **relative humidity** is **72%**.



Assuming the gas in the tank behaves as an ideal gas, calculate the **mass** of **water** vapor in the tank.

Read : The keys here are the definition of relative humidity and the relationship between mole fraction and partial pressure.

We need to assume the humid air behaves as an ideal gas in order to determine the partial pressure of water from the given $\mathbf{h_r}$. We can use the mole fraction of water in the gas to determine the mass of water in the gas.

Given:

m_{NCG} =	5	kg NCG		P_{tot} =	140	kPa
MW_{NCG} =	44.1	g NCG / mole NCG		h_r =	72%	
T =	80	^{o}C				

Find: m_{H2O} = ??? kg

Assumptions: 1- The gas in the tank behaves as an ideal gas. This must be verified.

Equations / Data / Solve:

Let's begin with the definition of relative humidity:

$$h_r = \frac{P_{H20}}{P^*_{H20}} \qquad \textbf{Eqn 1}$$

The vapor pressure is equal to the saturation pressure at the system temperature.

We can find this in the saturation temperature section of the steam **P^*_{H2O} (70^{o}C) 47.41 kPa**

Plug $\mathbf{P^*_{H2O}}$ and $\mathbf{h_r}$ into **Eqn 1** to get the partial pressure of water from the definition of relative humidity.

P_{H2O} 34.138 kPa

For an ideal gas:

$$P_{H2O} = y_{H2O}\ P_{tot} \qquad \textbf{Eqn 2}$$

or:

$$y_{H2O} = \frac{P_{H2O}}{P_{tot}} \qquad \textbf{Eqn 3}$$

Plugging values into **Eqn 3** yields: **y_{H2O} = 0.244 mol H_2O / mol wet gas**

For all gases, mole fraction is defined as: $$y_{H2O} = \frac{n_{H2O}}{n_{gas}} = \frac{n_{H2O}}{(n_{H2O} + n_{NCG})}$$ **Eqn 4**

Where : $n_i = m_i / MW_i$ **Eqn 5**

Now, we solve **Eqn 4** for n_{H2O} : $$n_{H2O} = y_{H2O}\left(n_{H2O} + n_{NCG}\right)$$ **Eqn 6**

$$n_{H2O} = n_{NCG}\frac{y_{H2O}}{1 - y_{H2O}}$$ **Eqn 7**

Now, we can plug the numbers into equations…

Eqn 5 yields :	n_{NCG} =	**113.38**	**moles NCG**
Eqn 7 yields :	n_{H2O} =	**36.56**	**moles H_2O**

Finally, Eqn 5 can be rewritten as : $m_i = n_i \; MW_i$ **Eqn 8**

We can answer the question posed by plugging numbers into **Eqn 8** :

Data: MW_{H20} = 18.016 g H_2O / mol H_2O m_{H2O} = 658.7 g H_2O

Verify: Test if ideal: Ideal Gas EOS : $$P\tilde{V} = RT$$ **Eqn 3**

Solve for the molar volume : $$\tilde{V} = \frac{RT}{P}$$ **Eqn 4**

V **20.97** **L/mole**

Therefore, since **V > 20 L/mole**, we can treat the wet gas as an ideal gas.

Answers : m_{H2O} = **659** **g H_2O**

2D-7 Humid Air and Relative Humidity 6 pts

Air is fed to a furnace at a **volumetric flow rate** of **745 m^3/h**. The **air** is at **50°C**, **100 kPa** and has a **relative humidity** of **37%**.

a.) Calculate the **molar flow rate** of **bone dry air** (**BDA**) into the furnace in **mole/h**.

b.) Calculate the **molar flow rate** of **water** (within the humid air) into the furnace in **mole/h**.

Read: The two keys to this problem is that the humid air behaves as an ideal gas and the definition of relative humidity. You can determine the mole fraction of water in humid air from the relative humidity. You can use the Ideal Gas EOS to determine the total molar flow rate from the volumetric flow rate. The product of the mole fraction of water and the total molar flow rate is the molar flow rate of water. Finally, the molar flow rate of BDA is just the difference between the total molar flow rate and molar flow rate of water.

Given:

T	**50**	**°C**
P	**100**	**kPa**
h_R	**37%**	
V_{dot}	**745**	**m^3/h**

Find :

n_{H2O}	**???**	**mol H_2O/h**
n_{BDA}	**???**	**mol BDA/h**

Diagram: None for this problem.

Assumptions : 1 - Humid air behaves as an ideal gas. This allows us to use the Ideal Gas EOS and tells us that the partial pressure is equal to the product of the mole fraction and the total pressure.

Equations / Data / Solve :

The molar flow rates of BDA and water are related to the total molar flow rate of humid air by the mole fractions. The equations are :

$$\dot{n}_{H2O} = y_{H2O}\,\dot{n}_{total} \qquad \textbf{Eqn 1}$$

$$\dot{n}_{BDA} = y_{BDA}\,\dot{n}_{total} \qquad \textbf{Eqn 2}$$

Because humid air is made up of bone dry air (BDA) and water, only :

$$y_{BDA} + y_{H2O} = 1 \qquad \textbf{Eqn 3}$$

Solve **Eqn 3** for **y_{BDA}** .

$$y_{BDA} = 1 - y_{H2O} \qquad \textbf{Eqn 4}$$

So, we need to determine **n_{total}** and **y_{H2O}** before we can use **Eqns 1**, **2 & 4** to answer this question.

Let's begin by evaluating the total molar flow

For this, we can use the rate version of the Ideal Gas EOS:

$$P\,\dot{V}_{total} = \dot{n}_{total}\,R\,T \qquad \textbf{Eqn 5}$$

Solve **Eqn 5** for the total molar flow rate.

$$\dot{n}_{total} = \frac{P\,\dot{V}_{total}}{R\,T} \qquad \textbf{Eqn 6}$$

R	**8.314**	**J/mol-K**	n_{total}	**27730**	**mol total/h**

The key to solving this problem is using the definition of relative humidity to determine the mole fraction of water in the humid air.

$$h_R = \frac{P_{H2O}}{P^*_{H2O}} = \frac{y_{H2O}\,P}{P^*_{H2O}} \qquad \textbf{Eqn 7}$$

We can now solve Eqn 7 for the mole fraction of water in the humid air.

$$y_{H2O} = h_R\,\frac{P^*_{H2O}}{P} \qquad \textbf{Eqn 8}$$

We can look up the vapor pressure of water at **50^{o}C** in the Saturated Temperature Table of the Steam Tables.

P^*_{H2O} **38.595** **kPa**

Now, plug values back into **Eqn 8** :

y_{H2O} **0.1428** **mol H_2O/mol**

Finally, plug values into **Eqns 1**, **4** & **2** :

n_{H2O}	**3960**	**mol H_2O/h**
y_{BDA}	**0.8572**	**mol BDA/mol**
n_{BDA}	**23770**	**mol BDA/h**

Verify : The ideal gas assumption needs to be verified.
We need to determine the specific volume and check if :

$$\tilde{V} > 20\ \text{L/mol}$$

$$\tilde{V}_{out} = \frac{R\,T_{out}}{P_{out}} \qquad \textbf{Eqn 9}$$

V **26.87** **L/mol**

The ideal gas assumption is valid because **V > 20 L/mole**.

Answers :

n_{H2O}	**3960**	**mol H_2O/h**

n_{BDA}	**23800**	**mol BDA/h**

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2D-8 Determining System Properties Using Thermodynamic Tables 10 pts

Complete the following table by determining the values of all the blank entries. The system contains only **R-134a**.

	T (°C)	P (kPa)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	x (kg vap/kg tot)
a.)	-15			369.85		
b.)	43	728				
c.)		250	0.049			
d.)	50	1547				
e.)		976			318.7	

Read : The key to this problem is to recognize that all of the variables in the table are state variables, or properties, and that they are all intensive properties. It is also important to assume that either one or two phases exist. The triple point of R-134a is not common knowledge, but it is pretty safe to assume that it does not appear in this table. We can verify this assumption later. Also, since we have no data availabe about solid R-134a, we can assume that we have either a subcooled liquid, a superheated vapor or an equilibrium mixture of saturated vapor and saturated liquid in the system. Gibbs Phase Rule tells us that for a pure substance in a single phase there are 2 degrees of freedom. If two phases are present, then there is just 1 degree of freedom. In either case, the two values of intensive properties given in each part of this problem will be sufficient to completely determine the values of all of the other intensive properties of the system. So, we are in good shape to move forward on solving this problem.

Given:

	T (°C)	P (kPa)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	x (kg vap/kg tot)
a.)	**-15**			**369.85**		
b.)	**43**	**728**				
c.)		**250**	**0.049**			
d.)	**50**	**1547**				
e.)		**976**		**318.7**		

Find: Fill in all the blank values in the table, above.

Assumptions: - No solid phase exists in any of these 5 systems

Equations / Data / Solve:

Part a.) **Given :** **T** **-15** **°C** **U** **369.85** **kJ/kg**

We again begin by determining the state of the system. In this case, it would be easiest to lookup the $\mathbf{U_{sat\ vap}}$ and

If :	$\mathbf{U > U_{sat\ vap}}$	Then :	The system contains a superheated vapor.
If :	$\mathbf{U < U_{sat\ liq}}$	Then :	The system contains a subcooled liquid.
If :	$\mathbf{U_{sat\ vap} > U > U_{sat\ liq}}$	Then :	The system contains an equilibrium mixture of saturated liquid and saturated vapor.

Data :

P*(kPa)	T_{sat} (°C)	$V_{sat\ liq}$ (m³/kg)	$V_{sat\ vap}$ (m³/kg)	$U_{sat\ liq}$ (kJ/kg)	$U_{sat\ vap}$ (kJ/kg)	$H_{sat\ liq}$ (kJ/kg)	$H_{sat\ vap}$ (kJ/kg)
163.94	-15	7.4469E-04	0.12067	180.02	369.85	180.14	389.63

Because **U = U$_{\text{sat vap}}$**, our system contains a saturated vapor. The bonus here is that the quality is **1** and all the

x	**1**	**kg vap/kg**
P	**163.94**	**kPa**

V	**0.12067**	**m^3/kg**
H	**389.63**	**kJ/kg**

Part B.) **Given :** **T** **43** **°C** **P** **728** **kPa**

We again begin by determining the state of the system. Unfortunately the system temperature is not listed in the

P*(kPa)	T$_{\text{sat}}$ (°C)	V$_{\text{sat liq}}$ (m^3/kg)	V$_{\text{sat vap}}$ (m^3/kg)	U$_{\text{sat liq}}$ (kJ/kg)	U$_{\text{sat vap}}$ (kJ/kg)	H$_{\text{sat liq}}$ (kJ/kg)	H$_{\text{sat vap}}$ (kJ/kg)
1016.6	40	8.7204E-04	0.019966	255.52	399.13	256.41	419.43
1159.9	45	8.8885E-04	0.0173439	262.91	401.40	263.94	421.52
700	26.71	8.3320E-04	0.0293654	236.41	392.64	236.99	413.20
750	29.08	8.3959E-04	0.027375	239.76	393.84	240.39	414.37

We could interpolate to determine the saturation properties at **728 kPa**, but there isn't much point ! Since the

Since the system temperature is greater than the saturation temperature at the system pressure, the system

x	**N/A - Superheated**

The Superheated Vapor Table includes tables for pressure of **700** and **800 kPa**, but not **728 kPa**. These two

The double interpolation can be done with the aid of tables like the ones developed in Lesson 2C on page 18.
The data required for the double interpolation tables are :

P*(kPa)	T (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)
700	40	0.031696	404.53	426.72
700	50	0.033322	413.35	436.67
800	40	0.027036	402.97	424.59
800	50	0.028547	412.00	434.84

Here is the double interpolation table for **V** :

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

	Pressure (kPa)		
T(°C)	700	728	800
40	0.031696	0.030391	0.0270357
43	0.032184	0.030869	0.027489
50	0.033322	0.031985	0.028547

V	**0.030869**	**m^3/kg**

Here is the double interpolation table for **U** :

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

	Pressure (kPa)		
T(°C)	700	728	800
40	404.53	404.09	402.97
43	407.18	406.76	405.68
50	413.35	412.97	412.00

U	**406.76**	**kJ/kg**

Here is the double interpolation table for **H** :

I chose to interpolate on pressure first and then to interpolate on temperature. If you do the interpolations in the opposite order, you will get a slightly different answer. Either method is satisfactory.

	Pressure (kPa)		
T(°C)	700	728	800
40	426.72	426.13	424.59
43	429.71	429.14	427.67
50	436.67	436.16	434.84

H	**429.14**	**kJ/kg**

Part c.) **Given :** P **250** **kPa** V **0.049** **m^3/kg**

We again begin by determining the state of the system. In this case, it would be easiest to lookup the $V_{sat\ vap}$ and

If : $V > V_{sat\ vap}$ Then : The system contains a superheated vapor.

If : $V < V_{sat\ liq}$ Then : The system contains a subcooled liquid.

If : $V_{sat\ vap} > V > V_{sat\ liq}$ Then : The system contains an equilibrium mixture of saturated liquid and saturated vapor.

Data :

P*(kPa)	T_{sat} (°C)	$V_{sat\ liq}$ (m^3/kg)	$V_{sat\ vap}$ (m^3/kg)	$U_{sat\ liq}$ (kJ/kg)	$U_{sat\ vap}$ (kJ/kg)	$H_{sat\ liq}$ (kJ/kg)	$H_{sat\ vap}$ (kJ/kg)
250	-4.28	7.6406E-04	0.080685	194.08	375.91	194.27	396.08

Because **V** lies between $V_{sat\ liq}$ and $V_{sat\ vap}$, the system is the two-phase envelope and $T = T_{sat}$.

T **-4.284** **°C**

In order to determine the values of the other properties of the system using the following equation, we will need to

$$M_{VLE} = M_{sat\,liq} + x\left(M_{sat\,vap} - M_{sat\,liq}\right) \qquad \textbf{Eqn 1}$$

We can determine **x** from the saturation data and the known value of u for the system using :

$$x = \frac{V - V_{sat\,liq}}{V_{sat\,vap} - V_{sat\,liq}} \qquad \textbf{Eqn 2}$$

x **0.604** **kg vap/kg**

Now, we can plug **x** back into **Eqn 1** and apply it to the unknown properties, **U** and **H**.

U **303.82** **kJ/kg**

H **316.07** **kJ/kg**

Part d.) **Given :** T **50** **°C** P **1547** **kPa**

The first step in solving each part of this problem is to determine the state of the system. Is it subcooled liquid, superheated vapor or a two-phase VLE mixture.

We could do this by determining the boiling point or saturation temperature at the system pressure. But, since **1547 kPa** does not appear in the Saturation Pressure Table for R-134a, this would require an interpolation. It is easier to determine the saturation pressure or vapor pressure based on the system temperature because **50°C** does appear in the Saturation Temperature Table and therefore does not require an interpolation.

P*(50°C) **1317.9** **kPa**

Since the actual system pressure is ABOVE the vapor pressure, the system contains a subcooled liquid.

The quality of a subcooled liquid is undefined. Therefore : **x =** **N/A - Subcooled**

The Subcooled Liquid Table for R-134 includes data fo **50°C** at both **1400 kPa** and **1600 kPa**.
Therefore, a single-interpolation is required for each unknown property in the problem statement.

P*(kPa)	T (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)
1400	50	9.0646E-04	270.32	271.59
1547	50	9.0517E-04	270.13	271.53
1600	50	9.0470E-04	270.06	271.51

V	**9.0517E-04**	**m^3/kg**
U	**270.13**	**kJ/kg**
H	**271.53**	**kJ/kg**

Part e.) **Given :** **P** **976** **kPa** **H** **318.7** **kJ/kg**

This part of the problem is very similar to **part a**.
We again begin by determining the state of the system. In this case, it would be easiest to lookup the $\mathbf{H_{sat\,vap}}$ and $\mathbf{H_{sat\,liq}}$ at the given temperature.

If :	$\mathbf{H > H_{sat\,vap}}$	Then :	The system contains a superheated vapor.
If :	$\mathbf{H < H_{sat\,liq}}$	Then :	The system contains a subcooled liquid.
If :	$\mathbf{H_{sat\,vap} > H > H_{sat\,liq}}$	Then :	The system contains an equilibrium mixture of saturated liquid and saturated vapor.

Data :

P*(kPa)	T_{sat} (°C)	$V_{sat\,liq}$ (m^3/kg)	$V_{sat\,vap}$ (m^3/kg)	$U_{sat\,liq}$ (kJ/kg)	$U_{sat\,vap}$ (kJ/kg)	$H_{sat\,liq}$ (kJ/kg)	$H_{sat\,vap}$ (kJ/kg)
950	37.50	8.6412E-04	0.0214415	251.86	397.95	252.69	418.32
1000	39.39	8.7007E-04	0.020316	254.63	398.85	255.50	419.16

Unfortunately, the system pressure of **976 kPa** does not appear in the Saturation Pressure Table.
So, we will have to interpolate between the two rows in the table shown here to determine the saturation properties at **976 kPa**.

P*(kPa)	T_{sat} (°C)	$V_{sat\,liq}$ (m^3/kg)	$V_{sat\,vap}$ (m^3/kg)	$U_{sat\,liq}$ (kJ/kg)	$U_{sat\,vap}$ (kJ/kg)	$H_{sat\,liq}$ (kJ/kg)	$H_{sat\,vap}$ (kJ/kg)
976	38.479	8.6721E-04	0.0208563	253.30	398.42	254.15	418.76

Because **H** lies between $\mathbf{H_{sat\,liq}}$ and $\mathbf{H_{sat\,vap}}$, the system is the two-phase envelope and $\mathbf{T = T_{sat}}$.

T **38.5** **°C**

In order to determine the values of the other properties of the system using the following equation, we will need to know the quality, **x**.

$$\mathbf{M_{VLE} = M_{sat\,liq} + x\left(M_{sat\,vap} - M_{sat\,liq}\right)}$$

Eqn 3

We can determine **x** from the saturation data and the known value of **U** for the system using :

$$\mathbf{x = \frac{h - h_{sat\,liq}}{h_{sat\,vap} - h_{sat\,liq}}}$$

Eqn 4

x **0.392** **kg vap/kg**

Now, we can plug **x** back into **Eqn 1** and apply it to the unknown properties, **V** and **U**.

V **8.6721E-04** **m^3/kg**

U **253.30** **kJ/kg**

Verify: The assumption that there is no ice in the is confirmed by the fact that the the states in **part (a)** and **part (b)** are both located in the R-134a Tables. The R-134a Tables only consider states in which no solid R-134a can exist at equilibrium.

Answers :

	T (°C)	P (kPa)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	x (kg vap/kg tot)
a.)	-15	163.9	0.120671	369.85	389.63	1
b.)	43	728	0.030869	406.76	429.14	N/A - Superheated
c.)	-4.284	250	0.049	303.82	316.07	0.604
d.)	50	1547	9.0517E-04	270.13	271.53	N/A - Subcooled
e.)	0	976	8.6721E-04	253.30	318.70	0.392

2D-9 Relative Humidity, Partial Pressure and Mole and Mass Fractions 6 pts

My classroom contains **250 m^3** of humid air at **26°C** and **44%** **relative humidity**. Calculate the **mass** of **dry air** (**BDA**) and the **mass** of **water** vapor in the room.

Read : The key to this problem is the definition of relative humidity. When the relative humidity and temperature are given, we can use data from the Steam Tables to determine the partial pressure and mole fraction of water in the gas phase. We can convert the mole fraction into a mass fraction. Then, by assuming the gas phase is an ideal gas, we can determine the total mass of air in the room. And, finally we can determine the mass of BDA and water in the gas in the room.

Given:

V_{tot}	250	m^3	T	26	°C
P_{tot}	105	kPa	h_r	44%	

Find:

m_{H2O}	???	kg H_2O	m_{BDA}	???	kg BDA

Assumptions: - The air-water gas mixture behaves as an ideal gas. At the end of the problem we will be able to determine the molar volume of the air-water gas mixture so we can verify this assumption.

Equations / Data / Solve:

Using the IG EOS and the known **P**, **T** and **V** of the room, we can determine the mass of air-water gas mixture in the room.

Ideal Gas EOS :

$$PV = nRT = \frac{m}{MW} RT \qquad \textbf{Eqn 1}$$

Solve **Eqn 1** for m_{gas} :

$$m_{gas} = MW_{gas} \frac{PV}{RT} \qquad \textbf{Eqn 2}$$

The following equation allows us to calculate the average molecular weight of a gas mixture using the mole fractions and molecular weights of its constituents.

$$\overline{MW}_{mix} = \sum_{i=1}^{\#\text{ of species}} y_i \, MW_i \qquad \textbf{Eqn 3}$$

For our system, **Eqn 3** becomes :

$$\overline{MW}_{gas} = y_{BDA} \, MW_{BDA} + y_{H2O} \, MW_{H2O} \qquad \textbf{Eqn 4}$$

MW_{BDA}	29	g BDA/mole BDA	MW_{H2O}	18.016	g H_2O/mole H_2O

So, in order to determine the average molecular weight of the gas, we need to know the composition of the gas. That is, we need to know the mole fractions of BDA and water in the gas mixture. Given the relative humidity and the temperature of an ideal gas mixture of air and water, we can determine the composition.

Begin with the definition of relative humidity :

$$h_r = \frac{P_{H20}}{P^*_{H20}}$$ **Eqn 5**

Since we know the temperature of the system is **26°C**, we can look up the vapor pressure of water at this temperature in the Saturation Temperature Table in the Steam Tables. Unfortunately, because **26°C** is not listed in the Saturation Temperature Table, interpolation is required.

T_{sat} (°C)	P_{sat} (kPa)
25	3.170
26	???
30	4.247

Interpolation yields : **P^*_{H2O}(26°C)** **3.385** **kPa**

We can plug the vapor pressure, along with the given value of the relative humidity into Eqn 5 to determine the partial pressure of water in the gas.

P_{H2O} 1.490 kPa

The last key relationship is the one between partial pressure and the mole fraction for ideal gases :

$$P_{H20} = y_{H2O}\ P_{tot}$$ **Eqn 6**

Solving for the mole fraction yields :

$$y_{H2O} = \frac{P_{H20}}{P_{tot}}$$ **Eqn 7**

Plugging numbers into **Eqn 7** yields : **y_{H2O} 0.0142 mol H_2O / mol gas**

We can calculate **y_{BDA}** because **$\Sigma y_i = 1$** : **y_{BDA} 0.986 mol BDA / mol gas**

At last, we can use these mole fractions in **Eqn 4** to determine the value of **MW_{gas}** and then use that in **Eqn 2** to determine the total mass of gas in the room.

MW_{gas} 28.844 g gas / mol gas **m_{gas} 304.43 kg gas**

Here we can either determine the mass fractions of BDA and water in the gas or we can determine the number of moles of BDA and water in the room. I will use both methods here.

$$n_{gas} = \frac{m_{gas}}{MW_{gas}}$$ **Eqn 8**

$$n_i = y_i\ n_{gas}$$ **Eqn 9**

n_{gas}	10.554	mol gas
n_{H2O}	0.150	mol H_2O
n_{BDA}	10.405	mol BDA

$$m_{gas} = n_{gas}\ MW_{gas}$$ **Eqn 10**

m_{H2O} 2.70 kg H_2O **m_{BDA} 302 kg BDA**

Alternate Ending

Convert mole fractions into mass fractions :

$$x_i = y_i \frac{MW_i}{MW_{gas}}$$ **Eqn 11**

The following unit analysis shows why **Eqn 11** is true.

$$\frac{\text{g of species i}}{\text{g of gas}} = \frac{\cancel{\text{mol of species i}}}{\cancel{\text{mol of gas}}}\left[\frac{\dfrac{\text{g of species i}}{\cancel{\text{mol of species i}}}}{\dfrac{\text{g of gas}}{\cancel{\text{mol of gas}}}}\right]$$ **Eqn 12**

x_{H2O}	**0.00886**	**kg H_2O /kg gas**	x_{BDA}	**0.991**	**kg BDA / kg gas**

Now, we can determine the mass of each species in the gas by multiplying the total mass by the mass fraction.

$$m_i = x_i \, m_{gas}$$ **Eqn 13**

m_{H2O}	**2.70**	**kg H_2O**	m_{BDA}	**302**	**kg BDA**

Verify : The ideal gas assumption needs to be verified.
We need to determine the specific volume and check if :

$$\tilde{V} > 20 \text{ L/mol}$$

$$\tilde{V}_{out} = \frac{R\,T_{out}}{P_{out}}$$ **Eqn 9**

R	8.134	**J/mol-K**
V	**23.17**	**L/mol**

The ideal gas assumption is valid because **V > 20 L/mole**.

Answers :

m_{H2O}	**2.70**	**kg H_2O**	m_{BDA}	**302**	**kg BDA**

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2E-1 Mass of Propane in a Rigid Tank 2 pts

Determine the **mass** of **propane** in a **10 L** tank if a **temperature** gauge on the tank reads **-20^{o}C** and a **pressure** gauge on the tank reads **10 kPa**. Assume **propane** behaves as an ideal gas, but **verify** this assumption.

Read : Apply the Ideal Gas EOS to determine the molar volume of the propane in the tank. If this value is greater than **20 L/mol**, the IG EOS is accurate to within **1%**.
Use the molar volume, the molecular weight and the volume of the tank to determine the mass of propane in the tank.

Given:	**V**	**10**	**L**	**P_{gauge}**	**10**	**kPa**
	T	**-20**	**^{o}C**			
Find:	**M_{C3}**	**???**	**g**			

Assumptions: 1- Assume the propane behaves as an ideal gas. Be sure to verify this assumption.

Equations / Data / Solve:

We could use the following equation to determine the mass of propane in the tank if we knew the specific volume.

$$M = \frac{V}{\hat{V}} \qquad \textbf{Eqn 1}$$

Equations of state are written in terms of the molar volume. So we need the following equation to get from molar volume to the specific volume that we need to make use of **Eqn 1**.

$$\hat{V} = \frac{\tilde{V}}{MW} \qquad \textbf{Eqn 2}$$

Now, we need to us the Ideal Gas EOS to determine the molar volume.

Ideal Gas EOS :

$$P\,\tilde{V} = R\,T \qquad \textbf{Eqn 3}$$

Let's solve for the molar volume because we know we will need this value to test whether the Ideal Gas EOS is applicable.

$$\tilde{V} = \frac{R\,T}{P} \qquad \textbf{Eqn 4}$$

We need to be careful with our units in **Eqn 2**. Let's begin by looking up the Universal Gas Constant in my favorite units, **J/mol-K**.

R 8.314 **J/mol-K**

Next we need to convert the temperature to **Kelvins** and the gauge pressure to absolute pressure in **Pascals**.

$$T(K) = T(^{o}C) + 273.15$$

T	**253.15**	**K**

$$P_{abs} = P_{gauge} + P_{atm}$$

Assume:	P_{atm}	**101.325**	**kPa**
	P_{abs}	**111.325**	**kPa**
		111,325	**Pa**

Plugging values into **Eqn 2** yields:

V	**0.018906**	m^3**/mol**
	18.91	**L/mol**

We can now look up the molecular weight of propane:

MW	**44.1**	**g/mol**

Now, we can plug values into **Eqn 2** and then **Eqn 1** to complete this solution.

(Watch the units here!)	**V**	**0.42870**	m^3**/kg**
	M_{C3}	**0.02333**	**kg**
		23.33	**g**

Verify: Propane is not a diatomic molecule. Therefore, it cannot be accurately treated as an ideal gas unless its molar volume is greater than **20 L/mol**.

Above, we found the molar volume of propane under the conditions in this problem is **18.91 L/mol**.

Therefore we conclude that it may not be accurate to use the Ideal Gas EOS to solve this problem.

You should probably try a more sophisticated **EOS** such as the Generalized Compressibility EOS in **this lesson** or one of the equations of state in the **next lesson**.

Answers : **M_{C3} 23.33 g**

(But this result is not reliable because the Ideal Gas assumption is not valid. A more accurate analysis yields **M_{C3} = 24.1 g**.)

2E-2 Ideal Gas or Not: Dioxide An Ideal Gas? 2 pts

Carbon Dioxide (**CO_2**) gas is held in a tank at **225 kPa** and **240 K**. Can the **carbon dioxide** be treated as an ideal gas with less than **1%** error?

Read : Apply the Ideal Gas EOS to determine the molar volume of the **CO_2** in the tank. If this value is greater than **20 L/mol**, the IG EOS is accurate to within **1%**.

Given:

T	240	K	P	225	kPa

Find: Can **CO_2** be accurate treated as an ideal gas?

Assumptions: 1- Assume the **CO_2** behaves as an ideal gas. Be sure to verify this assumption.

Equations / Data / Solve:

Ideal Gas EOS :

$$P\tilde{V} = R\,T \qquad \textbf{Eqn 1}$$

Let's solve for the molar volume because we know we will need this value to test whether the Ideal Gas EOS is applicable.

$$\tilde{V} = \frac{R\,T}{P} \qquad \textbf{Eqn 2}$$

We need to be careful with our units in **Eqn 2**. Let's begin by looking up the Universal Gas Constant in my favorite units, **J/mol-K**.

	R	8.314	J/mol-K
Next we need to convert the pressure from **kPa** to **Pascals**.	P	225,000	Pa
Plugging values into **Eqn 2** yields:	V	0.0088683	m^3/mol
		8.87	L/mol

Verify: **CO_2** is not a diatomic molecule. Therefore, it cannot be accurately treated as an ideal gas unless its molar volume is greater than **20 L/mol**.

Above, we found the molar volume of propane under the conditions in this problem is **8.87 L/mol**.

Therefore, we conclude that it may not be accurate to use the Ideal Gas EOS to solve this problem.

You should probably try a more sophisticated **EOS** such as the Generalized Compressibility EOS in **this lesson** or one of the equations of state in the **next lesson**.

Answers : **No. Under these conditions, CO_2 cannot be accurately treated as an ideal gas.**

2E-3 Compressibility Factor: Methanol 2 pts

Use the Generalized Compressibility Factor **EOS** to determine the **molar volume** of **methanol** at **725 K** and **27 MPa**.

Read : We can use **T** & **P** to calculate T_R and P_R and use the Generalized Compressibility Charts to determine **Z** & V_R^{ideal}. Then, we can use either **Z** or V_R^{ideal} to determine the molar volume. The values we get for the molar volume should agree within about **5%.**

Given:				**Find:**			
	T	725	**K**		**V**	**???**	**L/mol**
	P	27	**MPa**				

Assumptions: None.

Equations / Data / Solve:

Begin by collecting all of the constants needed for all Generalized Compressibility Equation of State.

R	8.314	**J/mol-K**	T_c	513.38	**K**
MW	32.042	**g NH_3 / mol NH_3**	P_c	8.216	**MPa**

Compressibility EOS :

Given T_R and the ideal reduced molar volume, use the compressibility charts to evaluate either P_R or the compressibility, **Z**. T_R, P_R and **Z** are defined as follows.

$$T_R = \frac{T}{T_c} \qquad \text{Eqn 1}$$

$$P_R = \frac{P}{P_c} \qquad \text{Eqn 2}$$

$$Z = \frac{P\tilde{V}}{RT} \qquad \text{Eqn 3}$$

Because we know both **T** and **P**, we can immediately plug values into **Eqns 1** & **2**.

T_R	**1.412**	P_R	**3.286**

Read the Generalized Compressibility Chart for $P_R = 0$ to **7** :

Z	**0.74**
V_R^{ideal}	**0.325**

We can solve **Eqn 3** for the molar volume and use the value of **Z** we just found to evaluate **V**.

$$\tilde{V} = \frac{ZRT}{P} \qquad \text{Eqn 4}$$

Plugging values into **Eqn 4** yields:

V	**0.0001652**	**m^3/mol**
	0.165	**L/mol**

Alternatively, we could us the **ideal reduced molar volume** that we found on the Generalized Compressibility Chart to determine the molar volume.

$$\tilde{V}_R^{ideal} = \frac{\tilde{V}}{R\,T_c\,/\,P_c}$$ **Eqn 5**

Now, we can solve **Eqn 5** and plug in values to determine **V**.

$$\tilde{V} = \tilde{V}_R^{ideal} \cdot \frac{R\,T_c}{P_c}$$ **Eqn 6**

V	**0.0001688**	**m^3/mol**
	0.169	**L/mol**

Verify: There are no assumptions to verify.

Answers : **Z:**

V	**0.165**	**L/mol**

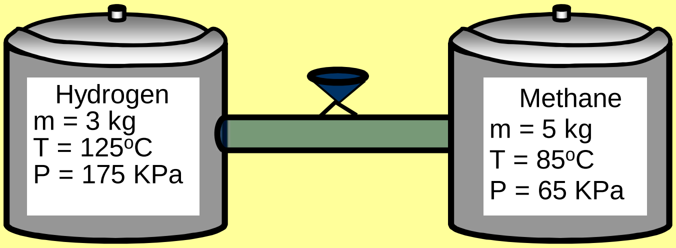
V_r^{ideal}:

V	**0.169**	**L/mol**

These values differ by less than **3%**. If you read the chart correctly, they should agree to within about **5%**.

2E-4 Equilibrium Pressure When Two Gases Are Mixed 5 pts

Two rigid tanks are connected by a large duct, as shown below. Tank **A** initially contains **3 kg** of **hydrogen** gas at **125°C** and **175 KPa**. Tank **B** initially contains **5 kg** of **methane** gas at **85°C** and **65 KPa**.



a.) Can either gas be treated as an ideal gas?

b.) The valve in the duct is opened and the gases in the tanks mix. If both tanks eventually cool to room temperature (**25°C**) after mixing, what will the final **pressure** be at equilibrium?

Read : The key here is to assume that both the pure gases and the final mixture of gases behave as ideal gases. We can immediately verify that the pure gases in their initial state are ideal gases, but we cannot verify that the final mixture of gases is ideal until we solve the problem and determine the final pressure.

Choose the contents of both tanks as the system. The fact that links the initial and final states of this system is that the total number of moles in the system does not change. This is a closed system.

Given:

m_{H2} =	3	kg	T_B =	85	°C
T_A =	125	°C		358.15	K
	398.15	K	P_B =	65	KPa
P_A =	175	KPa	T_{equ} =	25	°C
m_{CH4} =	5	kg		298.15	K

Find: P_{equ} = ??? kPa

Assumptions: - Both pure gases, as well as the final mixture, behave as ideal gases.

Equations / Data / Solve:

Part a.) A diatomic gas can be considered ideal when the following criterion is satisfied:

$$\tilde{V} = \frac{RT}{P} > 5\,\frac{L}{mol} \qquad \text{Eqn 1}$$

V_{N2} = 18.92 L/mol

V_{O2} = 45.81 L/mol

Where: R 8.314 J/mol K

Since both molar volumes are much greater than **5 L/mole**, it is safe to consider both gases to be ideal gases.

Part b.) The key to solving the problem is to ASSUME that the equilibrium mixture will be an ideal gas:

$$P_{eq} = \frac{n_{tot} R T_{eq}}{V_{tot}}$$ **Eqn 2**

Let's begin by determining how many moles of gas are initially in each tank.

$$n_i = \frac{m_i}{MW_i}$$ **Eqn 3**

Then we can determine the total moles of gas in the system :

$$n_{tot} = n_{O_2} + n_{N_2}$$ **Eqn 4**

MW_{N2} =	**28.01**	**g/mol**
MW_{O2} =	**32.00**	**g/mol**

n_{N2} =	**107.1**	**mole N_2**
n_{O2} =	**156.3**	**mole O_2**
n_{total} =	**263.4**	**mole total**

The total number of moles in the system does not changes as the gases mix!
The system, consisting of both tanks, is closed.

Next, we can use the IG EOS to determine the volume of each tank and then the total volume of the system.

$$V_{tot} = V_A + V_B$$ **Eqn 5**

Where (for ideal gases) :

$$V_A = n_{N2} \tilde{V}_{N2}$$ **Eqn 6**

and

$$V_B = n_{O2} \tilde{V}_{O2}$$ **Eqn 7**

V_A =	**2.03**	**m^3**
V_B =	**7.16**	**m^3**
V_{tot} =	**9.18**	**m^3**

From the IG EOS we derive the following equation for the equilibrium pressure :

$$P_{eq} = \frac{n_{tot} R T_{eq}}{V_{tot}}$$ **Eqn 8**

P_{eq} **71.08** **kPa**

Verify: Now, calculate the molar volume at the equilibrium state, just to be sure it is still safe to treat the gas as an ideal gas !

V_{eq} = **34.87** **L/mol**

Since **V_{eq}** is greater than **5 L/mole**, we were justified in using the Ideal Gas EOS for the equilibrium state as well.

Answers :

a.) Since both molar volumes are much greater than **5 L/mole**, it is safe to consider both gases to be ideal gases.

b.) **P_{eq} = 71.1 kPa**

2F-1 An Application of Equations of State 10 pts

Estimate the **pressure** of **ammonia** at a **temperature** of 27°C and a **specific volume** of 0.526 m^3/kg.

a.) Ideal Gas **EOS**
b.) Virial **EOS**
c.) van der Waal **EOS**
d.) Soave-Redlich-Kwong **EOS**
e.) Compressibility Factor **EOS**
f.) Steam Tables.

Read : Not much to say here.

Given:	**T**	27	**°C**	**Find:**	**P**	**???**	**kPa**
	V	0.526	**m^3/kg**				

Assumptions: None.

Equations / Data / Solve:

Begin by collecting all of the constants needed for all the Equations of State in this problem.

R	8.314	**J/mol-K**	**T_c**	405.55	**K**
MW	17.03	**g NH_3 / mol NH_3**	**P_c**	1.128E+07	**Pa**
			ω	0.250	

Part a.) **Ideal Gas EOS :**

$$P\tilde{V} = RT \quad \textbf{Eqn 1}$$

Solve for pressure :

$$P = \frac{RT}{\tilde{V}} \quad \textbf{Eqn 2}$$

We must determine the molar volume before we can use **Eqn 2** to answer the question.

Use the definition of molar volume:

$$\tilde{V} = \frac{V}{n} \quad \textbf{Eqn 3}$$

Where :

$$n = \frac{m}{MW} \quad \textbf{Eqn 4}$$

Therefore :

$$\tilde{V} = \frac{V}{m/MW} = \hat{V}\cdot MW \quad \textbf{Eqn 5}$$

Now, plug values back into **Eqn 2**.

Be careful with the units.

V	**8.958E-03**	**m^3/mol**
T	**300.15**	**K**
P	**2.786E+05**	**Pa**
P	**278.6**	**kPa**

Part b.) **Truncated Virial EOS :**

$$Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B}{\tilde{V}} \qquad \text{Eqn 6}$$

We can estimate B using :

$$B = \frac{RT_C}{P_C}\left(B_0 + \omega B_1\right) \qquad \text{Eqn 7}$$

$$B_0 = 0.083 - \frac{0.422}{T_R^{1.6}} \qquad \text{Eqn 8}$$

$$B_1 = 0.139 - \frac{0.172}{T_R^{4.2}} \qquad \text{Eqn 9}$$

Where :

$$T_R = \frac{T}{T_c} \qquad \text{Eqn 10}$$

We can solve **Eqn 6** for P :

$$P = Z\,\frac{RT}{\tilde{V}} \qquad \text{Eqn 11}$$

Plugging numbers into **Eqns 10, 8, 9, 7** and **11** (in that order) yields :

T_R	**0.740**	**B**	**-2.14E-04**	**m^3/mol**
B_0	**-0.6000**	**Z**	**9.76E-01**	
B_1	**-0.4698**	**P**	**271.9**	**kPa**

Part c.) **van der Waal EOS :**

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}^2} \qquad \text{Eqn 12}$$

We can determine the values of **a** and **b**, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = \frac{27 R^2 T_c^2}{64 P_c} \qquad \text{Eqn 13}$$

$$b = \frac{RT_c}{8P_c} \qquad \text{Eqn 14}$$

T_c	**405.55**	**K**	**a**	**0.4252**	**Pa-mol^2/m^6**
P_c	**1.128E+07**	**Pa**	**b**	**3.74E-05**	**m^3/mol**

Now, we can plug the constants a and b into **Eqn 12** to determine the pressure.

P	**274.4**	**kPa**

Part d.) **Soave-Redlich-Kwong EOS :**

$$P = \frac{R\,T}{\tilde{V}-b} - \frac{\alpha\,a}{\tilde{V}\left(\tilde{V}+b\right)} \qquad \text{Eqn 15}$$

We can determine the values of **a**, **b** and α, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = 0.42748\,\frac{R^2\,T_c^2}{P_c} \qquad \text{Eqn 16}$$

$$b = 0.08664\,\frac{R\,T_c}{P_c} \qquad \text{Eqn 17}$$

$$\alpha = \left[1 + m\left(1 - \sqrt{T_R}\right)\right]^2 \qquad \text{Eqn 18}$$

$$T_R = \frac{T}{T_c} \qquad \text{Eqn 19}$$

$$m = 0.48508 + 1.55171\,\omega - 0.1561\,\omega^2 \qquad \text{Eqn 20}$$

Where : ω **0.250**

Now, plug values into **Eqns 15 - 20** :

T_R	0.7401	a	0.43084	Pa-mol^2/m^6
m	0.8633	b	2.590E-05	m^3/mol
α	1.25575			

P	**272.7**	**kPa**

Part e.) **Compressibility EOS :**

Given T_R and the ideal reduced molar volume, use the compressibility charts to evaluate either P_R or the compressibility, **Z**

$$Z = \frac{P\,\tilde{V}}{R\,T} \qquad \text{Eqn 21}$$

From part c : T_R **0.7401**

Defiition of the ideal reduced molar volume :

$$\tilde{V}_R^{\,ideal} = \frac{\tilde{V}}{R\,T_c\,/\,P_c} \qquad \text{Eqn 22}$$

V_R^{ideal}	29.97
P_R	0.0228
Z	0.957

Read the Generalized Compressibility Chart for P_R = **0** to **1** :

We can use the definition of P_R to calculate P :

$$P_R = \frac{P}{P_c} \qquad \text{Eqn 23}$$

$$P = P_R\,P_c \qquad \text{Eqn 24}$$

P	**257.2**	**kPa**

Or, we can use Z and its definition to determine P :

$$P = Z\,\frac{R\,T}{\tilde{V}} = Z \cdot P^{IG} \qquad \text{Eqn 25}$$

P	**266.6**	**kPa**

Part f.) The **Ammonia Tables** provide the best available estimate of the pressure.

We begin by determining the state of the system. In this case, it would be easiest to lookup the $\mathbf{V_{sat\ vap}}$ and $\mathbf{V_{sat\ liq}}$ at the given temperature.

If : $\mathbf{V > V_{sat\ vap}}$ Then : The system contains a superheated vapor.

If : $\mathbf{V < V_{sat\ liq}}$ Then : The system contains a subcooled liquid.

If : $\mathbf{V_{sat\ vap} > V > V_{sat\ liq}}$ Then : The system contains an equilibrium mixture of saturated liquid and saturated vapor.

Data :

P*(kPa)	T (^{o}C)	$V_{sat\ liq}$ (m^3/kg)	$V_{sat\ vap}$ (m^3/kg)	$H_{sat\ liq}$ (kJ/kg)	$H_{sat\ vap}$ (kJ/kg)
1066.56	**27**	**1.67E-03**	**0.12066**	**308.11**	**1465.42**

Because $\mathbf{V > V_{sat\ vap}}$, the ammonia is superheated in this system.

At this point we can make use of the fact that we have a pretty good idea of what the actual pressure is in the tank (from **parts a-d**) or we can scan the superheated vapor tables to determine which two pressures bracket our known value of the specific volume. The given specific volume of **0.640 m^3/kg** lies between **200 kPa** and **250 kPa** and **T = 27^{o}C** lies between **25^{o}C** and **50^{o}C**. This is a tricky multiple interpolation problem !

The Superheated Ammonia Table gives us :

P*(kPa)	T (^{o}C)	V (m^3/kg)	H (kJ/kg)
200	20	0.6995	1510.1
200	30	0.7255	1532.5
250	25	0.5668	1518.2
250	50	0.6190	1574.7

We can now interpolate on this data to determine values of the specific volume at **T = 27^{o}C** at BOTH **200 kPa** and **250 kPa**. This will help us setup a second interpolation to determine the pressure that corresponds to **T = 27^{o}C** and **V = 0.640 m^3/kg**.

At **200 kPa** :

T (^{o}C)	V (m^3/kg)
20	0.6995
30	0.7255

$$\mathbf{V(27) = slope\cdot(27-20) + 0.6995}$$ **Eqn 26**

$$\mathbf{slope = \frac{0.7255 - 0.6995}{30 - 20}}$$ **Eqn 27**

slope **2.600E-03** **(m^3/kg)/^{o}C** **V** **0.71766** **m^3/kg**

At **300 kPa** :

T (^oC)	V (m^3/kg)
25	0.5668
50	0.6190

$$\mathbf{V(27) = slope \cdot (27 - 25) + 0.5668}$$ **Eqn 28**

$$\mathbf{slope = \frac{0.6190 - 0.5668}{50 - 25}}$$ **Eqn 29**

slope	**2.085E-03**	**(m^3/kg)/^{o}C**	**V**	**0.57102**	**m^3/kg**

Now, we must interpolate one more time to determine the pressure which, at **27^{o}C**, yields a spoecific volume of **0.640 m^3/kg** :

At **27^{o}C** :

P (kPa)	V (m^3/kg)
200	0.7177
250	0.5710

$$\mathbf{P(0.640) = slope \cdot (0.640 - 0.7177) + 200}$$ **Eqn 30**

$$\mathbf{slope = \frac{250 - 200}{0.5710 - 0.7177}}$$ **Eqn 31**

slope	**-340.96**	**(m^3/kg)/kPa**	**P**	**265.4**	**kPa**

Verify: No assumptions to verify.

Answers :

a.)	P	278.6	kPa	d.)	P	272.7	kPa
b.)	P	271.9	kPa	e.)	P	266.6	kPa
c.)	P	274.4	kPa	f.)	P	265.4	kPa

2F-2 An Application of Equations of State 6 pts

Steam is contained in a **203 L** tank at **600°C**. The **mass** of **steam** in the tank is **12.4 kg**. Determine the **pressure** in the tank using:

a.) Ideal Gas **EOS**
b.) Virial **EOS**
c.) van der Waal **EOS**
d.) Redlich-Kwong **EOS**
e.) Compressibility Factor **EOS**
f.) Steam Tables.

Read : Not much to say here.

Given:

m	12.4	kg	V	203	L
T	500	°C		0.203	m^3

Find: P ??? kPa

Assumptions: None.

Equations / Data / Solve:

Begin by collecting all of the constants needed for all the Equations of State in this problem.

R	8.314	J/mol-K	T_c	647.4	K
MW	18.016	g NH_3 / mol NH_3	P_c	2.21E+07	Pa

Part a.) **Ideal Gas EOS :**

$$P\tilde{V} = RT \qquad \text{Eqn 1}$$

Solve for pressure :

$$P = \frac{RT}{\tilde{V}} \qquad \text{Eqn 2}$$

We must determine the molar volume before we can use **Eqn 2** to answer the question.

Use the definition of molar volume:

$$\tilde{V} = \frac{V}{n} \qquad \text{Eqn 3}$$

Where :

$$n = \frac{m}{MW} \qquad \text{Eqn 4}$$

MW	18.016	g H_2O / mol H_2O	n	688.28	mol H_2O
			V	2.95E-04	m^3/mol

Now, plug values back into Eqn 2.
Be careful with the units.

R	8.314	J/mol-K
T	773.15	K
P	2.18E+07	Pa

P	**21.8**	**MPa**

Part b.) **van der Waal EOS :**

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}^2}$$ **Eqn 5**

We can determine the values of **a** and **b**, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = \frac{27 R^2 T_c^2}{64 P_c}$$ **Eqn 6**

$$b = \frac{R T_c}{8 P_c}$$ **Eqn 7**

T_c	647.4	K
P_c	2.21E+07	Pa
a	0.5530	Pa-mol^2/m^6
b	3.04E-05	m^3/mol

Now, we can plug the constants **a** and **b** into **Eqn 5** to determine the pressure.

P	17.9	MPa

Part c.) **Redlich-Kwong EOS :**

$$P = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}\left(\tilde{V} + b\right) T^{1/2}}$$ **Eqn 8**

We can determine the values of **a**, **b** and α, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = 0.42748 \frac{R^2 T_c^{5/2}}{P_c}$$ **Eqn 9**

$$b = 0.08664 \frac{R T_c}{P_c}$$ **Eqn 10**

Now, plug values into **Eqns 8 -10** :

a	14.25855	Pa-m^6-K$^{1/2}$/mol^2
b	2.110E-05	m^3/mol

P	18.0	MPa

Part d.) **Compressibility EOS :**

Given T_R and the ideal reduced molar volume, use the compressibility charts to evaluate either P_R or the compressibility, **Z**

$$Z = \frac{P \tilde{V}}{RT}$$ **Eqn 11**

$$T_R = \frac{T}{T_c}$$ **Eqn 12**

T_R	1.1942

Defiition of the ideal reduced molar volume :

$$\tilde{V}_R^{ideal} = \frac{\tilde{V}}{R T_c / P_c}$$ **Eqn 13**

V_R^{ideal}	1.2110

Read the Generalized Compressibility Chart for **P_R = 0 to 1** :

P_R	0.88
Z	0.885

We can use the definition of $\mathbf{P_R}$ to calculate **P** :

$$P_R = \frac{P}{P_c} \qquad \textbf{Eqn 14}$$

$$P = P_R\, P_c \qquad \textbf{Eqn 15}$$

P	19.4	MPa

Or, we can use **Z** and its definition to determine **P** :

$$P = Z\,\frac{R\,T}{\tilde{V}} = Z\,\tilde{V}^{IG} \qquad \textbf{Eqn 16}$$

P	19.3	MPa

Part e.) The **Steam Tables** provide the best available estimate of the pressure in the tank.

Because **T > T$_c$**, the properties of the water in the tank must be obtained from the superheated vapor table, even though the water is actually a supercritical fluid in this system.

At this point we can make use of the fact that we have a pretty good idea of what the actual pressure is in the tank (from **parts a-d**) or we can scan the spuerheated vapor tables to determine which two pressures bracket our known value of the specific volume.

In either case, we begin by converting the molar volume into a specific volume :

$$\hat{V} = \frac{\tilde{V}}{MW} \qquad \textbf{Eqn 17}$$

Using the **MW** of water from **part (a)** yields :

v	**1.637E-05**	**m^3/g**
v	**0.016371**	**m^3/kg**

The Superheated Steam Table gives us :

At **P =**	**20**	**MPa**	and	At **P =**	**40**	**MPa**
v =	**0.014793**	**m^3/kg**		**v =**	**0.005623**	**m^3/kg**

We can determine the pressure in our tank by interpolation :

P	**16.56**	**MPa**
P	**16.6**	**MPa**

Verify: No assumptions to verify.

Answers :

a.)	P	21.8	kPa
b.)	P	17.9	kPa
c.)	P	18.0	kPa
d.)	P	19.3	kPa
e.)	P	16.6	kPa

None of these Equations of State did very well because steam at high pressure behaves in a very non-ideal manner due to the high polarity of the molecules and the resulting stron electrostatic interactions.

2F-3 Determination of Pressure Inside a Tank Containing Ammonia 5 pts

Ammonia at 150°C is contained in a tank with a **volume** of 137 L. The **mass** of the **ammonia** in the tank is 7.4 kg. Determine the **pressure** in the tank by each of the following methods:

a.) Ideal Gas **EOS**
b.) Virial **EOS**
c.) van der Waal **EOS**
d.) Soave-Redlich-Kwong **EOS**
e.) Compressibility Factor **EOS**
f.) Ammonia Tables.

Data: T_c **= 405.55 K**, P_c **= 11,280 kPa**, **MW = 17.03 g** NH_3**/mol** NH_3, **Pitzer accentric factor = 0.256**.

Read : Not much to say here.

Given :

m	7.4	kg	V	137	L
T	150	°C		0.137	m^3
	423.15	K			

Find: P ??? kPa

Assumptions: None.

Equations / Data / Solve:

Begin by collecting all of the constants needed for all the Equations of State in this problem.

R	8.314	J/mol-K	T_c	405.55	K
MW	17.03	g NH_3 / mol NH_3	P_c	1.128E+07	Pa
			ω	0.256	

Part a.)

Ideal Gas EOS : $P\tilde{V} = RT$ **Eqn 1** Solve for pressure : $P = \frac{RT}{\tilde{V}}$ **Eqn 2**

We must determine the molar volume before we can use **Eqn 2** to answer the question.

Use the definition of molar volume: $\tilde{V} = \frac{V}{n}$ **Eqn 3** Where : $n = \frac{m}{MW}$ **Eqn 4**

n	434.5	mol NH_3	V	3.15E-04	m^3/mol
				0.3153	L/mol

Now, plug values back into **Eqn 2**.
Be careful with the units.

P 1.12E+07 Pa

P 11.2 MPa

However, since the molar volume is <u>FAR</u> less than **20 L/mole**, the Ideal Gas EOS is not applicable.
Choose any one of the following more sophisticated **EOS**'s to solve the problem.

Part b.)

Truncated Virial EOS :

$$Z = \frac{P\tilde{V}}{RT} = 1 + \frac{B}{\tilde{V}} \qquad \text{Eqn 5}$$

We can estimate **B** using :

$$B = \frac{R\,T_C}{P_C}\left(B_0 + \omega B_1\right) \qquad \text{Eqn 6}$$

$$B_0 = 0.083 - \frac{0.422}{T_R^{1.6}} \qquad \text{Eqn 7}$$

$$B_1 = 0.139 - \frac{0.172}{T_R^{4.2}} \qquad \text{Eqn 8}$$

Where :

$$T_R = \frac{T}{T_c} \qquad \text{Eqn 9}$$

We can solve **Eqn 5** for **P** :

$$P = Z\,\frac{R\,T}{\tilde{V}} \qquad \text{Eqn 10}$$

Plugging numbers into **Eqns 9, 7, 8, 6, 5** and **10** (in that order) yields :

T_R	1.043		B	-9.34E-05	m^3/mol
B_0	-0.3113		Z	7.04E-01	
B_1	-0.0049		P	7.85	MPa

Part c.)

van der Waal EOS :

$$P = \frac{R\,T}{\tilde{V} - b} - \frac{a}{\tilde{V}^2} \qquad \text{Eqn 11}$$

We can determine the values of **a** and **b**, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = \frac{27 R^2\, T_c^2}{64 P_c} \qquad \text{Eqn 12}$$

$$b = \frac{R\,T_c}{8 P_c} \qquad \text{Eqn 13}$$

a	0.4252	Pa-mol^2/m^6	b	3.74E-05	m^3/mol

Now, we can plug the constants **a** and **b** into **Eqn 11** to determine the pressure.

P	8.4	MPa

Part d.)

Redlich-Kwong EOS :

$$P = \frac{R\,T}{\tilde{V} - b} - \frac{a}{\tilde{V}\left(\tilde{V} + b\right)T^{1/2}} \qquad \text{Eqn 14}$$

We can determine the values of **a**, **b** and α, which are constants that depend only on the chemical species in the system, from the following equations.

$$a = 0.42748\,\frac{R^2\, T_c^{5/2}}{P_c} \qquad \text{Eqn 15}$$

$$b = 0.08664\,\frac{R\,T_c}{P_c} \qquad \text{Eqn 16}$$

Now, plug values into Eqns 14 -16 :

a	8.67636	Pa-m^6-K$^{1/2}$/mol^2
b	2.590E-05	m^3/mol

P	8.2	MPa

Part e.)

Compressibility EOS : Given $\mathbf{T_R}$ and the ideal reduced molar volume, use the compressibility charts to evaluate either $\mathbf{P_R}$ or the compressibility, **Z**

$$Z = \frac{P\tilde{V}}{RT}$$ **Eqn 17**

$$T_R = \frac{T}{T_c}$$ **Eqn 9**

T_R	1.0434

Defiition of the ideal reduced molar volume :

$$\tilde{V}_R^{ideal} = \frac{\tilde{V}}{R\,T_c\,/P_c}$$ **Eqn 18**

V_R^{ideal}	1.055
P_R	0.70
Z	0.73

Read the Generalized Compressibility Chart for $\mathbf{P_R = 0 \text{ to } 1}$:

We can use the definition of $\mathbf{P_R}$ to calculate **P** :

$$P_R = \frac{P}{P_c}$$ **Eqn 19**

$$P = P_R\,P_c$$ **Eqn 20**

P	**7.9**	**MPa**

Or, we can use **Z** and its definition to determine **P** :

$$P = Z\,\frac{R\,T}{\tilde{V}} = Z\,\tilde{V}^{IG}$$ **Eqn 21**

P	**8.1**	**MPa**

Part f.) **The Ammonia Tables provide the best available estimate of the pressure in the tank.**

Because $\mathbf{T > T_c}$, the properties of the ammonia in the tank must be obtained from the superheated vapor table, even though the it is actually a supercritical fluid in this system.

At this point we can make use of the fact that we have a pretty good idea of what the actual pressure is in the tank (from parts a-d) or we can scan the spuerheated vapor tables to determine which two pressures bracket our known value of the specific volume.

In either case, we begin by converting the molar volume into a specific volume :

$$\hat{V} = \frac{\tilde{V}}{MW}$$ **Eqn 22**

Using the **MW** of ammonia from **part (a)** yields :

V	**1.85E-05**	**m^3/g**
V	**0.018514**	**m^3/kg**

The Superheated Ammonia Table gives us :

At **P =**	**7.5**	**MPa**	and	At **P =**	**10**	**MPa**
v =	**0.020803**	**m^3/kg**		**v =**	**0.013381**	**m^3/kg**

We can determine the pressure in our tank by interpolation :

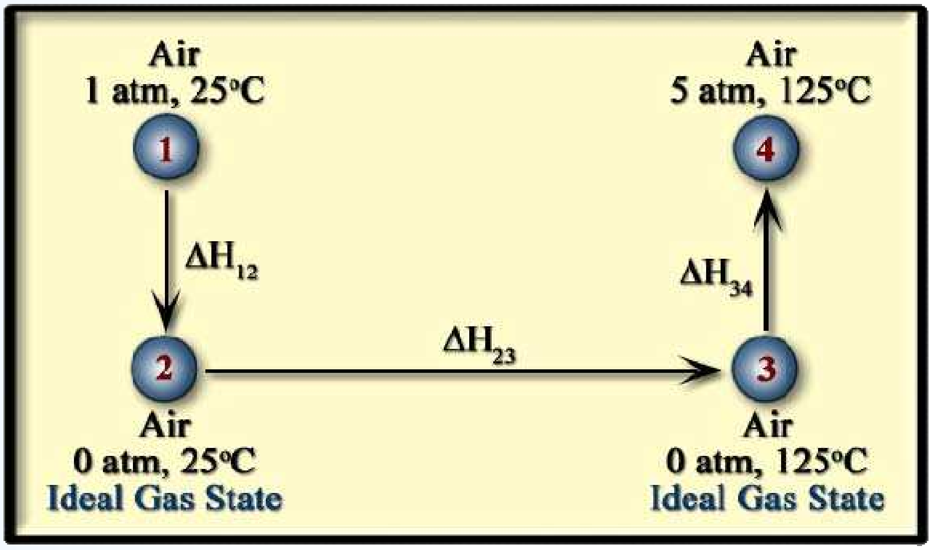
P	**8.3**	**MPa**

Verify: No assumptions to verify.

Answers :

a.)	**P**	**11.2**	**kPa**	**d.)**	**P**	**8.2**	**kPa**
b.)	**P**	**7.9**	**kPa**	**e.)**	**P**	**8.1**	**kPa**
c.)	**P**	**8.4**	**kPa**	**f.)**	**P**	**8.3**	**kPa**

Chapter 3



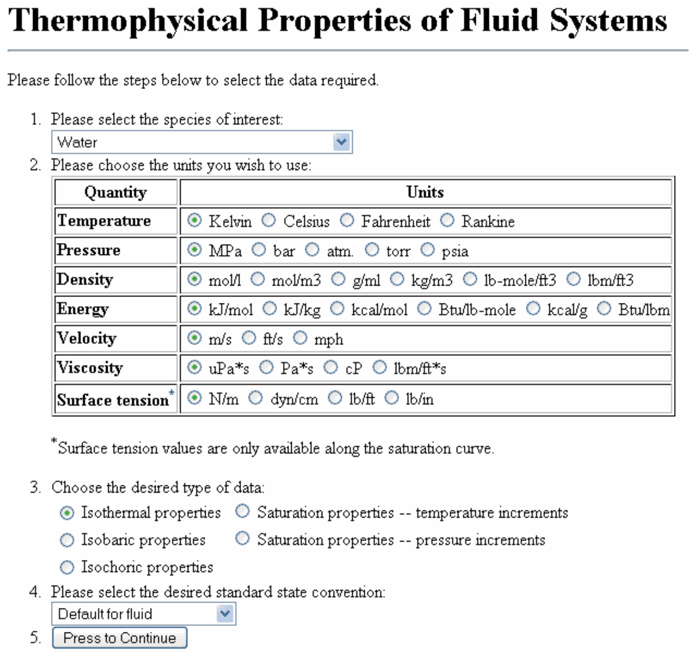
Heat Effects

In this chapter, you will learn about internal energy and enthalpy and how they depend on temperature and pressure, for real substances, ideal gases and incompressible substances. You will learn to obtain thermodynamic property data from the NIST Webbook. You will learn to work with tconstant volume and constant pressure heat capacities which relate changes in internal energy and enthalpy to changes in temperature. The chapter concludes with a discussion of hypothetical process paths (HPPs). HPPs allow you to determine the change in the value of a thermodynamic property using a convenient path instead of the actual path that a process follows. Phase changes are often key steps in HPPs because energy must be added or removed from a system to cause a phase change occur.

Internal Energy and Enthalpy

- **Internal Energy**
  - ◊ Isobaric – constant pressure
  - ◊ Non-nuclear energy stored within molecules
  - ◊ Sum of the vibrational, translational and rotational kinetic energies
  - ◊ U = strong fxn of T and a weak fxn of P
  - ◊ U ↑ sharply as T ↑ but U ↓ slightly as P ↑.
  - ◊ Ideal Gas, Incompressible Liquids, Solids
  - ◊ U = fxn(T) only
  - ◊ $U \neq$ fxn(P)
- **Enthalpy**
  - ◊ $\mathbf{H = U + P\,V}$ $\mathbf{dH = dH + d(PV)}$ $\mathbf{\Delta H = \Delta U + \Delta(PV)}$
  - ◊ H = strong fxn(T)
  - ◊ H = moderate fxn(P)
  - ◊ Ideal Gas: $H \neq$ fxn(P)S

- Atoms are in motion in all phases, even solids
- They vibrate, rotate and translate.
- This behavior is a strong function of temperature
- Internal energy is a weak function of P. U decreases slightly as P increases.
- At the same Temperature, $U_{gas} > U_{liq} > U_{solid}$
- Special Cases
  - ◊ Ideal Gases: U = fxn(T) only. $U \neq$ fxn(P)
  - ◊ Incompressible Liquids: U = fxn(T) only. $U \neq$ fxn(P)
  - ◊ Solids: U = fxn(T) only. $U \neq$ fxn(P)
- Enthalpy
  - ◊ $H = U + P\,V$
  - ◊ $\Delta H = \Delta U + \Delta(PV)$ is NOT always the same as $\Delta H = \Delta U + P\Delta V$
  - ◊ Specific and Molar forms of this equation are valid.
  - ◊ For ideal gases, $H \neq$ fxn(P)
  - ◊ For solids and incompressible liquids, H is a fxn(P)
    - $\Delta H = \Delta U + \Delta(PV) = \Delta(PV) = V\,\Delta(P)$ because V is constant !



Thermophysical Properties of Fluid Systems

Please follow the steps below to select the data required.

1. Please select the species of interest: Water
2. Please choose the units you wish to use:

Quantity	Units
Temperature	● Kelvin ○ Celsius ○ Fahrenheit ○ Rankine
Pressure	● MPa ○ bar ○ atm. ○ torr ○ psia
Density	● mol/l ○ mol/m3 ○ g/ml ○ kg/m3 ○ lb-mole/ft3 ○ lbm/ft3
Energy	● kJ/mol ○ kJ/kg ○ kcal/mol ○ Btu/lb-mole ○ kcal/g ○ Btu/lbm
Velocity	● m/s ○ ft/s ○ mph
Viscosity	● uPa*s ○ Pa*s ○ cP ○ lbm/ft*s
Surface tension*	● N/m ○ dyn/cm ○ lb/ft ○ lb/in

*Surface tension values are only available along the saturation curve.

3. Choose the desired type of data:
   - ● Isothermal properties ○ Saturation properties -- temperature increments
   - ○ Isobaric properties ○ Saturation properties -- pressure increments
   - ○ Isochoric properties
4. Please select the desired standard state convention: Default for fluid
5. Press to Continue

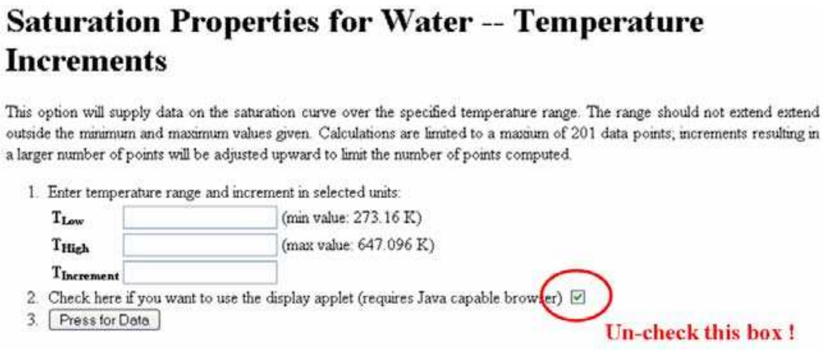
- Item 4 is VERY important !
- We will always use the "Default for Fluid" setting, unless I ask you to do otherwise.
- But, what is a "Standard State Convention" ?
- I call it a Reference State.

Reference State

- **We cannot determine an absolute U or H in the way we can determine an absolute T.**
- **We must choose a reference state and assign $\tilde{U}$ = 0 or $\tilde{H}$ = 0 at that state.**
- **Calculate all other values of $\tilde{U}$ and $\tilde{H}$ relative to the reference state.**
- **You cannot use thermodynamic data from different sources that are based on different reference states without correcting for the difference in reference state !!**

- A reference state requires the specification of two intensive variables (usually T and P) and the PHASE.
- The default reference state for water in the NIST Webbook is U = 0 for saturated liquid water at the triple point, 0.01$_o$C or 273.16 K.
- Try the NBP convention in the NIST Webbook and see what the reference state is.
- Very important point: we cannot assign BOTH U = 0 and H = 0 at the reference state !
  - ◊ Because H = U + PV
  - ◊ If BOTH U = 0 and H = 0 at the reference state, then either P or would need to be zero.
  - ◊ V is never zero.
  - ◊ So, P = 0
  - ◊ So, technically, if you choose a reference state at P = 0, then both U and H could be zero.
  - ◊ P = 0 is not a very practical reference state !

Generating a Saturated Temperature table



Saturation Properties for Water -- Temperature Increments

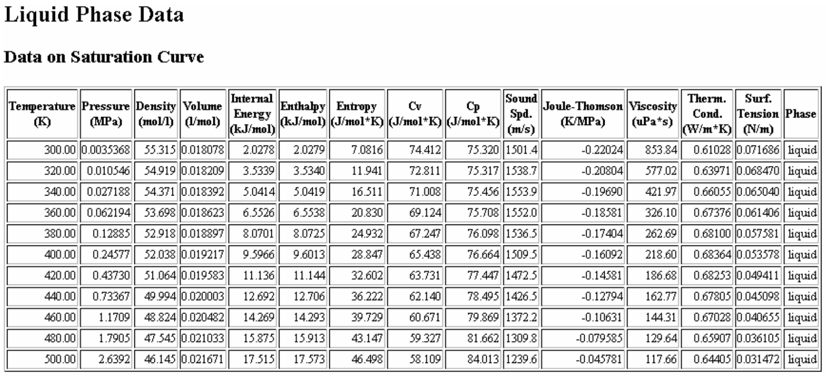
This option will supply data on the saturation curve over the specified temperature range. The range should not extend extend outside the minimum and maximum values given. Calculations are limited to a maxium of 201 data points; increments resulting in a larger number of points will be adjusted upward to limit the number of points computed.

1. Enter temperature range and increment in selected units:
   T_{Low} (min value: 273.16 K)
   T_{High} (max value: 647.096 K)
   $T_{Increment}$
2. Check here if you want to use the display applet (requires Java capable browser) ☑
3.

Un-check this box !

- So, given any T and P you can get properties without interpolating !
- But you still need to interpolate if you are given values of other intensive properties.
- Example:
  - ◊ What is the temperature water at 50 kPa that has a specific volume of 4.00 m$_3$/kg ?
- So, the NIST Webbook is very helpful, but it doesn't completely eliminate the need to interpolate.

Saturated Liquid Properties



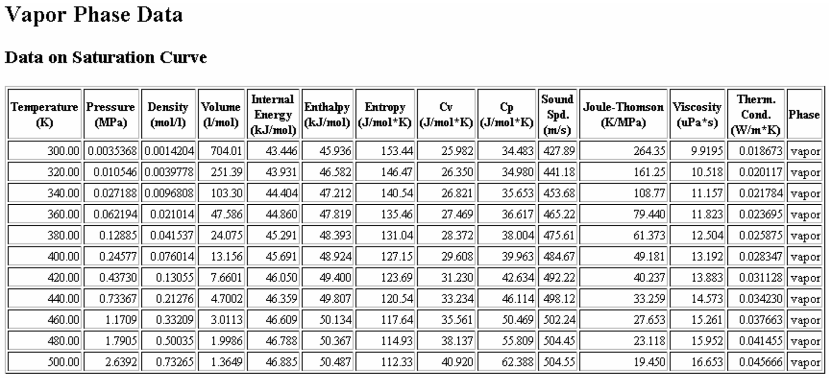
Liquid Phase Data

Data on Saturation Curve

Temperature (K)	Pressure (MPa)	Density (mol/l)	Volume (l/mol)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Cv (J/mol*K)	Cp (J/mol*K)	Sound Spd. (m/s)	Joule-Thomson (K/MPa)	Viscosity (uPa*s)	Therm. Cond. (W/m*K)	Surf. Tension (N/m)	Phase
300.00	0.0035368	55.315	0.018078	2.0278	2.0279	7.0816	74.412	75.320	1501.4	-0.22024	853.84	0.61028	0.071686	liquid
320.00	0.010546	54.919	0.018209	3.5339	3.5340	11.941	72.811	75.317	1538.7	-0.20804	577.02	0.63971	0.068470	liquid
340.00	0.027188	54.371	0.018392	5.0414	5.0419	16.511	71.008	75.456	1553.9	-0.19690	421.97	0.66055	0.065040	liquid
360.00	0.062194	53.698	0.018623	6.5526	6.5538	20.830	69.124	75.708	1552.0	-0.18581	326.10	0.67376	0.061406	liquid
380.00	0.12885	52.918	0.018897	8.0701	8.0725	24.932	67.247	76.098	1536.5	-0.17404	262.69	0.68100	0.057581	liquid
400.00	0.24577	52.038	0.019217	9.5966	9.6013	28.847	65.438	76.664	1509.5	-0.16092	218.60	0.68364	0.053578	liquid
420.00	0.43730	51.064	0.019583	11.136	11.144	32.602	63.731	77.447	1472.5	-0.14581	186.68	0.68253	0.049411	liquid
440.00	0.73367	49.994	0.020003	12.692	12.706	36.222	62.140	78.495	1426.5	-0.12794	162.77	0.67805	0.045098	liquid
460.00	1.1709	48.824	0.020482	14.269	14.293	39.729	60.671	79.869	1372.2	-0.10631	144.31	0.67028	0.040655	liquid
480.00	1.7905	47.545	0.021033	15.875	15.913	43.147	59.327	81.662	1309.8	-0.079585	129.64	0.65907	0.036105	liquid
500.00	2.6392	46.145	0.021671	17.515	17.573	46.498	58.109	84.013	1239.6	-0.045781	117.66	0.64405	0.031472	liquid

- Nice ! You get MANY properties and MANY digits !
- The EOS's used by the NIST Webbook are the best available.

Saturated Vapor Properties

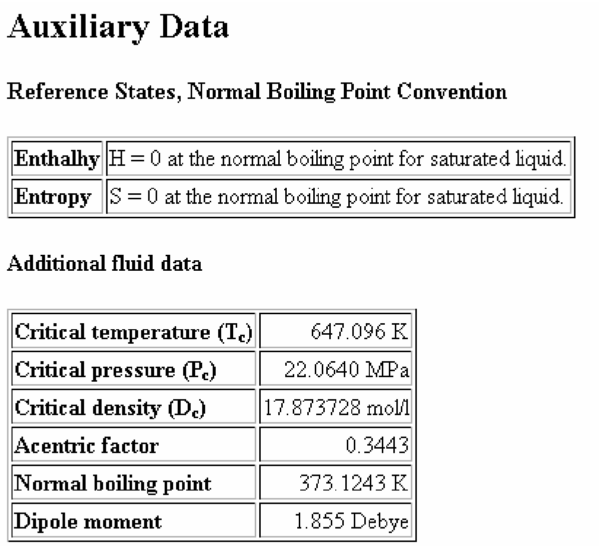


Vapor Phase Data

Data on Saturation Curve

Temperature (K)	Pressure (MPa)	Density (mol/l)	Volume (l/mol)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Entropy (J/mol*K)	Cv (J/mol*K)	Cp (J/mol*K)	Sound Spd. (m/s)	Joule-Thomson (K/MPa)	Viscosity (uPa*s)	Therm. Cond. (W/m*K)	Phase
300.00	0.0035368	0.0014204	704.01	43.446	45.936	153.44	25.982	34.483	427.89	264.35	9.9195	0.018673	vapor
320.00	0.010546	0.0039778	251.39	43.931	46.582	146.47	26.350	34.980	441.18	161.25	10.518	0.020117	vapor
340.00	0.027188	0.0096808	103.30	44.404	47.212	140.54	26.821	35.653	453.68	108.77	11.157	0.021784	vapor
360.00	0.062194	0.021014	47.586	44.860	47.819	135.46	27.469	36.617	465.22	79.440	11.823	0.023695	vapor
380.00	0.12885	0.041537	24.075	45.291	48.393	131.04	28.372	38.004	475.61	61.373	12.504	0.025875	vapor
400.00	0.24577	0.076014	13.156	45.691	48.924	127.15	29.608	39.963	484.67	49.181	13.192	0.028347	vapor
420.00	0.43730	0.13055	7.6601	46.050	49.400	123.69	31.230	42.634	492.22	40.237	13.883	0.031128	vapor
440.00	0.73367	0.21276	4.7002	46.359	49.807	120.54	33.234	46.114	498.12	33.259	14.573	0.034230	vapor
460.00	1.1709	0.33209	3.0113	46.609	50.134	117.64	35.561	50.469	502.24	27.653	15.261	0.037663	vapor
480.00	1.7905	0.50035	1.9986	46.788	50.367	114.93	38.137	55.809	504.45	23.118	15.952	0.041455	vapor
500.00	2.6392	0.73265	1.3649	46.885	50.487	112.33	40.920	62.388	504.55	19.450	16.653	0.045666	vapor

Extra Info from the NIST Webbook



Auxiliary Data

Reference States, Normal Boiling Point Convention

Enthalhy	H = 0 at the normal boiling point for saturated liquid.
Entropy	S = 0 at the normal boiling point for saturated liquid.

Additional fluid data

Critical temperature (T_c)	647.096 K
Critical pressure (P_c)	22.0640 MPa
Critical density (D_c)	17.873728 mol/l
Acentric factor	0.3443
Normal boiling point	373.1243 K
Dipole moment	1.855 Debye

- Now, let's go to a computer lab and take the NIST Webbook out for a spin.
- A link to the NIST Webbook appears on the course homepage in the box near the center of the page.

Example Problems

- **Determine the T_{sat}, $\hat{U}$ and $\hat{H}$ of saturated liquid ammonia at 300 kPa.**
  (Default ref. state)
- **Determine $\hat{U}$ $\hat{H}$ and $\hat{V}$ of butane at 14.696 psia and 77^{o}F in units of Btu, lb$_m$ and ft^3.**
  (Default ref. state)
- **Determine $\hat{U}$ $\hat{H}$ and $\hat{V}$ of a saturated mixture of R-123 at –40^{o}C and x = 0.30.**
  (Default ref. state, kJ, mole, m^3)

- Problem 1
  - T_{sat} = -9.2243 ^{o}C
  - U = 300.25 kJ/kg
  - H = 300.71 kJ/kg
- Problem 2
  - V = 6.5394 ft^3/lb$_m$
  - U = 251.92Btu/lb$_m$
  - H = 269.71 Btu/lb$_m$
- Problem 3
  - P_{sat} = 3.5752 KPa
  - $U_{sat\ liq}$ = 24.660 kJ/mol , $U_{sat\ vap}$ = 52.800 kJ/mol , $U_{x=0.03}$ = 33.102 kJ/mol
  - $H_{sat\ liq}$ = 24.660 kJ/mol, $H_{sat\ vap}$ = 54.731 kJ/mol, $H_{x=0.03}$ = 33.681 kJ/mol
  - $V_{sat\ liq}$ = 9.4405 x 10^{-5} m^3/mol , $V_{sat\ vap}$ = 0.54014 m^3/mol , $V_{x=0.03}$ = 0.16211 m^3/mol

Heat Capacity

- **Definition**
  - ◊ Amount of energy (J, Btu) that must be added to 1 mole or lbmole of a substance to increase its temperature by 1 degree (^oC , K, ^oF or ^oR).
  - ◊ Units: J/mole-K Btu/lbmole-^oF
- **Specific Heat**
  - ◊ Amount of energy (J, Btu) that must be added to 1 gram , kilogram or lb_m of a substance to increase its temperature by 1 degree (^oC , K, ^oF or ^oR).
  - ◊ Units: kJ/kg-K Btu/lb_m-^oF
- **Constant Pressure Heat Capacity / Specific Heat:** $\tilde{C}_P$ $\hat{C}_P$
  - ◊ Definition: $\tilde{C}_P = \left(\frac{\partial \tilde{H}}{\partial T}\right)_P$ $\hat{C}_P = \left(\frac{\partial \hat{H}}{\partial T}\right)_P$
- **Constant Volume Heat Capacity / Specific Heat:** $\tilde{C}_V$ $\hat{C}_V$
  - ◊ Definition: $\tilde{C}_V = \left(\frac{\partial \tilde{U}}{\partial T}\right)_P$ $\hat{C}_V = \left(\frac{\partial \hat{U}}{\partial T}\right)_P$
- **Heat Capacity Ratio :** $\gamma = \frac{\tilde{C}_P}{\tilde{C}_V} = \frac{\hat{C}_P}{\hat{C}_V}$

- Some people use the terms heat capacity and specific heat interchangeably

Why is Heat Capacity Useful ?

- **Isobaric Processes :** $\tilde{H}_2 - \tilde{H}_1 = \int_{T_1}^{T_2} \tilde{C}_P \, dT$

- **Isochoric Processes :** $\tilde{U}_2 - \tilde{U}_1 = \int_{T_1}^{T_2} \tilde{C}_V \, dT$

Ideal Gases

- $\tilde{H}$ is a function of T <u>only</u>, not P
- $\tilde{U}$ is a function of T <u>only</u>, not P
- Therefore: $\Delta\tilde{H} = \int_{T_1}^{T_2} \tilde{C}_P^o \, dT$ $\Delta\tilde{U} = \int_{T_1}^{T_2} \tilde{C}_V^o \, dT$

  are valid for ALL processes for ideal gases
- IG Heat Capacities: $\tilde{C}_P^o$ $\tilde{C}_V^o$
- NIST: Shomate Equation: $\tilde{C}_P^o = A + Bt + Ct^2 + Dt^3 + E/t^2$

  where: $t = T(K)/1000$

  $\tilde{C}_P^o \, [=] \, J/mol \cdot K$

-

Solids & Incompressible Liquids

- $\tilde{V}$ ≈ constant : $d\tilde{H} = d\tilde{U} + d(P\,\tilde{V}) = d\tilde{U} + \tilde{V}\,dP$
- $\tilde{V}$ is very small for most solids and many liquids (far from the critical point).
- Therefore: $d\tilde{U} \gg \tilde{V}\,dP$ $d\tilde{H} \approx d\tilde{U}$

  $$\frac{d\tilde{H}}{dT} \approx \frac{d\tilde{U}}{dT}$$

  $$\boxed{\tilde{C}_P \approx \tilde{C}_V}$$

- This approximation is good for solids.
- It is pretty rough for liquids except for P > P_C.
- Good for water except near the critical point.

Gibbs Phase Rule

- **^{o}Freedom = C – P + 2**
  - ◊ ^{o}Free = number of intensive variables you can independently specify.
  - ◊ C = number of chemical species in the system
  - ◊ P = number of phases within the system.
- **Consider a pure substance in a single phase**
  - ◊ C = 1, P = 1 … ^{o}Free = 2
  - ◊ Therefore, we must specify both T & P to completely determine the state of subcooled liquids and superheated vapors !

- Remember that P > 3 is possible as long as C > 1.
- For subcooled liquids and superheated vapors we cannot independently specify T, P and the molar volume !

Hypothetical Process Paths

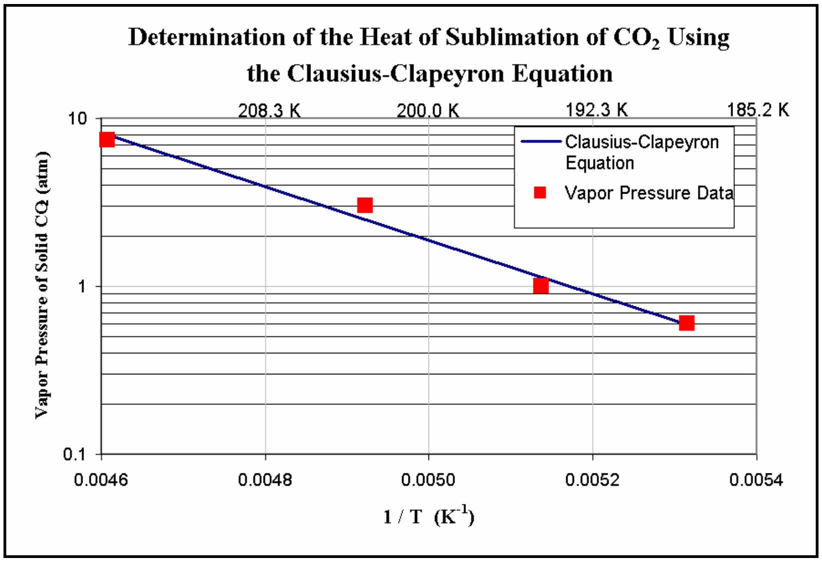
- **Actual Process Path**
  - ◊ The series of states that a system actually passes through during a process
- **Hypothetical Process Path**
  - ◊ A convenient path connecting the initial and final states of a process that makes it easy to evaluate changes in state variables that occur during the process
- **Goal: Construct an HPP in which just T or P changes or the phase changes in each step that makes up the path.**
- **Good news:**
  - ◊ We will use thermodynamic tables (only) for real gases in this course
  - ◊ We will use the IG heat capacity (Shomate Eqn) for ideal gases
  - ◊ If a liquid is not in the thermodynamic tables in the NIST Webbook, we will consider it to be incompressible

Phase Changes

- **Latent Heat of Vaporization:** $\Delta\tilde{H}_{vap}$
  - ◊ Best choice is to look up the value
  - ◊ Estimate the value
    - Clausius-Clapeyron Equation : $\mathbf{Ln\,P^* = \left(-\dfrac{\Delta\tilde{H}_{vap}}{R}\right)\dfrac{1}{T} + C}$
- **Vapor Pressure**
  - ◊ Best choice is to look up the value
  - ◊ Estimate the value
    - Antoine Equation : $\mathbf{Log_{10}\,P^* = A - \dfrac{B}{T+C}}$

- The Antoine Eqn often takes a slightly different form depending on where you look up the constants A, B and C
- NIST: P* = bar, T = Kelvin
- Note: 1 bar = 100 kPa

ΔH_{sub} from the Clausius-Clapeyron Eqn

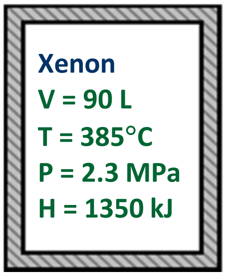


3A-1 Enthalpy and Internal Energy for Ideal Gases 2 pts

A rigid tank contains **90 L** of **xenon** gas at **385°C** and **2.3 MPa**. The **xenon** gas has a total **enthalpy** of **1350 kJ**. Assuming the **xenon** behaves as an ideal gas, determine its **specific internal energy**.

Read : Given the temperature, pressure and volume of xenon in an ideal gas state, we can calculate the mass of xenon in the system using the Ideal Gas EOS. This allows us to convert the enthalpy into specific enthalpy. We can use the definition of enthalpy or specific enthalpy to relate **U** to **H** and **PV** and then eliminate **PV** using the Ideal Gas EOS again. The units may get tricky.

Diagram:



Given: V =	90	L	P =	2.3	MPa
T =	385	°C	H =	1350	kJ

Find: U = ??? kJ/kg

Assumptions:

1 - Equilibrium conditions exist inside the tank.
2 - Xenon is an ideal gas at this **T** and **P**.

Equations / Data / Solve:

Since xenon behaves as an ideal gas, the definition of specific **enthalpy** can be modified as follows:

$$\hat{H} = \hat{U} + P\hat{V} \quad \text{Eqn 1} \qquad P\hat{V} = \frac{RT}{MW} \quad \text{Eqn 2} \qquad \hat{U} = \hat{H} - \frac{RT}{MW} \quad \text{Eqn 3}$$

But :

$$\hat{H} = \frac{H}{m} \quad \text{Eqn 4} \qquad \hat{U} = \frac{H}{m} - \frac{RT}{MW} \quad \text{Eqn 5}$$

For ideal gases :

$$PV = \frac{m}{MW} RT \quad \text{Eqn 6} \qquad m = MW \frac{PV}{RT} \quad \text{Eqn 7}$$

Molecular weight of xenon :	(NIST WebBook)	MW	131.29	g / mol
Universal Gas Constant values :		R	0.08205	atm L/gmol K
		R	8.314	J/mol K or Pa m^3/mol K

Note: To convert ^{o}**C** to **K**, add **273.15** to ^{o}**C**.

T	**658.15**	**K**
m	**4.9667**	**kg**
RT / MW	**41.7**	**kJ/kg**
H	**271.8**	**kJ/kg**
U	**230.1**	**kJ/kg**

Verify:

Answers : **U** **230** **kJ/kg**

(Rounded to 3 significant digits.)

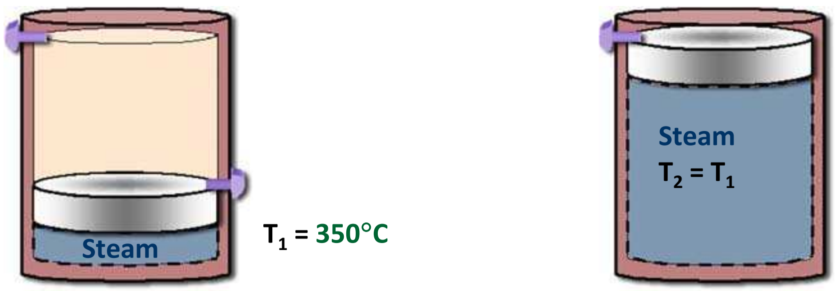
3B-1 ΔU and ΔH for Isothermal Expansion of Superheated Water Vapor 2 pts

A piston-and-cylinder device contains superheated **steam** at **350°C** and **10 atm**. The piston is held in place by a pin. When the pin is removed, the **steam** expands isothermally until the piston hits a second pin.

At this point, the **pressure** is **5 atm**. Determine the change in the molar enthalpy and molar internal energy of the steam in units of KJ/mol.

Read : Because the water vapor is superheated, it has **2** degrees of freedom. In this case both the **T** and **P** must be specified to completely determine the state. Because the state is completely determined, we can use the given **T** and **P** values to look up properties like **U** and **H** in the Superheated Tables in the Steam Tables.

Diagram:



Given:

T_1	350	°C		P_1	8	atm
T_2	350	°C		P_2	2	atm

Find:

ΔU	???	kJ/mol
ΔH	???	kJ/mol

Assumptions: None.

Equations / Data / Solve:

Use the **NIST WebBook** to determine the properties of superheated water vapor at the initial and final pressures. As always, use the **ASHRAE** convention. A portion of the thermodynamic table used in this problem is provided below.

Temp. (°C)	Pressure (atm)	Internal Energy (kJ/mol)	Enthalpy (kJ/mol)	Phase
350	1	**52.041**	**57.213**	vapor
350	2	**52.015**	**57.178**	vapor
350	3	**51.989**	**57.143**	vapor
350	4	**51.963**	**57.107**	vapor
350	5	**51.936**	**57.071**	vapor
350	6	**51.910**	**57.036**	vapor
350	7	**51.883**	**57.000**	vapor
350	8	**51.857**	**56.964**	vapor

The internal energy and enthalpy at the given pressures are:

P = 8 atm			**P = 1 atm**		
U_1	**51.857**	KJ/mol	U_2	**52.015**	KJ/mol
H_1	**56.964**	KJ/mol	H_2	**57.178**	KJ/mol

Remember that the change in any property is defined as the final state minus the initial state.

$\Delta U = U_2 - U_1 =$ 0.158 KJ/mol

$\Delta H = H_2 - H_1 =$ 0.214 KJ/mol

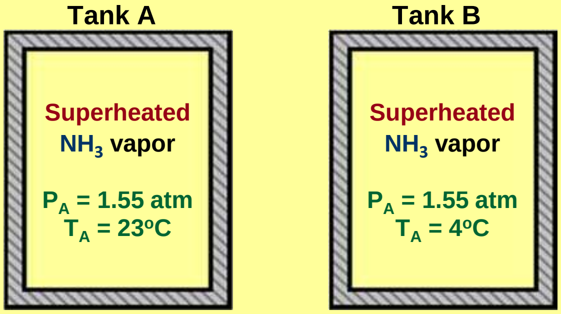
Verify: No assumptions to verify this time.

Answers :

ΔU	**0.158**	**kJ/mol**
ΔH	**0.214**	**kJ/mol**

3B-2 Internal Energy of Superheated Water Vapor 2 pts

Superheated **ammonia** vapor is stored in two rigid tanks, as shown below. Can you determine, by observation and reasoning alone, which has the higher **molar internal energy**, **A** or **B**?



Calculate the difference in **molar internal energy** between the two tanks using data from the **NIST WebBook**.

Read : Because the water vapor is superheated, it has **2** degrees of freedom. In this case both the **T** and **P** must be specified to completely determine the state. Because the state is completely determined, we can use the given **T** and **P** values to look up properties like **U** and **H** in the Superheated Tables in the Steam Tables.

Diagram: Given in the problem statement.

Given:

P_A = 1.55 atm
T_A = 23 °C
P_B = 1.55 atm
T_B = 4 °C

Find: $\Delta U = U_A - U_B$ = ??? kJ/mol

Assumptions: None.

Equations / Data / Solve:

The internal energy of a substance is the sum of the kinetic energies stored in the vibrational, rotational, and translational motion of the molecules. Tank A has more energy by virtue of its higher temperature. Therefore, it must have the higher intern

We must look up the isobaric properties of superheated water in the **NIST WebBook**. Use the **ASHRAE** convention. A portion of the thermodynamic table used in this problem is given below.

T (°C)	P (atm)	U (kJ/mol)	Phase
2	1.55	**22.849**	vapor
3	1.55	**22.878**	vapor
4	1.55	**22.908**	vapor
5	1.55	**22.937**	vapor
21	1.55	**23.401**	vapor
22	1.55	**23.430**	vapor
23	1.55	**23.459**	vapor
24	1.55	**23.487**	vapor

The internal energies at the two given temperatures are:

T = 23^{o}C				**T = 4^{o}C**		
U_A	**23.459**	**KJ/mol**		**U_B**	**22.908**	**KJ/mol**

As we predicted, the internal energy of the water vapor in **Tank A** is greater than in **Tank B**.
The **U** of **Tank A** is greater by:

$\Delta U = U_A - U_B =$ 0.551 KJ/mol

Verify: No assumptions to verify this time.

Answers : **ΔU 0.551 KJ/mol**

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3C-1 Enthalpy Change of Ammonia Using the IG Heat Capacity 5 pts

Ammonia gas is heated from **325 K** to **750 K**. Using the ideal gas heat capacity given by the Shomate Equation, calculate the **ΔU** and **ΔH** in **J/mole**.

Read : The Shomate Equation will yield the most accurate estimate of the **enthalpy change** for an ideal gas.

Diagram: A diagram is not needed for this problem.

Given:	T_1	325	K	T_2	750	K
Find:	$\Delta H_{1\text{-}2}$ =	???	J/mole	$\Delta U_{1\text{-}2}$ =	???	J/mole

Assumptions: 1 - Assume ammonia behaves as an ideal gas.

Equations / Data / Solve:

Let's begin by collecting the data we will need from the **NIST Webbook** :

Temp (K)	298. - 1400.
A	19.99563
B	49.77119
C	-15.376
D	1.921168
E	0.189174

Part a.) The enthalpy change associated with a temperature change for an ideal gas can be determined from :

$$\Delta\tilde{H}_{1-2} = \int_{T_1}^{T_2} \tilde{C}_P^o \, dT \qquad \text{Eqn 1}$$

The Shomate Equation for the ideal gas **heat capacity** is :

$$\tilde{C}_P^o = A + Bt + Ct^2 + Dt^3 + E/t^2 \qquad \text{Eqn 2}$$

where :

$$t = T(K)/1000 \qquad \text{Eqn 3}$$

and :

$$\tilde{C}_P^o [=] J/mol \cdot K$$

Combining Eqns 1, 2 and 3 and integrating yields :

$$\Delta\tilde{H} = A(T_2 - T_1) + \frac{B/2}{1000}(T_2^2 - T_1^2) + \frac{C/3}{1000^2}(T_2^3 - T_1^3) + \frac{D/4}{1000^3}(T_2^4 - T_1^4) - \frac{E}{1000^{-2}}(T_2^{-1} - T_1^{-1})$$

Eqn 4

Plug in values for the temperatures and the constants to get : ΔH 18358 J/mol

We can determine Δ**U** using the definition of enthalpy :

$$\Delta\tilde{H} = \Delta\tilde{U} + \Delta\left(P\tilde{V}\right)$$ **Eqn 7**

For ideal gases, **Eqn 7** becomes :

$$\Delta\tilde{H}^{IG} = \Delta\tilde{U}^{IG} + \Delta\left(R\,T\right) = \Delta\tilde{U}^{IG} + R\,\Delta T$$ **Eqn 8**

We can then solve **Eqn 8** for Δ**U** :

$$\Delta\tilde{U}^{IG} = \Delta\tilde{H}^{IG} - R\left(T_2 - T_1\right)$$ **Eqn 9**

R	8.314	**J/mol-K**	**ΔU**	**14824**	**J/mol**

Verify: Without knowing the pressure, it is not possible to verify this assumption.

Answers :

ΔH	**18400**	**J/mol**	**ΔU**	**14800**	**J/mol**

3C-2 Application of the Gibbs Phase Rule to the Triple Point 2 pts

R-134a at its triple point is held in a rigid vessel. How many intensive variables can be independently specified?

Read : At the triple point, all three phases exist in equilibrium.
This is the unique aspect of the triple point and this problem.

Diagram: A diagram is not necessary for this problem.

Given:

Number of chemical species present:	**C**	**1**
Number of phases present at equilibrium:	**P**	**3**

Find: **^{o}Free = ???**

Assumptions: None.

Equations / Data / Solve:

Gibbs Phase Rule : **^{o}Free = C - P + 2** **^{o}Free = 0**

NO intensive variables can be independently specified at the triple point !

This means that there is just **one** triple point and all of the properties of all of the phases are fixed !

The triple point is unique.

Verify: No assumptions were made in the solution of this problem.

Answers : **^{o}Free 0**

3C-3 Liquid Heat Capacities and Specific Heats 2 pts

The Thermophysical Properties of Fluid Systems page in the **NIST WebBook** lists the heat capacity of liquid **heptane** at various **temperatures**.

Use this resource to determine the **constant volume specific heat** of liquid **heptane** at **7°C** and **1 atm**. How many intensive properties of liquid **heptane** can be independently specified?

Read : Looking up the heat capacity in the **NIST WebBook** is straightforward.
Gibbs Phase Rule will tell us how many intensive variables can be independently specified.

Diagram: A diagram is not necessary for this problem.

Given: T **5** °C P **1** atm

Find: C_V **???** **J/kg K** °Free = **???**

Assumptions: - The mixture is at equilibrium

Equations / Data / Solve:

First, we determine the **constant volume specific heat** at of liquid heptane.

From the **NIST WebBook**, we can obtain C_p and C_v for heptane at **1 atm** and **5 °C**. Use the isobaric option for a range of temperatures including **5°C** or use the isothermal option including a pressure of **1 atm**. Selecting the correct units makes this task easier. Use temperature in degrees Celsius and pressure in atmospheres.

From the **NIST WebBook**, I obtained the following data :

Temp. (C)	Pressure (atm)	Density (kg/m^3)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)
2	1	698.87	0.0014309	-226.32	-226.18	-0.70175	1.6984	2.159
3	1	698.04	0.0014326	-224.16	-224.01	-0.69391	1.7016	2.1624
4	1	697.2	0.0014343	-222.00	-221.85	-0.68609	1.7048	2.1657
5	**1**	**696.37**	**0.001436**	**-219.83**	**-219.68**	**-0.67828**	**1.7080**	**2.1690**
6	1	695.54	0.0014377	-217.66	-217.51	-0.67049	1.7112	2.1724
7	1	694.7	0.0014395	-215.48	-215.34	-0.66272	1.7144	2.1758
8	1	693.87	0.0014412	-213.31	-213.16	-0.65496	1.7177	2.1792

C_P = 2169 J/kg K **C_V = 1708 J/kg K**

Degrees of Freedom: Gibbs Phase Rule is: $^{o}\mathbf{Free = 2 + C - P}$

^{o}**Free =** Degrees of freedom or the number of intensive properties that can be independently specified

C = Number of chemical species within the system
C = **1** species

P = Number of phases
P = **1** liquid phase

$^{o}\mathbf{Free = 2 + 1 - 1 =}$ **2**

Note:

We only need 2 intensive properties, such as: $\mathbf{T, P, \tilde{V}, \tilde{U}, \tilde{H}}$

to completely determine the state of the system.

Verify: The equilibrium assumption cannot be verified from the data available in this problem.

Answers :

C_V	**1708**	**J/kg K**

^{o}**Free**	**2**

3C-4 Enthalpy Change of N2 Using the IG Heat Capacity 5 pts

Nitrogen gas is heated from **450 K** to **1120 K**. Determine **ΔU** and **ΔH** in **kJ/kg** by…

a.) Integrating the Shomate Equation

b.) Treating the C_P value as a constant, determined using the Shomate Equation at the average temperature, **800 K**

c.) Treating the C_P value as a constant, determined using the Shomate Equation at room temperature, **25^{oC}**

Read : The Shomate Equation will yield the most accurate estimate of the enthalpy change. Assuming a constant value of C_p determined at the average temperature should yield a reasonable estimate of ΔH as well. Using the C_p value at room temperature should not be very accurate. We can compare this result to the value we get in **part (a)**.

Diagram: A diagram is not necessary for this problem.

Given: T_1 **600** **K** T_2 **1000** **K**

Find: $\Delta H_{1\text{-}2}$ = **???** **kJ/kg**

a.) Nitrogen behaves as an ideal gas

b.) Nitrogen behaves as an ideal gas with a linear relationship between C_p and **T**. This is equivalent to using a constant value of C_p that is equal to the average of $C_p(T_1)$ and $C_p(T_2)$.

c.) Nitrogen behaves as an ideal gas with a constant **heat capacity**.

Assumptions: The assumptions are part of the problem statement.

Equations / Data / Solve:

Let's begin by collecting the data we will need from the **NIST Webbook** :

MW **28.01** **g/mole**

Temp (K)	298. - 6000.
A	26.092
B	8.218801
C	-1.976141
D	0.159274
E	0.044434

Part a.) The **enthalpy change** associated with a **temperature change** for an ideal gas can be determined from :

$$\Delta\tilde{H}_{1-2} = \int_{T_1}^{T_2} \tilde{C}_P^o \, dT \qquad \text{Eqn 1}$$

The Shomate Equation for the ideal gas **heat capacity** is :

$$\tilde{C}_P^o = A + Bt + Ct^2 + Dt^3 + E/t^2 \qquad \text{Eqn 2}$$

where :

$$t = T(K)/1000 \qquad \text{Eqn 3}$$

and :

$$\tilde{C}_P^o \, [=] \, J/mol \cdot K \qquad \text{Eqn 4}$$

Combining **Eqns 1**, **2** and **3** and integrating yields :

$$\Delta\tilde{H} = A(T_2 - T_1) + \frac{B/2}{1000}(T_2^2 - T_1^2) + \frac{C/3}{1000^2}(T_2^3 - T_1^3) + \frac{D/4}{1000^3}(T_2^4 - T_1^4) - \frac{E}{1000^{-2}}(T_2^{-1} - T_1^{-1})$$

Eqn 5

Plug in values for the temperatures and the constants to get : **ΔH 12615 J/mol**

$$\Delta\hat{H}(kJ/kg) = \Delta H(J/mole) \frac{\cancel{1000}\, g/kg}{\left[\cancel{1000}\, J/kJ\right]\left[MW(g/mole)\right]} \qquad \text{Eqn 6}$$

ΔH 450.4 kJ/kg

We can determine ΔU using the definition of enthalpy : $\Delta\tilde{H} = \Delta\tilde{U} + \Delta\left(P\tilde{V}\right)$ **Eqn 7**

For ideal gases, Eqn 7 becomes : $\Delta\tilde{H}^{IG} = \Delta\tilde{U}^{IG} + \Delta\left(RT\right) = \Delta\tilde{U}^{IG} + R\,\Delta T$ **Eqn 8**

We can then solve Eqn 8 for ΔU : $\Delta\tilde{U}^{IG} = \Delta\tilde{H}^{IG} - R\left(T_2 - T_1\right)$ **Eqn 9**

R **8.314** **J/mol-K** **ΔU 9289 J/mol**

$$\Delta\hat{U}(kJ/kg) = \Delta\tilde{H}(J/mole) \frac{\cancel{1000}\, g/kg}{\left[\cancel{1000}\, J/kJ\right]\left[MW(g/mole)\right]} \qquad \text{Eqn 10}$$

ΔU 331.6 kJ/kg

Part b.) First we need to use the Shomate Equation, **Eqns 2** & **3**, to evaluate $\mathbf{C_p(T_1)}$ and $\mathbf{C_p(T_2)}$:

t_1	**0.6**	$C_p(T_1)$	**30.470**	**J/mol-K**
t_2	**1**	$C_p(T_2)$	**32.538**	**J/mol-K**

Therefore, the average value of $\mathbf{C_p}$ is : $C_{p,avg}$ **31.504** **J/mol-K**

When the **heat capacity** is a constant, **Eqn 1** simplifies to :

$$\Delta \tilde{H}_{1-2} = \tilde{C}^o_{P,avg}\left(T_2 - T_1\right) \qquad \textbf{Eqn 11}$$

ΔH	12602	J/mol
ΔH	**449.9**	**kJ/kg**
ΔU	**331.2**	**kJ/kg**

This amounts to about **0.1%** error relative to the result in **part (a)**.

Part c.) We can use **Eqns 2** & **3** to evaluate the heat capacity at **25°C** or **298.15 K** :

t_1	**0.29815**	$C_p(298.15K)$	**28.871**	**J/mol-K**

ΔH	**11548**	**J/mol**
ΔH	**412.3**	**kJ/kg**
ΔU	**293.6**	**kJ/kg**

This amounts to almost **9%** error relative to the result in **part (a)**.
That is not acceptable.

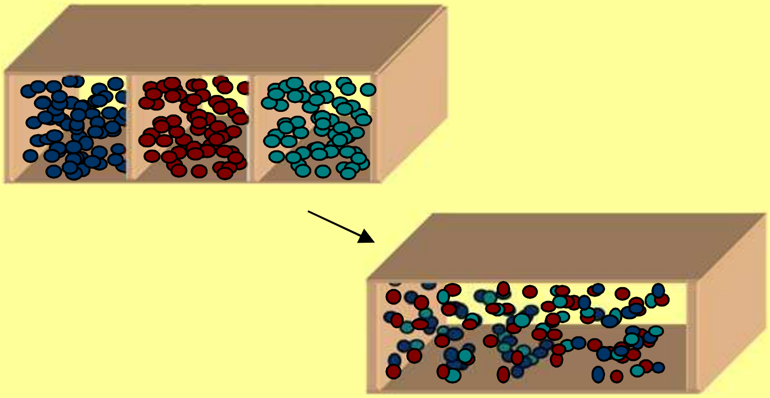
Verify: No assumptions were made other than the ones in the problem statement.

Answers :

a.)	**ΔU**	**332**	**kJ/kg**	**ΔH**	**450**	**kJ/kg**
b.)	**ΔU**	**331**	**kJ/kg**	**ΔH**	**450**	**kJ/kg**
c.)	**ΔU**	**294**	**kJ/kg**	**ΔH**	**412**	**kJ/kg**

3D-1 Calculating and Using the Heat Capacities of Ideal Gas Mixtures 4 pts

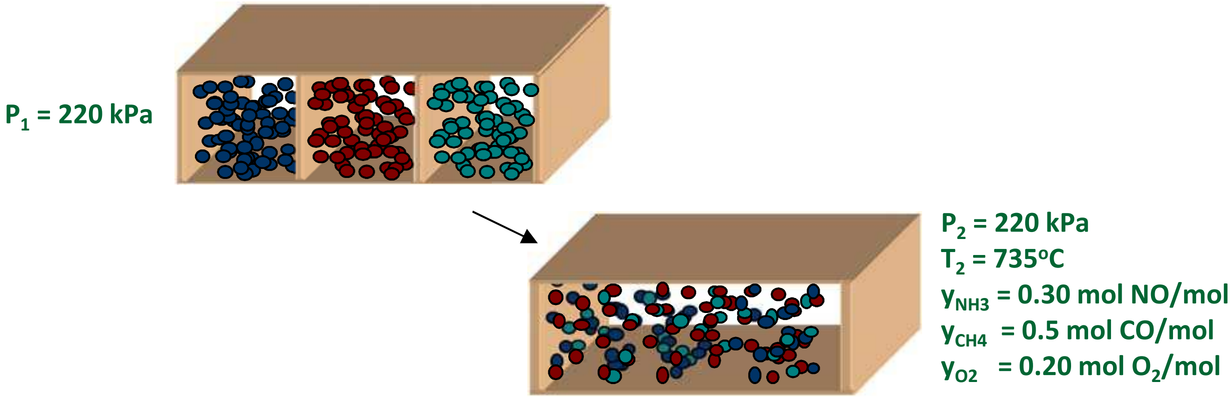
Three ideal gases, **Nitric Oxide** (**NO**), **Carbon Monoxide** (**CO**), and **Oxygen** (**O_2**), at **220 kPa** and **350°C** are held in a tank with three chambers, as shown below.



The dividers between the chambers are removed and the three gases are allowed to mix.The mixture contains **30 mole% NO**, **50 mole% CO**, and **20 mole% O_2**. The mixture is then heated to **735°C**.
Calculate the **ΔU**, in **J/mole**, of the mixture for the heating process. Assume the mixture is an ideal gas.

Read : The key to this problem is that **enthalpy** does not depend on **pressure** for an ideal gas. So, the initial and final pressures are not relevant. We want to determine the change in the **internal energy**, but only the **constant pressure heat capacities** are tabulated. We can either use **$C_v = C_p - R$** and then integrate **C_v** with respect to **T** to get Δ**U** or we can integrate **C_p** with respect to **T** to get Δ**H** and then use the definiition of enthalpy to get Δ**U**. The final aspect of the problem is that the system contains a mixture. We can either use the mole fractions to determine the constants of the heat capacity polynomial for the mixture and then integrate **C_p** with respect to **T** one time, or we can integrate **C_p** for each chemical component with respect to **T** and sum the resulting Δ**H** values to get Δ**H** for the mixture. Either way, once we have Δ**H**, we use the definition of **enthalpy** to determine Δ**U**.

Diagram: The figure given in the problem statement is adequate. Just include the initial and final temperatures.



Given:

P_1 = 220 kPa
T_1 = 350 °C = 623.15 K
T_2 = 735 °C = 1008.15 K

y_{NH3} = 0.30 mol NO/mol
y_{CH4} = 0.50 mol CO/mol
y_{O2} = 0.20 mol O_2/mol

Find: ΔU = ??? J/mole

Assumptions:

1 - The initial state and the final state are equilibrium states.
2 - There is no change in **internal energy** or **enthalpy** due to mixing of the gases.
3 - The pure components and the mixture behave as ideal gases.

Equations / Data / Solve:

The **internal energy** of an ideal gas does not depend on **pressure**, only on **temperature**.
Therefore, the question becomes, what is the change in **internal energy** from T_1 = 400 °C, to T_2 = 600 °C.

$$\Delta\tilde{H}_{1-2} = \int_{T_1}^{T_2} \tilde{C}_P^o \, dT \qquad \text{Eqn 1}$$

The Shomate Equation for the ideal gas **heat capacity** is :

$$\tilde{C}_P^o = A + Bt + Ct^2 + Dt^3 + E/t^2 \qquad \text{Eqn 2}$$

where :

$$t = T(K)/1000 \qquad \text{Eqn 3}$$

and :

$$\tilde{C}_P^o \, [=] \, J/mol \cdot K \qquad \text{Eqn 4}$$

Combining **Eqns 1**, **2** and **3** and integrating yields :

$$\Delta\tilde{H} = A(T_2 - T_1) + \frac{B/2}{1000}(T_2^2 - T_1^2) + \frac{C/3}{1000^2}(T_2^3 - T_1^3) + \frac{D/4}{1000^3}(T_2^4 - T_1^4) - \frac{E}{1000^{-2}}(T_2^{-1} - T_1^{-1}) \qquad \text{Eqn 5}$$

$$\Delta\tilde{U}^{IG} = \Delta\tilde{H}^{IG} - R\left(T_2 - T_1\right) \qquad \text{Eqn 6}$$

T in Kelvin !

Heat Capacity Constants from the **NIST WebBook**:

	Nitric Oxide	Carbon Monoxide	Oxygen
	298. - 1200.	298. - 1300.	298. - 6000.
A	23.83491	25.56759	29.659
B	12.58878	6.09613	6.137261
C	-1.139011	4.054656	-1.186521
D	-1.497459	-2.671301	0.09578
E	0.214194	0.131021	-0.219663

R = 8.314 J/mol K

Method #1: Calculate the constants for the heat capacity polynomial for the gas mixture and then integrate to determine **ΔH** for the mixture.

	Mixture			
$A_{mix} = y_{NO} A_{NO} + y_{CO} A_{CO} + y_{O2} A_{O2} =$	**25.86607**			
$B_{mix} = y_{NO} B_{NO} + y_{CO} B_{CO} + y_{O2} B_{O2} =$	**8.05215**			
$C_{mix} = y_{NO} C_{NO} + y_{CO} C_{CO} + y_{O2} C_{O2} =$	**1.44832**			
$D_{mix} = y_{NO} D_{NO} + y_{CO} D_{CO} + y_{O2} D_{O2} =$	**-1.765732**	**ΔH_{mix} =**	**12528**	**J/mol**
$E_{mix} = y_{NO} E_{NO} + y_{CO} E_{CO} + y_{O2} E_{O2} =$	**0.085836**	**ΔU_{mix} =**	**9327**	**J/mol**

Method #2: Calculate **ΔH** and then **ΔU** for EACH gas and then compute the molar average **ΔU** and **ΔH** using the following equations:

$$\Delta \tilde{U}_{mix} = \sum_i y_i \, \Delta \tilde{U}_i \qquad \textbf{Eqn 7}$$

$$\Delta \tilde{H}_{mix} = \sum_i y_i \, \Delta \tilde{H}_i \qquad \textbf{Eqn 8}$$

	NO	CO	O_2	Mixture	
ΔH =	12633	12307	12923	12528	J/mol
ΔU =	9433	9106	9722	**9327**	**J/mol**

Verify: Assumptions **1** & **2** cannot be verified from the data given in the problem.

The ideal gas assumption needs to be verified.

We need to determine the specific volume and check if : $\tilde{V} > 20 \ L/mol$

$$\tilde{V} = \frac{R\,T}{P} \qquad \textbf{Eqn 9}$$

V_1 **23.55** **L/mol**

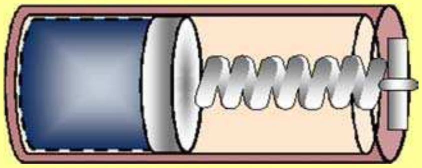
V_2 **38.10** **L/mol**

The ideal gas assumption is valid because **V > 20 L/mole** For both the initial and final states.

Answers : **ΔU_{mix} = 9327 J/mol**

3D-2 Heating Liquid Methanol in a Piston-and-Cylinder Device 3 pts

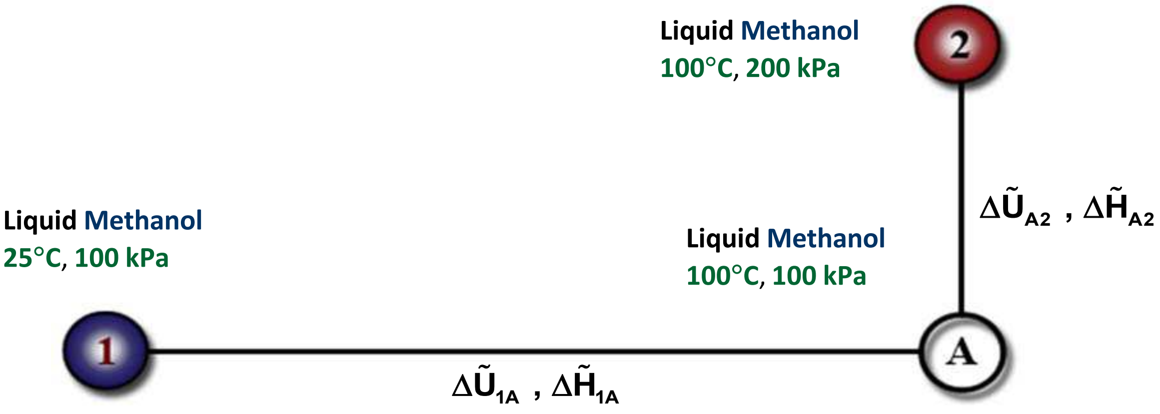
Liquid **methanol** is heated from **25°C** to **100°C** in the piston-and-cylinder device shown below. The initial **pressure** is **100 kPa** and the spring causes the **pressure** to increase during the process to **200 kPa**.



Calculate **ΔU** and **ΔH** in **J/mol**. Assume C_P is a constant and has a value of **83.4 J/mol-K**. Assume the **molar volume** is also constant and has a value of **0.01848 mol/L**.

Read : Construct a good **HPP** for this process. Treat the liquid methanol as an incompressible fluid and verify this assumption at the end. This will simplify determining **ΔU** and **ΔH** for changes in **pressure**. Use the given **heat capacity** to determine **ΔU** and **ΔH** for changes in **temperature**.

Diagram:



Given:	T_1	25	°C	T_2	125	°C
	P_1	100	kPa	P_2	200	kPa
	C_P	83.4	J/mol-K	V	0.01848	mol/L
Find:	ΔU_{12}	???	J/mol	ΔH_{12}	???	J/mol

Assumptions: 1 - Liquid methanol is incompressible. The **molar volume** is constant throughout this process.

Equations / Data / Solve:

The reason we use a hypothetical process path is to break a complex process into a series of simpler steps.
In this problem, **step 1-A** is isobaric and **step A-2** is isothermal.
Because **U** and **H** are state variables, they are <u>additive</u>, as follows.

$$\Delta\tilde{H}_{12} = \Delta\tilde{H}_{1A} + \Delta\tilde{H}_{A2} \qquad \text{Eqn 1}$$

$$\Delta\tilde{U}_{12} = \Delta\tilde{U}_{1A} + \Delta\tilde{U}_{A2} \qquad \text{Eqn 2}$$

ΔH for **Step 1-A** can be determined as follows because the **heat capacity** is a constant.

$$\Delta\tilde{H}_{1A} = \int_{T_1}^{T_A} \tilde{C}_P \; dT = \tilde{C}_{P,avg}\left(T_A - T_1\right) \qquad \textbf{Eqn 3}$$

Plugging values into **Eqn 2** yields: **ΔH_{1A} 8340.0 J/mol**

Now we can use the definition of **enthalpy** to help us determine **ΔU**.

$$\Delta\tilde{H}_{1A} = \Delta\tilde{U}_{1A} + \Delta(P\,\tilde{V})_{1A} = \Delta\tilde{U}_{1A} + P\,\Delta\tilde{V}_{1A} + \tilde{V}\,\Delta P_{1A} \approx \Delta\tilde{U}_{1A} \qquad \textbf{Eqn 4}$$

Since **$P_A = P_1$, $\Delta P = 0$**.

We boldly assumed the **molar volume** of the liquid methanol was constant throught this process, **$\Delta V_{1A} = 0$**.

The result is easy to compute! **ΔU_{1A} 8340.0 J/mol**

For an incompressible liquid, as we have assumed liquid **methanol** to be, **U** is not a function of **P**.

Therefore : **ΔU_{A2} 0 J/mol**

To determine **ΔH**, we must return to the definition of **enthalpy**.

$$\Delta\tilde{H}_{A2} = \Delta\tilde{U}_{A2} + \Delta(P\,\tilde{V})_{A2} = P\,\Delta\tilde{V}_{A2} + \tilde{V}\,\Delta P_{A2} \approx \tilde{V}\,\Delta P_{A2} \qquad \textbf{Eqn 5}$$

Plugging values into **Eqn 5** yields : **ΔH_{A2} 1.848 J/mol**

You can see that **ΔH_{A2}** is very small compared to **ΔH_{1a}**. It is often neglected unless the change in **T** is very small or the change in **P** is very large indeed.

Now, we can plug values back into **Eqn 1** and **Eqn 2** to complete the solution of this problem.

ΔU_{12} 8340 J/mol

ΔH_{12} 8342 J/mol

Verify: We cannot verify the incompressibility of liquid methanol using only the information given in the problem statement.

However, the **NIST Webbook** yields the following data for the molar volume of liquid methanol.

T (C)	P (MPa)	V (L/mol)
25	0.1	0.017358
100	0.1	0.018479

T (C)	P (MPa)	V (L/mol)
25	0.2	0.018068
100	0.2	0.018797

The data show that the molar volume changes by about **6%** during **Step 1-A** and about **4%** during **Step A-2**.
This seems like a lot of error, but it does not translate into as much error in **ΔU** or **ΔH**.

The first place this assumption matters is in determining **ΔU_{1A}**.

$$\Delta\tilde{H}_{1A} = \Delta\tilde{U}_{1A} + \Delta(P\,\tilde{V})_{1A} = \Delta\tilde{U}_{1A} + P\,\Delta\tilde{V}_{1A} \qquad \textbf{Eqn 6}$$

$$\Delta\tilde{U}_{1A} = \Delta\tilde{H}_{1A} - P\,\Delta\tilde{V}_{1A} \qquad \textbf{Eqn 7}$$

$P\,\Delta V_{1A}$ 0.071 J/mol

This is less than **1%** of **ΔH_{1A}** ! This is not significant.

The next place the incompressibility assumption matters is in **ΔH_{A2}**.

$$\Delta\tilde{H}_{A2} = \Delta\tilde{U}_{A2} + \Delta(P\,\tilde{V})_{A2} = P\,\Delta\tilde{V}_{A2} + \tilde{V}\,\Delta P_{A2} \approx \tilde{V}\,\Delta P_{A2} \qquad \text{Eqn 5}$$

$P\ \Delta V_{A2}$	**0.0636**	**J/mol**

This is less than **4%** of **ΔH_{A2}** but it is less than 1% of **ΔH_{12}** ! This is not significant either.

So for determining changes in **U** and **H** the constant **molar volume** assumption for liquid **methanol** was reasonable for these conditions.

When solving a problem for chemical that **is** in the **NIST Webbook** database, you should use the best available information.

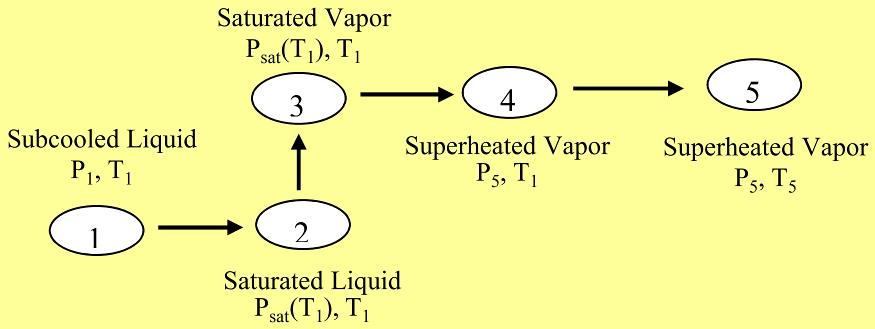
But in the absence of extensive data about the **molar volume** of liquids, it is very common and often accurate to assume they are incompressible over **pressure** ranges of **1 MPa** or even more.

Answers :

ΔU_{12}	**1.85**	**J/mol**
ΔH_{12}	**8340**	**J/mol**

3E-1 Hypothetical Process Paths and the Latent Heat of Vaporization 7 pts

Use the hypothetical process path (**HPP**) shown below to help you determine the **ΔH** in **J/mole** for **propane** (**C_3H_8**) as it changes from a subcooled liquid at **P_1 = 300 kPa** and **T_1 = 250 K** to a superheated vapor at **P_5 = 100 kPa** and **T_5 = 300 K**. Calculate the molar **ΔH** for each step in the **HPP**. Assume the **propane** vapor behaves as an ideal gas and a constant **heat capacity** of **69.0 J/mole-K**.



Do not use tables of thermodynamic properties, except to check your answers. Use the Antoine and Clausius-Clapeyron Equations to estimate the heat of vaporization of **propane** at **T_1**.

Note: The molar volume of saturated liquid **propane** at **250 K** is **7.8914 x 10^{-5} m^3/mole**.

Read : **Step 1-2** is straightforward because we will assume that the liquid propane is incompressible.
We can use the Antoine Equation with the Clausius-Clapeyron Equation to estimate **ΔH_{vap}** for **step 2-3**.
Step 3-4 is easy because we were instructed to assume the propane is an ideal gas and the **enthalpy** of an ideal gas is not a function of **pressure**.
Step 4-5 is straightforward because the problem instructs us to use a constant **C_p** value.

Diagram: The diagram in the problem statement is adequate.

Given:			
	P_1	**300**	**kPa**
	T_1	**250**	**K**
	T_5	**300**	**K**
	P_5	**100**	**kPa**
	V_{liq}	**7.8914E-05**	**m^3/mole**
	C^o_P	**69.0**	**J/mole-K**

Find:			
	ΔH_{1-2}	**???**	**J/mol**
	ΔH_{2-3}	**???**	**J/mol**
	ΔH_{3-4}	**???**	**J/mol**
	ΔH_{4-5}	**???**	**J/mol**
	ΔH_{1-5}	**???**	**J/mol**

Assumptions:

1 - Clausius-Clapeyron applies:
- The saturated vapor is an ideal gas
- The **molar volume** of the saturated vapor is **much, much** greater than the **molar volume** of the saturated liquid.
- The latent heat of vaporization is constant over the temperature range of interest.

2 - The superheated vapor also behaves as an ideal gas.

3 - Liquid propane is incompressible.

Equations / Data / Solve:

Step 1-2 involves a change in **pressure** on an incompressible liquid at constant **temperature**.

Since neither the **internal energy** nor the **molar volume** of an incompressible liquid are functions of **pressure** :

$$\Delta\tilde{H}_{12} = \Delta\tilde{U}_{12} + \Delta\left(P\,\tilde{V}\right)_{12} = \tilde{V}\,\Delta P_{12} \qquad \textbf{Eqn 1}$$

We can use the Antoine Equation to determine the vapor or saturation pressure of propane at T_1.

$$\log_{10}(P^*) = A - (B / (T + C)) \qquad \textbf{Eqn 2}$$

P is in **bar** **T** is in **Kelvin**

The Antoine constants from the **NIST WebBook** are:

A = **4.53678**
B = **1149.36**
C = **24.906**

$P_2 = P^*(T_1)$ **Eqn 3** P_2 **226.9** **kPa**

Now, we can plug numbers into **Eqn1**, but be careful with the units.

ΔH_{12} **-5.768** **J/mole**

Next, we can observe that ΔH_{23} = Latent Heat of Vaporization at **250 K**.

We can estimate the heat of vaporization using the Clausius -Clapeyron Equation.

$$\ln P^* = \left(-\frac{\Delta\tilde{H}_{vap}}{R}\right)\frac{1}{T} + C \qquad \textbf{Eqn 4}$$

If we plot **Ln P*** vs. **1/T(K)**, the slope is **-** $\Delta H_{vap}/R$.

We can calculate the vapor pressures at two different **temperatures** using the Antoine Equation. Use temperatures near the temperature of interest, **250 K**. Use the two points to estimate the slope over this small range of temperatures.

$$\text{Slope} = \frac{\ln P_b^* - \ln P_a^*}{1/T_b - 1/T_a} \qquad \textbf{Eqn 5}$$

From the Antoine Equation:

T_a	249.9	K	P_a	226.12	kPa
T_b	250.1	K	P_b	227.71	kPa

Slope = **-2188.7** K

Next we use this slope with **Eqn 4** to determine the latent heat of vaporization at **250 K** :

R = **8.314** **J/mol K** ΔH_{vap} **18197** **J/mole**

ΔH_{23} **18,197** **J/mole**

Next, we need to determine the **enthalpy change** from **state 3** to **4**, in which the **pressure** of the saturated vapor is reduced. This causes the vapor to become a superheated vapor.

Recall the assumption that the vapor behaves as an ideal gas. Because **enthalpy** is only a function of **T** for ideal gases, and since $T_3 = T_4$:

ΔH_{34} **0** **J/mole**

Next, let's consider the **enthalpy change** from **state 4** to **5**.
Because we assumed the vapor phase is an ideal gas with constant C_P, we can evaluate ΔH using:

$$\Delta\tilde{H}_{45} = \tilde{C}_p \, \Delta T_{15} \qquad \textbf{Eqn 6}$$

Plugging numbers into **Eqn 6** yields : **ΔH_{45} = 3,450 J/mole**

Finally, put them all together: **$\Delta H_{15} = \Delta H_{12} + \Delta H_{23} + \Delta H_{34} + \Delta H_{45}$ = 21,641 J/mole**

Notice that ΔH_{12} is very small compared to ΔH_{23} and ΔH_{45}. In fact ΔH_{12} is negligible.

This shows why it is often acceptable to approximate the **enthalpy** of a subcooled liquid using the **enthalpy** of the saturated liquid at the same **TEMPERATURE**. It is NOT accurate to approximate the **enthalpy** of a subcooled liquid using the **enthalpy** of the saturated liquid at the same **PRESSURE**.

Verify: 1 - We can test the validity of the ideal gas assumption for state 3 as follows.

$$\tilde{V}_3 = \frac{R\,T_3}{P_3} = \frac{R\,T_2}{P_2} = \frac{R\,T_1}{P^*(T_1)}$$

V_3 9.160 L/mol

Because **V_3 < 20 L/mole**, the Clausius-Clapeyron Equation is not very accurate. This issue makes the results from this analysis somewhat unreliable.

It is not as easy to test the 2nd assumption that underpins the Clausius-Clapeyron Equation.
We can use the **NIST Webbook** to determine the molar volume of saturated liquid and saturated vapor at **250 K**.

$V_{sat\ vap}$ 8.9258 L/mol
$V_{sat\ liq}$ 0.078977 L/mol

$V_{sat\ vap}$ / $V_{sat\ liq}$ = 113.02

Since **$V_{sat\ vap}$** is more than **100** times greater than **$V_{sat\ liq}$** this assumption underpinning the use of the Clausius-Clapeyron Equation is **valid**.

Because we considered a very narrow temperature range, just **0.2°C**, the last assumption underpinning the use of the Clausius-Clapeyron Equation is almost certainly valid.

2 - Is the superheated vapor be accurately treated as an ideal gas?

$$\tilde{V}_4 = \frac{R\,T_4}{P_4} = \frac{R\,T_1}{P_5}$$

V_3 20.785 L/mol

Because **V_4 > 20 L/mole**, the Clausius-Clapeyron Equation can be applied.

3 - Since **ΔH_{12}** is negligible, this assumption is not very important.

Nonetheless, we can use the **NIST Webbook** to determine the molar volume of liquid at **P_1 = 100 kPa** and at **P_2 = 226.9 kPa** at **250 K**. and see if the molar volume changes significantly.

V_1 0.0625 L/mol 10.887059 **V_2 0.0693 L/mol**

We find that **V_2** differs from **V_1** by about **11%.** So it is not very accurate to treat liquid propane as an incompressible liquid under these conditions.

This may be ok in this problem since **~ΔH_{12}** is so small that an **11%** error in its value will still not matter.

Answers :

ΔH_{1-2}	-5.77	J
ΔH_{2-3}	18,197	J
ΔH_{3-4}	0	J
ΔH_{4-5}	3,450	J
ΔH_{1-5}	21,600	J

(Rounded to 3 significant digits)

The assumption that the saturated vapor can be accurately treated as an ideal gas is not valid and, as a result, $\mathbf{\Delta H_{23}}$ and $\mathbf{\Delta H_{15}}$ are not reliable.

3E-2 Determination of the Vapor Pressure of Ammonia 4 pts

Use the Clausius-Clapeyron equation to estimate the vapor pressure, in **kPa**, of **ammonia** at **-25°C**. The normal boiling point of **ammonia** is **-33.34°C** and the latent heat of vaporization at this temperature is **1370 kJ/kg**.

Read : The keys here are to know that the normal boiling point is the boiling point at **1atm** and that the Clausius-Clapeyron Equation provides a relationship between the **rate** at which vapor pressure **changes** and the latent heat of vaporization. Knowing that **P*(-33.34°C) = 101.325 kPa** and the latent heat of vaporization at this temperature allows us to evaluate both the slope and the intercept in the Clausius-Clapeyron Equation and then use the result to estimate the vapor pressure at any other **temperature**. We should keep in mind that this estimate is only reasonably accurate at **temperatures** close to the one known value, **-33.34°C** in this case.

Diagram: A diagram is not needed in the solution of this problem.

Given:

T_1	-33.34	°C	T_2	-25	°C
	239.81	K		248.15	K
P_1*	101.325	kPa	ΔH_{vap}	1370	kJ/kg
MW	17.03	g/mole	R	8.314	J/mole-K

Find: P_2* ??? kPa

Assumptions: 1 - Clausius-Clapeyron applies:

- The saturated vapor is an ideal gas
- The **molar volume** of the saturated vapor is **much, much** greater than the **molar volume** of the saturated liquid.
- The latent heat of vaporization is constant over the temperature range of interest.

Equations / Data / Solve:

We can estimate the latent heat of vaporization using the Clausius -Clapeyron Equation.

$$\mathrm{LnP^*} = \left(-\frac{\Delta\tilde{H}_{vap}}{R}\right)\frac{1}{T} + C$$ **Eqn 1**

If we plot **Ln P*** vs. **1/T(K)**, the slope is **- ΔH_{vap}/R**. Don't forget to use **T** in **Kelvins** in **Eqn 1**.

So, the next thing we need to do is use the given value of the latent heat to estimate this slope.

$$\Delta\tilde{H}_{vap} = \Delta\hat{H}_{vap}\ \mathrm{MW}$$ **Eqn 2**

ΔH_{vap}	23331	J/mole
Slope	-2806.242	K

Next, we can use the one known value of the vapor pressure (at **-33.34°C**) to evaluate the constant (**C**) in the Clausius-Clapeyron Equation.

$$C = \ln P^*(239.81\,K) + \left(-\frac{\Delta\tilde{H}_{vap}}{R}\right)\frac{1}{T}$$ **Eqn 3** **C** **16.320**

Evaluating **C** in this manner has a catch. This value of **C** only applies as long as the same units of **pressure** are used in **Eqn 1**. Since we used **P_1*** in **kPa**, we must always use **P** in **kPa** whenever we use this value of **C**.

Now, we can use the values of the slope and intercept that we have determined to subsititute back into **Eqn 1** to estimate the vapor pressure of ammonia at a temperature other than **-33.34°C**, in this case **-25°C**.

P_2* **150.1** **kPa**

Verify: Only the ideal gas assumption can be verified using the data in the problem statement.

Ideal Gas EOS :

$$P\tilde{V} = RT$$ **Eqn 4**

Solve for **molar volume** :

$$\tilde{V} = \frac{RT}{P}$$ **Eqn 5**

Plug in values based upon the results we obtained above :

V_1	**1.97E-02**	**m^3/mol**	**V_2**	**1.37E-02**	**m^3/mol**
	19.7	**L/mol**		**13.7**	**L/mol**

Because the **molar volume** of the saturated vapor at both (**-33.34°C**, **101.325 kPa**) and (**-25°C**, **150.1 kPa**) is less than **20 L/mole**, it is not accurate to treat the saturated vapors as ideal gases. This is a more serious problem at **-25°C** and **150.1 kPa**.

The 2nd and 3rd assumptions required to use the Clausius-Clapeyron Equation cannot be verified with the information provided in the problem statement. However, based on data available in the Ammonia Tables, these two assumption are valid under the conditions in this problem.

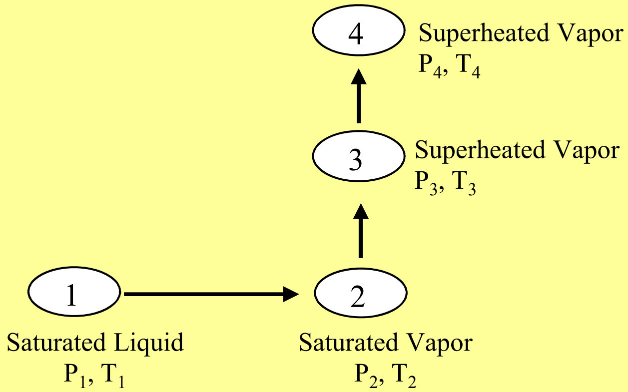
The Ammonia Tables also tell us that: **P_2*** **151.5** **kPa**

Our answer based on the Clausius-Clapeyron Equation is accurate to within about **1%**. This is surprisingly good in light of the fact that the ideal gas assumption for the saturated vapor is not valid!

Answers: **P_2*** **150.1** **kPa**

3E-3 Hypothetical Process Paths and the Latent Heat of Vaporization 4 pts

Use the hypothetical process path (**HPP**) shown here to help you determine **ΔH** in **Joules** for **32.5 g** of **heptane** (**C_7H_{16}**) as it changes from a saturated liquid at **300 K** to a **temperature** of **370 K** and a **pressure** of **58.7 kPa**.



Calculate the **ΔH** for each step in the **HPP**. Do not use tables of thermodynamic properties, except to check your answers. Instead, use the Antoine Equation to estimate the **heat of vaporization** of **heptane** at **300 K**.

Use the average **heat capacity** of **heptane** gas over the **temperature** range of interest. Assume **heptane** gas is an ideal gas at the relevant **temperatures** and **pressures**.

Read : **Step 1-2** is a bit tricky. We can use the Antoine Equation with the Clausius-Clapeyron Equation to estimate ΔH_{vap}.

Step 2-3 is straightforward because the problem instructs us to use an average C_p value. The only difficulty will be that C_p values may not be available at the temperatures of interest.

Step 3-4 is cake because we were instructed to assume the heptane gas is ideal. As a result, **enthalpy** is not a function of **pressure** and $\Delta H_{3\text{-}4} = 0$.

Diagram: The hypothetical process path diagram in the problem statement is adequate.

Given:

m	**32.5**	**g**
$T_1 = T_2$	**300**	**K**
x_1	**0**	**kg vap/kg (sat'd liq)**
$T_3 = T_4$	**370**	**K**
P_4	**58.7**	**kPa**

Find:

$\Delta H_{1\text{-}2}$	**???**	**J**
$\Delta H_{2\text{-}3}$	**???**	**J**
$\Delta H_{3\text{-}4}$	**???**	**J**
$\Delta H_{1\text{-}4}$	**???**	**J**

Assumptions:

1 - Clausius-Clapeyron applies:
  - 1a- The saturated vapor is an ideal gas
  - 1b- The **molar volume** of the saturated vapor is **much, much** greater than the **molar volume** of the saturated liquid.
  - 1c- The latent heat of vaporization is constant over the temperature range of interest.

2 - The superheated vapor also behaves as an ideal gas.

3 - The **heat capacity** of the superheated vapor is nearly linear with respect to temperature over the temperature range of interest so that using the average value is a reasonable approximation.

Equations / Data / Solve:

First we can observe that:

ΔH_{1-2} = Latent heat of vaporization at **300 K**

We can estimate the latent heat of vaporization using the Clausius - Clapeyron Equation.

$$\mathrm{LnP^*} = \left(-\frac{\Delta\tilde{H}_{vap}}{R}\right)\frac{1}{T} + C \qquad \textbf{Eqn 1}$$

If we plot **Ln P*** vs. **1/T(K)**, the slope is **- $\Delta H_{vap}/R$**.

We can calculate the vapor pressures at <u>two different</u> **temperatures** using the Antoine Equation. Use temperatures near the temperature of interest, **300 K**. Use the two points to estimate the slope over this small range of temperatures.

$$\mathrm{Slope} = \frac{\mathrm{LnP_b^*} - \mathrm{LnP_a^*}}{1/T_b - 1/T_a} \qquad \textbf{Eqn 2}$$

Antoine Equation:

$$\mathrm{Log_{10}(P^*)} = A - (B / (T + C)) \qquad \textbf{Eqn 3}$$

P is in **bar** **T** is in **Kelvin**

The Antoine constants from the **NIST WebBook** are:

A = **4.02832**
B = **1268.636**
C = **-56.199**

From the Antoine Equation:

$T_1 = T_2$	300	K	$P_1 = P_2 = P_3$	6.68	kPa
T_a	299.5	K	$P_a =$	6.52	kPa
T_b	300.5	K	$P_b =$	6.85	kPa
			Slope	-4423.1	K

Next we use this slope with **Eqn 1** to determine the latent heat of vaporization at **300 K** :

R = **8.314** **J/mol K**

ΔH_{vap} **=** **36773** **J/mol**

$$n = \frac{m}{MW} \qquad \textbf{Eqn 4}$$

MW **100.20** **g/mol**
n **0.3244** **mol**

$$\boxed{\Delta H_{(1\text{-}2)} = 11{,}928 \ \mathrm{J}}$$

Next, let's consider the enthalpy change from **states 2** to **3**, saturated vapor to superheated vapor.

The **enthalpy change** associated with a **temperature change** for an ideal gas can be determined from :

$$\Delta\tilde{H}_{2-3} = \int_{T_2}^{T_3} \tilde{C}_P^o \, dT \qquad \textbf{Eqn 5}$$

Because we assumed a <u>constant</u> **heat capacity**, **Eqn 4** simplifies to:

$$\Delta\tilde{H}_{2-3} = \tilde{C}_{P,avg}^o \left(T_3 - T_2\right) \qquad \textbf{Eqn 6}$$

The **heat capacities** are tabulated in the **NIST WebBook**, under the Name Search option. <u>Interpolate</u> to estimate C_p at both T_1 and T_2. Then, <u>average</u> these two values of C_p to obtain the **average heat capacity**. This is equivalent to determining a <u>linear</u> equation between T_1 and T_2 and integrating.

Gas phase heat capacity data from the **NIST WebBook**:

Temperature (K)	Cp,gas (J/mol*K)
300	165.98
400	210.66
500	252.09

There are many different ways to estimate $C_p(T_1)$ and $C_p(T_2)$.

$C_p(T_1)$ =	166.0	J/mole-K
$C_p(T_2)$ =	197.3	J/mole-K
$C_{p,\,avg}$ =	181.6	J/mole-K

$\Delta H_{(2-3)}$ = 12,713 J/mol

Now, just multiply by the number of moles, **n**, to get ΔH_{2-3} :

$\Delta H_{(2-3)}$ = **4,124 J**

Last, we need to determine the **enthalpy change** from **states 3** to **4**, in which the **pressure** of the superheated vapor is increased.

Recall the assumption that the vapor behaves as an ideal gas. Because **enthalpy** is only a function of **T** for ideal gases, and since $T_3 = T_4$:

$\Delta H_{(3-4)}$ = **0 J**

Finally: $\Delta H_{1-4} = \Delta H_{1-2} + \Delta H_{2-3} + \Delta H_{3-4}$ = **16,051 J**

Verify: The problem statement instructed us to make all of the assumptions that we used.

Nonetheless, we will verify the assumptions as well as we can from the given information.

1a - Is the saturated vapor is an ideal gas ?

$$\tilde{V}_2 = \frac{R\,T_1}{P_1}$$

T_1	300	K
P_1	6.68	kPa
V_1	373.4	L/mol

Use the Antoine Equation to determine P_1 :

Since V_1 **> 21 L/mole** this ideal gas assumption is **valid**.

1b - Is the **molar volume** of the saturated vapor is **much, much** greater than the **molar volume** of the saturated liquid.

1c - Is the latent heat of vaporization is constant over the temperature range of interest.

We cannot assess the validity of assumptios **1b** and **1c** from the data given in the problem.

2 - Does the superheated vapor also behave as an ideal gas.

$$\tilde{V}_4 = \frac{R\,T_4}{P_4}$$

$T_3 = T_4$	370	K
P_4	58.70	kPa
V_4	52.4	L/mol

Since V_4 **> 21 L/mole** this ideal gas assumption is **valid**.

3 - Is the **heat capacity** of the superheated vapor is nearly constant over the temperature range of interest ?

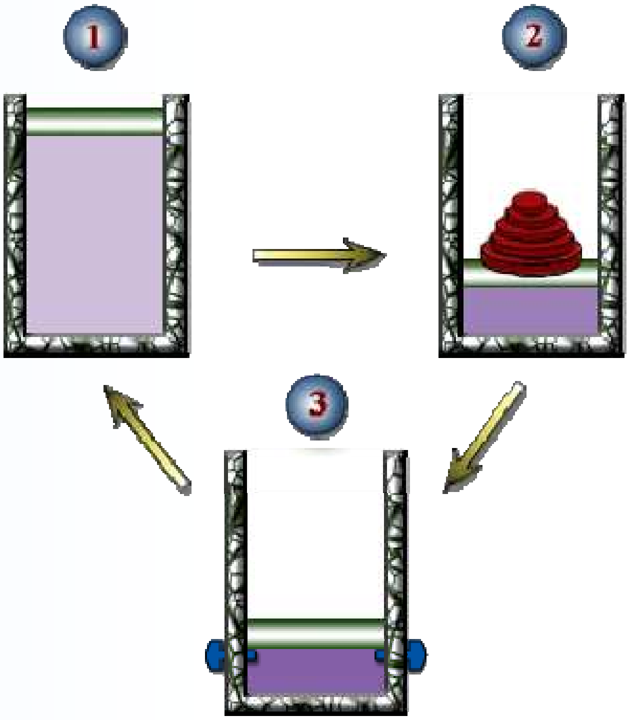
We cannot verify this assumption with the data provided in the problem statement.

We have no evidence that any of the assumptions are invalid.

Answers : (All rounded to 3 significant digits)

ΔH_{1-2}	11,900	J
ΔH_{2-3}	4,120	J
ΔH_{3-4}	0	J
ΔH_{1-5}	16,100	J

Chapter 4



The First Law of Thermodynamics:

We begin this chapter by introducing the concepts of work and heat. This leads to the study of multi-step processes in which a system interacts with the surroundings by exchanging heat and work.

When heat and work are exchanged with the surroundings, the P, V and T of the system can change. We can show the process path on a PV Diagram. Several special types of process paths are discussed in this chapter.

The First Law of Thermodynamics is introduced and used to analyze processes that take place in closed systems (no mass crosses the system boundary). The chapter concludes with a discussion of heat engines and their thermal efficiency and refrigeration and heat pump cycles and their coefficients of performance.

Work

- **Definition**
  - ◊ A force acting through a distance
  - ◊ A restraining force is overcome to move an object

$$W_{12} = \int_{\text{State 1}}^{\text{State 2}} F\,dx \;[=]\; N\cdot m \;[=]\; J$$

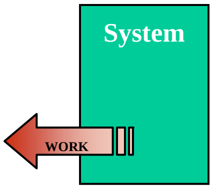
- **Boundary Work or PV Work: F = P A**

$$W_b = \int_{\text{State 1}}^{\text{State 2}} P\,A\,dx = \int_{\text{State 1}}^{\text{State 2}} P\,dV$$

- **Thermodynamic Definition of Work**
  - ◊ Work is done by a system on its surroundings if the sole effect of a process on its surroundings could have been raising a weight.
  - ◊ This definition allows for other forms of work, such as spring work, electrical work, gravitational work and acceleration work.

Sign Convention for Work

- **$\hat{U}$ increases as T increases, so changes in have a natural sign.**
- **This is not true for work**
- **A system can do work on the surroundings or the surroundings can do work on the system**
- **We choose which is positive and which is negative. We choose a sign convention.**
- **In this course we choose work done BY the system on the surroundings to be positive**
- **Always include an arrow for work on our sketches to indicate the sign convention we are using.**



Power & Path Variables

- **Power**: the rate at which work is done

$$\dot{W} = \frac{\delta W}{dt} \; [=] \; \mathbf{W, Btu/s, ft \cdot lb_f/s, hP}$$

- **Exact Differentials**
  - State variables: $U \Rightarrow dU$
  - Changes in state variables, like U, do not depend on which process path the system follows between 2 states

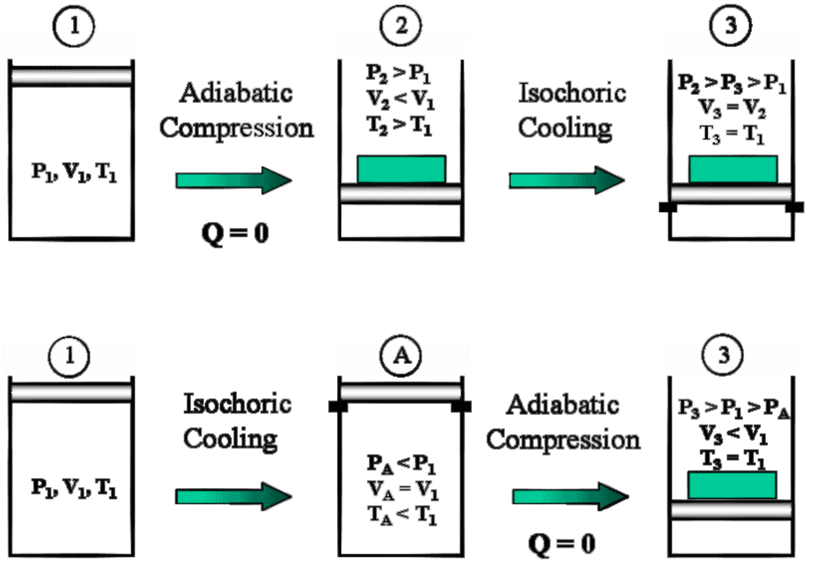
$$\int_1^2 dU = U_2 - U_1 = \Delta U$$

- **Inexact Differentials**
  - SI: L, m^3, $mL=cm^3$
  - Path Variables: $W \Rightarrow \delta W$
  - Systems do not have work
  - Work is a form of energy that only exists as it moves across a system boundary.

$$\int_1^2 \delta W = W_{12}$$

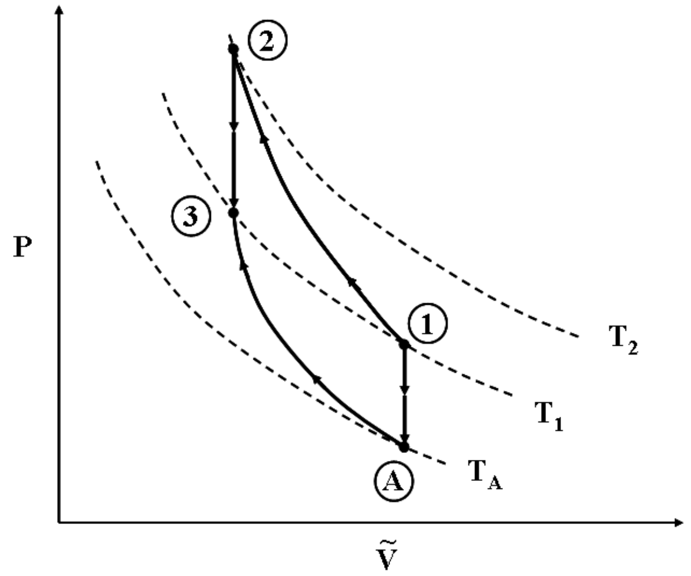
  - W_{12} depends on the path the process follows from state 1 to state 2.
  - Use δ instead of d for inexact differentials of path variables

Boundary Work and Process Paths

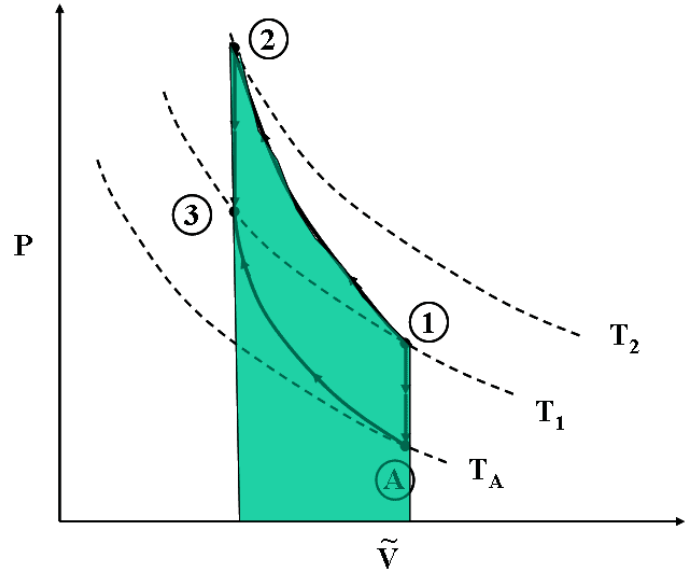


- Consider the two processes shown here: 1-2-3 and 1-A-3
- ΔU, ΔH and ΔV are the same for each of these processes because they begin and end at the same states.
- But, is the amount of boundary work the same for both processes ?
- The easiest way to tell is to plot the process path on a PV Diagram
- Then, make use of the fact that boundary work is the integral of P dV to determine if the boundary work done by the two processes are the same.

Process Paths on a PV Diagram

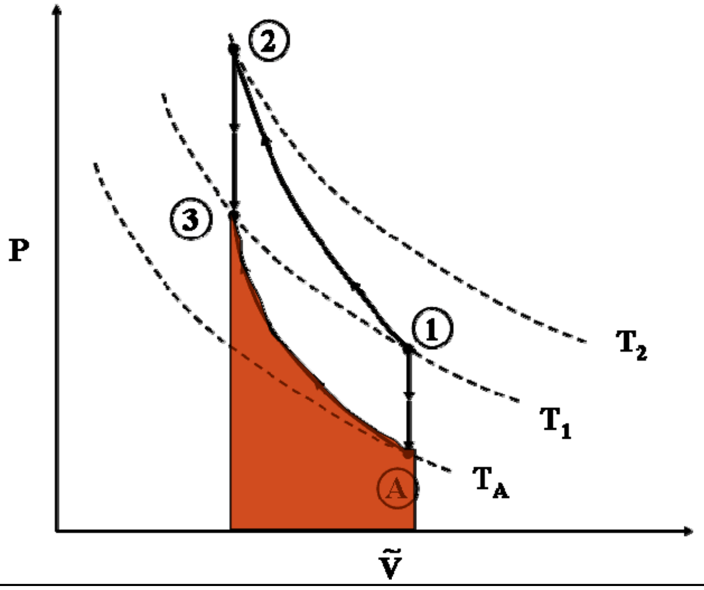


Boundary Work on a PV Diagram



- The shaded area is the boundary work done during the process 1-2-3.
- Is this work positive or negative under our sign convention ?

Boundary Work on a PV Diagram



- The shaded area is the boundary work done during the process 1-A-3.
- Is this work positive or negative under our sign convention ?
- The amount of boundary work is NOT equal for the two processes !
- This is because work depends on the process path.
- Work is a PATH variable, NOT a property or state variable like V, U and H.
- What about heat ? Is the heat transfer for the two processes the same ?
- Nope. Heat is also a PATH variable.

Quasi-Equilibrium Processes

- **Does it matter how rapidly we compress the gas in steps 1-2 and A-3 ? Yes !**

$$W_b = \int_{\text{State 1}}^{\text{State 2}} F_{resist}\, dx = \int_{\text{State 1}}^{\text{State 2}} P_{resist}\, dV$$

- **When a gas is rapidly compressed…**
  - The molecules cannot get out of the way of the piston rapidly enough
  - As a result, the local pressure right in front of the piston is greater than the pressure in the bulk of the gas.
  - $P_{resist} > P_{bulk}$
  - As a result, $P_{fast} > P_{slow}$ and $-W_{fast} > -W_{slow}$
- **Quasi-Equilibrium Processes**
  - Infinitely slow
  - Always in an equilibrium state, $P_{resist} = P_{bulk}$

- For a turbine, $W_{fast} < W_{slow}$

W_b for Special Types of Processes

- **Isobaric:** $$\tilde{W}_{12} = \int_1^2 P\,d\tilde{V} = P\int_1^2 d\tilde{V} = P\left(\tilde{V}_2 - \tilde{V}_1\right) = P\,\Delta\tilde{V}$$
- **Isothermal & IG:** $$\tilde{W}_{12} = \int_1^2 P\,d\tilde{V} = \int_1^2 \frac{R\,T}{\tilde{V}}\,d\tilde{V} = R\,T\,\mathrm{Ln}\left(\frac{\tilde{V}_2}{\tilde{V}_1}\right) = R\,T\,\mathrm{Ln}\left(\frac{P_1}{P_2}\right)$$
- **Polytropic:**
  - ◊ $\delta = 1$ & IG: isothermal !
  - ◊ $\delta \neq 1$: $$\tilde{W}_{12} = \int_1^2 P\,d\tilde{V} = C\int_1^2 \frac{d\tilde{V}}{\tilde{V}^{\delta}} = \frac{P_2\,\tilde{V}_2 - P_1\,\tilde{V}_1}{1-\delta}$$
- **Polytropic & IG:** $$\tilde{W}_{12} = \frac{R\,T_2 - R\,T_1}{1-\delta} = \frac{R}{1-\delta}\left(T_2 - T_1\right) = \frac{R}{1-\delta}\,\Delta T$$

- **Isobaric** is the easiest type of process when it comes to evaluating the boundary work.
- Evaluating W_b for an **isothermal** process isn't easy unless the fluid in the system is an ideal gas. Then, it isn't bad at all
- Notice that the last equality is true because $P_1V_1 = P_2V_2$ for an IG undergoing a polytropic process.

Heat : Q

- **Another form of energy in transition across a system boundary, like work.**

$$Q\,[=]\,J \qquad \hat{Q}\,[=]\,kJ/kg \qquad \tilde{Q}\,[=]\,J/mole \qquad \dot{Q}\,[=]\,J/s \text{ or } W$$

- **Flows spontaneously from "hot" to "cold"**
- **Heat is the flow of thermal energy while U is the amount of thermal energy a system holds.**
- **Heat is comparable to electrical current while U is comparable to electrical potential or voltage.**
- **Sign Convention**:
  - ◊ Heat flow <u>into</u> a system > 0

Heat : A Few Details

- **Heat is a <u>path variable</u> and the differential of heat is <u>inexact</u>, so we use δ:**

$$\int_1^2 \delta Q = Q_{12}$$

- **In an <u>adiabatic</u> process Q = 0**
- **If the heat transfer rate, $\dot{Q}$, is constant, then:**

$$Q = \dot{Q}\,\Delta t$$

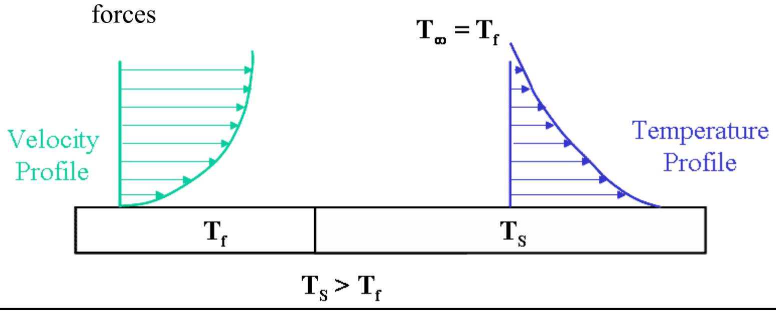
- **Heat Flux:** $\dot{q} = \dfrac{\dot{Q}}{A}\ [=]\ W/m^2$

Conduction

- **Fourier's Law**: $\dot{q} = -k\dfrac{dT}{dx}$
- **k = thermal conductivity [=] W/m-K**
- **If k = constant:** $\dfrac{dT}{dx} = \dfrac{\Delta T}{\Delta x}$
- **Magnitude of k:**
  - Metals: k ≈ 100 W/m-K
  - Non-metals: k ≈ 1 - 10 W/m-K
  - Liquids: k ≈ 0.1 - 10 W/m-K
  - Gases: k ≈ 0.01 – 0.1 W/m-K
  - Insulation: k ≈ 0.01 – 0.1 W/m-K

Convection Heat Transfer

- **Convection is the combination of conduction and fluid motion**
- **For the same fluid and conditions: $Q_{conv} > Q_{cond}$**
- **Forced Convection**
  - ◊ Fluid motion is driven by an external force, such as pressure
- **Free or Natural Convection**
  - ◊ Fluid motion is driven by density differences and buoyant forces



Newton's Law of Cooling

- **Hot surface:** $\dot{q} = h\left(T_S - T_f\right)$
- **Cold surface:** $\dot{q} = h\left(T_f - T_S\right)$
- **h = convection heat transfer coefficient [=] W/m^2-K**
  - ◊ Depends on fluid and surface properties
  - ◊ Depends on the nature of the fluid velocity profile
- **Magnitude of h:**

◊ Free convection, gases:	h ≈ 2 - 25	W/m^2-K
◊ Free convection, liquids:	h ≈ 50 - 1000	W/m^2-K
◊ Forced convection, gases:	h ≈ 25 - 250	W/m^2-K
◊ Forced convection, liquids:	h ≈ 50 – 20,000	W/m^2-K
◊ Boiling phase change:	h ≈ 2500 – $1x10^5$	W/m^2-K

- Remember that q = heat <u>flux</u> [=] W/m^2.

Radiation Heat Transfer

- **Atoms emit photons in the infrared part of the spectrum. The photons carry thermal energy to the surface that absorbs them.**

$$\dot{Q}_{emit,max} = \sigma A T_S^4$$

$$\dot{Q}_{emit} = \varepsilon \sigma A T_S^4 \qquad \varepsilon = \text{emissivity}$$

$$\dot{Q}_{absorb} = \alpha \dot{Q}_{emit} \qquad \text{We usually assume } \alpha = 1$$

- **Radiation exchange between a body its surroundings**
  - ◊ If $\alpha = 1$: $\dot{Q}_{net} = \dot{Q}_{emit,body} - \dot{Q}_{emit,surr} = \varepsilon_{body} \sigma A T_{body}^4 - \varepsilon \sigma_{surr} A T_{Surr}^4$
  - ◊ Boldly assume $\varepsilon_{body} = \varepsilon_{surr} = \varepsilon$: $\dot{Q}_{net} = \varepsilon_{body} \sigma A \left(T_{body}^4 - T_{Surr}^4\right)$

- T must be expressed in Kelvins or Rankine (an absolute T-scale.
- Emissivity is a measure of the ability of a surface to emit thermal radiation
- σ = Stefan-Boltzmann Constant
  = 5.67×10^{-8} W/m^2-K^4.

First Law of Thermodynamics

- **Definition**
  - ◊ Energy is neither created nor destroyed.
- **General Conservation Principle**

IN - OUT + GEN = ACC

- **1st Law, Integral Form**

$$Q - W = \Delta E = \Delta\left[U + E_{kin} + E_{pot}\right]$$

- **1st Law Differential Form**

$$\dot{Q} - \dot{W} = \frac{dE}{dt} = \frac{d}{dt}\left[U + E_{kin} + E_{pot}\right]$$

- This statement of the 1st Law applies to systems that do not undergo nuclear reactions.
- Many quantities are conserved.
- The integral form of the 1st Law will help you analyze a process that takes place between two given times.
- These are often BATCH processes.
- The differential form of the 1st Law will help you analyze continuous processes.
- This form is sometimes called the RATE form of the 1st Law because the terms are all rates.
- The heat transfer rate, the rate at which work is done, the rate at which the energy of the system changes.

Special Cases

- **Special Case: No change in E_{kin} or E_{pot}**
  - ◊ $\Delta E_{kin} = 0$ **and** $\Delta E_{pot} = 0$

$$Q - W = \Delta U$$

  - ◊ $\frac{dE_{kin}}{dt} = 0$ **and** $\frac{dE_{pot}}{dt} = 0$

$$\dot{Q} - \dot{W} = \frac{dU}{dt}$$

- **Handy Relationships:** $\dot{m} = \frac{\dot{V}}{\hat{V}}$

$$\dot{V} = v\ A_{cross}$$

- In many problems changes in potential energy are negligible
- Equipment isn't usually so LARGE that changes in potential energy are significant
- Changes in kinetic energy are often negligible when …
- Pipe diameters do not change and
- No phase change occurs
- We need velocity data or flow rate and pipe diameter to calculate E_{kin}
- When we don't have this data, we usually assume changes in E_{kin} are negligible.
- These two relationships will be used again and again, whenever a fluid flows through the system.

Total Work and Heat

- **Work :** $W_{total} = W_b + W_s + W_e + ...$
- **Heat :** $Q_{total} = Q_{cond} + C_{conv} + Q_{rad}$

- There are MANY different types of work, but only one type of heat
- There are three mechanisms for heat transfer
- Conduction: Fourier's Law
- Convection: Newton's Law of Cooling
- Radiation: The Radiation Equation
- W_{Sh} = shaft work
- This is very important. It is work that involves rotating machinery and is the most common form of work in continuous processes

Problem Solving Procedure

- **Read Carefully**
- **Draw a Diagram**
- **List Given Information**
- **List All Assumptions**
- **Write Equations and Lookup Data**
- **Solve Equations**
- **Verify Assumptions**

- The problem solving procedure is a strategy to help organize your thoughts and your work so you can avoid mistakes and communicate effectively with other engineers and scientists.
- Read
  - ◊Get all the important info out of the problem statement and onto your paper.
- Draw a Diagram
  - ◊Draw a process schematic, include a system boundary
  - ◊Draw a phase diagram
- List All Given
  - ◊List all given values
  - ◊Assign variable names, including units
  - ◊List objectives (what are you trying to find)
- List All Assumptions
  - ◊State your sign convention
  - ◊You will add to this list as you work through the problem
- Write Equations and Lookup Data
- Count the unknown variables and the INDEPENDENT equations
- Work backwards from the answer you are looking for
- When # of equations = # of unknowns, the problem is well-posed and you can solve it !
- Solve Equations
- This usually turns out to be the EASY part !
- Work backwards from the answer you are looking for.
- Repeat the last two steps until you have gotten all the answers you need.
- Verify Assumptions
- In this course, the main assumption you may have to verify is that a gas is ideal.
- Answer Questions
- Hard to believe, but students routinely forget to answer

Isobaric Processes

- **P = constant**
- **1st Law**
  - ◊ closed system, integral form
  - ◊ Assume: $\Delta E_{pot} = 0$ and $\Delta E_{kin} = 0$

$$Q - W = \Delta U$$

- **IF boundary work is the only form of work:**

$$W_{total} = W_b + \cancel{W_{sh}} + \cancel{W_e} = W_b$$

- **Definition of boundary work:**

$$W_b = \int_1^2 P\,dV = P\int_1^2 dV = P\left(V_2 - V_1\right) = P\,\Delta V$$

- **1st Law becomes:**

$$Q - P\,\Delta V = \Delta U \quad \Rightarrow \quad Q = \Delta U + P\,\Delta V$$

$$\boxed{Q = \Delta H}$$

- This form of the 1st Law applies when the following assumptions are true.
- Closed system
- Quasi-equilibrium
- Only form of work is boundary work
- Isobaric
- $\Delta E_{pot} = 0$
- $\Delta E_{kin} = 0$

Isochoric Processes

- **V = constant**
- **1st Law**
  - ◊ closed system, integral form
  - ◊ Assume: **$\Delta E_{pot} = 0$** and **$\Delta E_{kin} = 0$**

$$Q - W = \Delta U$$

- **IF boundary work is the only form of work:**

$$W_{total} = W_b + \cancel{W_{sh}} + \cancel{W_e} = W_b$$

- **Definition of boundary work:**

$$W_b = \int_1^2 P\,dV = 0$$

- **1st Law becomes:**

$$Q = \Delta U$$

- This form of the 1st Law applies when the following assumptions are true.
- Closed system
- Quasi-equilibrium
- Only form of work is boundary work
- Isochoric
- $\Delta E_{pot} = 0$
- $\Delta E_{kin} = 0$

Cycle Types and Characterization

- **Three types of cycles**
  - ◊ Power Cycles or Heat Engines
  - ◊ Refrigeration Cycles
  - ◊ Heat Pump Cycles
- **How to Categorize Systems that Undergo Cycles**
  - ◊ Open vs. Closed Cycles
    - Closed Cycles: Use a working fluid that never leaves the system
    - Open Cycles: Take in fresh fluid and reject the used fluid when the cycle is complete. Usually the fluid is air.
  - ◊ Gas vs. Vapor Cycles
    - Vapor Cycles: Take advantage of a phase change in the fluid
    - Gas Cycles: The fluid always remains in the gas phase. The fluid is usually air or an air-fuel mixture (as in an internal combustion engine).

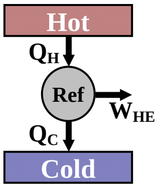
- Power Cycle and Heat Engine are two names for cycles that take in a net amount of heat and produce a net amount of work.
- Refrigeration and heat pump cycles are really the same cycle, but they have different OBJECTIVES.
- Closed Cycles: home refrigerator, heat pump and air-conditioners
- Open Cycles: Automobile engines, jet engines, turbine engines (warships, some tanks like the US M1 Abrams)

Power Cycles

- **1st Law:** $Q_{cycle} - W_{cycle} = \cancel{\Delta E_{cycle}}$

  $$\boxed{Q_{cycle} = W_{cycle}}$$

- **Tie-fighter Diagrams**
  - ◊ No sign convention !
    All quantities are positive.
  - ◊ 1st Law : $\boxed{Q_H - Q_C = W_{HE}}$



- **Thermal Efficiency:** $\eta = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{W_{HE}}{Q_H}$

$$\eta = \frac{Q_H - Q_C}{Q_H}$$

$$\boxed{\eta = 1 - \frac{Q_C}{Q_H}}$$

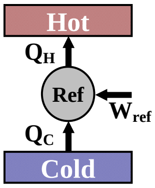
- Because the initial state and final state of a cycle are the same, $\Delta E_{cycle} = 0$, always.
- It is easy to get confused about the sign convention.
- We just got used to using it and now we (temporarily) discard it.
- With TFD's it is essential to put arrows for all heat and work interactions
- NOTES:
- ALL heat engines must reject heat to a low-temperature reservoir.
- ALL heat engines must reject heat to a low-temperature reservoir.
- No heat engine can completely convert heat into work.
- We will study why this is true in great detail in Chapters 6 and 7.
- Thermal Efficiency is at the heart of this course.
- It gives us our first way to compare and evaluate the performance of different cycles.
- Remember that the goal or desired output of a heat engine is the WORK.
- The required input or the stuff you have to buy in order to run the heat engine is the heat source or fuel: Q_H.

Refrigeration Cycles

- **1st Law:** $Q_{cycle} - W_{cycle} = \cancel{\Delta E_{cycle}}$

  $$\boxed{Q_{cycle} = W_{cycle}}$$

- **Tie-fighter Diagram**
  - ◊ No sign convention !
    All quantities are positive.
  - ◊ 1st Law : $\boxed{Q_H - Q_C = W_{ref}}$



- **Coefficient of Performance:**

$$\beta_R = COP_R = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{Q_C}{W_{ref}} = \frac{Q_C}{Q_H - Q_C}$$

$$\boxed{\beta_R = COP_R = \frac{1}{\frac{Q_H}{Q_C} - 1}}$$

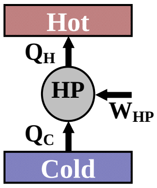
- It's cool that the 1st Law reduces to exactly the same form for the refrigerator as it did for the heat engine.
- The goal or desired output for a refrigerator is to make your food cold, that is Q_C.
- The stuff you need to buy in order to run a refrigerator is the required input, that is the WORK.
- You know that your refrigerator will not keep your food cold unless you plug it into the electrical socket and pay your electric bill !

Heat Pump Cycles

- **1st Law:** $Q_{cycle} - W_{cycle} = \cancel{\Delta E_{cycle}}$

$$\boxed{Q_{cycle} = W_{cycle}}$$

- **Tie-fighter Diagram**
  - ◊ No sign convention ! All quantities are positive.
  - ◊ 1st Law : $\boxed{Q_H - Q_C = W_{HP}}$



- **Coefficient of Performance:**

$$\beta_{HP} = COP_{HP} = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{Q_H}{W_{HP}} = \frac{Q_H}{Q_H - Q_C}$$

$$\boxed{\beta_{HP} = COP_{HP} = \frac{1}{1 - \frac{Q_C}{Q_H}}}$$

- Once again, the 1st Law reduces to exactly the same form.
- The goal or desired output for a heat pump is to keep your house warm, that is Q_H.
- The stuff you need to buy in order to run a heat pump is the required input, that is the WORK.
- Your heat pump heating system will not keep your house warm unless you connect it to the electrical circuit and pay your electric bill !

4A-1 Work for a Cycle Carried Out in a Closed System 6 pts

Ten kilograms of **carbon dioxide** (**CO_2**) is held in a piston-and-cylinder device. The **CO_2** undergoes a thermodynamic cycle consisting of three processes. The processes are :

Process 2-3: constant volume
Process 3-1: constant temperature compression
a.) Sketch the cycle path on a **PV Diagram**
b.) Calculate the **net work** done in **kJ**
Data: $T_1 = 145^oC$, $T_2 = 440^oC$, $P_1 = 150$ kPa

Read : Work your way around the cycle, step by step. The **work** for the cycle is the sum of the **work** for each step.
Assume the **CO_2** behaves as an ideal gas throughout all three process steps.
Apply the definition of boundary work or PV work to each step in the cycle.

Diagram: See the solution to part (a).

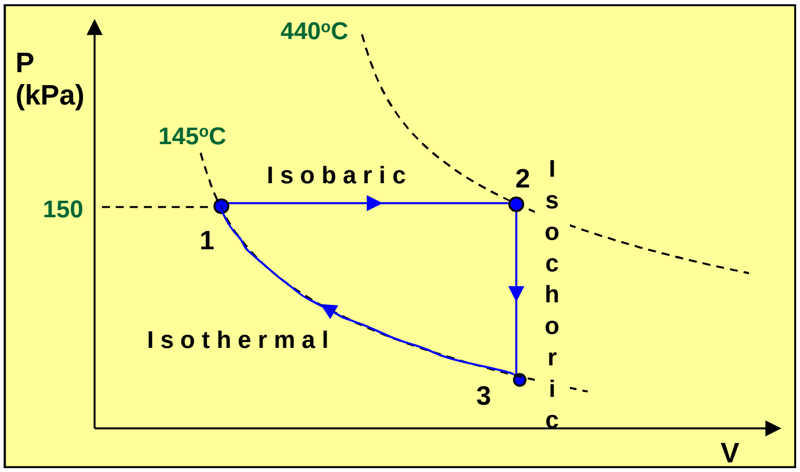
Given:				**Find:**			
	m	**10**	**kg**		**a.)**	Sketch cycle on a PV Diagram.	
	T_1	**145**	**^oC**		**b.)**	**W_{cycle} =**	**???** **kJ**
	T_2	**440**	**^oC**				
	P_1	**150**	**kPa**				

Assumptions:

1 - The gas is held in a closed system.
2 - Boundary work is the only form of work interaction
3 - Changes in kinetic and potential energies are negligible.
4 - **CO_2** behaves as an ideal gas. This must be verified at all three states.

Equations / Data / Solve:

Part a.)



Part b.) Since **$W_{cycle} = W_{12} + W_{23} + W_{31}$**, we will work our way around the cycle and calculate each **work** term along the way.

Step 1-2 is isobaric, therefore, the definition of **boundary work** becomes:

$$W_{12} = \int_{V_1}^{V_2} P\,dV = P_1\left(V_2 - V_1\right)$$ **Eqn 1**

We can simplify **Eqn 1** using the fact that **$P_2 = P_1$** and the Ideal Gas EOS :

$$P V = n R T$$ **Eqn 2**

$$W_{12} = P_2 V_2 - P_1 V_1 = nR\left(T_2 - T_1\right)$$ **Eqn 3**

We can determine the number of moles of **CO_2** in the system from the given **mass** of **CO_2** and its **molecular weight**.

$$n = \frac{m}{MW}$$ **Eqn 4**

MW_{CO2}	44.01	**g/mole**	**n**	**227.22**	**mole**

Plug values into **Eqn 3** :

R	8.314	**J/mole-K**
W_{12}	**557.29**	**kJ**

Because the volume is constant in **step 2-3**: **W_{23}** **0** **kJ**

Step 3-1 is isothermal, therefore, the definition of **boundary work** becomes:

$$W_{31} = \int_{3}^{1} P\,dV = nRT_1\,Ln\left(\frac{V_1}{V_3}\right) = nRT_1\,Ln\left(\frac{P_3}{P_1}\right)$$ **Eqn 5**

The problem is that we don't know either **P_3** or **V_3**. Either one would be useful in evaluating **W_{31}** because we know **P_1** and we can determine **V_1** from the Ideal Gas EOS, **Eqn 2**.

We can evaluate **V_3** using the fact that **$V_3 = V_2$**. Apply the the Ideal Gas EOS to **state 2**.

$$V_3 = V_2 = \frac{nRT_2}{P_2} = \frac{nRT_2}{P_1}$$ **Eqn 6**

V_3 **8.981** **m^3**

Next, we can apply **Eqn 6** to **state 1** : **V_1** **5.266** **m^3**

Now, we can plug values into **Eqn 4** to evaluate **W_{13}** : **W_{31}** **-421.71** **kJ**

Sum the **work** terms for the three steps to get **W_{cycle}** : **W_{cycle} = 135.6 kJ**

Verify : Only the ideal gas assumption can be verified.

We need to determine the **specific volume** and check if : $\widetilde{V} > 20\ L/mol$

$$\widetilde{V} = \frac{RT}{P}$$ **Eqn 7**

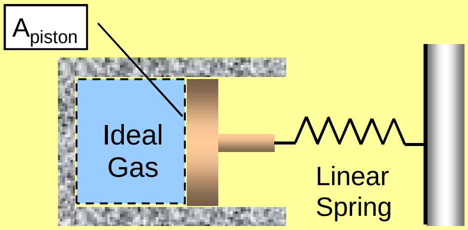
R	8.314	**J/mol-K**	**V_1**	**23.18**	**L/mol**
			$V_2 = V_3$	**39.53**	**L/mol**

The ideal gas assumption is valid because **V > 20 L/mole** in all three states.

Answers : **a.)** See the sketch, above. **b.)** **W_{cycle} = 135.6 kJ**

4A-2 Quasi-Equilibrium Expansion of a Gas 4 pts

A piston-and-cylinder device is oriented horizontally and a compressed spring exerts a force on the back of the piston, as shown below. The gas inside the cylinder is cooled slowly and, as the cooling proceeds, the spring pushes the piston farther into the cylinder. During the cooling process, the spring exerts a force that decreases linearly with position. Initially, the gas occupies a **volume** of **8.4 L** while the spring exerts a **force** of **1.2 kN** on the back of the piston. At the end of the process, the gas occupies **3.7 L** and the spring exerts a **force** of **0 kN** on the piston. The **area** of the piston face is **200 cm^2** and atmospheric **pressure** is **100 kPa**.



Assume the gas behaves as an ideal gas and friction is negligible. Determine…

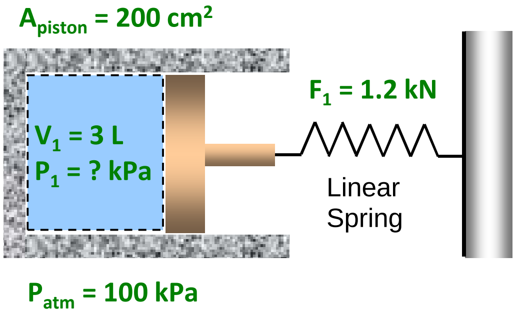
a.) The initial and final **pressures** inside the cylinder, in **kPa**

b.) The boundary work, in **kJ**.

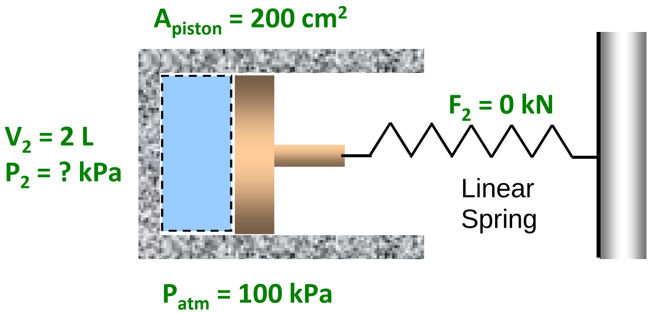
Read : The key to solving this problem is to determine the slope and intercept for the linear relationship between the **force** exerted by the spring on the piston and the **volume** that the gas occupies. This relationship is linear because, for a cylinder of uniform **diameter**, gas **volume** varies linearly with respect to the position of the piston.

Diagram:

Initial State



Final State



Given:

V_1 = 8.4 L
V_2 = 3.7 L
F_1 = 1200 N
F_2 = 0 N

P_{atm} 100 kPa
A_{piston} 0.020 m^2

Find:

P_1 = ??? kPa
P_2 = ??? kPa

W = ??? kJ

Assumptions:

1 - The gas in the cylinder is a closed system.
2 - The process occurs slowly enough that it is a quasi-equilibrium process.
3- There is <u>no</u> friction between the piston and the cylinder wall.
4- The spring **force** varies linearly with position.

Equations / Data / Solve:

In the initial and final states, the piston is not accelerating. In fact, it is not moving. Therefore, there is no unbalanced force acting on it. This means that the vector sum of all the forces acting on the piston must be zero.

$$P_1 = P_{atm} + \frac{F_1}{A_{piston}} \qquad \text{Eqn 1}$$

Initial State: **P_1 160 kPa**

$$P_2 = P_{atm} + \frac{F_2}{A_{piston}} \qquad \text{Eqn 2}$$

Final State: **P_2 100 kPa**

For a quasi-equilibrium process, **boundary** or **PV work** is defined by:

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \left[P_{atm} + \frac{F_{spring}}{A_{piston}}\right] dV \qquad \text{Eqn 3}$$

Because $\mathbf{F_{spring}}$ varies linearly with the position of the piston **AND volume** also varies linearly with the position of the piston, we can conclude that $\mathbf{F_{spring}}$ <u>must</u> vary linearly with respect to the **volume** !

$$F_{spring} = mV + b = mV + (F_1 - mV_1) = \left(\frac{F_2 - F_1}{V_2 - V_1}\right) V + (F_1 - mV_1) \qquad \text{Eqn 4}$$

m **2.5532E+05** **N/m^3**

b **-944.68** **N**

$$W = \int_{V_1}^{V_2} \left[P_{atm} + \frac{mV + b}{A_{piston}}\right] dV = \left(P_{atm} + \frac{b}{A_{piston}}\right)(V_2 - V_1) + \frac{m}{A_{piston}} \frac{\left(V_2^2 - V_1^2\right)}{2} \qquad \text{Eqn 5}$$

W -611.0 J

Verify: None of the assumptions can be verified using only the information given in the problem statement.

Answers :

P_1 160 kPa
P_2 100 kPa

W -611 J

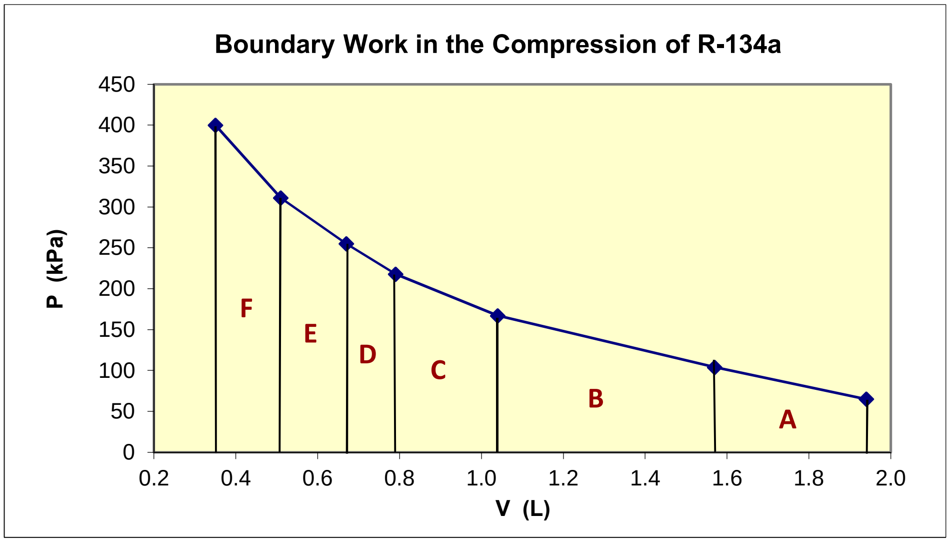
4A-3 Quasi-Equilibrium Compression of R-134a 4 pts

R-134a vapor is held in a piston-and-cylinder device at **30°C** and **65 kPa**. The **R-134a** vapor is compressed as small weights are slowly added to the back of the piston until the **pressure** inside the cylinder reaches **400 kPa**.
Determine...
a.) The boundary work done by the **R-134a**
b.) The final **temperature** of the **R-134a**
The following measurements have been made during the process:
P(kPa), V(L) = (65,1.94), (104,1.57), (167,1.04), (218,0.79), (255,0.67), (311,0.51), (350,0.35)

Read : The key concept here is that **boundary** or **PV work** is represented by the area under the process path curve on a **PV Diagram**. So, once we plot the given data on a **PV Diagram**, all we need to do is numerically integrate to determine the area under the curve and we will know the **work** !

We can use the R-134a Tables in the **NIST Webbook** to determine the final **temperature** because we know both $\mathbf{P_{final}}$ and $\mathbf{V_{final}}$. But we don't know the number of **moles** in the system. Fortunately, we can use the initial state $\mathbf{P_1}$, $\mathbf{V_1}$, and $\mathbf{T_1}$ to determine the number of **moles** in this closed system.

Diagram:



Given:

P (kPa)	V (L)
65	1.94
104	1.57
167	1.04
218	0.79
255	0.67
311	0.51
400	0.35

$\mathbf{T_1}$ 30 °C

Find:

a.)	$\mathbf{W_{total}}$	???	J
b.)	$\mathbf{T_2}$	???	°C

Assumptions:

1 - Each state in the data table is an equilibrium state.
2 - The process is a quasi-equilibrium process.
3 - The system is a closed system.
4 - The trapezoidal rule gives an acceptable estimate of the area under the process path in the **PV Diagram**.

Equations / Data / Solve:

Part a.) The area of each trapezoid under the process path in the **PV Diagram** is the product of the average **pressure** for that trapezoid and the change in **volume** across the trapezoid.

$$W_{trap} = \left(\frac{P_b + P_a}{2}\right) \cdot \left(V_a - V_b\right)$$ **Eqn 1**

Where subscript **a** refers to the left side of each trapezoid and subscript **b** refers to the right side of each trapezoid in the **PV Diagram**.

Trapezoid	P_{avg} (kPa)	ΔV (L)	W (J)
A	85	-0.37	-31.3
B	136	-0.53	-71.8
C	193	-0.25	-48.1
D	237	-0.12	-28.4
E	283	-0.16	-45.3
F	356	-0.16	-56.9

W_{total} = -281.7 J

Part b.) If we knew the **specific volume** of the R-134a in the final state, we could use the R-134a tables to determine the **temperature**. But, at this point, all we know is the **total volume** in the final state.
The trick here is that the **mass** is the same in the initial and final states.
Therefore, we need to determine the **mass** of R-134a in the system.
We can use the information we have for the initial state to determine the **mass** of R-134a in the system.

First, look up the **specific volume** of R-134a in the initial state in the isothermal thermodynamic tables:

V_1 0.37538 m^3/kg

Then, to calculate **m_{R134a}**, use:

$$m_{R134a} = \frac{V_1}{\hat{V}_1}$$ **Eqn 1**

m_{R134a} 0.00517 kg

Finally, calculate **V_2** using:

$$\hat{V}_2 = \frac{V_2}{m_{R134a}}$$ **Eqn 2**

V_2 0.067723 m^3/kg

Now, use the isobaric thermodynamic tables at a **pressure** of **400 kPa**. Specify a **temperature** range that you are sure brackets **V_2**. Here is a table that I cut-and-pasted from the **NIST WebBook**.

Temperature (C)	Pressure (MPa)	Volume (m^3/kg)	Phase
72	0.4	0.066907	vapor
73	0.4	0.067138	vapor
74	0.4	0.067369	vapor
75	0.4	0.067599	vapor
76	0.4	0.067830	vapor
77	0.4	0.068060	vapor

V_2 falls between **75°C** and **76°C**
So, now, we must interpolate.

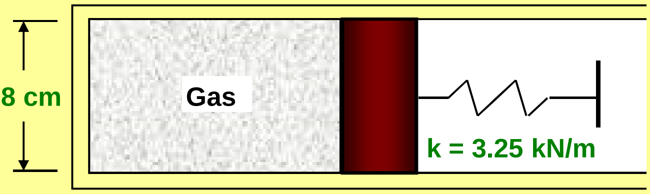
T_2 75.54 °C

Verify: None of the assumptions can be verified from the data given in the problem statement.

Answers : **a.)** **W_{total} = -282 J** **b.)** **T_2 75.5 °C**

4A-4 Expansion of a Gas in a Cylinder Against a Spring 5 pts

A gas is held in a horizontal piston-and-cylinder device, as shown below.

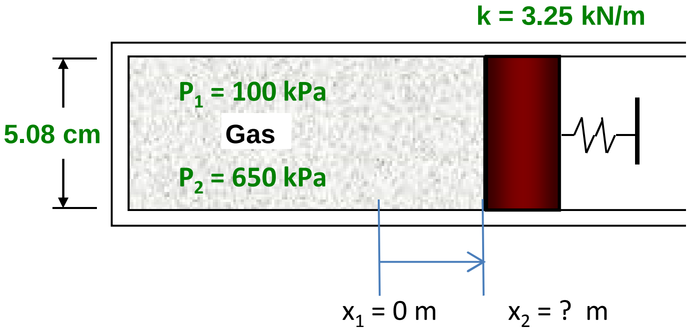


A spring is attached to the back of the frictionless piston. Initially, the spring exerts no force on the piston.

The gas is heated until the **pressure** inside the cylinder is **650 kPa**. Determine the boundary work done by the gas on the piston. Assume **P_{atm} = 100 kPa**.

Read : The key to solving this problem is to determine the slope and intercept for the linear relationship between the **force** exerted by the spring on the piston and the **pressure** within the gas. This relationship is linear because the **pressure** within the cylinder is atmospheric **pressure** plus the spring **force** divided by the cross-sectional area of the piston.

Diagram:



Given: P_2	650	kPa	D_{piston}	0.0508	m
$P_1 = P_{atm}$	100	kPa	k	3.25	kN/m

Find: W = ??? kJ

Assumptions:

1 - The gas in the cylinder is a closed system.
2 - The process occurs slowly enough that it is a quasi-equilibrium process.
3 - There is no friction between the piston and the cylinder wall.
4 - The spring **force** varies linearly with position.

Equations / Data / Solve:

For a quasi-equilibrium process, **boundary** or **PV work** is defined by:

$$\mathbf{W} = \int_{x_1=0}^{x_2} \mathbf{F_{total}\,dx} = \int_{x_1=0}^{x_2} \left(\mathbf{F_{spring}} + \mathbf{F_{atm}}\right)\mathbf{dx} \qquad \text{Eqn 1}$$

It is critical to note that the gas must overcome the **force** due to atmospheric **pressure** AND the **force** of the spring during this expansion process. Because the **force** exerted by the linear spring on the piston increases linearly as the gas expands, we can write the following equation relating the **force** exerted by the gas on the piston to the **displacement** of the piston from its original, unstretched position.

$$F_{spring} = k\,x \qquad \textbf{Eqn 2}$$

Where **x** is the displacement of the piston from its initial position.

Plug **Eqn 2** into **Eqn 1** and integrate to get :

$$W = \left(\frac{k\,x^2}{2} + F_{atm}\,x \right) \Bigg|_0^{x_2} = \frac{k}{2} x_2^2 + F_{atm}\,x_2 \qquad \textbf{Eqn 3}$$

Where x_2 is the displacement of the spring in the final state.
So, our next objective is to determine how far the piston moved during this process.

In the initial and final states, the piston is not accelerating. In fact, it is not moving. Therefore, there is no unbalanced force acting on it. This means that the vector sum of all the **forces** acting on the piston must be **zero**.

Initial State:

$$P_1 = P_{atm} \qquad \textbf{Eqn 4}$$

P_1 **100** **kPa**

The relationship between **force** and **pressure** is:

$$F_{atm} = P_{atm}\,A_{piston} \qquad \textbf{Eqn 5}$$

Where :

$$A_{piston} = \frac{\pi}{4} D_{piston}^2 \qquad \textbf{Eqn 6}$$

A_{piston} **2.03E-03** m^2

F_{atm} **0.2027** **kN**

Final State:

$$P_2 = P_{atm} + \frac{F_2}{A_{piston}} \qquad \textbf{Eqn 7}$$

or :

$$F_2 = (P_2 - P_{atm})\,A_{piston} \qquad \textbf{Eqn 8}$$

Now, plug numbers into **Eqn 8** :

F_2 **1.1148** **kN**

Because the spring is linear :

$$F_2 = k\,x_2 \qquad \textbf{Eqn 9}$$

or :

$$x_2 = \frac{F_2}{k} \qquad \textbf{Eqn 10}$$

x_2 **0.3430** **m**

Finally, substitute back into **Eqn 3** to evaluate the **work** done by the gas in the cylinder on its surroundings during this process :

W = **0.26070** **kJ**

W	**260.7**	**J**

Verify: None of the assumptions can be verified using only the information given in the problem statement.

Answers :

W	**261**	**J**

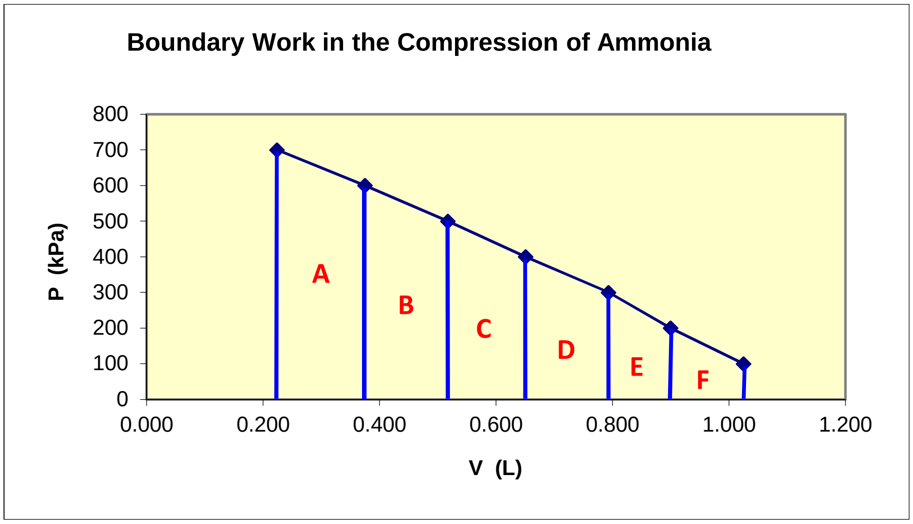
4A-5 Quasi-Equilibrium Expansion of a Gas 4 pts

A gas is compressed in a piston-and-cylinder device from **100 kPa** to **700 kPa**. Estimate the boundary work done **by** the gas during the process. The following measurements were made during the process.

P (kPa)	100	200	300	400	500	600	700
V (cm^3)	1025	900	793	651	517	375	224

Read : The key concept here is that **boundary** or **PV work** is represented by the area under the process path curve on a **PV Diagram**. So, once we plot the given data on a **PV Diagram**, all we need to do is numerically integrate to determine the area under the curve and we will have the **work** !

Diagram:



Given:

P (kPa)	V (L)
100	1.025
200	0.900
300	0.793
400	0.651
500	0.517
600	0.375
700	0.224

Find: **W** **???** **J**

Assumptions:

1 - Each state in the data table is an equilibrium state.
2 - The process is a quasi-equilibrium process.
3 - The system is a closed system.
4 - The trapezoidal rule gives an acceptable estimate of the area under the process path in the **PV Diagram**.

Equations / Data / Solve:

The area of each trapezoid under the process path in the **PV Diagram** is the product of the average **pressure** for that trapezoid and the change in **volume** across the trapezoid.

$$W_{trap} = \left(\frac{P_b + P_a}{2}\right) \cdot (V_a - V_b) \qquad \textbf{Eqn 1}$$

Where subscript **a** refers to the left side of each trapezoid and subscript **b** refers to the right side of each

Trapezoid	P_{avg} (kPa)	ΔV (L)	W (J)
A	650	-0.151	-98
B	550	-0.142	-78
C	450	-0.134	-60
D	350	-0.142	-50
E	250	-0.107	-27
F	150	-0.125	-19

W_{total}	**-331.8**	**J**

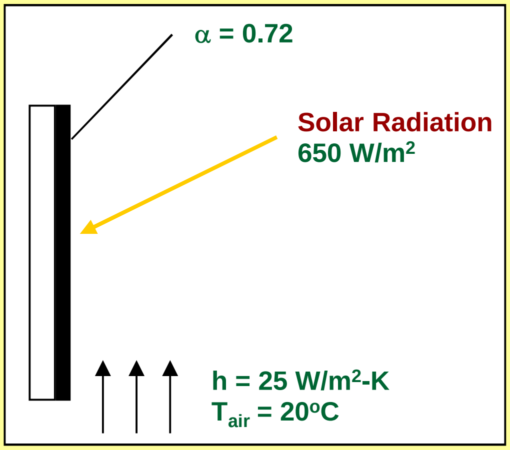
Verify: None of the assumptions can be verified from the data given in the problem statement.

Answers :

W_{total}	**-332**	**J**

4B-1 Radiation Heating and Convective Cooling of a Flat Plate 3 pts

The sun shines on the front of a thin plate that is insulated on the back surface, as shown below. The surface of the plate exposed to the sun has an **absorptivity** of 0.72 for solar radiation.



The total solar radiation incident on the front of the plate is 650 W/m^2. The front of the plate loses heat to thesurroundings which are at 20^{o}C but radiation heat loss from the plate is negligible.

The **convection heat transfer coefficient** is 25 W/m^2-K. The plate warms up until the solar heat gained is balanced by the convection heat lost. Determine the **temperature** of the front surface of the plate at steady-state.

Read : The key to this problem is to recognize that at steady-state, the rate at which heat is transferred into the plate from the sun by radiation must be equal to the rate at which heat is lost from the plate to the surrounding air by convection.

Diagram: The diagram in the problem statement is adequate.

Given:

α	0.72		T_{air}	20	^{o}C
q_{max}	650	W/m^2	h	25	W/m^2-K

Find: T_s ??? ^{o}C

Assumptions:

1 - Radiation heat losses from the plate are negligible.

2 - Heat losses through the edges of the plate are negligible.

3 - The back of the plate is perfectly insulated. Thus, at steady-state, the **temperature** of the plate is uniform.

4 - The incident radiation, the **convection heat transfer coefficient** and the **absorptivity** of the surface are all uniform over the surface of the plate.

Equations / Data / Solve:

The key to this problem is to recognize that at steady-state, the rate at which heat is transferred into the plate from the sun by radiation must be equal to the rate at which heat is lost from the plate to the surrounding air by convection.

$$q_{rad} = q_{conv} \qquad \text{Eqn 1}$$

Absorptivity, α, is the fraction of the incident radiation that is absorbed by a surface. Therefore:

$$q_{rad} = \alpha\, q_{max} \qquad \text{Eqn 2}$$

Newton's Law of Cooling gives us the **convection heat transfer rate** at the surface of the plate.

$$q_{conv} = h\left(T_s - T_{air}\right) \qquad \text{Eqn 3}$$

Set **Eqn 1** equal to **Eqn 2** and solve for **T_s**:

$$T_s = T_{air} + \frac{\alpha\, q_{max}}{h} \qquad \text{Eqn 4}$$

Plug numbers into **Eqn 3** to answer the question:

T_s	38.72	^oC

Verify: The assumptions cannot be verified from the information in the problem statement alone.

Answers :

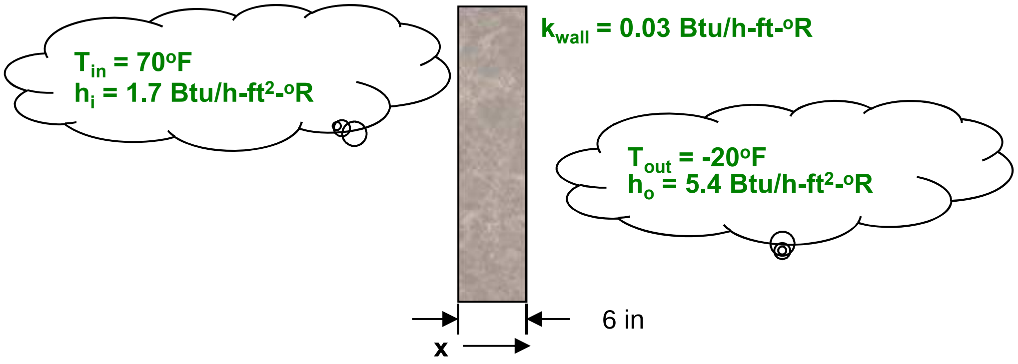
T_s	38.7	^oC

4B-2 Heat Transfer Through the Wall of a House 4 pts

The walls of a well-insulated home in the U.S. are about **6 in** thick and have a **thermal conductivity** of **0.03 Btu/f-ft-^{o}R**. A comfortable indoor **temperature** is **70^{o}F** and on a cold day the outdoor **temperature** is **-20^{o}F**.
The **convection heat transfer coefficient** on the inside surface of the wall is **1.7 Btu/h-ft^{2}-^{o}R** while wind makes the **convection heat transfer coefficient** on the outside of the wall **5.4 Btu/h-ft^{2}-^{o}R**.
Calculate the steady-state heat transfer rate through the wall in **Btu/h**. Assume radiation heat losses are negligible and the **area** of the wall is **100 ft^{2}**.

Read : The key here is to recognize that, at steady state, the **convection heat transfer rate** into the wall must be equal to the rate at which heat is conducted through the wall and that must be equal to the rate at which heat is removed from the wall by convection on the outside. We can write **3 eqns** in **3 unknowns**: Newton's Law of Cooling for the inside and outside surfaces and Fourier's Law of Conduction for heat transfer through the wall. The **three unknowns** are the inside and outside wall surface **temperatures** and the heat transfer rate.

Diagram:



Given:

k =	0.03	Btu/h-ft-°R	T_{in}	70	^{o}F
L	0.5	ft	h_{in}	1.7	Btu/h-ft^{2}-°R
A	100	ft^{2}	h_{out}	5.4	Btu/h-ft^{2}-°R
			T_{out}	-20	^{o}F

Find: q = ??? Btu/h

Assumptions:

1 - The system operates at steady-state.

2 - Newton's Law of Cooling applies for convection heat transfer on both the inside and outside surfaces of the wall.

3 - The **thermal conductivity** within the wall is constant. This is a weak assumption, but it lets us approximate **dT/dx** as **ΔT/Δx**.

Equations / Data / Solve:

The key here is to recognize that, at steady state, the **convection heat transfer rate** into the wall must be equal to the rate at which heat is conducted through the wall and that must be equal to the rate at which heat is removed from the wall by convection.

Inside convection:

$$\dot{q}_i = h_i A \left(T_i - T_{wi}\right) \qquad \textbf{Eqn 1}$$

Outside convection:

$$\dot{q}_o = h_i A \left(T_{wo} - T_o\right) \qquad \textbf{Eqn 2}$$

Conduction through the wall:

$$\dot{q}_{wall} = -kA\frac{dT}{dx} \approx -kA\frac{\left(T_{wo} - T_{wi}\right)}{L} \qquad \textbf{Eqn 3}$$

Now, we have three equations in **3** unknowns: **q**, T_{wi} and T_{wo}.
We must algebraically solve the equations simultaneously for the three unknowns.

Solve **Eqn 1** for T_{wi} :

$$T_{wi} = T_i - \frac{\dot{q}}{h_i A} \qquad \textbf{Eqn 4}$$

Solve **Eqn 2** for T_{wo} :

$$T_{wo} = T_o + \frac{\dot{q}}{h_o A} \qquad \textbf{Eqn 5}$$

Replace T_{wi} and T_{wo} in **Eqn 3** using **Eqn 4** and **Eqn 5**:

$$\dot{q} = kA\frac{\left(T_{wi} - T_{wo}\right)}{L} = \frac{kA}{L}\left[\left(T_i - T_o\right) - \frac{q}{A}\left(\frac{1}{h_i} + \frac{1}{h_o}\right)\right] \qquad \textbf{Eqn 6}$$

Manipulate **Eqn 6** algebraically to get:

$$\frac{\dot{q}}{A}\left(\frac{1}{h_i} + \frac{k}{L} + \frac{1}{h_o}\right) = \left(T_i - T_o\right) \qquad \textbf{Eqn 7}$$

Solve **Eqn 7** for **q** :

$$\dot{q} = \frac{A\left(T_i - T_o\right)}{\left(\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}\right)} \qquad \textbf{Eqn 8}$$

q = 516.1 Btu/h

The positive sign of **q** indicates that heat flows in the positive x-direction, as defined in the **diagram**.

We could now evaluate T_{wi} and T_{wo}, using **Eqns 4** and **5**, but it is not required.

T_{wi} 67.0 °F **T_{wo} -19.0 °F**

Verify: The first two assumptions cannot be verified, but we can shed some light on the last assumption.

The wall temperature varies from **-19°F** on the outside to **67°F** on the inside. This seems like a wide range of temperatures. I was <u>not</u> able to find data specific to this temperature range, but at higher temperatures, I found data that indicated the **thermal conductivity** of wood changed by about **20%** over a range of **100°C** <u>above room temperature</u>. This leads me to believe the answer below may <u>not</u> be accurate to **2 significant digits**.

Answers : **q = 520 Btu/h**

4B-3 Surface Temperature of a Spacecraft 4 pts

The International Space Station (**ISS**) absorbs solar radiation and radiates heat into space. At steady-state, the radiation emitted is equal to the solar energy absorbed.

The surface of the **ISS** has an **emissivity** of 0.74 and an **absorptivity** of 0.25 for solar radiation. Incident solar radiation on the **ISS** is 1500 W/m^2. Determine the surface **temperature** of the **ISS** at steady-state.

Read : The key to solving this problem is to recognize that solar radiation is incident on the spacecraft and the spacecraft radiates heat to deep space, which is at an average **temperature** of **3 K**.

Diagram:

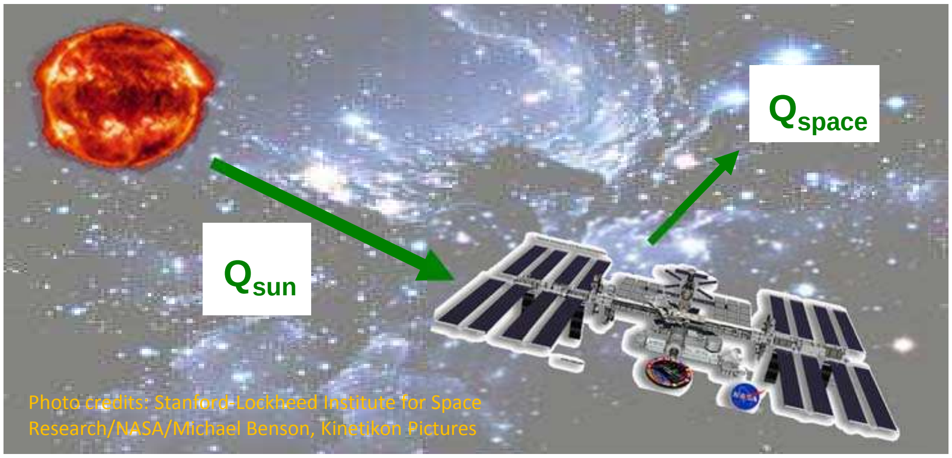


Photo credits: Stanford-Lockheed Institute for Space Research/NASA/Michael Benson, Kinetikon Pictures

Given:

Q_{sun}	1500	W/m^2	T_{space}	3	K
α	0.25		σ	5.67E-08	W/m^2-K^4
ε	0.74				

Find: $T_{surface}$??? ^{o}C

Assumptions:

1 - Incident solar radiation on the spacecraft is uniform.
2 - The spacecraft radiates thermal energy to its surroundings which are at an average **temperature** of **3 K**.

Equations / Data / Solve:

The spacecraft absorbs **30%** of the incident radiation from the sun, because:

$$Q_{in} = \alpha Q_{sun}$$ **Eqn 1** Q_{in} 375 W/m^2

The space craft radiates to deep space and deep space radiates to the spacecraft as well.

$$\underset{net}{Q_{out}} = \varepsilon \sigma \left(T^4_{surface} - T^4_{space} \right)$$ **Eqn 2**

When the **temperature** on the surface of the spacecraft reaches a steady-state value, there will be no net amount of **heat transfer** to or from the surface :

$$Q_{in} = Q_{\substack{out\\net}} \qquad \textbf{Eqn 3}$$

Next, we can combine Eqns 2 and 3 and solve for the surface temperature of the spacecraft.

$$T_{surface} = \left[T_{space}^4 + \frac{Q_{in}}{\varepsilon\,\sigma}\right]^{1/4} \qquad \textbf{Eqn 4}$$

$T_{surface}$	**307.47**	**K**
$T_{surface}$	**34.32**	**°C**

Verify: None of the assumptions can be verified from the data given in the problem statement.

Answers :

$T_{surface}$	**34**	**°C**

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4C-1 Application of the 1st Law to a Stone Falling Into Water 5 pts

A circus performer drops a cannonball with a **mass** of **50 kg** from a platform **12 m** above a drum containing **25 kg** of **water**. Initially, the cannonball and the **water** are at the same **temperature**, **state1**.

Calculate **ΔU**, **ΔE_{kin}**, **ΔE_{pot}**, **Q** and **W** for each of the following changes of state and for the entire process.

a.) From **state 1** until the cannonball is about to enter the **water**, **state 2**.

b.) From **state 2** until the instant the cannonball comes to rest on the bottom of the drum, **state 3**.

c.) From **state 3** until heat has been transferred to the surroundings in such an amount that the cannonball and **water** in the drum have returned to their initial **temperature**, **state 4**, **$T_4 = T_1$**. Assume **$g = 9.8066\ m/s^2$**.

Read : Choose the combination of the stone and the water as the system.

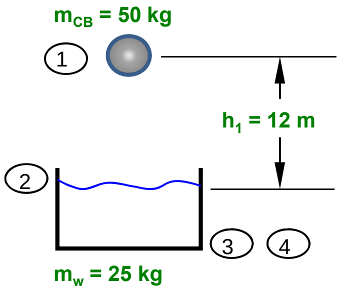
In **step 1-2**, if no friction or heat transfer exist, potential energy is converted into kinetic energy.

In **step 2-3**, if the water has negligible depth, kinetic energy is converted into internal energy by friction between the stone and the water.

In **step 3-4**, heat transfer from the system to the surroundings reduces the internal energy of the system back to its initial value.

Given:	**$T_1 = T_4$**			**Find:**	**ΔU**	**???**	**kJ**
	h_1	**12**	**m**		**ΔE_{kin}**	**???**	**kJ**
	m_w	**25**	**kg**		**ΔE_{pot}**	**???**	**kJ**
	m_{CB}	**50**	**kg**		**Q**	**???**	**kJ**

Diagram:



Assumptions:

1 - Friction between the air and the stone is negligible.

2 - The air and the stone are at the same **temperature**.

3 - The depth of the water is very small, compared to **h_1**.

4 - **g** **9.8066** **m/s^2**

Equations / Data / Solve:

The starting point for this problem is the integral form of the1st Law :

$$Q - W = \Delta U + \Delta E_{kin} + \Delta E_{pot} \qquad \textbf{Eqn 1}$$

Step 1-2 As the stone falls through the air, it experiences some air friction, but we can assume that this is negligible. Consequently, there is no change in the **temperature** or **internal energy** of the stone. If we further assume that the air and stone are at the same **temperature**, then no heat transfer occurs during **step 1-2**. Finally, if we consider the stone and the water to be our system, then no **work** has crosses the system boundary either.

$\Delta U_{12} = 0$ **Eqn 2** $Q_{12} = 0$ **Eqn 3**

$W_{12} = 0$ **Eqn 4**

This allows us to simplify the 1st Law to : $\Delta E_{kin} + \Delta E_{pot} = 0$ **Eqn 5**

Next, we can evaluate ΔE_{pot} from its definition. $\Delta E_{pot} = m \frac{g}{g_c} \Delta z$ **Eqn 6**

When we apply this equation to our problem, **$\Delta z = h_1 = -12$ m**. So we can now plug values into **Eqn 3**.

g_c	1	kg-m/N-s^2	ΔE_{pot}	-5884.0	J

Now, we can use **Eqn 2** to evaluate ΔE_{kin} :	ΔE_{kin}	5884.0	J

Step 2-3 Apply the 1st Law, **Eqn 1**, to a process from **State 2** to **State 3**, again using the stone and the water as our system.

Assume that the depth of the water is negligible so that: $\Delta E_{pot} = 0$ **Eqn 7**

Because the water and stone are at the same **temperature**, no heat transfer occurs, therefore :

$Q_{23} = 0$ **Eqn 8**

Just as in **Step 1-2**, no work crosses the boundary of the system (the stone and the water) :

$W_{23} = 0$ **Eqn 9**

Now, use **Eqns 7 - 9** to simplify the 1st Law, **Eqn 1** to : $\Delta U + \Delta E_{kin} = 0$ **Eqn 10**

Eqn 10 tells us that in **Step 2-3**, all of the kinetic energy of the stone is converted into internal energy in both the stone and the water.

Since the kinetic energy of the stone in **state 1** is **zero**:	$E_{kin,1}$	0	J
We conclude from part (a) that:	$E_{kin,2}$	5884.0	J
After the stone hits the bottom of the tank, it has **zero** kinetic energy:	$E_{kin,3}$	0	J
Therefore, for **Step 2-3**:	ΔE_{kin}	-5884.0	J
Plug this value of ΔE_{kin} into **Eqn 10** to get:	ΔU	5884.0	J

Step 3-4 Apply the 1st Law, **Eqn 1**, to a process from **State 3** to **State 4**, again using the stone and the water as our system.

In **Step 3-4**, there is no change in either the kinetic or the potential energy of the system. No **work** crosses the boundary of the system. Therefore :

$\Delta E_{kin} = 0$ **Eqn 11** $\Delta E_{pot} = 0$ **Eqn 12**

$W_{34} = 0$ **Eqn 13**

This allows us to simplify the 1st Law to : $Q_{34} = \Delta U$ **Eqn 14**

Because in **Step 3-4** the system returns to its original **temperature**:

$$\Delta U_{34} = U_4 - U_3 = -\left(U_3 - U_1\right) = -\left(U_3 - U_2\right) = -\Delta U_{23}$$ **Eqn 15**

ΔU	**-5884.0**	**J**

Finally, we can plug this value for **ΔU** back into **Eqn 14** to evaluate **Q_{34}**:

Q_{34}	**-5884.0**	**J**

Note that the negative value for **Q_{34}** means that heat is transferred from the system to the surroundings.

Step 1-4 We can determine the values **Q**, **W**, **ΔU**, **ΔE_{kin}** and **ΔE_{pot}** for the process from **state 1** to **state 4** by adding the results from parts (a) through (c).

Q_{14}	**-5883.96**	**J**
W_{14}	**0.0**	**J**
ΔU_{14}	**0.0**	**J**

ΔE_{kin}	**0.0**	**J**
ΔE_{kin}	**-5883.96**	**J**

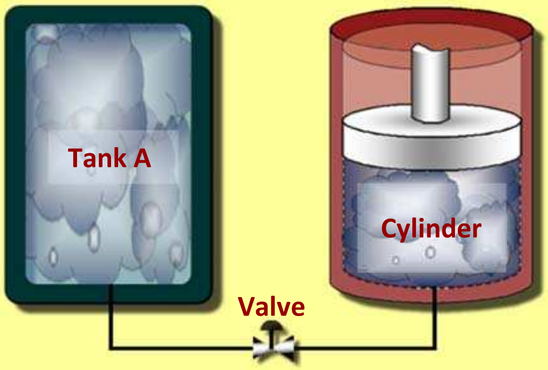
Verify: The assumptions made in this problem solution cannot be verified.

Answers :

	Q (J)	W (J)	ΔU (J)	ΔE_{kin} (J)	ΔE_{pot} (J)
a.) 1-2	0	0	0	5884	-5884
b.) 2-3	0	0	5884	-5884	0
c.) 3-4	-5884	0	-5884	0	0
d.) 1-4	-5884	0	0	0	-5884.0

4C-2 Equilibration of a Tank and a Piston-and-Cylinder Device 5 pts

Each of two vessels contains of **steam** at a different **temperature** and **pressure**. Vessel **A** is a rigid tank with a **volume** of **0.9 m^3** and vessel **B** is a piston-and-cylinder device that holds **0.7 m^3** of **steam**.
They are connected by a pipe with a closed valve in the line. Initially, tank **A** contains saturated steam at **150 kPa** while cylinder **B** contains superheated steam at **350°C** and **400 kPa**.



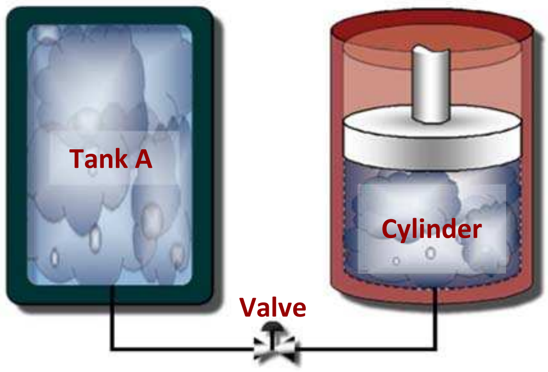
When the valve is opened, the **steam** in the two vessels is allowed to come to equilibrium.
a.) Determine the **mass** of **steam** in each vessel before the valve is opened: m_{A1} and m_{B1}.

b.) If the equilibrium **temperature** is **T_2 = 240°C**, calculate **Q** and **W** for the equilibration process.

Read : The key aspect of this problem is whether ANY water remains in the cylinder, **B**, at equilibrium. If there is water left in **B** at the final state, it will exist at T_2 and $P_2 = P_{B1}$ because the piston would still be "floating". The other key is that this is a closed system, so the **mass** of water in the entire system remains constant. We can use the Steam Tables and the given initial **volumes** to answer **part (a)**. In **part (b)**, the **work** is done at constant **pressure** as the piston descends. So it is not difficult to compute. Finally, solve the 1st Law to determine **Q**. This is possible because we know the initial and final states and the **work** !

Diagram:

X_{A1} = 1 kg vap/kg
P_{A1} = 150 kPa
V_A = 0.9 m^3
T_{A2} = 240 °C



T_{B1} = 350 °C
P_{B1} = 400 kPa
V_{B1} = 0.7 m^3
T_{B2} = 240 °C

Given:

P_{A1}	150	kPa	V_{B1}	0.7	m^3
V_A	0.9	m^3	T_{B1}	350	°C
x_{A1}	1	kg vap/kg	P_{B1}	400	kPa
T_2	240	°C			

Find: a.) m_{A1} ??? kg b.) Q ??? kJ
m_{B1} ??? kg W ??? kJ

Assumptions:
1 - - The initial and final states are **equilibrium** states.
2 - - The process is a **quasi-equilibrium** process.

Equations / Data / Solve:

Part a.) We can determine $\mathbf{m_{A1}}$ because we know the **volume** of the tank and we can look up the **specific volume** of the saturated vapor that it contains.

$$m_{A1} = \frac{V_A}{\hat{V}_{A1}}$$

Eqn 1

NIST WebBook: V_{A1} **1.1593** m^3/kg

m_{A1}	0.7763	kg

We can use the same approach to determine $\mathbf{m_{B1}}$, but first we must determine its state.

At **350 kPa**, $\mathbf{T_{sat} = 133.52^oC}$. Since $\mathbf{T_{B1} > T_{sat}}$, tank **B** initially contains superheated vapor.

NIST WebBook: V_{B1} **0.71396** m^3/kg

m_{B1}	0.9805	kg

Part b.) In **part b**, there are two possibilities. At equilibrium, either **B** contains some water or it is completely empty.

Case 1 - **B** is not empty: $\mathbf{P_{B2} = P_{B1}}$ because the piston is still floating.

Case 2 - **B** is empty: $\mathbf{V_{B2} = 0}$ and ALL of the water is in tank **A**.

Let's test **Case 1** first. Since $\mathbf{T_2 > T_{sat}}$ at **500 kPa**, the water would still be superheated vapor and the **specific volume** would be:

V **0.58314** m^3/kg

Therefore the total **volume** occupied by this superheated vapor would be:

$$V_2 = (m_{A1} + m_{B1})\hat{V}_2$$

Eqn 2 V_2 **1.0245** m^3

Since this **volume**, which the total **mass** of water in the system occupies at $\mathbf{P_{B1}}$, is greater than the **volume** of tank **A**, we can conclude that all of the water could not fit into tank **A** at $\mathbf{P_{B1}}$. If $\mathbf{P_2}$ were less than $\mathbf{P_{B1}}$, the water would occupy even more **volume** and again would not fit into tank **A**. Some water must remain in the cylinder and, therefore, $\mathbf{P_{B2} = P_{B1}}$.

Therefore: $V_{B2} = V_2 - V_{A1} =$ **0.1245** m^3

Calculate the **PV** or **boundary work** from:

$$W = \int_{V_1}^{V_2} P\,dV$$

Eqn 3

But, since this process is isobaric:

$$W = P\,(V_2 - V_1)$$

Eqn 4

W	-230.2	kJ

Finally, we need to apply the 1st Law to determine **Q**. Use all of the water in both vessels as the system.

$$Q - W = \Delta U$$

Eqn 5

Or:

$$Q = U_2 - U_1 + W = m_{tot}\hat{U}_2 - m_{A1}\hat{U}_{A1} - m_{B1}\hat{U}_{B1} + W$$

Eqn 6

Use the **NIST WebBook** and the **ASHRAE Convention** to determine all of the **specific internal energies**.

m_{tot}	**1.7568**	**kg**
U_2	**2651.0**	**kJ/kg**

U_{A1}	**2519.2**	**kJ/kg**
U_{B1}	**2884.4**	**kJ/kg**
ΔU	**-126.6**	**kJ**
Q	**-356.8**	**kJ**

Verify: None of the assumptions can be verified from the data given in the problem statement.

Answers :

a.)

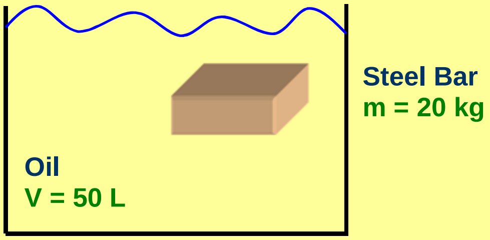
m_{A1}	**0.776**	**kg**
m_{B1}	**0.980**	**kg**

b.)

W	**-230**	**kJ**
Q	**-357**	**kJ**

4C-3 Quenching a Steel Bar in Oil 4 pts

A hot **steel** bar weighing **20 kg** is submerged in an insulated bath holding **50 L** of heavy oil. The **steel** bar and the **oil** are allowed to equilibrate thermally without exchanging heat with the surroundings.



Initially, the **steel** bar and the **oil** are at **700°C** and **25°C**, respectively. Determine the final **temperature** of the **steel** bar and the **oil**.

Data: **steel**: **ρ = 8000 kg/m^3**, **C_V = 0.42 kJ/kg-K**, **oil**: **ρ = 890 kg/m^3**, **C_V = 2.1 kJ/kg-K**.

Read : The easiest way to solve this problem is to choose the entire contents of the tank, both the oil and the steel, as our system.

If we assume that this system is adiabatic and does not have any **work** interactions with its surroundings, then the **internal energy** of the system must remain constant as the steel bar cools and the oil becomes warmer.

If we further assume that the steel and oil are incompressible, then this is a constant **volume** process. For solids and liquids it is often reasonable to assume the **heat capacity** is a constant over a fairly wide **temperature** range. The only unknown left in the **1st Law** is the final system **temperature** !

Diagram: The diagram in the problem statement is adequate.

Given:

m_{steel}	20	kg	ρ_{steel}	8000	kg/m^3
$T_{steel,1}$	700	°C	$C_{V,steel}$	0.42	kJ/kg-K
V_{oil}	0.05	m^3	ρ_{oil}	890	kg/m^3
$T_{oil,1}$	25	°C	$C_{V,oil}$	2.1	kJ/kg-K

Find: T_2 = ??? °C

Assumptions:

1 - - Steel and oil have constant **heat capacities**.
2 - - No heat is exchanged with the surroundings by either the steel or the oil.
3 - - Steel and the oil are both incompressible, so this process is a constant **volume** process.

Equations / Data / Solve:

We begin by writing the 1st Law and we choose as our system the oil and the steel.

$$Q - W = \Delta U = m_{steel}\left(\hat{U}_{2,steel} - \hat{U}_{1,steel}\right) + m_{oil}\left(\hat{U}_{2,oil} - \hat{U}_{1,oil}\right) = 0 \qquad \text{Eqn 1}$$

By cleverly selecting our system, **Q = 0** and **W = 0**. This makes the solution <u>simpler</u>.

$$m_{oil} = \rho_{oil} V_{oil}$$ **Eqn 2**

Therefore: m_{oil} **44.5** **kg**

Because both oil and steel are assumed to be incompressible with constant **heat capacities**:

$$\Delta \hat{U} \approx \hat{C}_V \Delta T$$ **Eqn 3**

$$m_{steel} \hat{C}_{V,steel} \left(T_2 - T_{1,steel}\right) + m_{oil} \hat{C}_{V,oil} \left(T_2 - T_{1,oil}\right) = 0$$ **Eqn 4**

Now, solve for T_2:

$$\left(m_{steel} \hat{C}_{V,steel} + m_{oil} \hat{C}_{V,oil}\right) T_2 = m_{steel} \hat{C}_{V,steel} T_{1,Cu} + m_{oil} \hat{C}_{V,oil} T_{1,oil}$$ **Eqn 5**

$$T_2 = \frac{m_{steel} \hat{C}_{V,steel} T_{1,steel} + m_{oil} \hat{C}_{V,oil} T_{1,oil}}{m_{steel} \hat{C}_{V,steel} + m_{oil} \hat{C}_{V,oil}}$$ **Eqn 6**

T_2	**80.67**	**°C**

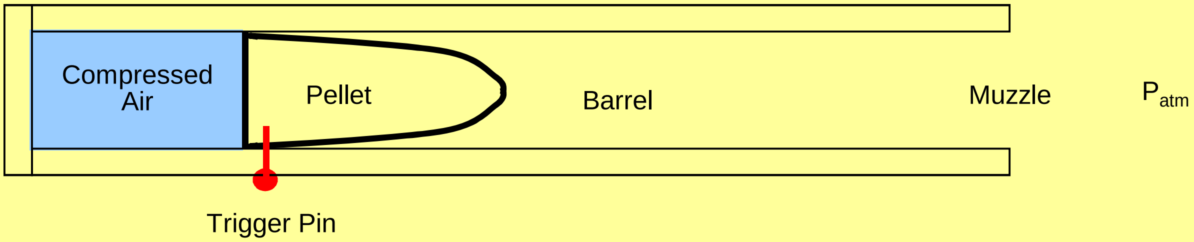
Verify: None of the assumptions can be verified from the data given in the problem statement.
Assumptions **1** & **3** are very nearly true for solids over the temperature range covered in this problem.
Assumption **2** could be made nearly true with sufficient insulation.

Answers :

T_2	**80.7**	**°C**

4C-4 Muzzle Velocity of a Pellet Fired From an Air Gun 6 pts

An **air**-powered pellet gun uses **1.5 mL** of compressed **air** at **1.2 MPa** and **29^{o}C** in a small tank to propel a projectile. Assume the pellet, with a **mass m_p = 0.02 kg**, seals against the walls of the cylinder and behaves like a piston.



The pellet is held by a pin until the trigger is pulled, removing the pin and releasing the pellet. Assume the **air** expands isothermally as the pellet moves along the barrel of the gun and the **pressure** in the barrel drops all the way to ambient **pressure**, **100 kPa**, just as the bullet leaves the gun barrel. Estimate…

a.) The **mass** and **volume** of the **air** in the cylinder as the pellet reaches the end of the barrel
b.) The **work** done by the **air** within the cylinder on the pellet and
the **work** done ON the ambient **air** (outside the gun) by the bullet
c.) The **velocity** of the bullet when it leaves the gun barrel (muzzle velocity)
Boldly assume that the process is a quasi-equilibrium process.

Read : - We must assume that the process is a quasi-equilibrium process. This is not a great assumption, but it does yield a reasonable 1st approximation of the muzzle velocity of the bullet.

- We can use the Generalized Compressibility EOS to show that, despite the **molar volume**, the gas behind the bullet actually behaves as an ideal gas. This fact allows us to solve the problem using the Ideal Gas EOS.

- We can determine the **work** done on the bullet by the air behind it using the relationship for **boundary work** done by an ideal gas as it expands isothermally.

- The bullet does **work** on the surrounding air in a constant **pressure** process. So, we can evaluate this **work** term using the formula for isobaric compression of an ideal gas.

- Finally, we can apply the 1st Law to the bullet. There is no heat exchanged and no change in the internal energy or potential energy of the bullet. The only remaining terms are the two **work** terms we already know how to determine and the change in kinetic energy. The initial **velocity** is **zero**, so the only unknown left in the 1st Law equation is the final **velocity** of the bullet as it leaves the barrel of the gun !

Diagram: The diagram in the problem statement is adequate.

Given:	V_1	1.5E-06	m^3	P_2	100	kPa
	P_1	1200	kPa			
	T_1	29	^{o}C	T_c	132.5	K
	m_B	0.02	kg	P_c	3770	kPa
Find:	V_2	???	m^3	$W_{on\ surr}$	???	J
	$W_{on\ bullet}$	???	J	v_2	???	m/s

Assumptions:

- For purposes of computing the **work** done on the bullet, you may treat the air inside the cylinder as an ideal gas. This is not entirely accurate because the initial **pressure** is so high.

- Assume that the air in the barrel is initially in an equilibrium state.

- Assume that the air in the barrel as the bullet leaves the gun is also in an equilibrium state.

- Assume the process is isothermal.

- For estimation purposes, assume that the process is a quasi-equilibrium process. This assumption will yield the maximum muzzle velocity that the bullet could attain.

- Assume that the air within the system is a closed system until the bullet leaves the gun.

Part c.)

- If the **temperature** of the bullet remains constant, then its internal energy does not change.

- Changes in the gravitational potential energy of the bullet are negligible, especially if the gun is fired horizontally !

- Heat transfer to or from the bullet is negligible if the process is isothermal.

Equations / Data / Solve:

Part a.) Begin by using the initial state to determine the number of moles of air inside the barrel. This remains constant until the bullet leaves the gun and that is the time interval in which we are interested.

Use the Generalized Ccompressibility EOS :

$$z = \frac{P\tilde{V}}{RT} \qquad \textbf{Eqn 1}$$

or :

$$n = \frac{PV}{zRT} \qquad \textbf{Eqn 2}$$

Reduced **temperature** and **pressure** are required in order to use the compressibility charts to determine the **compressibility**, **z** :

$$T_R = \frac{T}{T_c} \qquad \textbf{Eqn 3}$$

$$P_R = \frac{P}{P_c} \qquad \textbf{Eqn 4}$$

R	8.314	**J/mol-K**	z_1	**1**	(Because $\mathbf{P_c}$ is so high)
T_{R1}	**2.28**				
P_{R1}	**0.318**		**n**	**7.17E-04**	**moles**
T_{R2}	T_{R1}		V_1	**2.1E-03**	m^3/mol
P_{R2}	**0.027**		z_2	**1**	(Because $\mathbf{P_c}$ is so high)

Since **Z = 1** throughout the process, it is safe to treat air as an ideal gas throughout this process.

This is a surprise since the **molar volume** is **2.5 L/mol** and that is less than **5 L/mol**.

The process is assumed to be isothermal and we discovered that the air could be treated as an ideal gas. Therefore:

$$P_1 V_1 = P_2 V_2 \qquad \textbf{Eqn 5}$$

or :

$$V_2 = V_1 \frac{P_1}{P_2} \qquad \textbf{Eqn 6}$$

$$\boxed{V_2 \quad 1.80\text{E-}05 \quad m^3}$$

m = n MW **Eqn 7**

MW_{air} 28.97 **g/mole**

$$\boxed{m \quad 0.0208 \quad g}$$

Part b.) Next, we can calculate the **work** done by the air, on the bullet using the **work** equation derived for isothermal processes like this one:

$$W = \int_{V_1}^{V_2} P\,dV = P_1 V_1 \mathrm{Ln}\left(\frac{V_2}{V_1}\right) = P_1 V_1 \mathrm{Ln}\left(\frac{P_1}{P_2}\right) \qquad \textbf{Eqn 8}$$

$W_{on\ bullet}$	4.473	J

The bullet does **work** on the surrounding air against a constant restraining **pressure**, P_{atm}.
Therefore:

$$W_{on\,surr} = \int_{V_1}^{V_2} P\,dV = P_{atm}\left(V_2 - V_1\right) \qquad \textbf{Eqn 9}$$

$W_{on\ surr}$	1.650	J

Part c.) In order to determine the muzzle **velocity** of the bullet, we must determine the change in the kinetic energy of the bullet as a result of the net amount of **work** done on it.

We can do this by applying the 1st Law, using the bullet as our system.

$$Q - W_{net} = \Delta U + \Delta E_{kin} + \Delta E_{pot} \qquad \textbf{Eqn 10}$$

The net **work** is the difference between the **work** done on the surroundings and the **work** done by the bullet.

$$W_{net} = W_{on\ surr} + W_{on\ bullet} = -\Delta E_{kin} = -\frac{m_B}{2\,g_c}\left(v_2^2 - v_1^2\right) = -\frac{m_B}{2\,g_c} v_2^2 \qquad \textbf{Eqn 11}$$

W_{net} -2.823 J

Solving for v_2 :

$$v_2 = \sqrt{-W_{net}\frac{2\,g_c}{m_B}} \qquad \textbf{Eqn 12}$$

v_2	16.80	m/s

Verify: The ideal gas assumptions were shown to be valid using the Generalized Compressibility EOS.

The remaining assumptions are reasonable, but cannot be verified using only the information given in the problem statement.

Answers :

V_2	1.80E-05	m^3
$W_{on\ bullet}$	4.47	J
$W_{on\ surr}$	1.65	J
v_2	16.8	m/s

4E-1 Isobaric Expansion of Steam in a Closed System 6 pts

Steam is contained in a piston and cylinder device with a free-floating piston. Initially, the **steam** occupies a **volume** of **0.18 m^3** at a [pressure of **500 kPa**.

The **steam** is slowly heated until the **temperature** is **400°C**, while the **pressure** remains constant. If the cylinder contains **0.65 kg** of **steam**, determine the **heat transfer** and the **work** in **kJ** for this process.

Read : We know the values of **two** intensive variables for **state 1**: **P** and **specific volume**, so we can determine the values of all other properties in this state.

We know the values of **two** intensive variables for **state 2**: **T** and **P**, so we can determine the values of all other properties in this state.

Therefore we can calculate Δ**U** directly.

We can also use the definition of **work** for an isobaric process to evaluate W_{12}.

Once we know W_{12} and Δ**U**, we can use the 1st Law to evaluate Q_{12}.

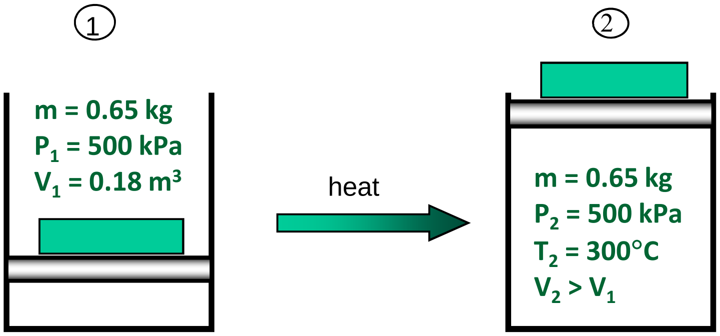
Given:

V_1	0.18	m^3
m	0.65	kg
P_1	500	kPa
P_2	500	kPa
T_2	300	°C

Find:

Q_{12}	???	kJ
W_{12}	???	kJ

Diagram:



Assumptions:

1 - Changes in kinetic and potential energies are negligible.
2 - The process is a quasi-equilibrium process.

Equations / Data / Solve:

Choose the water inside the cylinder as the system.

Apply the integral form of the 1st Law to the process:

$$Q_{12} - W_{12} = \Delta U + \Delta E_{kin} + \Delta E_{pot}$$ **Eqn 1**

If we assume that changes in kinetic and potential energies are negligible, then **Eqn 1** simplifies to :

$$Q_{12} - W_{12} = \Delta U = U_2 - U_1 = m\left(\hat{U}_2 - \hat{U}_1\right) \qquad \textbf{Eqn 2}$$

We can evaluate **W_{12}** from the definiton of **work** applied to an isobaric process.

$$W_{12} = \int_1^2 P\,dV = P\int_1^2 dV = P\left(V_2 - V_1\right) = mP\left(\hat{V}_2 - \hat{V}_1\right) \qquad \textbf{Eqn 3}$$

Let's combine **Eqns 2** and **3**:

$$Q_{12} = m\left[\left(\hat{U}_2 - \hat{U}_1\right) + P\left(\hat{V}_2 - \hat{V}_1\right)\right] = m\left(\hat{H}_2 - \hat{H}_1\right) \qquad \textbf{Eqn 4}$$

We still need to lookup the same amount of data in the Steam Tables, **V** and **H**, but the calculations are just a little bit simpler and faster using **H** than using **U**.

Before we can look up **H**, we need to determine the state of the water in the cylinder.

Calculate **V_1** from : $\tilde{V}_1 = \dfrac{V_1}{m_1}$ **V_1** **0.2769** **m^3/kg**

At **500 kPa** :

$V_{sat\ liq}$ **0.0010925** **m^3/kg**
$V_{sat\ vap}$ **0.37481** **m^3/kg**

Since **$V_{sat\ liq} < V_1 < V_{sat\ vap}$**, we conclude that a saturated mixture exists in the cylinder at **state 1**.

So, we must next evaluate the **quality** of the steam.

$$x = \frac{V - V_{sat\,liq}}{V_{sat\,vap} - V_{sat\,liq}} \qquad \textbf{Eqn 5}$$

x **0.7381** **kg vap/kg**

Then, we can use the **quality** to evaluate the **specific enthalpy** :

$$\hat{H}_1 = x_1\hat{H}_{sat\,vap} + \left(1 - x_1\right)\hat{H}_{sat\,liq} \qquad \textbf{Eqn 6}$$

$H_{sat\ liq}$ **640.09** **kJ/kg**
$H_{sat\ vap}$ **2748.1** **kJ/kg**

H_1 **2196.0** **kJ/kg**

Next, we need to determine the **phases** present in **State 2**. We can do this by comparing **T_2** to **$T_{sat}(P_2)$**.

In the saturation pressure table of the Steam Tables we find: **$T_{sat}(P_2)$** **151.8** **°C**

Because **$T_2 > T_{sat}(P_2)$**, **state 2** is a superheated vapor.

From the **NIST Webbook** or the Superheated Tables of the Steam Tables we obtain the following data:

V_2 **0.52261** **m^3/kg** **H_2** **3064.6** **kJ/kg**

Now, we can plug values back into **Eqns 3** and **4** to evaluate **Q_{12}** and **W_{12}**:

W_{12} **79.8** **kJ** **Q_{12}** **564.6** **kJ**

Verify: The assumptions made in this problem solution cannot be verified.

Answers :

W_{12}	**80**	**kJ**	**Q_{12}**	**565**	**kJ**

4F-1 Heat and Work for a Cycle Carried Out in a Closed System 6 pts

A gas in a piston and cylinder device undergoes **three** quasi-equilibrium processes to complete a thermodynamic cycle. The following information is known about the **three** steps that make up the cycle.

Process 1-2: constant volume, **V = 37 L**, **ΔU_{12} = 31.6 kJ**

Process 2-3: expansion with **PV = constant** and **ΔU_{23} = 0**

Process 3-1: constant pressure, **P = 155 kPa**, **W_{31} = -15.1 kJ**

Assume changes in kinetic and potential energies are negligible.

a.) Sketch the path for the cycle on a **PV Diagram**
b.) Calculate the **total boundary work** for the cycle in **kJ**
c.) Calculate **Q_{23}** in **kJ**
d.) Calculate **Q_{31}** in **kJ**
e.) Determine whether this cycle is a power cycle or a refrigeration/heat-pump cycle and calculate the **COP** or **thermal efficiency**.

Read : Work your way around the cycle, step by step. Sum the boundary **work** for the **three** steps to determine **W_{cycle}**.
Write and solve the 1st Law for **steps 2-3** and **3-1** to determine **Q_{23}** and **Q_{31}**.
Write and solve the 1st Law for **steps 2-3** and **3-1** to determine **Q_{12}** and sum the **Q**'s to evaluate **Q_{cycle}**.
Check your work using **$Q_{cycle} = W_{cycle}$** because **$\Delta U_{cycle} = 0$**.
Power cycle is **$W_{cycle} > 0$**. Refrigeration or HP cycle of **$W_{cycle} < 0$**.

Given:

Step 1-2:	$V_1 = V_2$	0.037	m^3	**Step 3-1:**	$P_3 = P_1$	155	kPa
	$U_2 - U_1$	31.6	kJ		W_{31}	-15.1	kJ
Step 2-3	$P_2 V_2 = P_3 V_3$						
	$U_3 = U_2$						

Diagram: See the answer to **part (a)**, below.

Find:

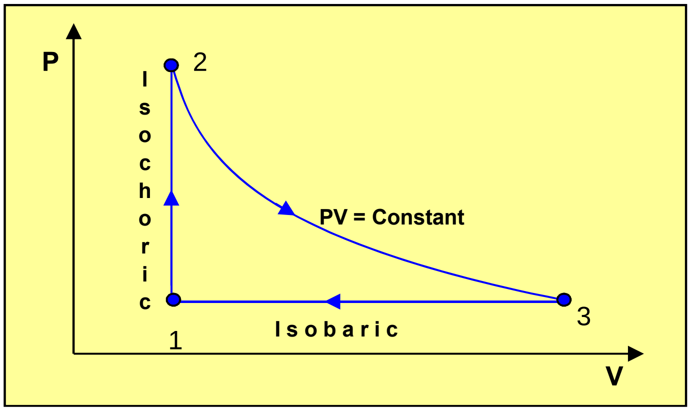
a.)	**Sketch the cycle on a PV Diagram.**			**d.)**	Q_{31}	???	kJ
b.)	W_{cycle}	???	kJ	**e.)**	**Power or Refrigeration Cycle ?**		
c.)	Q_{23}	???	kJ				

Assumptions:

1 - The gas is a closed system
2 - **Boundary work** is the only form of **work** interaction
3 - Changes in kinetic and potential energies are negligible.

Equations / Data / Solve:

Part a.)



Part b.) Since $\mathbf{W_{cycle} = W_{12} + W_{23} + W_{31}}$, we will work our way around the cycle and calculate each **work** term along the way.

Because the **volume** is constant in **step 1-2**: W_{12} 0 kJ

In **step 2-3**: **P V = C** , therefore, the definition of boundary **work** becomes:

$$W_{23} = \int_{V_2}^{V_3} P\,dV = C\int_{V_2}^{V_3}\frac{dV}{V} = C\,\mathrm{Ln}\frac{V_3}{V_2}$$ **Eqn 1**

But, we don't know V_3 ! Perhaps we can use W_{31} to detemine V_3.

Step 3-1 is isobaric, therefore, the definition of boundary **work** becomes:

$$W_{31} = \int_{V_3}^{V_1} P\,dV = P_3\left(V_1 - V_3\right)$$ **Eqn 2**

Solve this equation for V_3 :

$$V_3 = V_1 - \frac{W_{31}}{P_3}$$ **Eqn 3**

V_3 0.1344 m^3

Now, plug V_3 and $C = P_3V_3$ into **Eqn 1** to determine W_{23}:

W_{23} 26.9 kJ

Sum the work terms for the three steps to get W_{cycle}: **W_{cycle} 11.78 kJ**

Part c.) Write the 1st Law for **step 2-3**: $Q_{23} - W_{23} = U_3 - U_2 = 0$ **Eqn 4**

$Q_{23} = W_{23}$ 26.88 kJ

Part d.) Write the 1st Law for **step 3-1**: $Q_{31} - W_{31} = U_1 - U_3$ **Eqn 5**

But, $U_2 = U_3$: $Q_{31} - W_{31} = U_1 - U_3 = U_1 - U_2 = -(U_2 - U_1)$ **Eqn 6**

Solve for Q_{31} : $Q_{31} = W_{31} - (U_2 - U_1)$ **Eqn 7**

Plug in the given values : **Q_{31} -46.70 kJ**

Part e.) First, we should determine $\mathbf{Q_{12}}$ from the 1st Law:

$$Q_{12} - W_{12} = U_2 - U_1 = 0$$ **Eqn 8**

$$Q_{12} = U_2 - U_1$$ **Eqn 9**

Q_{12}	31.6	kJ

Define: $$Q_{cycle} = Q_{12} + Q_{23} + Q_{31}$$ **Eqn 10**

Q_{cycle}	11.78	kJ

Since $Q_{cycle} > 0$ and $W_{cycle} > 0$, this is a power cycle !

Notice that $\mathbf{Q_{cycle} = W_{cycle}}$ because $\Delta \mathbf{U_{cycle} = 0}$.

Thermal Efficiency is defined by :

$$\eta_{th} = \frac{W_{cycle}}{Q_H} = \frac{W_{cycle}}{Q_{12} + Q_{23}} = \frac{Q_{cycle}}{Q_{12} + Q_{23}}$$ **Eqn 11**

η_{th}	20.14	%

Verify: The assumptions made in this problem solution cannot be verified. But all of these assumptions are pretty solid.

Answers :

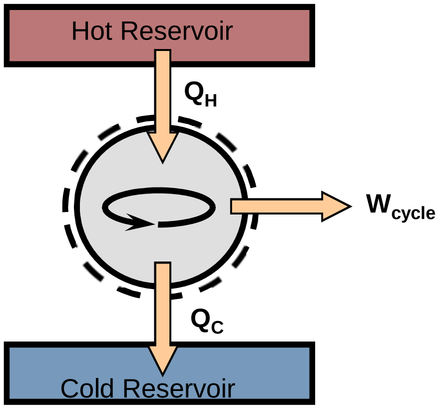
a.)	See the sketch, above.			d.)	Q_{31}	-46.7	kJ
b.)	W_{cycle}	11.8	kJ	e.)	This is a Power Cycle.		
c.)	Q_{23}	26.9	kJ		η_{th}	20.1	%

4F-2 Efficiency of a Power Cycle 2 pts

A power cycle produces **2.4 x 10^8 Btu** and rejects **7.1 x 10^8 Btu** of heat to the surroundings. Calculate the thermal efficiency of the cycle.

Read : This one is a straightforward application of the definition of the the 1st Law and the efficiency of a power cycle.

Diagram :



Given:	W_{cycle}	2.4E+08	Btu	Q_C	7.1E+08	Btu
Find:	η_{cycle}	???				

Assumption: **1 -** The cycle only exchanges heat with the **two** thermal reservoirs.

Equations / Data / Solve:

1st Law applied to the heat engine:

$$W_{cycle} = Q_H - Q_C \qquad \textbf{Eqn 1}$$

Definition of thermal efficiency :

$$\eta = \frac{W_{cycle}}{Q_H} \qquad \textbf{Eqn 2}$$

Degree of freedom analysis: **2 eqns** in **2 unknowns**: Q_H and η.

Solve **Eqn 1** for Q_H :

$$Q_H = W_{cycle} + Q_C \qquad \textbf{Eqn 3}$$

Q_H 9.5E+08 Btu

Next, plug numbers into **Eqn 2** :

η	**0.2526**

Verify assumption: The only assumption cannot be verified.

Answer questions:

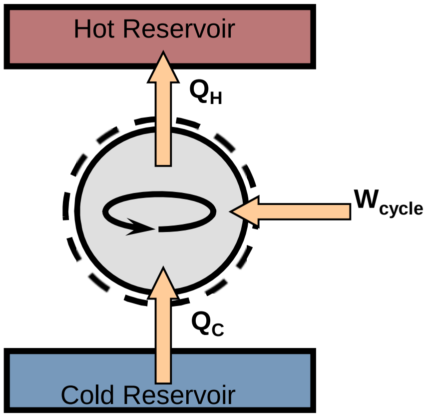
η	**25.3**	**%**

4F-3 Coefficient of Performance of a Refrigeration Cycle 3 pts

An industrial refrigerator rejects heat at a rate of **24,750 kJ/min** to the surroundings. If the refrigeration cycle has a **COP** of β = **3.3**, determine Q_C and W_{cycle}, each in **kJ/min**.

Read : This one is a straightforward application of the definition of the the 1st Law and the COP of a refrigeration cycle.

Diagram :



Given:	COP = β	3.3		**Find:**	Q_C	??? kJ/min
	Q_H	24,750	kJ/min		W_{cycle}	??? kJ/min

Assumption: - The cycle only exchanges heat with the **two** thermal reservoirs.

Equations / Data / Solve:

1st Law applied to the refrigerator:

$$Q_H = W_{cycle} + Q_C \qquad \text{Eqn 1}$$

Definition of **COP** for a refrigerator :

$$\beta = \frac{Q_C}{W_{cycle}} \qquad \text{Eqn 2}$$

Degree of freedom analysis: **2 eqns** in **2 unknowns**: Q_C and W_{cycle}.

Solve **Eqn 2** for Q_C and use the result to eliminate Q_C from **Eqn 1** :

$$Q_C = \beta\, W_{cycle} \qquad \text{Eqn 3} \qquad\qquad Q_H = W_{cycle} + \beta\, W_{cycle} \qquad \text{Eqn 4}$$

Next, solve **Eqn 4** for W_{cycle} in terms of the known quantities Q_H and β.

$$W_{cycle} = \frac{Q_H}{\beta + 1} \qquad \text{Eqn 5}$$

Plug numbers into **Eqn 5** : W_{cycle} 5755.8 kJ/min

Now, use this value for W_{cycle} and the given value of β in **Eqn 3** to evaluate Q_C :

Q_C 18994.2 kJ/min

Verify: The only assumption cannot be verified.

Answers : Q_C = 19000 kJ/min W_{cycle} = 5760 kJ/min

4F-4 Heat and Work for a Cycle Executed in a Closed System Containing R-134a 8 pts

Ammonia in a piston-and-cylinder device undergoes a 3-step thermodynamic cycle made up of the following **three** quasi-equilibrium processes.

Step 1-2: Isochoric heating from **-20°C** and **150 kPa** up to **50°C**
Step 2-3: Isothermal compression until the **quality** is **0.55 kg vap/kg**, **Q_{23} = -91.7 kJ**
Step 3-1: Adiabatic expansion

a.) Sketch the process path for this cycle on a **PV Diagram**. Put a point on the diagram for each state and label it. Be sure to include and label **all** the important features for a complete **PV Diagram** for this system

b.) Calculate **Q_{cycle}** and **W_{cycle}** in **kJ/kg**

c.) Determine whether this cycle is a power cycle or a refrigeration/heat-pump cycle? Explain your reasoning.

Read : We are given **T_1** and **P_1**, so we can determine any and all properties of the system using the Ammonia Tables. In particular, we can evaluate the **specific volume** and we know that this does not change in **step 1-2**. This gives us a **2nd** intensive property for **state 2** and allows us to evaluate all of the properties of **state 2**. We expect **$T_2 > T_1$**. **Step 2-3** is an isothermal compression to a **quality** of **x_3 = 0.55**. Because **$T_3 = T_2$**, we will be able to evaluate all of the properties of **state 3**, again using the Ammonia Tables. In each of the three steps, we know the value of either the **heat** or the **work**. **W_{12} = 0** because the process is **isochoric**. **Q_{23}** is given and **Q_{31} = 0** because the process is adiabatic. So, when we apply the 1st Law to each step, there is just one unknown and we can evaluate it. Once we know **Q** and **W** for each step, we can determine **Q_{cycle}** and **W_{cycle}** because they are the sum of the **Q**'s and **W**'s for the steps that make up the cycle, respectively.

Given :

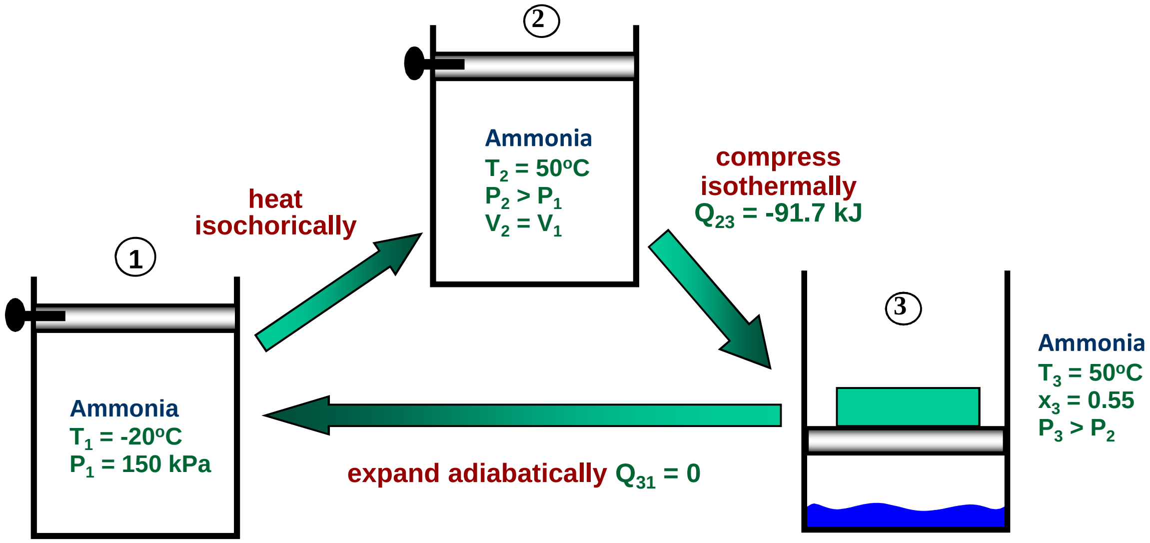
T_1	-20	°C
P_1	150	kPa
T_2	50	°C
T_3	50	°C
x_3	0.55	kg vap/kg
Q_{23}	-91.7	kJ/kg
Q_{31}	0	kJ

Find :

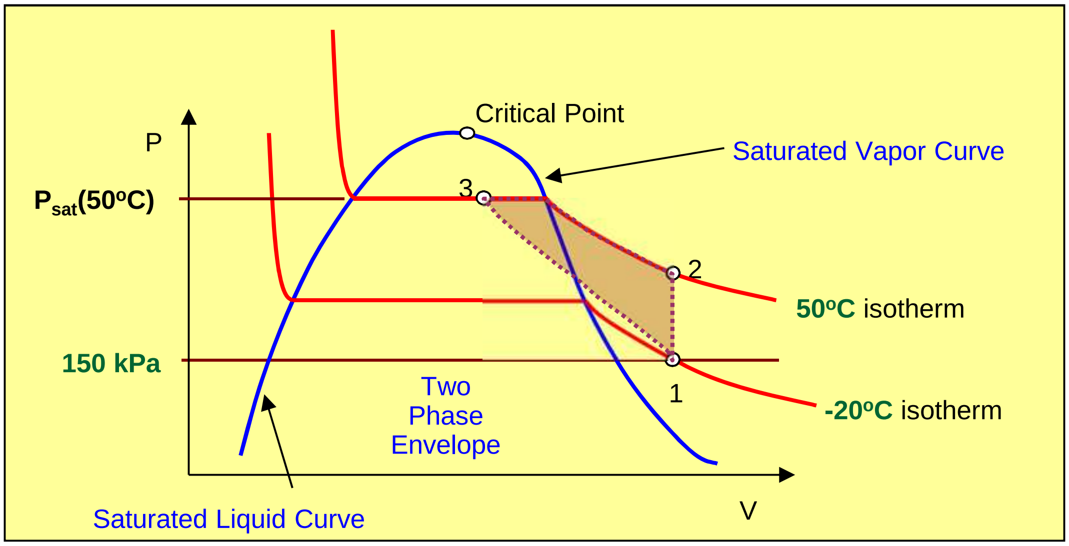
Q_{12}	???	kJ/kg
W_{23}	???	kJ/kg
W_{31}	???	kJ/kg
Q_{cycle}	???	kJ/kg
W_{cycle}	???	kJ/kg

Power or Refrigeration Cycle ?

Diagrams :



Part a.)



Assumptions :

1 - Changes in kinetic and potential energies are negligible.

2 - Boundary **work** is the only form of **work** that crosses the system boundary.

Equations / Data / Solve :

Part b.) Let's begin by writing the 1st Law for each of the **three steps** that make up the cycle, assuming that changes in potential and kinetic energies are negligible.

Step 1-2 :

$$\hat{Q}_{12} - \cancel{\hat{W}_{12}} = \Delta\hat{U} = \hat{U}_2 - \hat{U}_1$$

Eqn 1

Step 2-3 :

$$\hat{Q}_{23} - \hat{W}_{23} = \Delta\hat{U} = \hat{U}_3 - \hat{U}_2$$

Eqn 2

Step 3-1 :

$$\cancel{\hat{Q}_{31}} - \hat{W}_{31} = \Delta\hat{U} = \hat{U}_1 - \hat{U}_3$$

Eqn 3

Step 1-2 is isochoric, so no boundary **work** occurs. If we assume that boundary **work** is the only form of **work** interaction in this cycle, then $\mathbf{W_{12} = 0}$. $\mathbf{Q_{31} = 0}$ because **step 3-1** is adiabatic.

We can solve **Eqns 1 - 3** to evaluate the unknowns $\mathbf{Q_{12}}$, $\mathbf{W_{23}}$ and $\mathbf{W_{31}}$.

Step 1-2 :

$$\hat{Q}_{12} = \hat{U}_2 - \hat{U}_1$$

Eqn 4

Step 2-3 :

$$\hat{W}_{23} = \hat{Q}_{23} + \hat{U}_2 - \hat{U}_3$$

Eqn 5

Step 3-1 :

$$\hat{W}_{31} = \hat{U}_3 - \hat{U}_1$$

Eqn 6

Our next step must be to determine the value of the **specific internal energy** at **states 1**, **2** and **3** because, once we know these, we can use **Eqns 4 - 6** to evaluate the unknowns $\mathbf{Q_{12}}$, $\mathbf{W_{23}}$ and $\mathbf{W_{31}}$.

Let's begin with **state 1**. First, we must determine the phase or phases that exist in **state 1**. We can accomplish this by comparing $\mathbf{P_1}$ to $\mathbf{P_{sat}(T_1)}$.

$\mathbf{P_{sat}(T_1)}$ **190.08** **kPa**

Since $\mathbf{P_1 < P_{sat}(T_1)}$, we conclude that a superheated vapor exists in the cylinder at **state 1**.

We can determine $\mathbf{U_1}$ from the Superheated Ammonia Tables. We can also determine $\mathbf{V_1}$ because we know $\mathbf{V_2 = V_1}$ and the knowledge of this **2nd** intensive variable for **state 2** will allow us to evaluate $\mathbf{U_2}$.

$\mathbf{V_1 = V_2 =}$ **0.79779** $\mathbf{m^3/kg}$

$\mathbf{U_1}$ **1303.8** **kJ/kg**

Next, let's work on **state 2**. We know the value of **2** intensive variables, $\mathbf{T_2}$ and $\mathbf{V_2}$, and we know that if a superheated vapor expands at constant **volume**, it must still be a superheated vapor. Consequently, we can use the Superheated Ammonia Tables to determine $\mathbf{U_2}$ (and any other properties at **state 2** that we want).

At $T_2 = 50^oC$, it turns out that $V_2 = 0.79779\ m^3/kg$ falls between **100 kPa** and **200 kPa**, so we must interpolate to determine U_2.

At $T_2 = 50^oC$:

V (m^3/kg)	U (kJ/kg)	P (kPa)
1.56571	1425.2	100
0.79779	U_2	P_2
0.77679	1421.7	200

U_2	1421.8	kJ/kg
P_2	197.3	kPa

Now, let's work on **state 3**. We know the **temperature** and the **quality**, so we can determine U_3 using :

$$\hat{U}_3 = x_3\,\hat{U}_{sat\,vap} + (1 - x_3)\,\hat{U}_{sat\,liq} \qquad \text{Eqn 7}$$

We can use the Saturated Ammonia Tables to determine $U_{sat\ vap}$ and $U_{sat\ liq}$ at 50^oC and then we can plug numbers into **Eqn 7** to evaluate U_3.

$U_{sat\ liq}$	171.41	kJ/kg
$U_{sat\ vap}$	263.69	kJ/kg
U_3	222.2	kJ/kg

Now, we can go back and plug the values of the **specific internal energies** into **Eqns 4 - 6** to evaluate the unknowns Q_{12}, W_{23}, and W_{31}.

Q_{12}	118.04	kJ/kg
W_{23}	1107.94	kJ/kg
W_{31}	-1081.60	kJ/kg

Next, we need to evaluate the **specific work** and **specific heat transfer** for the entire cycle.
The **specific work** for the cycle is the sum of the **specific work** for each **step**.
The **specific heat transfer** for the cycle is the sum of the **specific heat transfer** for each **step**.

Q_{cycle}	26.34	kJ/kg
W_{cycle}	26.34	kJ/kg

Verify: The assumptions cannot be verified from the information in the problem statement alone.

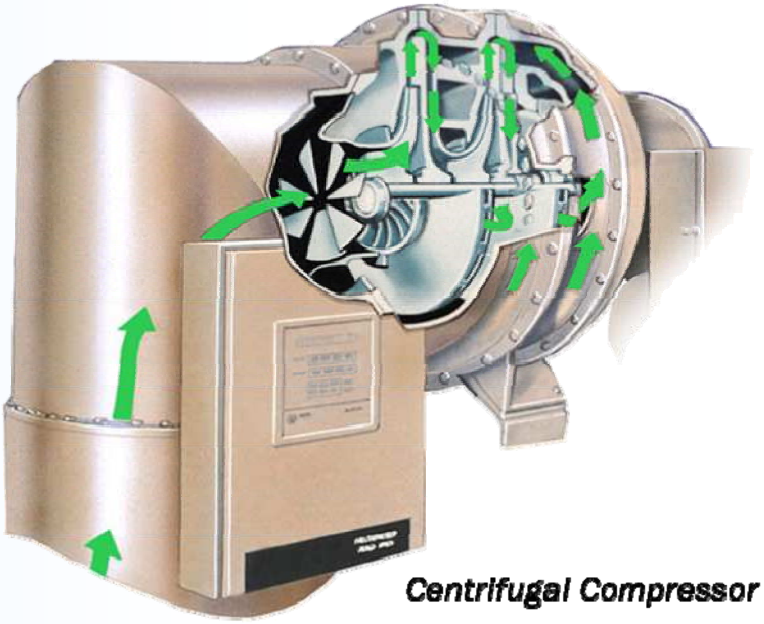
Answers : **Part a.)** See the diagram, above.

Part b.)

	Q (kJ/kg)	W (kJ/kg)
Step 1 - 2	118.0	0
Step 2 - 3	-91.7	1107.9
Step 3 - 1	0	-1081.6
Cycle	26.3	26.3

Part c.) **Because $W_{cycle} > 0$, this is a power cycle !**

Chapter 5



Centrifugal Compressor

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The First Law of Thermodynamics: Open Systems

Many important processes take place in open systems. Application of the First Law of Thermodynamics to an open system leads to the **rate** form of the First Law. The concept of **flow work** is introduced and this allows us to eliminate internal energy terms in favor of enthalpy terms in the First Law.

The next part of Chapter 5 shows how the First Law can be used to analyze a wide variety of steady-state processes including pumps, compressors, turbines, nozzles and heat exchangers.

Chapter 5 concludes with a discussion of transient or unsteady-state processes. We learn how to analyze transient uniform state, uniform flow processes.

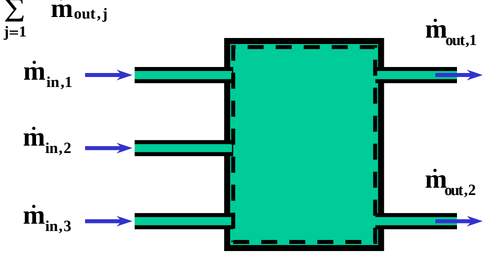
Conservation of Mass

- **Mass is neither created nor destroyed**
- **Integral Mass Balance on an Open System**

$$\Delta \mathbf{m}_{sys} = \sum_{i=1}^{\#feeds} \mathbf{m}_{in,i} - \sum_{j=1}^{\#effluents} \mathbf{m}_{out,j}$$

- **Differential Mass Balance on an Open System**

$$\frac{d}{dt}\mathbf{m}_{sys} = \sum_{i=1}^{\#feeds} \dot{\mathbf{m}}_{in,i} - \sum_{j=1}^{\#effluents} \dot{\mathbf{m}}_{out,j}$$



- Rate of change of mass in the system = total rate at which mass enters the system - total rate at which mass leaves the system
- A feed stream enters the system
- An effluent stream leaves the system
- These equations look complicated, but, in practice, they are pretty simple.

Flow Rates and Velocity

- **Specific Volume:** $\hat{V} = \frac{V}{m}$ **or:** $\hat{V} = \frac{\dot{V}}{\dot{m}}$
  - ◊ Therefore: $\dot{m} = \frac{\dot{V}}{\hat{V}}$
- **Volumetric Flow Rate:** $\dot{V} = \int v \, dA = 2\pi \int_0^R v \, dr = \langle v \rangle A_{cross}$
  - ◊ Where: <v> = average fluid velocity
    A_{cross} = cross-sectional area for flow
- **Conclusion:** $\boxed{\dot{m} = \frac{\dot{V}}{\hat{V}} = \frac{\langle v \rangle A_{cross}}{\hat{V}}}$

- The 2nd equality in the equation for volumetric flow rate only applies to conduits with circular cross-sectional area.

1^{st} Law, Open Systems

- **SISO = Single Inlet, Single Outlet**

$$\frac{d}{dt}E_{sys} = \dot{Q} - \dot{W} + \dot{m}_{in}\left[\hat{U}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right] - \dot{m}_{out}\left[\hat{U}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]$$

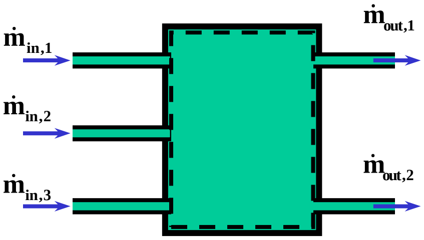
$$\frac{d}{dt}m_{sys} = \dot{m}_{in} - \dot{m}_{out}$$

- **At Steady-State <u>nothing</u> changes with respect to <u>time</u>**

$$\frac{d}{dt}m_{sys} = 0 \qquad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$\frac{d}{dt}E_{sys} = 0$$

$$\boxed{\dot{Q} - \dot{W} = \dot{m}\left[\Delta\hat{U} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right]}$$



- 1^{st} Law, SS, SISO system
- No tricks.

Flow Work

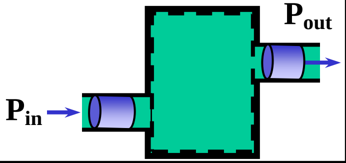
- **Three main types of work in this course:**
  - ◊ W_b is boundary work
  - ◊ W_s is shaft work
    - Associated with the rotating shafts in fluid processing equipment
  - ◊ W_{flow} is flow work or inection work
    - Work done to force fluid to flow into and out of the system
- **Evaluating Net Flow Work**

$$\dot{W}_{flow} = \dot{W}_{flow,in} + \dot{W}_{flow,out} = -\left(P_{in}\,A_{in}\right)v_{in} + \left(P_{out}\,A_{out}\right)v_{out}$$

$$\dot{W}_{flow} = \left(P\,\dot{V}\right)_{out} - \left(P\,\dot{V}\right)_{in} = \dot{m}\left[\left(P\,\hat{V}\right)_{out} - \left(P\,\hat{V}\right)_{in}\right] = \dot{m}\,\Delta\left(P\,\hat{V}\right)$$

- **1^{st} Law :** $\dot{Q} - \dot{W}_s - \dot{m}\,\Delta\left(P\,\hat{V}\right) = \dot{m}\left[\Delta\hat{U} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right]$

$$\boxed{\dot{Q} - \dot{W}_s = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right]}$$



- It is unusual to have boundary work in an open system.
- One boundary of the system would be required to move.
- This might be accomplished by a balloon or a bladder or a piston-and-cylinder device, but would be MOST unusual.
- The flow work gets combined with the change in internal energy and becomes the change in enthalpy.
- Shaft work is the work you need to put into a system to make a process proceed or it is the useful work you get out of a process.
- Flow work is not especially useful or interesting, so we split it away from the shaft work.
- In fact, the 1st Law is so often applied to SS, SISO systems that we tabulate a function that already takes flow work into account…and we call it enthalpy.
- Flow work is always present in flow systems, but we don't pay much attention to it because we cannot recover this work in a useful form.
- The bottom line is that for SS, SISO systems, enthalpy is the key property, not internal energy.
- It just saves you time, over and over again, to look up specific H instead of looking up BOTH specific U and specific V.

MIMO Processes

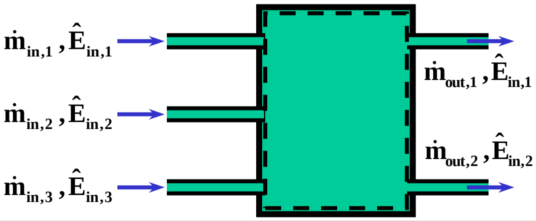
- **MIMO = Multiple Inlet, Multiple Outlet**
- **1^{st} Law :**

$$\frac{d}{dt}E_{sys} = \dot{Q} - \dot{W} + \sum_{i=1}^{\#inlets} \dot{m}_{in,i}\left[\hat{U}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right]_i - \sum_{j=1}^{\#outlets} \dot{m}_{out,j}\left[\hat{U}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]_j$$

- **1^{st} Law, SS :**

$$\dot{Q} - \dot{W} = \sum_{j=1}^{\#outlets} \dot{m}_{out,j}\left[\hat{U}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]_j - \sum_{i=1}^{\#inlets} \dot{m}_{in,i}\left[\hat{U}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right]_i$$

$$\dot{Q} - \dot{W}_S = \sum_{j=1}^{\#outlets} \dot{m}_{out,j}\left[\hat{H}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]_j - \sum_{i=1}^{\#inlets} \dot{m}_{in,i}\left[\hat{H}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right]_i$$



- MIMO processes are common, but not too common in this course.
- Mixers and splitters are the only MIMO devices we will use in this course.
- We will learn more about them later in this class.
- MIMO processes are not nearly as challenging as the long equations on this page ight make you believe.

Steady-State Processes

- **In the real world, the goal is usually to operate flow processes at steady-state.**
- **Common steady-state flow processes:**
  - ◊ Nozzles & Diffusers
  - ◊ Turbines
  - ◊ Pumps and Compressors
  - ◊ Throttling Devices
  - ◊ Heat Exchangers
  - ◊ Mixing Chambers
  - ◊ Pipes

- These are the processes we will analyze in this course.
- With these devices, engineers have built an amazing array of processes from jet airplanes to artificial hearts !

Nozzles & Diffusers

- **Common Assumptions:** $\Delta \hat{E}_{pot} \approx 0$

  $\dot{Q} \approx 0$

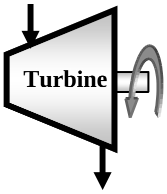
  $\dot{W}_S \approx 0$

- **1st Law :** $\boxed{\Delta \hat{H} + \frac{\Delta v^2}{2 g_C} = 0}$

- Nozzles and diffusers are usually relatively small and the fluid moves through it quickly
- Therefore, little change in potential energy occurs
- There is little area or time for significant heat transfer to occur
- Still, these assumptions are not always valid, so you need to pay attention and <u>think</u> !

Turbines

- **A turbine is a device which converts the energy of a flowing fluid into shaft work.**



- **Common Assumptions:** $\Delta \hat{E}_{pot} \approx 0$

  $\Delta \hat{E}_{kin} \approx 0$

  $\dot{Q} \approx 0$

- **1st Law :** $\boxed{\dot{W}_s = -\dot{m}\,\Delta \hat{H}}$

Changes in potential energy across a turbine are almost always negligible.

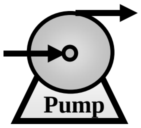
Changes in kinetic energy across a turbine are often important. Be careful !

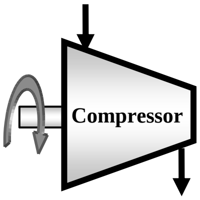
Heat losses from a turbine are OFTEN significant as we will see later. Real turbines are not adiabatic.

As always, you need to pay attention and <u>think</u> !

But, when these assumptions are valid, the 1st Law reduces to a nice, simple form.

Pumps & Compressors





- **Pumps cause liquids to flow by raising the pressure**
- **Compressors cause gases to flow by raising the pressure**
- **Common Assumptions:** $\Delta\hat{E}_{pot} \approx 0$

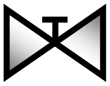
  $\Delta\hat{E}_{kin} \approx 0$

  $\dot{Q} \approx 0$
- **1**

- Changes in potential energy across a pump or compressor are almost always negligible.
- Changes in kinetic energy across a pump are negligible if the inlet and outlet pipe diameters are the same if the liquid is incompressible over the range of pressure from P_{in} to P_{out}.
- Changes in kinetic energy across a compressor are often significant because the specific volume of a gas changes a great deal as the pressure changes. Be careful !
- Heat losses from a compressor are OFTEN significant as we will see later. Real compressors are not adiabatic.
- Pumps are usually assumed to be essentially adiabatic.
- As always, you need to pay attention and think !
- But, when these assumptions are valid, the 1st Law reduces to a nice, simple form that should look familiar.

Throttling Devices

- **A common throttling device is a partially closed valve**

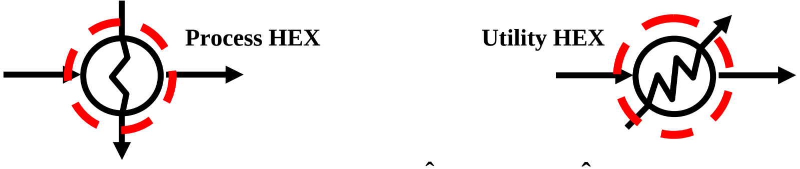


- **Other throttling devices include capillary tubes and porous plugs.**
- **Common Assumptions :** $\Delta\hat{E}_{pot} \approx 0$ $\Delta\hat{E}_{kin} \approx 0$

  $\dot{Q} \approx 0$ $\dot{W}_S \approx 0$
- **1st law :** $\boxed{\Delta\hat{H} = 0}$

- The purpose of a throttling device is to drop the pressure by causing a lot of friction.
- I usually draw a valve when there is a throttling device in a system, whether the throttling device is a valve or not !
- In a home refrigerator, the throttling device is actually a long , small-diameter tube called a capillary tube. I still draw a throttling valve because they are easy to draw and it is very clear what the purpose of the valve is: to drop the pressure !
- Throttling devices are not always particularly close to adiabatic, so Q ~0 may not always be a good assumption.
- The other assumptions are pretty solid unless the throttling device is enormous (unlikely) or the pipe diameter changes (possible) or you put a turbine of some sort in a capillary tube (silly).

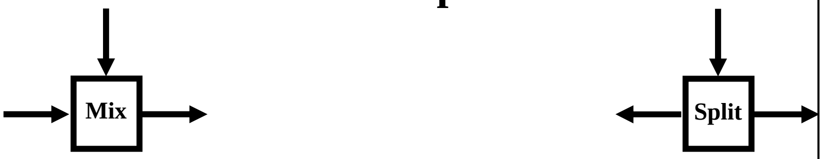
Heat Exchangers



- **Common Assumptions:** $\Delta\hat{E}_{pot} \approx 0$ $\Delta\hat{E}_{kin} \approx 0$ $\dot{Q}_{overall} \approx 0$ $\dot{W}_S \approx 0$
- **1^{st} law**
  - ◊ Hot Side: $\dot{Q} = \dot{m}_{hot}\,\Delta\hat{H}_{hot}$
  - ◊ Cold Side: $\dot{Q} = \dot{m}_{cold}\,\Delta\hat{H}_{cold}$
  - ◊ Overall: $\boxed{\dot{m}_{cold}\,\Delta\hat{H}_{cold} = \dot{m}_{hot}\,\Delta\hat{H}_{hot}}$

- In a process HEX, heat is transferred between two streams that are both important parts of an overall process
- In a utility HEX, heat is exchanged between a process stream and a utility stream that is not an important stream in the overall process
- Utility streams are usually steam, air or cooling water
- These assumptions are not always valid, so you need to pay attention and think !

Mixers & Splitters



- **Mixers and Splitters are tees in a pipe where streams join and mix or split, depending on the direction of flow into and out of the tee.**
- **Common Assumptions:** $\dot{Q} \approx 0$ $\dot{W}_S \approx 0$ $\Delta\hat{E}_{pot} \approx 0$
- **1^{st} Law:** $\boxed{\sum_{i=1}^{\#inlets} \dot{m}_{in,i}\left[\hat{H}_{in} + \hat{E}_{kin,in}\right]_i = \sum_{j=1}^{\#outlets} \dot{m}_{out,j}\left[\hat{H}_{out} + \hat{E}_{kin,out}\right]_j}$
- **Conservation of Mass:** $\sum_{i=1}^{\#inlets} \dot{m}_{in,i} = \sum_{j=1}^{\#outlets} \dot{m}_{out,j}$

- Tees in pipes are small and offer little area for heat transfer.
- The fluid spends very little time in these devices.
- So, it is generally true that Q ~ 0 and ΔE_{pot} ~ 0.
- There are no moving parts in these devices, so W_s really is zero.
- Occasionally, ΔE_{kin} ~ 0, but it is not all that common. The flow rates and pipe diameters must be carefully matched to make this true.
- Still, these assumptions are not always valid, so you need to pay attention and think !

Pipe Flow: Bernoulli Equation

- **Assumptions:** $\dot{Q}_{overall} \approx 0$ $\dot{W}_S \approx 0$
  - ◊ Incompressible fluid: $\Delta\hat{V} \approx 0$
  - ◊ No friction: $\Delta\hat{U} \approx 0$
- **1st Law:** $\cancel{\dot{Q}} - \cancel{\dot{W}_s} = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right]$

$$0 = \dot{m}\left[\cancel{\Delta\hat{U}} + \hat{V}\,\Delta P + P\,\cancel{\Delta\hat{V}} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right]$$

- **Bernoulli Equation :** $\boxed{\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z = 0}$

- Use of the Bernoulli Equation requires that the following assumptions apply:
- Adiabatic
- No shaft work
- No friction
- Incompressible fluid
- This is not often applicable because friction is often significant.

Transient Processes

- If **ANY** variable associated with the process changes over time, then it is a transient process.
- Transient processes are very complex and a computer is often required to solve the equations that describe these processes.
- We will only consider special transient processes in which the following assumptions are true.
- **Uniform Flow**
  - ◊ The properties and flow rates of all inlet and outlet streams are constant or uniform over the cross-sectional area for flow and are also constant with respect to time.
- **Uniform State**
  - ◊ The state of the mass within the system is uniform.
  - ◊ At all times, the properties of the outlet stream are exactly the same as the properties of the system at that time.

- A pot of water boiling on a stove is a transient process.
  - ◊ Although the T, P and other properties of the saturated liquid and saturated water vapor in the pot remain constant
  - ◊ The MASS of water inside the pot changes as water vapor flows out into the kitchen.

Transient Mass Balances

- **Differential Mass Balance:** $\frac{d}{dt} m_{sys} = \dot{m}_{in} - \dot{m}_{out}$
  - ◊ Integrate : $\int_{m_1}^{m_2} dm_{sys} = \int_{t_1}^{t_2} \dot{m}_{in}\, dt - \int_{t_1}^{t_2} \dot{m}_{out}\, dt$
  - ◊ Uniform Flow : $\dot{m}_{in} \neq fxn(t) \qquad \dot{m}_{out} \neq fxn(t)$

$$m_2 - m_1 = \dot{m}_{in}\left(t_2 - t_1\right) - \dot{m}_{out}\left(t_2 - t_1\right)$$

$$\boxed{\Delta m_{sys} = m_{in} - m_{out}}$$

  - ◊ MIMO : $m_2 - m_1 = \left(t_2 - t_1\right) \sum_{i=1}^{inlets} \dot{m}_{in,i} - \left(t_2 - t_1\right) \sum_{j=1}^{outlets} \dot{m}_{out,j}$

$$\boxed{\Delta m_{sys} = m_{in} - m_{out}}$$

- The uniform flow assumption allows us to easily evaluate the integrals on the right-hand side of the mass balance equation without the help of a computer.
- The key here is that the mass flow rate must be the same over the cross-sectional area for flow.
- The results are very simple.
- The boxed equations are true even if the mass flow rate varies with respect to time.

Integrating the 1st law

- **The 1st Law for Open Systems:**

$$\frac{d}{dt} E_{sys} = \dot{Q} - \dot{W}_s + \dot{m}_{in}\left[\hat{H}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right] - \dot{m}_{out}\left[\hat{H}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]$$

  - ◊ W_s is work other than flow work, usually shaft work.

- **Integrate from t_1 to t_2 :**

$$\Delta E_{sys} = Q - W_s + \int_{t_1}^{t_2} \dot{m}_{in}\left[\hat{H}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right] dt - \int_{t_1}^{t_2} \dot{m}_{out}\left[\hat{H}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right] dt$$

- It is important to remember that flow work is included in the enthalpy terms on the right-hand side of this equation.
- When we integrate heat transfer rate in Watts, we get a total amount of heat transfer in Joules.
- When we integrate the rate at which work is done in Watts, we get a total amount of work in Joules.
- When we integrate the rate at which work is done in Watts, we get a total amount of work in Joules.
- When we integrate the rate at which work is done in Watts, we get a total amount of work in Joules.
  - ◊ ΔE_{sys} has units of energy, say J or Btu.

Uniform Flow and the 1st Law

$$\Delta E_{sys} = Q - W_s + \int_{t_1}^{t_2} \dot{m}_{in}\left[\hat{H}_{in} + \hat{E}_{kin,in} + \hat{E}_{pot,in}\right]dt - \int_{t_1}^{t_2} \dot{m}_{out}\left[\hat{H}_{out} + \hat{E}_{kin,out} + \hat{E}_{pot,out}\right]dt$$

- **Uniform Flow Assumption:**

$$\left[\hat{H}, \hat{E}_{kin} \text{ and } \hat{E}_{pot}\right]_{in,out} \neq \text{fxns}(t)$$

$$\boxed{\Delta E_{sys} = Q - W_s + m_{in}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{in} - m_{out}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{out}}$$

- The uniform flow assumption tells us that the properties of the inlet and outlet streams do not change with respect to time.
- This lets us pull them out of the integrals on the right-hand side of the 1st law equation.
- The remaining integrals of the mass flow rate were evaluated 2 slides back.
  - ◊ They are the total mass entering between t_1 and t_2 and the total mass leaving between t_1 and t_2 .

Uniform State and the 1st Law

$$\Delta E_{sys} = Q - W_s + m_{in}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{in} - m_{out}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{out}$$

- **Uniform State Assumption:**

$$\Delta E_{sys} = m_{sys,2}\left[\hat{U}_{sys} + \hat{E}_{\substack{kin\\sys}} + \hat{E}_{\substack{pot\\sys}}\right]_2 - m_{sys,1}\left[\hat{U}_{sys} + \hat{E}_{\substack{kin\\sys}} + \hat{E}_{\substack{pot\\sys}}\right]_1$$

- **If changes in kinetic and potential energies are negligible:**

$$\Delta E_{sys} = m_{sys,2}\hat{U}_{sys,2} - m_{sys,1}\hat{U}_{sys,1}$$

$$\boxed{m_{sys,2}\hat{U}_{sys,2} - m_{sys,1}\hat{U}_{sys,1} = Q - W_s + m_{in}\hat{H}_{in} - m_{out}\hat{H}_{out}}$$

- The uniform state assumption allows us to evaluate the total energy of the system as just the total mass of the system times the specific energy of the system.
- Here we only consider our three favorite forms of energy: internal, kinetic and potential.
- We can substitute this result into the 1st law to get the 1st boxed equation.
- This is the 1st Law for Uniform Flow, Uniform State systems.
- If changes in kinetic and potential energies are negligible, then the much simpler equation in the last box is applicable.
- This is the equation we will use to solve most of the transient problems in this course.
- But you need to know where it came from and when it is applicable.

Transient 1st Law for MIMO Processes

- **SISO:**

$$\Delta E_{sys} = Q - W_s + m_{in}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{in} - m_{out}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{out}$$

- **MIMO:**

$$\Delta E_{sys} = Q - W_s + \sum_{i=1}^{inlets} m_{in,i}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{in,i} - \sum_{j=1}^{outlets} m_{out,j}\left[\hat{H} + \hat{E}_{kin} + \hat{E}_{pot}\right]_{out,j}$$

◊ If changes in kinetic and potential energies are negligible:

$$\boxed{m_{sys,2}\hat{U}_{sys,2} - m_{sys,1}\hat{U}_{sys,1} = Q - W_s + \sum_{i=1}^{inlets} m_{in,i}\,\hat{H}_{in,i} - \sum_{j=1}^{outlets} m_{out,j}\,\hat{H}_{out,j}}$$

- As before, MIMO processes are more tedious than SISO processes, but are conceptually almost identical.

5A-1 Volumetric and Mass Flow Rates 2 pts

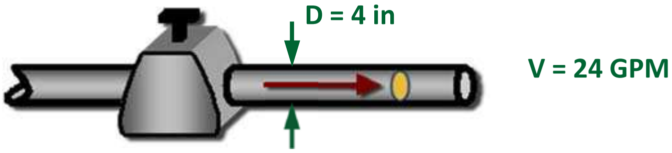
Water is flowing through a pipe with an inside **diameter** of **4 inches**. The **volumetric flow rate** is **24 GPM** (**gallons per minute**). Determine ...

a.) The **mass flow rate** in **lb$_m$/min**. Assume the **density** of **water** is **1000 kg/m^3**

b.) The average **velocity** of the **water** in the pipe in **ft/s.**

Read : This is a straightforward application of the relationships between **mass** and **volumetric flow rates**, **density** or **specific volume**, **velocity** and **cross-sectional area** for flow.

Diagram:



Given:	V	24	gal/min	D	4	in
	ρ	64.4	lb$_m$/ft^3		0.333	ft

Find:	**a.)**	m	???	lb$_m$/min	**b.)**	v	???	ft/s

Assumptions: 1 - The **density** of the water is uniform and constant.

Equations / Data / Solve:

Part a.) The key relationship for this part of the problem is :

$$\dot{m} = \rho \cdot \dot{V}$$ **Eqn 1**

Before we can use **Eqn 1**, we need to convert the units on the **volumetric flow rate** to **ft^3/min.**

$$\dot{V} = 24\,\frac{\text{gal}}{\text{min}} \cdot \frac{1\ \text{ft}^3}{7.4805\ \text{gal}}$$ **Eqn 2** V 3.208 ft^3/min

Now, we plug values into **Eqn 1** to get : m 206.6 lb$_m$/min

Part b.) The key relationship for this part of the problem is :

$$\dot{m} = \rho \cdot \dot{V} = \rho\, v\, A = \frac{v\, A}{\hat{V}} \qquad \text{Eqn 3}$$

We can solve **Eqn 3** for the average water **velocity**, **v** :

$$v = \frac{\dot{V}}{A} = \frac{\dot{m}}{\rho\, A} = \frac{\dot{m}\,\hat{V}}{A} \qquad \text{Eqn 4}$$

Where :

$$A = \frac{\pi}{4} D^2$$

Plugging values into **Eqns 4 & 5** yields :

A	**0.08727**	**ft^2**
v	**0.6127**	**ft/s**

Verify: We cannot verify the constant density assumption.

Answers : **a.)** **m** **207** **lb_m/min** **b.)** **v** **0.613** **ft/s**

5B-1 Flow Work in a Steam Pipe 2 pts

Steam enters a system at **200 kPa** and **350°C** and leaves the system at the same **mass flow rate** at **180 kPa** and **350°C**.

Calculate the **flow work** in **kJ/kg** for this process assuming the usual sign convention in which **work** done by the system is considered to be positive.

Read : This problem reinforces the point that it takes energy to **push** fluid **into** a system and it takes energy to **push** a fluid **out** of a system into the surroundings. This energy is called **flow work** and it can be very significant.

In this problem, we apply the definition of **flow work** to evaluate the net amount of **flow work** in a simple flow system, perhaps a long pipe.

Diagram:

Given:

P_1	200	kPa	P_2	180	kPa
T_1	350	°C	T_2	350	°C

Find: W_{flow} ??? kJ/kg

Assumptions: None.

Equations / Data / Solve:

The **net** amount of **PV** or **flow work** in a flow process is the **flow work** done **by** the system through exiting streams **minus** the **flow work** done **on** the system through entering streams.

$$\hat{W}_{flow,net} = \hat{W}_{flow,exit} - \hat{W}_{flow,inlet} = (P\hat{V})_2 - (P\hat{V})_1 \qquad \text{Eqn 1}$$

We know the inlet and outlet **pressures**, so now we need to determine the inlet and outlet **specific volumes**. We can get this information from the Steam Tables or the **NIST Webbook**.

V_1	1.4330	m^3/kg	V_2	1.5927	m^3/kg

Now, we can plug values into **Eqn 1** to evaluate the **flow work**.

$W_{flow,exit}$	286.69	kJ/kg			
$W_{flow,inlet}$	286.59	kJ/kg	$W_{flow,net}$	0.1006	kJ/kg
				100.6	J/kg

Verify: No assumptions.

Answers : $W_{flow,net}$ 101 J/kg

5B-2 Heat Transfer Required to Keep the Energy in a Flow System Constant 4 pts

Water vapor enters a tank at a rate of **32.4 kg/min** at **250°C** and **140 kPa** and leaves the tank at the same rate at **180°C** and **110 kPa**. The **diameter** of the inlet and outlet pipes are **6 cm** and **15 cm**, respectively.

No form of **work** enters or leaves the tank other than **flow work**. Calculate...

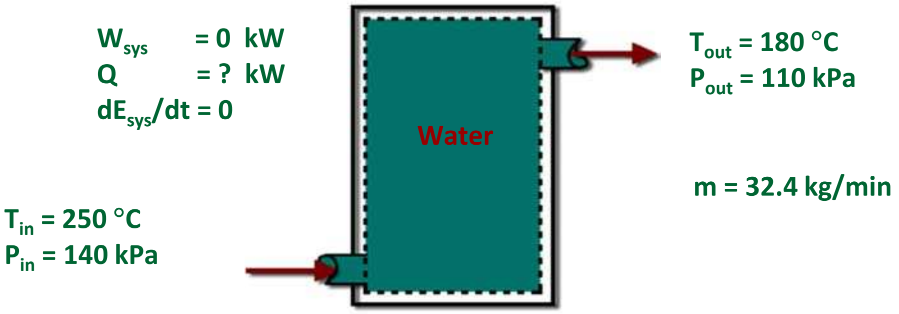
a.) The total rate at which energy is entering the tank in the feed stream

b.) The total rate at which energy is entering the tank in the effluent stream

c.) The heat transfer rate required to keep the total energy of the **water** inside the tank constant.

Read : The key to this problem is the enthalpy form of the 1st Law for open systems. Once you assume that gravitational potential energy is negligible in this problem, the solution is straightforward. Kinetic energy changes are not negligible. The relationships amoung **velocity**, **density**, **specific volume**, **volumetric flow rate** and **mass flow rate** are also important.

Diagram:



Given:	m	32.4	kg/min		W_{sys}	0	kW
		0.54	kg/s		T_{out}	180	°C
	T_{in}	250	°C		P_{out}	110	kPa
	P_{in}	140	kPa		D_{out}	15	cm
	D_{in}	6	cm			0.15	m
		0.06	m	c.)	dE_{sys}/dt	0	kW

Find:	a.)	E_{in}	???	kW	c.)	Q	???	kW
	b.)	E_{out}	???	kW				

Assumptions: 1 - Gravitational potential energy is negligible in computing the energy entering and leaving the system. Without this assumption, we would add the same arbitrary amount of energy to both the feed and effluent streams and then assume that changes in potential energy are negligible. Either way, potential energy is negligible in this problem.

Equations / Data / Solve:

The key equation for this problem in the enthalpy form of the 1st Law for open systems.

$$\frac{d}{dt}E_{sys} = \dot{Q} - \dot{W}_{sys} + \dot{m}_{in}\left[\hat{H}_{in} + \frac{\langle v_{in}\rangle^2}{2\,g_C} + \frac{g}{g_C}\,z_{in}\right] - \dot{m}_{out}\left[\hat{H}_{out} + \frac{\langle v_{out}\rangle^2}{2\,g_C} + \frac{g}{g_C}\,z_{out}\right] \quad \text{Eqn 1}$$

Eqn 1 can be simplified for this problem because $\mathbf{W_{sys} = 0}$ and we have assumed that <u>changes</u> in potential energy are <u>negligible</u>.

$$\frac{d}{dt}E_{sys} = Q + \dot{E}_{in} - \dot{E}_{out} = \dot{Q} + \dot{m}\left[\hat{H}_{in} + \frac{\langle v_{in}\rangle^2}{2\,g_C}\right] - \dot{m}\left[\hat{H}_{out} + \frac{\langle v_{out}\rangle^2}{2\,g_C}\right] \quad \text{Eqn 2}$$

Part a) & **Part b)**

In order to evaluate $\mathbf{E_{in}}$ and $\mathbf{E_{out}}$, we first need to lookup the **enthalpies** of the inlet and outlet streams. The Steam Tables or the **NIST Webbook** provide the information we need.

H_{in}	2973.2	kJ/kg	H_{out}	2835.4	kJ/kg

Next, we need to evaluate the **specific kinetic energies** at the inlet and outlet.

$$\hat{E}_{kin} = \frac{v^2}{2\,g_C} \quad \text{Eqn 3}$$

We can determine the velocity from the mass flow rate as follows :

$$v = \frac{\dot{V}}{A} = \frac{\dot{m}}{\rho\,A} = \frac{\dot{m}\,\hat{V}}{A} \quad \text{Eqn 4}$$

Where : $A = \frac{\pi}{4}D^2$ **Eqn 5**

We still need the **specific volumes** of the water at the inlet and outlet conditions to make use of **Eqn 4**. The Steam Tables or the **NIST Webbook** provide the information we need.

V_{in}	1.7163	m^3/kg	V_{in}	1.8883	m^3/kg

Now, we can plug values into **Eqns 5**, **4** & **3**, in that order.

A_{in}	0.002827	m^2	A_{out}	0.017671	m^2
v_{in}	327.8	m/s	v_{out}	57.7	m/s
g_c	1	$kg\text{-}m/N\text{-}s^2$			
$E_{kin,in}$	53.72	kJ/kg	$E_{kin,out}$	1.66	kJ/kg

We can now use the right-hand portion of **Eqn 2** to complete **parts (a)** and **(b)** of this problem.

E_{in}	1634.5	kW	E_{out}	1532.0	kW

Part c.)

Eqn 2 can be simplified because $\mathbf{dE_{sys}/dt = 0}$. The result can be solved for **Q** to obtain the following equation.

$$Q = \dot{E}_{out} - \dot{E}_{in} = \dot{m}\left[\hat{H}_{out} + \frac{\langle v_{out}\rangle^2}{2\,g_C}\right] - \dot{m}\left[\hat{H}_{in} + \frac{\langle v_{in}\rangle^2}{2\,g_C}\right] \quad \text{Eqn 6}$$

Plugging values into **Eqn 6** yields : **Q** **-102.5** **kW**

Verify: The assumption made in this solution cannot be verified with the given information.

Answers :

a.)	E_{in}	1630	kW	c.)	Q	-103	kW
b.)	E_{out}	1530	kW				

5C-1 Cross-Sectional Area Requirement for an Adiabatic Nozzle 6 pts

Steam at **3.5 MPa** and **350°C** flows steadily into an adiabatic nozzle at a **mass flow rate** of **3.15 kg/s**. The **steam** leaves the nozzle at **500 kPa** with a **velocity** of **571 m/s**.

If the inlet **velocity** of the **steam** is **5 m/s**, determine the exit **diameter** of the nozzle in **cm**.

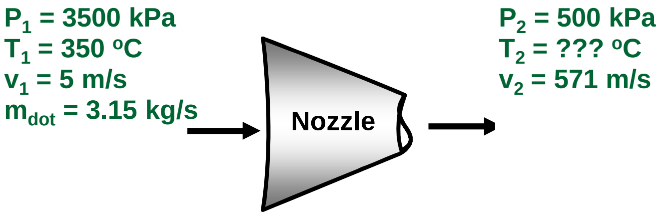
Read : The key here is that we know both the **mass flow rate** and **velocity** of the effluent stream. If we can determine the **specific volume** of the effluent, we can determine the **cross-sectional area** for flow at the effluent, **A_2**. We are given the value of **one** intensive variable for the effluent, **P_2**, but we need to know another in order to completely determine the state of the effluent. Once know the state of the effluent, we can use the Steam Tables to determine the **specific volume** and then the **cross-sectional area**. We must apply the steady-state form of 1st Law for open systems to this process. If we assume that heat transfer and changes in potential energy are negligible and that no **shaft work** occurs, we can solve for the **specific enthalpy** of the effluent and thereby fix the state of the system. This allows us to complete the problem.

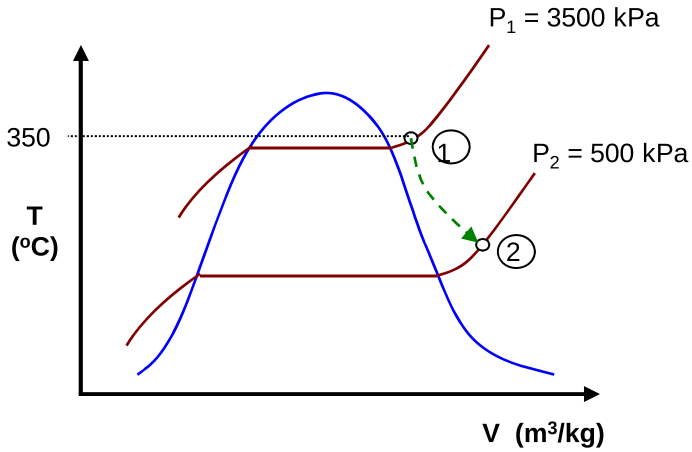
Given:

P_1	**3500**	**kPa**	**P_2**	**500**	**kPa**
T_1	**350**	**°C**	**v_2**	**571**	**m/s**
v_1	**5**	**m/s**	**m_{dot}**	**3.15**	**kg/s**

Find: **D_2** **???** **cm**

Diagram:





Assumptions:

1 - The nozzle operates at steady-state.

2 - Heat transfer is negligible.

3 - No **shaft work** crosses the system boundary.

4 - The change in the potential energy of the fluid from the inlet to the outlet is negligible.

Equations / Data / Solve:

Let's begin by writing the steady-state form of the 1st Law for open systems.

$$\cancel{\dot{Q}} - \cancel{\dot{W}_S} = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \cancel{\Delta\hat{E}_{pot}}\right] \qquad \textbf{Eqn 1}$$

Based on the assumptions listed above, we can simplify **Eqn 1** as follows :

$$\Delta\hat{H} + \Delta\hat{E}_{kin} = \hat{H}_2 - \hat{H}_1 + \frac{v_2^2 - v_1^2}{2\,g_C} = 0 \qquad \textbf{Eqn 2}$$

The only unknown in **Eqn 2** is H_2 because we can lookup H_1 and the **velocities** are both **given**.

So, let's look up H_1 and solve **Eqn 2** for H_2 :

$$\hat{H}_2 = \hat{H}_1 - \frac{v_2^2 - v_1^2}{2\,g_C} \qquad \textbf{Eqn 3}$$

H_1	3104.8	kJ/kg
H_2	2941.8	kJ/kg

We could use H_2 and P_2 to determine T_2 using the Steam Tables, but we are more interested in V_2 because :

$$\dot{m} = \frac{\dot{V}_2}{\hat{V}_2} = \frac{v_2\,A_2}{\hat{V}_2} \qquad \textbf{Eqn 4} \qquad \text{or :} \qquad A_2 = \frac{\dot{m}\,\hat{V}_2}{v_2} \qquad \textbf{Eqn 5}$$

Once we know the **specific volume** at state 2, we can use **Eqn 5** to determine the **cross-sectional area** of the effluent pipe.

Interpolating on the Steam Tables at **500 kPa** :

T (^{o}C)	H (kJ/kg)	V (m^3/kg)
200	2855.8	0.42503
T_2	2941.8	V_2
250	2961.0	0.47443

T_2	240.9	^{o}C
V_2	0.46541	m^3/kg
A_2	2.568E-03	m^2

Now, plug V_2 into **Eqn 5** :

Determine D_2 from A_2 :

$$A_2 = \frac{\pi}{4} D_2^2 \qquad \textbf{Eqn 6}$$

Solving for D_2 yields:

$$D_2 = \sqrt{\frac{4}{\pi} A_2} \qquad \textbf{Eqn 7}$$

Plug values into **Eqn 7**:

D_2	0.05718	m
	5.718	cm

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

D_2	5.72	cm^2

5C-2 Heat Losses From a Steam Turbine 5 pts

At steady-state, a **steam** turbine produces **1,050 MW** by letting the **pressure** down from **50 bar** to **1 bar**. The **steam** enters the turbine at **320^{o}C** with a **velocity** of **8 m/s** and a **mass flow rate** of **150 kg/min**.

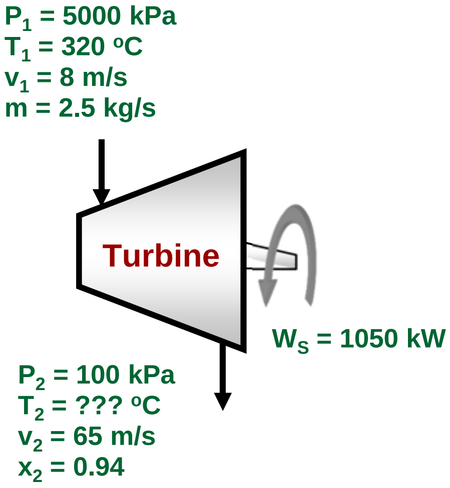
The **steam** leaves the turbine with a quality of **0.94 kg vapor/kg** and a **velocity** of **65 m/s**. Determine the **rate of heat loss** from the turbine to the surroundings in **kW**.

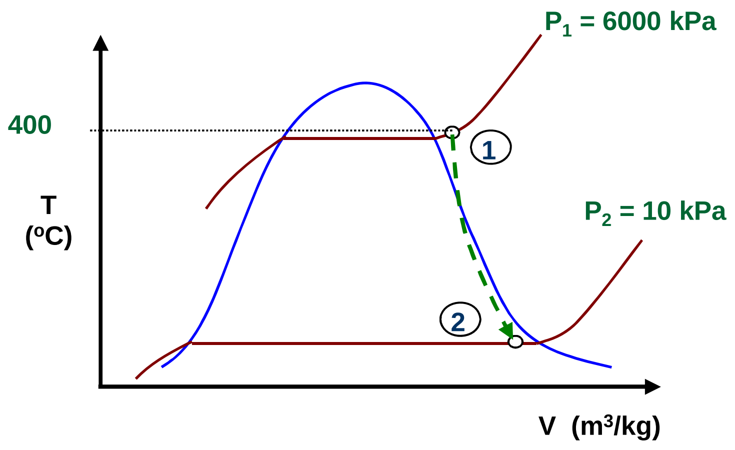
Read : Apply the steady-state form of the 1st Law for open systems and solve for **Q**. Assume changes in potential energy are negligible. We know the values of **two** intensive variables for state 1, so we can look up $\mathbf{H_1}$. We know the **pressure** and **quality** for state 2, so we can also determine $\mathbf{H_2}$. Then, just plug back into the 1st Law to get **Q** !

Given:	m	150	kg/min	v_1	8	m/s
		2.500	kg/s	P_2	100	kPa
	W_s	1050	kW	x_2	0.94	
	P_1	5000	kPa	v_2	65	m/s
	T_1	320	^{o}C			

Find: Q ??? kW

Diagram:





Assumptions:
1 - The turbine operates at steady-state.
2 - The change in the potential energy of the fluid from the inlet to the outlet is negligible.

Equations / Data / Solve:

Let's begin by writing the steady-state form of the 1st Law for open systems.

$$\dot{Q} - \dot{W}_s = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \cancel{\Delta\hat{E}_{pot}}\right] \qquad \textbf{Eqn 1}$$

Solve **Eqn 1** for **Q** :

$$\dot{Q} = \dot{W}_s + \dot{m}\left[\hat{H}_2 - \hat{H}_1 + \frac{v_2^2 - v_1^2}{2\,g_C}\right] \qquad \textbf{Eqn 2}$$

We must use the Steam Tables to determine $\mathbf{H_2}$ and $\mathbf{H_1}$:

$$\hat{H}_2 = x_2\,\hat{H}_{sat\,vap} + (1 - x_2)\,\hat{H}_{sat\,liq} \qquad \textbf{Eqn 3}$$

H_1	2986.2	kJ/kg

At P_2 = 100 kPa :

$H_{sat\ liq}$	417.50	kJ/kg
$H_{sat\ vap}$	2674.95	kJ/kg
H_2	2539.5	kJ/kg

Now, we can plug values into **Eqn 2** to evaluate **Q** :

Q	-61.62	kW

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

Q	-61.6	kW

5C-3 Shaft Work Requirement for an Air Compressor 6 pts

A compressor, operating at steady-state, increases the **pressure** of an **air** stream from **1 bar** to **10 bar** while losing **4.2 kW** of heat to the surroundings.

At the compressor inlet, the **air** is at **25°C** and has a **velocity** of **14 m/s**. At the compressor outlet, the **air** is at **350°C** and has a **velocity** of **2.4 m/s**.

If the compressor inlet has a **cross-sectional area** of **500 cm^2** and the **air** behaves as an ideal gas, determine the **power** requirement of the compressor in **kW**.

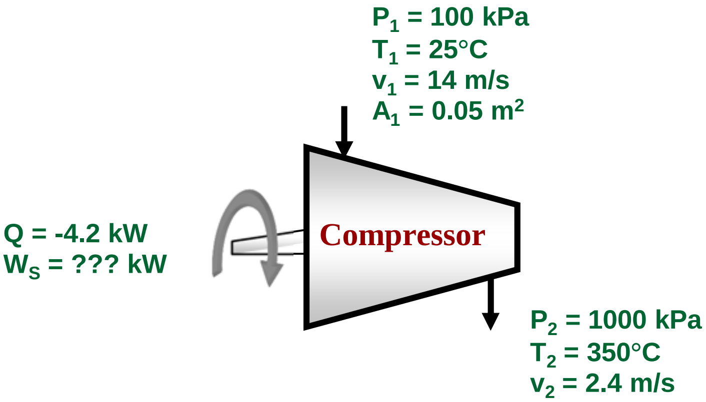
Read : The keys here are the 1st Law, the Ideal Gas EOS and the Ideal Gas Property Tables. Since we know the **velocity**, **temperature** and **pressure** of both the feed and effluent, we can determine the change in the **specific enthalpy** (using the Ideal Gas Properties Table for air) and the **specific kinetic energy**. The problem is the **mass flow rate**. Use the Ideal Gas EOS to determine the **specific volume**. Then, use the relationship between **velocity**, **cross-sectional area** for flow, **specific volume** and **mass flow rate** to determine the **mass flow rate**. After that, plug all the values back into the 1st Law and solve for the **shaft work**.

Given:

P_1	1	bar	P_2	10	bar
	100	kPa		1000	kPa
T_1	25	°C	T_2	350	°C
	298.15	K		623.15	K
v_1	14	m/s	v_2	2.4	m/s
A_1	500	cm^2	Q	-4.20	kW
	0.050	m^2			

Find: W_s ??? kW

Diagram:



Assumptions:

1 - The compressor operates at steady-state.
2 - The change in the potential energy of the fluid from the inlet to the outlet is negligible.
3 - The air behaves as an ideal gas throughout this process.

Equations / Data / Solve:

Let's begin by writing the steady-state form of the 1st Law for open systems.

$$\dot{Q} - \dot{W}_s = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \cancel{\Delta\hat{E}_{pot}}\right] \tag{Eqn 1}$$

Solve for W_S :

$$\dot{W}_s = \dot{Q} - \dot{m}\left[\hat{H}_2 - \hat{H}_1 + \frac{v_2^2 - v_1^2}{2\,g_C}\right] \tag{Eqn 2}$$

We know the inlet and outlet **velocities** and we can lookup the inlet and outlet **specific enthalpies** in the Ideal Gas Properties Table. So, the only remaining obstacle to evaluating the **shaft work** using **Eqn 2** is the **mass flow rate**.

The following relationship will let us evaluate the **mass flow rate** :

$$\dot{m} = \frac{\dot{V}_1}{\hat{V}_1} = \frac{v_1\,A_1}{\hat{V}_1} \tag{Eqn 3}$$

Next, we must use the Ideal Gas Equation of State to determine the **specific volume** of the air feed.

$$P\,\tilde{V}_1 = R\,T_1 \tag{Eqn 4}$$

Solve for **V** :

$$\tilde{V}_1 = \frac{R\,T_1}{P_1} \tag{Eqn 5}$$

Convert **molar volume** to **specific volume** :

$$\hat{V}_1 = \frac{\tilde{V}_1}{MW_{air}} \tag{Eqn 6}$$

Plugging values into **Eqns 5 & 6** yields :

			V_1	0.02479	m^3/mole
R	8.314	J/mol-K	V_1	0.85565	m^3/kg
MW_{air}	28.97	g/mole	m_{dot}	0.8181	kg/s

Next we need to look up the **specific enthalpy** of air at the inlet and outlet **temperature** in the Ideal Gas Properties Table for air. Remember that the **enthalpy** of an ideal gas does NOT depend on the **pressure** !

At 25°C or 298.15 K, no interpolation is required : H^o_1 85.565 kJ/kg

At 200°C or 473.15 K, interpolation is required :

T (K)	H^o (kJ/kg)
620	418.55
623.15	H^o_2
630	429.25

H^o_2 421.92 kJ/kg

Finally, we can plug values back into **Eqn 2** to evaluate the **shaft work** : W_s -279.29 kW

Verify:

Only the ideal gas assumption can be verified. Use **Eqn 5** for both state 1 and state 2.

V_1	24.79	L/mole	V_2	5.181	L/mole

Because air is made up of diatomic gases, the test for the applicability of the Ideal Gas EOS is whether the **molar volume > 5 L/mole**.

This considition is satisfied at both the inlet and outlet conditions, so using the Ideal Gas EOS and the Ideal Gas Properties Tables will yield results accurate to at least **2** significant figures.

Answers :

W_s	-279	kW

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5C-4 Expansion of Steam Through a Throttling Valve 5 pts

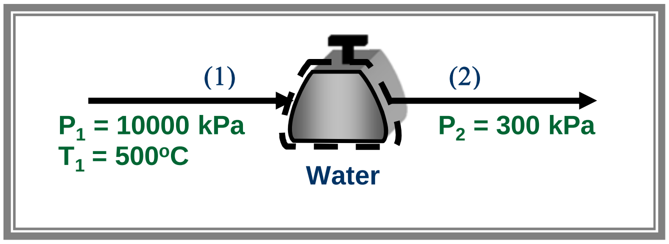
A throttling valve is used to reduce the **pressure** in a **steam** line from **10 MPa** to **300 kPa**. If the **steam** enters the throttling valve at **500°C**, determine…

a.) The **steam temperature** at the outlet of the throttling valve

b.) The **area ratio**, A_2/A_1, required to make the **kinetic energy** the same at the inlet and the outlet.

Read : We know the values of **two** intensive variables for the inlet steam, so we can determine the values of all of its other properties, including the **specific enthalpy**, from the Steam Tables. If changes in kinetic and potential energy are negligible and the throttling device is adiabatic, then the throttling device is isenthalpic. In this case, we then know the **specific enthalpy** of the outlet stream. The **pressure** of the outlet stream is **given**, so we now know the values of **two** intensive properties of the outlet stream and we can determine the values of any other property using the Steam Tables. **Part (b)** is an application of the 1st Law. The **area** must be greater at the outlet in order to keep the **velocity** the same because the steam expands as the **pressure** drops across the throttling device.

Diagram:



Given: P_1	**10000**	**kPa**	**Find:**	T_1	**???**	**°C**
T_1	**500**	**°C**		A_2 / A_1	**???**	
P_2	**300**	**kPa**				

Assumptions:

1 - The throttling device is adiabatic.

2 - Changes in potential energy are negligible.

3 - Changes in kinetic energy are negligible because the **cross-sectional area** for flow in the feed and effluent lines have been chosen to make the fluid **velocity** the same at the inlet and the outlet.

Equations / Data / Solve:

Begin by looking up the specific enthalpy of the feed in the steam tables.

At a pressure of **10,000 kPa**, the saturation **temperature** is : T_{sat} **311.00** °C

Because $T_1 > T_{sat}$, we conclude that the feed is superheated steam and we must consult the Superheated Steam Tables. Because **10,000 kPa** is listed in the table, interpolation is not required.

V_1 **0.032811** m^3/kg H_1 **3375.1** kJ/kg

The 1st Law for a throttling device that is adiabatic and causes negligible changes in kinetic and potential energies is :

$$\hat{H}_2 = \hat{H}_1 \qquad \text{Eqn 1}$$

Because the **pressure** drops in the throttling device and the feed is a superheated vapor, the effluent must also be a superheated vapor. So, to answer **part (a)**, we must use the Superheated Steam Tables to determine the **temperature** of **300 kPa** steam that has a **specific enthalpy** equal to H_2.

At **300 kPa** :

H (kJ/kg)	T (°C)	V (m³/kg)
3275.5	400	1.0315
3375.1	447.2	1.1048
3486.6	500	1.1867

T_2 447.2 °C

V_2 1.1048 m^3/kg

Part b.) We need to use the definition of kinetic energy to determine how much the **area** of the outlet pipe must be greater than the **area** of the inlet pipe in order to keep the kinetic energy (and therefore the **velocity**) constant.

$$E_{kin} = \frac{\dot{m}\,v^2}{2\,g_c} \qquad \text{Eqn 2}$$

$$E_{kin,1} = \frac{\cancel{\dot{m}}\,v_1^2}{2\,\cancel{g_c}} = \frac{\cancel{\dot{m}}\,v_2^2}{2\,\cancel{g_c}} = E_{kin,2} \qquad \text{Eqn 3}$$

Because the **mass flow rate** at the inlet and outlet is the same, **Eqn 3** simplifies to :

$$v_2 = v_1 \qquad \text{Eqn 4}$$

Next, we need to consider the relationship between **velocity**, **specific volume** and **cross-sectional area**.

$$v = \frac{\dot{m}\cdot\hat{V}}{A} \;[=]\; \frac{(kg/s)(m^3/kg)}{m^2} \;[=]\; \frac{m}{s} \qquad \text{Eqn 5}$$

Now, substitute **Eqn 5** into **Eqn 4** to get :

$$\frac{m\cdot\hat{V}_2}{A_2} = \frac{m\cdot\hat{V}_1}{A_1} \qquad \text{Eqn 6}$$

Solve for the **area ratio**, A_2 / A_1 :

$$\frac{A_2}{A_1} = \frac{\hat{V}_2}{\hat{V}_1} \qquad \text{Eqn 7}$$

Plugging values into **Eqn 7** yields : A_2 / A_1 **33.672**

Verify: None of the assumptions made in this problem solution can be verified.

Answers : T_2 **447** °C A_2 / A_1 **33.7**

5C-5 Open Feedwater Heater 6 pts

Steam at **350°C** and **650 kPa** is mixed with subcooled water at **30°C** and **650 kPa** in an **open feedwater heater** (**FWH**) as a way to produce saturated liquid **water** at the same pressure.

Assuming the open **FWH** is adiabatic, determine the **mass flow rate** of **steam** required per **kilogram** of subcooled liquid **water** fed to the open **FWH**.

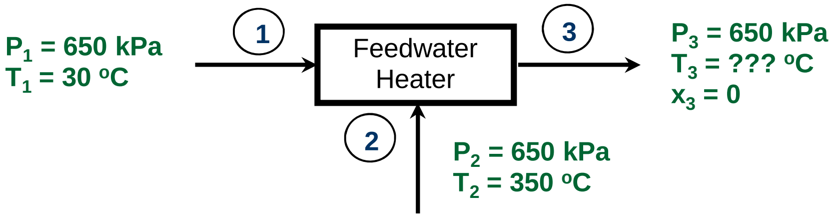
Read : The feedwater heater is just a fancy mixer. When we write the MIMO form of the 1st Law at steady-state, there are **three** unknowns: the **three mass flow rates**. The states of all three streams are fixed, so we can determine the **specific enthalpy** of each of them.
Mass conservation tells us that $\mathbf{m_3 = m_1 + m_2}$. We can use this to eliminate $\mathbf{m_3}$ from the 1st Law. Then we can solve the 1st Law for $\mathbf{m_1 / m_2}$!

Given:

T_1	30	°C	P_3	650	kPa
P_1	650	kPa	x_3	0	kg vap/kg
T_2	350	°C	Q	0	
P_2	650	kPa			

Find: $m_{dot1} / m_{dot,2}$ = ???

Diagram:



Assumptions:

1 - The feedwater heater operates at steady-state.
2 - Changes in potential and kinetic energies are negligible.
3 - Heat transfer is negligible.
4 - No **shaft work** crosses the system boundary in this process.

Equations / Data / Solve:

An open feedwater heater is essentially a mixer in which superheated vapor is used to raise the **temperature** of a subcooled liquid. We can begin our analysis with the steady-state form of the 1st Law.

$$\cancel{\dot{Q}} - \cancel{\dot{W}_S} = \sum_{j=1}^{\#\,\text{outlets}} \dot{m}_{out,j}\left[\hat{H}_{out} + \cancel{\hat{E}_{kin,out}} + \cancel{\hat{E}_{pot,out}}\right]_j - \sum_{i=1}^{\#\,\text{inlets}} \dot{m}_{in,i}\left[\hat{H}_{in} + \cancel{\hat{E}_{kin,in}} + \cancel{\hat{E}_{pot,in}}\right]_i$$

Eqn 1

The assumptions in the list above allow us to simplify the 1st Law considerably:

$$\dot{m}_1\hat{H}_1 + \dot{m}_2\hat{H}_2 = \dot{m}_3\hat{H}_3$$

Eqn 2

Conservation of mass on the feedwater heater operating at steady-state tells us that :

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Eqn 3

We can solve **Eqn 3** for $\mathbf{m_{dot,3}}$ and use the result to eliminate $\mathbf{m_{dot,3}}$ from **Eqn 2**. The result is:

$$\dot{m}_1\hat{H}_1 + \dot{m}_2\hat{H}_2 = \left(\dot{m}_1 + \dot{m}_2\right)\hat{H}_3$$

Eqn 4

The easiest way to determine $\mathbf{m_{dot,1} / m_{dot,2}}$ is to divide **Eqn 4** by $\mathbf{m_{dot,2}}$.

$$\left(\frac{\dot{m}_1}{\dot{m}_2}\right)\hat{H}_1 + \hat{H}_2 = \left(\frac{\dot{m}_1}{\dot{m}_2} + 1\right)\hat{H}_3$$

Eqn 5

Now, we can solve **Eqn 5** for $\mathbf{m_{dot,1} / m_{dot,2}}$:

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_3 - \hat{H}_1}$$

Eqn 6

Now, all we need to do is to determine the **specific enthalpy** of all **three** streams and plug these values into **Eqn 6** to complete the problem.

First we must determine the phase(s) present in each

$\mathbf{T_{sat}(650kPa) =}$ **161.98** $\mathbf{{}^oC}$

Therefore:

Stream 1 is a subcooled liquid because $\mathbf{T_1 < T_{sat}}$

Stream 2 is a superheated vapor because $\mathbf{T_2 > T_{sat}}$

$\mathbf{T_3 = T_{sat}}$ because it is a saturated liquid.

Data from the Steam Tables of the **NIST Webbook** (using the default reference state) :

$\mathbf{H_1}$	**126.32**	**kJ/kg**
$\mathbf{H_2}$	**3165.1**	**kJ/kg**
$\mathbf{H_3}$	**684.1**	**kJ/kg**

Now, plug these values into **Eqn 6** to obtain :

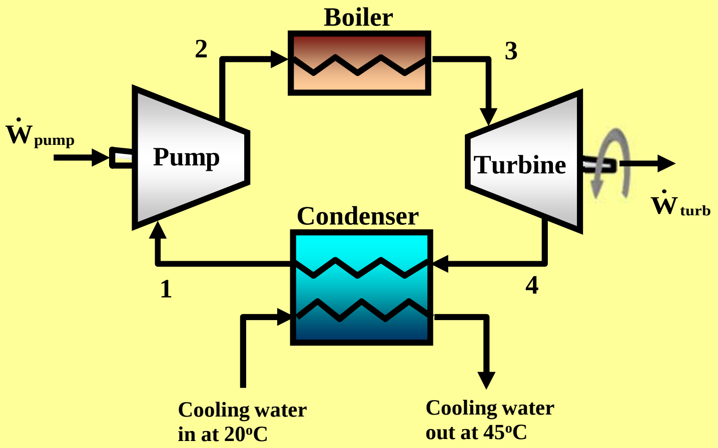
$\mathbf{m_{dot1} / m_{dot,2} =}$ **4.448**

Verify: None of the assumptions made in this problem solution can be verified.

Answers : $\mathbf{m_{dot1} / m_{dot,2} =}$ **4.45**

5C-6 Analysis of a Steam Power Cycle 8 pts

The **steam** power plant, shown below, operates at steady-state with negligible heat losses to the surroundings and negligible **pressure** drops due to friction in the boiler and condenser.



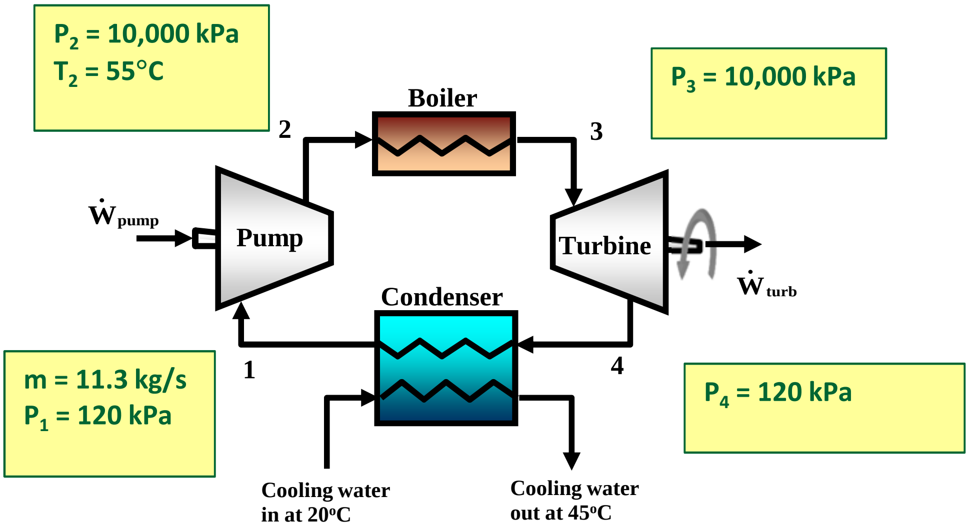
If the **mass flow rate** of the **steam** is **11.3 kg/s**, determine…

a.) The **power** of the turbine and the pump
b.) The **velocity** at the outlet of the pump
c.) The **heat transfer rates** in the boiler and in the condenser
d.) The **mass flow rate** of cooling water required in the condenser
e.) The **thermal efficiency** of the power cycle

Data: **P_1 = 120 kPa, T_1 = 55°C, P_2 = 10,000 kPa, T_2 = 50°C, D_2 = 0.05 m, P_3 = 10,000 kPa, T_3 = 700°C, P_4 = 120 kPa, x_4 = 0.95 kg vap/kg, $T_{cw,in}$ = 20°C, $T_{cw,out}$ = 45°C**

Read : Cycle problems of this type usually require you to work your way around the cycle, process by process until you have determined the values of all of the unknowns. This is a good approach here because the problem statement asks us to determine the values of unknowns in every process in the cycle. The only decision is where to begin. We can begin with the turbine because that is the 1st question and also because we have enough information to answer **part (a)**. We know **T_3** and **P_3**, so we can determine **H_3**. **Stream 4** is saturated mixture with known **P_4** and **x_4**, so we can also determine **H_4**. With the usual assumtions about kinetic and potential energy, we can determine **W_{turb}**. In fact, because we know the **T** and **P** of **streams 1** and **2** as well, we can analyze the processes in this cycle in any convenient order. So, we will let the questions posed in the problem determine the order in which we analyze the processes. We will apply the 1st Law to the pump, the boiler and the condenser, in that order. Use the Steam Tables in the **NIST Webbook**.

Diagram:



Given:	m	11.3	kg/s	P_3	10000	kPa
	P_1	120	kPa	T_3	700	^oC
	T_1	55	^oC	P_4	120	kPa
	P_2	10000	kPa	x_4	0.95	kg vap/kg
	T_2	55	^oC	$T_{cw,in}$	20	^oC
	D_2	0.05	m	$T_{cw,out}$	45	^oC

Find:	W_{turb}	???	MW	Q_{boil}	???	MW
	W_{pump}	???	kW	Q_{cond}	???	MW
	v_2	???	m/s	m_{cw}	???	kg/s
				η_{th}	???	

Assumptions:

1 - Changes in kinetic and potential energy are negligible in all the processes in the cycle.

2 - The pump and turbine are adiabatic.

3 - All of the heat that leaves the working fluid in the condenser is transferred to the cooling water. No heat is lost to the surroundings.

Equations / Data / Solve:

Part a.) Begin by writing the 1st Law for the turbine, assuming that changes in kinetic and potential energy are negligible. This makes sense because we have no **elevation**, **velocity** or pipe **diameter** information to use.

$$\cancel{\dot{Q}} - \dot{W}_{s,turb} = \dot{m}\,\Delta\hat{H}$$ **Eqn 1**

If we assume that the turbine is adiabatic, we can solve **Eqn 1** for the **shaft work** of the turbine :

$$\dot{W}_{s,turb} = \dot{m}\left(\hat{H}_3 - \hat{H}_4\right)$$ **Eqn 2**

Now, we must use the Steam Tables to determine H_3 and H_4. Let's begin with stream 3.

At a **pressure** of **10,000 kPa**, the saturation **temperature** is : T_{sat} **311.00** ^oC

Because $T_3 > T_{sat}$, we conclude that stream 3 is superheated steam and we must consult the Superheated Steam Tables. Fortunately, there is an entry in the table for **10,000 kPa** and **700°C**, so no interpolation is necessary.

H_3 **3870.0 kJ/kg**

Stream 4 is a saturated mixture at **120 kPa**, so we need to use the properties of saturated liquid and saturated vapor at **120 kPa** in the following equation to determine H_4 :

At **120 kPa** :

$$\hat{H}_4 = x_4\,\hat{H}_{sat\,vap} + \left(1 - x_4\right)\hat{H}_{sat\,liq}$$ **Eqn 3**

$H_{sat\,liq}$ **439.36 kJ/kg**

$H_{sat\,vap}$ **2683.1 kJ/kg**

H_4 **2570.9 kJ/kg**

Now, we can plug H_3 and H_4 back into **Eqn 2** to answer **part (a)** : W_{turb} **14.680 MW**

Part b.) Write the 1st Law for the pump, assuming that changes in kinetic and potential energy are negligible. This makes sense because we have no **elevation** or **velocity** data and we are given only the outlet pipe **diameter**. Also, assume the pump is adiabatic, $Q_{pump} = 0$.

$$\cancel{\dot{Q}} - \dot{W}_{s,pump} = \dot{m}\,\Delta\hat{H}$$ **Eqn 4**

$$\dot{W}_{s,pump} = \dot{m}\left(\hat{H}_1 - \hat{H}_2\right)$$ **Eqn 5**

Now, we must determine H_1 and H_2. We know the **T** and **P** for both of these streams, so we should have no difficulty determining the **H** values.

$T_{sat}(P_1)$ **104.78** ^oC $T_1 < T_{sat}$, therefore we must consult the Subcooled Water Tables.

$T_{sat}(P_2)$ **311.00** ^oC $T_2 < T_{sat}$, therefore we must consult the Subcooled Water Tables.

The **NIST Webbook** provides these **enthalpy** values without interpolation.

H_1 **230.34 kJ/kg** H_2 **238.74 kJ/kg**

Now, we can plug H_1 and H_2 back into **Eqn 5** to answer **part (b)** : W_{pump} **-94.857 kW**

Part c.) Here, we need to consider the relationship between velocity, specific volume and cross-sectional area.

$$v_2 = \frac{\dot{m} \cdot \hat{V}_2}{A_2} \; [=] \; \frac{(kg/s)(m^3/kg)}{m^2} \; [=] \; \frac{m}{s} \qquad \text{Eqn 6}$$

where :
$$A_2 = \frac{\pi}{4} D_2^2 \qquad \text{Eqn 7}$$

From the **NIST Webbook** :

A_2	0.0019635	m^2
V_2	0.0010101	m^3/kg

Now, we can plug values into **Eqn 6** to answer **part (c)** :

v_2	5.813	m/s

Part d.) Write the 1st Law for the boiler, assuming that changes in kinetic and potential energy are negligible. This makes sense because we have no **elevation**, **velocity** or pipe **diameter** data. There is no **shaft work** in a boiler.

$$\dot{Q}_{boil} - \cancel{\dot{W}_{s,boil}} = \dot{m} \, \Delta \hat{H} \qquad \text{Eqn 8}$$

$$\dot{Q}_{boil} = \dot{m} \left(\hat{H}_3 - \hat{H}_2 \right) \qquad \text{Eqn 9}$$

We determined H_2 in **part (b)** and H_3 in **part (a)**, so all we need to do is plug numbers into **Eqn 9**.

Q_{boil}	41.033	MW

Part e.) Write the 1st Law for the condenser assuming that changes in kinetic and potential energy are negligible. This makes sense because we have no **elevation**, **velocity** or pipe **diameter** data. Use the working fluid as the system so that Q_{cond} is the amount of heat transferred to the cooling water. There is no **shaft work** in a **condenser**.

$$\dot{Q}_{cond} - \cancel{\dot{W}_{s,cond}} = \dot{m} \, \Delta \hat{H} \qquad \text{Eqn 10}$$

$$\dot{Q}_{cond} = \dot{m} \left(\hat{H}_1 - \hat{H}_4 \right) \qquad \text{Eqn 11}$$

We determined H_1 in **part (b)** and H_4 in **part (a)**, so all we need to do is plug numbers into **Eqn 11**.

Q_{cond}	-26.448	MW

Part f.) In order to determine the **mass flow rate** of the cooling water, we must write the 1st Law using the cooling water as our system. For this system, $\mathbf{Q_{cw} = - Q_{cond}}$ because heat leaving the working fluid for the cycle enters the cooling water.

Q_{cw}	**26.448**	**MW**

Assume that changes in kinetic and potential energy are negligible. This makes sense because we have no **elevation**, **velocity** or pipe **diameter** data. There is no **shaft work** for the cooling water system.

$$\dot{Q}_{cw} - \cancel{\dot{W}_{s,cw}} = \dot{m}\,\Delta\hat{H} \qquad \text{Eqn 12}$$

We cannot use the Steam Tables to determine the **enthalpy** of the cooling water because we do not know the **pressure** in either stream. The next best thing we can do is to use the **specific heat** of the cooling water to determine ΔH_{cw} using:

$$\Delta\hat{H}_{cw} = \int_{T_{cw,in}}^{T_{cw,out}} \hat{C}_P \, dT \qquad \text{Eqn 13}$$

If we further assume that the **specific heat** of liquid water is constant over the **temperature** range **20^oC - 45^oC**, than **Eqn 13** simplifies to:

$$\Delta\hat{H}_{cw} = \hat{C}_P\left(T_{cw,out} - T_{cw,in}\right) \qquad \text{Eqn 14}$$

We can then combine **Eqn 14** with **Eqn 12** to obtain :

$$\dot{Q}_{cw} = \dot{m}_{cw}\,\hat{C}_P\left(T_{cw,out} - T_{cw,in}\right) \qquad \text{Eqn 15}$$

Finally, we can solve **Eqn 15** for $\mathbf{m_{cw}}$:

$$\dot{m}_{cw} = \frac{\dot{Q}_{cw}}{\hat{C}_P\left(T_{cw,out} - T_{cw,in}\right)} \qquad \text{Eqn 16}$$

All we need to do is look up the average **heat capacity** of water between **20^oC** and **45^oC**.

NIST Webbook :	$C_{P,cw}(50^oC)$	**4.1813**	**kJ/kg-K**			
	$C_{P,cw}(20^oC)$	**4.1841**	**kJ/kg-K**	$C_{P,cw}$	**4.1827**	**kJ/kg-K**

Let's use :	$C_{P,cw}$	**4.182**	**kJ/kg-K**
Then :	m_{cw}	**252.97**	**kg/s**

Part g.) The **thermal efficiency** of this power cycle can be determined directly from its definition.

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{W}_{turb} + \dot{W}_{pump}}{\dot{Q}_{boil}}$$

Eqn 17 η_{th} **0.3555**

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

a.)	W_{turb}	14.68	MW	e.)	Q_{cond}	-26.4	MW
b.)	W_{pump}	-94.9	kW	f.)	m_{cw}	253	kg/s
c.)	v_2	5.81	m/s	g.)	η_{th}	35.5	%
d.)	Q_{boil}	41.0	MW				

5C-7 Heat Losses From a Steam Compressor 6 pts

A **25 kW** compressor is used to increase the **pressure** of saturated **steam** at **140°C** to **1.2 MPa**. The compressor effluent is at **280°C**. If the **steam** flow rate is **3.7 kg/min**…

a.) Calculate the **rate of heat loss** from the compressor

b.) Assume the **steam** behaves as an ideal gas and calculate the **% error** in the **heat loss** that results from the ideal gas assumption.

Read : This is a straightforward application of the steady-state form of the 1st Law.
In **part (a)**, we can lookup properties in the Steam Tables.
In **part (b)**, we must use the Ideal Gas Heat Capacity from the **NIST Webbook** to evaluate Δ**H**.

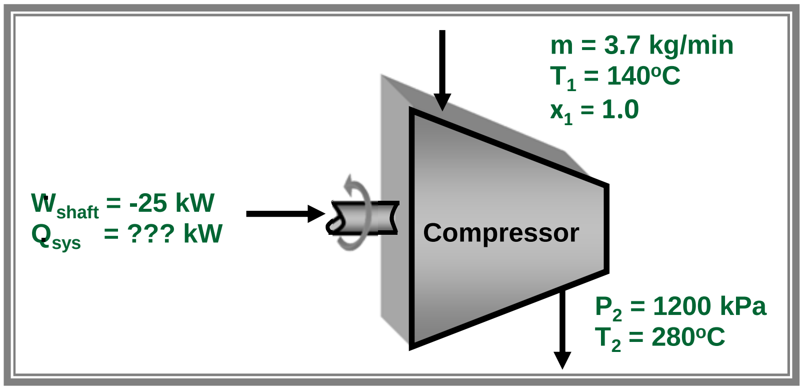
Given:

m	**3.7**	**kg/min**	**P_2**	**1200**	**kPa**
	0.0617	**kg/s**	**T_2**	**280**	**°C**
x_1	**1.00**	**kg vap/kg total**	**W**	**-25**	**kW**
T_1	**140**	**°C**			

Find:

Q	**???**	**kW**

Diagram:



Assumptions: **1 -** Changes in kinetic and potential energy are negligible.

Equations / Data / Solve:

Part a.) Begin by writing the steady-state form of the 1st Law for open systems in which changes in kinetic and potential energy are negligible.

$$\dot{Q} - \dot{W}_S = \dot{m}\,\Delta\hat{H}$$ **Eqn 1**

We can solve **Eqn 1** for **Q** :

$$\dot{Q} = \dot{W}_S + \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$ **Eqn 2**

Now, we need to determine H_1 and H_2. We can lookup H_1 in the **NIST Webbook**.

H_1 2733.4 kJ/kg

For state 2, we must first determine the phase.

$P_{sat}(T_2)$ 6416.6 kPa $P_2 < P_{sat}$, therefore we must consult the Superheated Steam Tables.

From the **NIST Webbook**, we can obtain : H_2 3002.6 kJ/kg

Now, we can plug values into **Eqn 2** : Q -8.404 kW

Part b.) **Eqn 2** still applies if the steam is treated as an ideal gas.

The difference from **part (a)** lies in how we evaluate the change in the **specific enthalpy** of the steam.

In **part (b)** we evaluate the change in the **enthalpy** using :

$$\Delta\tilde{H}_{1-2} = \int_{T_1}^{T_2} \tilde{C}_P^o \, dT \qquad \textbf{Eqn 3}$$

The Shomate Equation for the ideal gas heat capacity is :

$$\tilde{C}_P^o = A + Bt + Ct^2 + Dt^3 + E/t^2 \qquad \textbf{Eqn 4}$$

NIST Webbook :

where : $$t = T(K)/1000 \qquad \textbf{Eqn 5}$$

and : $$\tilde{C}_P^o \,[=]\, J/mol \cdot K \qquad \textbf{Eqn 6}$$

Temp (K)	500. - 1700.
A	30.092
B	6.832514
C	6.793435
D	-2.53448
E	0.082139

Let's use these Shomate constants even though **50°C** is outside of the recommended **temperature** range. These are the best values available to us.

Combining Eqns 1, 2 and 3 and integrating yields :

$$\Delta\tilde{H} = A(T_2 - T_1) + \frac{B/2}{1000}(T_2^2 - T_1^2) + \frac{C/3}{1000^2}(T_2^3 - T_1^3)\ldots$$
$$+ \frac{D/4}{1000^3}(T_2^4 - T_1^4) - \frac{E}{1000^{-2}}(T_2^{-1} - T_1^{-1}) \qquad \textbf{Eqn 7}$$

Plug in values for the **temperatures** and the constants to get : ΔH 4908 J/mol

MW_{H2O} 18.016 g/mole ΔH 272.4 kJ/kg

Now, plug this value into **Eqn 2** to evaluate Q_{IG} : Q_{IG} -8.200 kW

We can calculate the **%error** due to assuming that the steam is an ideal gas using :

$$\%error = \frac{Q - Q_{IG}}{Q} \times 100\% \qquad \textbf{Eqn 8}$$

%error 2.42%

Verify: The assumption made in this problem solution cannot be verified.

Answers :

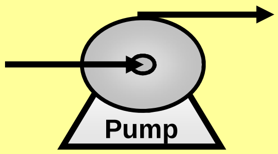
Q	-8.40	kW
Q_{IG}	-8.20	kW
%error	2.4%	

Note that the negative sign indicates that heat transfer is from the compressor to the surroundings.

5C-8 Pump Horsepower Requirment 6 pts

The pump shown below increases the **pressure** in liquid **water** from **100 kPa** to **6000 kPa**. What is the **minimum horsepower** motor required to drive the pump for a flow rate of **25 L/s** ?

P_1 = 100 kPa
D_1 = 4 cm
V_{dot} = 25 L/s



P_2 = 6000 kPa
D_2 = 6 cm

Assume the liquid **water** is incompressible and that its **specific volume** is equal to that of saturated liquid at **25°C**.

Read : The minimum **horsepower** that a motor must supply to this pump is the value of **W_s** that we can determine by applying the 1st Law. We can assume that heat transfer and changes in potential energy are negligible. We still have a problem because without any **temperature** data, we cannot look up the properties of the water in the Steam Tables. However, if we assume that the liquid water is **incompressible**, the problem gets much simpler. For starters, the **volumetric flow rate** in and out of the pump must be equal at steady-state. If we also assume that the **temperature** of the water does not change significantly in the process, then **$\Delta U = 0$** because **U** of an incompressible liquid is a function of **temperature** only. These assumptions and the relationship between **mass flow rate**, **volumetric flow rate** and **specific volume** will dramatically simplify the 1st Law and allow us to evaluate **W_s**. Unfortunately, we will still need to assume a value for the **specific volume**.

Diagram: The diagram in the problem statement is adequate.

Given:

P_1	100	**kPa**	**D_1**	4	**cm**
P_2	6000	**kPa**		0.040	**m**
V_{dot}	25	**L/s**	**D_2**	6	**cm**
	0.025	**m^3/s**		0.060	**m**

Find: **W_s** ??? **hP**

Assumptions:

1 - The pump operates adiabatically and nearly isothermally.
2 - Changes in potential energy are negligible.
3- Water behaves as an incompressible fluid in this process.

Equations / Data / Solve:

Write the 1st Law for the pump, assuming that changes in potential energy are negligible. This makes sense because we have no elevation data. Also, assume the pump is adiabatic, $Q_{pump} = 0$.

$$\cancel{\dot{Q}} - \dot{W}_{s,pump} = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin}\right] \qquad \textbf{Eqn 1}$$

$$\dot{W}_{s,pump} = -\dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin}\right] \qquad \textbf{Eqn 2}$$

The problem is that we cannot lookup the **specific enthalpy** and we do <u>not</u> know the **mass flow rate**.
We can use the <u>definition</u> of **enthalpy** to work around this :

$$\Delta\hat{H} = \Delta\hat{U} + \Delta\left(P\hat{V}\right) \qquad \textbf{Eqn 3}$$

For an incompressible liquid, **U = fxn(T)** <u>only</u>. Since we assumed $T_1 = T_2$, $\Delta U = 0$.
Also, **specific volume** is a constant for an incompressible liquid at constant **temperature**, so **V** pops out of the Δ brackets in the last term of **Eqn 3**.

$$\Delta\hat{H} = \hat{V}\,\Delta P \qquad \textbf{Eqn 4}$$

Eqn 4 is a pretty simple result, but we still <u>cannot</u> evaluate the **specific volume**.

Now, let's consider the kinetic energy term :

$$\hat{E}_{kin} = \frac{v^2}{2\,g_c} \qquad \textbf{Eqn 5}$$

Velocity is related to the **volumetric flow rate** and the **cross-sectional area** for flow by :

$$v = \frac{\dot{V}}{A} \qquad \textbf{Eqn 6}$$

where :

$$A = \frac{\pi}{4}\,D^2 \qquad \textbf{Eqn 7}$$

We can evaluate A_1 and A_2 :

A_1	0.0012566	m^2
A_2	0.0028274	m^2

Now, we can use **Eqn 6** and then **Eqn 5** to determine the **velocities** and the <u>change</u> in the **specific kinetic energy** :

v_1	19.89	m/s			
v_2	8.84	m/s	ΔE_{kin}	-158.803	J/kg

Now, we need to think about the relationship between **mass flow rate**, **volumetric flow rate** and the **specific volume**.

$$\dot{V} = \dot{m}\,\hat{V} \qquad \textbf{Eqn 8}$$

or :

$$\dot{m} = \frac{\dot{V}}{\hat{V}} \qquad \textbf{Eqn 9}$$

So, for an isothermal, adiabatic pump working on an incompressible fluid, with <u>negligible</u> <u>changes</u> in potential energy, the 1st Law simplifies from **Eqn 2** to :

$$\dot{W}_{s,pump} = -\dot{V}\,\Delta P - \dot{m}\,\Delta\hat{E}_{kin} \qquad \textbf{Eqn 10}$$

Unfortunately, in the end we still need to <u>assume</u> a value for the **specific volume**.
We will use the value of **specific volume** of saturated liquid water at **25°C** :

	V	0.001003	m^3/kg
The, we can use **Eqn 9** to determine m_{dot} :	m_{dot}	24.93	kg/s
Finally, we can plug numbers into **Eqn 10** to evaluate W_s :	W_s	-143.54	kW
Now, all we need to to do is convert to units of **horsepower**, **hP** :	1 hp =	745.7	W
Therefore :	W_s	-192.49	hP

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

W_s	-192	hP

5C-9 Outlet Temperature From a Steam Diffuser 5 pts

A diffuser is used to reduce the velocity of **steam** from **525 ft/s** to **64 ft/s**. The inlet **steam** is saturated vapor at **285°F** and the effluent **pressure** is **60 psia**. Calculate the **temperature** of the effluent.

Read : If we can determine the **enthalpy** of the effluent, we can use the Steam Tables and the known value of the **pressure** to determine the **temperature**.

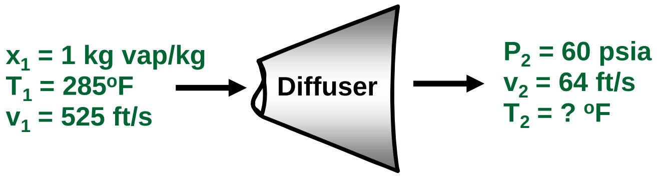
We can simplify the 1st Law if we assume the process is adiabatic with no **shaft work**. Changes in potential energy are negligible.

We can evaluate ΔE_{kin} because we know both the inlet and outlet velocities.

We can evaluate H_1 from the Saturated Temperature Table of the Steam Tables.

This leaves only **one** unknown in the 1st Law, H_2. Once we evaluate H_2, by solving the 1st Law, we can use H_2 and P_2 and the Steam Tables to determine T_2.

Diagram:



Given:

x_1	1	kg vap/kg	P_2	60	psia
T_1	285	°F	v_2	64	ft/s
v_1	525	ft/s			

Find: T_2 ??? °F

Assumptions:

1 - The fluid passes quickly through the diffuser so that heat exchange with the surroundings is negligible.

2 - Assume changes in potential energy are negligible. Unless the diffuser is very long and oriented vertically, this is a pretty good assumption.

3- No **shaft work** occurs in this process.

Equations / Data / Solve:

We could use the Steam Tables to determine T_2 if we knew the value of one more intensive variable in state 2.

The most likely choice is to find H_2. This is the right choice because H_2 appears in the 1st Law.

Begin by writing the 1st Law for an open system :

$$\dot{Q} - \dot{W}_s = \dot{m}\left[\Delta\hat{H} + \Delta\hat{E}_{kin} + \Delta\hat{E}_{pot}\right] \qquad \text{Eqn 1}$$

Assume that the diffuser is adiabatic, there is no **shaft work** and that changes in potential energy are negligible. This allows us to simplify **Eqn 1** to :

$$\Delta \hat{H} + \Delta \hat{E}_{kin} = 0 \qquad \textbf{Eqn 2}$$

We can lookup $\mathbf{H_1}$ because we know the water is a saturated vapor at **285°F**.

$\mathbf{P_1}$	53.266	psia	$\mathbf{H_1}$	1176.3	$\mathbf{Btu/lb_m}$

The **specific kinetic energy** is defined as :

$$\hat{E}_{kin} = \frac{v^2}{2\,g_c} \qquad \textbf{Eqn 3}$$

Since we know both **velocities**, we can evaluate both the inlet and outlet **specific kinetic energies** and the change as well :

$\mathbf{g_c}$	32.174	$\mathbf{ft\text{-}lb_m / lb_f\text{-}s^2}$	**1 Btu =**	778.170	$\mathbf{ft\text{-}lb_f}$
$\mathbf{E_{kin,1}}$	4283.3	$\mathbf{ft\text{-}lb_f / lb_m}$	$\mathbf{E_{kin,1}}$	5.504	$\mathbf{Btu/lb_m}$
$\mathbf{E_{kin,2}}$	63.7	$\mathbf{ft\text{-}lb_f / lb_m}$	$\mathbf{E_{kin,2}}$	0.082	$\mathbf{Btu/lb_m}$
$\mathbf{\Delta E_{kin}}$	-4219.7	$\mathbf{ft\text{-}lb_f / lb_m}$	$\mathbf{\Delta E_{kin}}$	-5.423	$\mathbf{Btu/lb_m}$

Now, we can solve **Eqn 2** for $\mathbf{H_2}$:

$$\hat{H}_2 = \hat{H}_1 - \Delta \hat{E}_{kin} \qquad \textbf{Eqn 4}$$

Now, we can plug numbers into **Eqn 4** and evaluate $\mathbf{H_2}$: $\mathbf{H_2}$ **1181.7** $\mathbf{Btu/lb_m}$

Next, we need to determine the state of the water in state 2.

At **60 psia** :	$\mathbf{H_{sat\ liq}}$	262.38	$\mathbf{Btu/lb_m}$	$\mathbf{H_{sat\ vap}}$	1178.6	$\mathbf{Btu/lb_m}$

Since $\mathbf{H_2 > H_{sat\ vap}}$, we conclude that stream 2 is a superheated vapor. Therefore, we must use the Superheated Steam Tables to determine the **temperature** of steam at **60 psia** that has a **specific enthalpy** of **1181.7** $\mathbf{Btu/lb_m}$.

From the **NIST Webbook** at **60 psia**:

T (°F)	H ($\mathbf{Btu/lb_m}$)
292.7	1178.6
$\mathbf{T_2}$	1181.7
300	1182.7
350	1209.2

Interpolating between $\mathbf{T_{sat}}$ **= 292.7°F** and **300°F** yields $\mathbf{T_2}$: $\mathbf{T_2}$ **298.25** **°F**

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

$\mathbf{T_1}$	298	°F

5C-10 Thermal Equilibration of a Copper Block with an Iron Block 5 pts

When a block of cold **carbon steel** touches a hot **aluminum** block in an insulated chamber, both blocks eventually reach thermal equilibrium at a **temperature** between their initial **temperatures**.

Determine the equilibrium **temperature** if the **aluminum** block has a **mass** of **10 kg** and an initial **temperature** of **380°C** and the **carbon steel** block has a **mass** of **25 kg** and an initial **temperature** of **10°C**.

Read : The **mass** and initial **temperature** of each block are **given**. We know that the equilibrium **temperature** of the blocks must lie between the two intial **temperatures**. We will need to lookup the **heat capacity** or **specific heat** of both steel and aluminum. Then, we can apply the 1st Law to a system made up of the two blocks. No **work** or **heat transfer** crosses the boundary of this system during the approach to equilibrium, so the only unknown in the equation is the final, equilibrium **temperature**. So, we can solve for it and evaluate it.

Diagram: A diagram is optional here. The system and process are fairly simple.

Given:

m_{steel}	25	kg	m_{Al}	10	kg
$T_{steel,1}$	10	°C	$T_{Al,1}$	380	°C

Find: T_2 ??? °C

Assumptions:

1 - Steel and aluminum have constant **heat capacities**.
2 - No heat is exchanged with the surroundings by either the steel or the aluminum.
3 - Steel and aluminum are both incompressible, so this process is a constant **volume** process.

Equations / Data / Solve:

We begin by writing the 1st Law and we choose as our system the steel and the aluminum.

$$Q - W = \Delta U = m_{St}\left(\hat{U}_{2,St} - \hat{U}_{1,St}\right) + m_{Al}\left(\hat{U}_{2,Al} - \hat{U}_{1,Al}\right) = 0$$

Eqn 1

By cleverly selecting our system, **Q = 0** and **W = 0**. This makes the solution simpler.

Because both steel and aluminum are assumed to be incompressible with constant **heat capacities**:

$$\Delta\hat{U} \approx \hat{C}_V\,\Delta T$$

Eqn 2

Substitute **Eqn 2** into **Eqn 1** twice, once for steel and once for aluminum to get :

$$m_{St}\,\hat{C}_{V,St}\left(T_2 - T_{1,St}\right) + m_{Al}\,\hat{C}_{V,Al}\left(T_2 - T_{1,Al}\right) = 0$$

Eqn 3

Notice that there is only one T_2 because in the final, equilibrium state, the steel and the aluminum are both at the same **temperature** !

Now, solve **Eqn 3** for $\mathbf{T_2}$:

$$\left(m_{Cu}\,\hat{C}_{V,Cu} + m_{oil}\,\hat{C}_{V,oil}\right) T_2 = m_{Cu}\,\hat{C}_{V,Cu}\,T_{1,Cu} + m_{oil}\,\hat{C}_{V,oil}\,T_{1,oil} \qquad \textbf{Eqn 4}$$

Now, solve **Eqn 4** for **T2**.

$$T_2 = \frac{m_{Cu}\,\hat{C}_{V,Cu}\,T_{1,Cu} + m_{oil}\,\hat{C}_{V,oil}\,T_{1,oil}}{m_{Cu}\,\hat{C}_{V,Cu} + m_{oil}\,\hat{C}_{V,oil}} \qquad \textbf{Eqn 5}$$

Before we can evaluate $\mathbf{T_2}$, we must lookup the **specific heat** of steel and of aluminum.

For carbon steel, I found :	$C_{P,st}$	**0.49**	**kJ/kg-K**
For incompressible solids, $\mathbf{C_V = C_P}$, so :	$C_{V,st}$	**0.49**	**kJ/kg-K**
For aluminum, I found :	$C_{P,Al}$	**0.91**	**kJ/kg-K**
	$C_{V,Al}$	**0.91**	**kJ/kg-K**
Now, we can finally plug numbers into **Eqn 5** and evaluate $\mathbf{T_2}$:	T_2	**167.7**	^{o}C

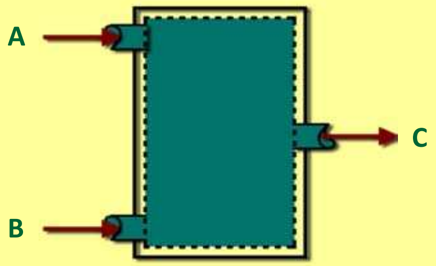
Verify: None of the assumptions can be verified.

Answers :

T_2	**168**	^{o}C

5D-1 Charging a Water Tank 3 pts

Water flows into a tank from **two** different pipes, **A** and **B**, and leaves the tank through pipe **C**. The **volumetric flow rates** in the pipes are **7.4**, **3.9** and **8.1 L/min**, respectively.



If the tank is initially empty, how much **time** is required for **1500 kg** of **water** to accumulate in the tank? Assume the **density** of **water** is **985 kg/m^3**.

Read :

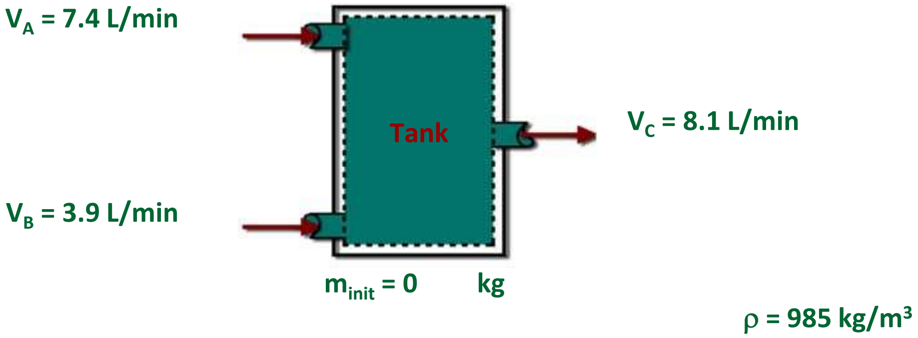


Diagram:

Given:	V_A	7.4	L/min	m_{init}	0	kg
	V_B	3.9	L/min	m_{final}	1500	kg
	V_C	8.1	L/min	ρ	985	kg/m^3
					0.985	kg/L
Find:	Δt	???	hr			

Assumptions:

1 - The **density** of the water is uniform and constant.

2 - All of the **volumetric flow rates** are constant: uniform-flow process.

Equations / Data / Solve:

The key equation in the solution of this problem is the Differential or Rate Mass Balance Equation., with constant inlet and outlet **mass flow rates**.

$$\Delta m_{sys} = m_{sys}(t_2) - m_{sys}(t_1) = (t_2 - t_1) \sum_i^{in} \dot{m}_i - (t_2 - t_1) \sum_j^{out} \dot{m}_{out} \qquad \textbf{Eqn 1}$$

Applying **Eqn 1** to this problem yields :

$$\Delta m_{sys} = \left(\dot{m}_A + \dot{m}_B - \dot{m}_C\right) \cdot \Delta t \qquad \textbf{Eqn 2}$$

Eqn 2 is useful because we can determine the **time** required to accumulate **mass**, **m**, in the tank using the following equation.

$$\Delta t = \frac{\Delta m_{sys}}{\dot{m}_A + \dot{m}_B - \dot{m}_C} \qquad \textbf{Eqn 3}$$

Next, we need to determine the **mass flow rates** from the **volumetric flow rates** of the **three** streams.

$$\dot{m} = \rho \cdot \dot{V} \qquad \textbf{Eqn 4}$$

Now we can plug numbers into **Eqns 4**, **2** and **3**, in that order to solve the problem.

m_A	7.289	Δm_{sys}	**3.152**	**kg/min**
m_B	3.8415			
m_C	7.9785	Δt	**475.9**	**min**
			7.931	**hr**

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers :

Δt	**7.93**	**hr**

5E-1 Charging an Evacuated Vessel From a Steam Line 6 pts

An evacuated tank will be filled with **steam** that flows into the tank from a **steam** supply line where the **pressure** is **2.8 MPa** and the **temperature** is **350°C**.

The **steam** from the supply line passes through a valve before it enters the tank. When the valve is opened, the tank fills with **steam** and the **pressure** increases until it reaches **2.8 MPa**.

At this point, **steam** flow stops and the valve is closed. If the process is adiabatic and changes in **kinetic** and **potential energies** are negligible, determine the final **temperature** of the **steam** in the tank.

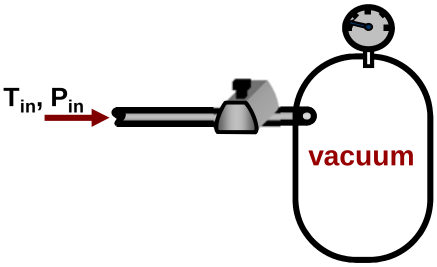
Read : Use the contents of the tank as the system. Most of the key assumptions for this problem are given in the problem statement. One additional assumption is that no shaft work crosses the boundary of the system during the process. Another crucial assumption is that this is a uniform flow, uniform state process. These assumptions allow us to simplify the 1st Law dramatically. We can use a transient mass balance to show that the **mass** in the tank in the final state is equal to the **mass** that was added to the tank. This probably seems obvious since the tank was initially empty. We can determine the **specific enthalpy** of the steam entering the tank from the Steam Tables because we know both $\mathbf{T_{in}}$ and $\mathbf{P_{in}}$. We will be able to determine the **internal energy** of the steam in the tank in the final state from the 1st Law. Then, we can use the Steam Tables to determine the value of $\mathbf{T_2}$ using $\mathbf{U_2}$ and the **given** value of $\mathbf{P_2}$.

Given:					
P_{in}	1800	kPa	Q	0	kJ
T_{in}	350	°C	W_S	0	kJ
P_2	1800	kPa			

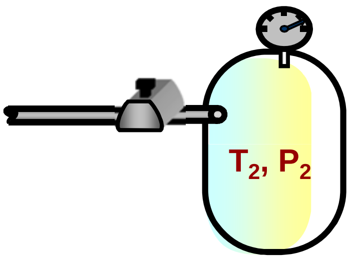
Find: T_2 ??? °C

Diagram:

Initial :



Final :



Assumptions:

1 - Although this is a transient process, it can be analyzed as a uniform flow, uniform state problem because the properties of the steam entering the tank are constant.

2 - Changes in kinetic and potential energies are negligible.

3 - No **shaft work** crosses the boundary of the system, which consists of the contents of the tank.

4 - The process is adiabatic.

Equations / Data / Solve:

The integral form of the transient energy balance equation for a single-input, single-output system in which kinetic and potential energies are negligible is :

$$m_2\,\hat{U}_2 - \cancel{m_1}\,\hat{U}_1 = \cancel{Q} - \cancel{W}_{shaft} + m_{in}\,\hat{H}_{in} - \cancel{m}_{out}\,\hat{H}_{out} \qquad \textbf{Eqn 1}$$

In our process, no **shaft work** occurs, no heat transfer occurs, there is no **mass** leaving the system and there is no **mass** inside the system initially, so **Eqn 1** can be simplified a great deal.

$$m_2\,\hat{U}_2 = m_{in}\,\hat{H}_{in} \qquad \textbf{Eqn 2}$$

The integral form of the transient mass balance on the tank is :

$$\Delta m_{sys} = m_{in} - \cancel{m}_{out} = m_2 \cancel{- m_1} \qquad \textbf{Eqn 3}$$

Eqn 3 can be simplified because there is no **mass** leaving the sytstem and there is no **mass** initially inside the system.

$$m_{in} = m_2 \qquad \textbf{Eqn 4}$$

We can use **Eqn 4** to further simplify **Eqn 2** :

$$\hat{U}_2 = \hat{H}_{in} \qquad \textbf{Eqn 5}$$

Next, we need to determine $\mathbf{H_{in}}$. First, we need to determine the state of the system.

In the **NIST Webbook**, we can find the $\mathbf{P_{sat}(T_{in})}$ for steam: $\mathbf{P_{sat}(T_{in})}$ **16.529** **MPa**

Because $\mathbf{P_{in} < P_{sat}(T_{in})}$, we must consult the Superheated Steam Tables to evaluate $\mathbf{H_{in}}$.

From the **NIST Webbook**, we can obtain : $\mathbf{H_{in}}$ **3141.8** **kJ/kg**

Plugging values into **Eqn 5** gives us : $\mathbf{U_2}$ **3141.8** **kJ/kg**

Now, we know the values of **two** intensive properties at state 2: **pressure** and **specific internal energy**. The state is completely determined and we can use the Steam Tables to evaluate any other intensive property, such as $\mathbf{T_2}$. To do so, we must first detemine which phase or phases are present in state 2.

At **P = 1.4 MPa**, the **NIST Webbook** tells us that :

$\mathbf{U_{sat\ liq}}$	**882.37**	**kJ/kg**	$\mathbf{U_{sat\ vap}}$	**2597.2**	**kJ/kg**

Because $\mathbf{U_2 > U_{sat\ vap}(P_2)}$, we must consult the Superheated Steam Tables to evaluate $\mathbf{T_2}$.

At **P = 1.8 MPa**, the **NIST Webbook** tells us that :

T (^oC)	U (kJ/kg)
510	3135.7
T_2	3141.8
520	3152.9

Interpolation yields : **T_2** **513.56** **^oC**

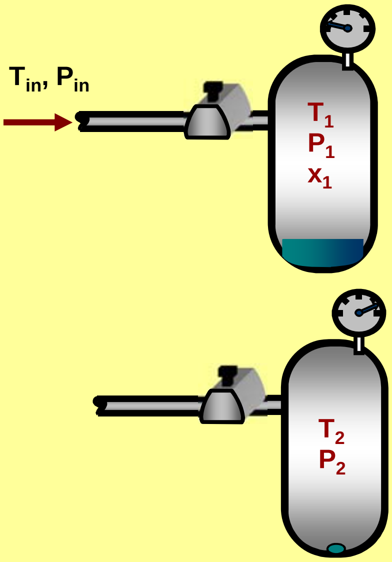
Verify: None of the assumptions made in this problem solution can be verified.

Answers : **T_2** **513.6** **^oC**

The **temperature** of the steam in the tank in the final state is greater than the **temperature** of the steam in the feed line because the surroundings did **flow work** on the system as it filled with steam. This **flow work** caused the **internal energy** of the steam in the tank to exceed the **internal energy** of the steam in the feed line. Consequently, the **temperature** of the steam in the tank in the final state must be greater than the **temperature** of feed.

5E-2 Charging a Tank With R-134a 7 pts

A rigid tank contains **ammonia** at **-20°C** with a **quality** of **0.50 kg vap/kg**. Superheated **ammonia** vapor at **800 kPa** and **80°C** slowly and steadily enters the tank from a supply line through a pipe with a valve.



When the **pressure** in the tank reaches **600 kPa**, the valve is closed. At this point, the last drop of liquid **ammonia** in the tank vaporizes and the tank contains only saturated **ammonia** vapor.

The total **volume** of the tank is **350 L**. Determine...

a.) The final **temperature** of the **ammonia** in the tank
b.) The **mass** of **ammonia** that has entered the tank
c.) The **heat transfer** between the tank and the surroundings that has taken place during this process

Read : **Part (a)** is straightforward because the vapor in the system is saturated at 800 kPa. We can then determine the initial and final **mass** of ammonia in the tank. A mass balance between the initial and final states of the system (the tank) tells us that the **mass** added to the tank is just the final **mass** minus the initial **mass**. This process is a transient process because the **mass** of ammonia inside the system (the tank) changes with time. We will need to use the integral form of the transient 1st Law Equation to answer **part (c)**.

Given:	V	350	L	P_{in}	800	kPa
		0.35	m^3	T_{in}	40	°C
	T_1	-20	°C	P_2	600	kPa
	x_1	0.50	kg vap/kg total	x_2	1.00	kg vap/kg total
Find:	T_2	???	°C	Q	???	kJ
	m_{in}	???	kg			

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - Although this is a transient process, it can be analyzed as a uniform flow problem because the properties of the R-134a entering the tank are constant.

2 - Kinetic and potential energies are negligible.

3 - No shaft work crosses the boundary of the system, which consists of the content of the tank.

Equations / Data / Solve:

Part a.) The vapor inside the tank in the final state is saturated. Therefore, it is at the saturation **temperature** of ammonia at a **pressure** of **600 kPa**. We can obtain this **temperature** from the **NIST Webbook**.

T_2 **9.285** **^oC**

Part b.) The integral form of the transient mass balance on the tank is :

$$\Delta m_{sys} = m_{in} - \cancel{m}_{out} = m_2 - m_1 \qquad \textbf{Eqn 1}$$

We can determine the intial and final mass of R-134a in the system using :

$$m = \frac{V}{\hat{V}} \qquad \textbf{Eqn 2}$$

Our next step is to determine the intial and final **specific volume** of the ammonia.

In the initial state : At **-20^{o}C** : **P_1** **190.08** **kPa**

$$\hat{V}_1 = x_1\,\hat{V}_{sat\,vap} + (1 - x_1)\,\hat{V}_{sat\,liq} \qquad \textbf{Eqn 3}$$

$V_{sat\ liq}$ **0.0015035** **m^3/kg**

$V_{sat\ vap}$ **0.62373** **m^3/kg**

V_1 **0.31262** **m^3/kg**

The final state is simpler because the ammonia in the tank is a saturated vapor.

At **600 kPa** : **$V_{sat\ vap}$** **0.21035** **m^3/kg** **V_2** **0.21035** **m^3/kg**

Now, we can use **Eqn 2** to to determine the initial and final **mass** of ammonia in the tank.

m_1 **1.12** **kg** **m_2** **1.66** **kg**

Plug these values back into **Eqn 1** to determine the **mass** of ammonia that was added to the tank during this process.

m_{in} **0.544** **kg**

part c.) The integral form of the transient energy balance equation for a single-input, single-output system in which kinetic and potential energies are negligible is :

$$m_2 \hat{U}_2 - m_1 \hat{U}_1 = Q - W_{shaft} + m_{in} \hat{H}_{in} - m_{out} \hat{H}_{out} \qquad \textbf{Eqn 4}$$

In our process, no **shaft work** occurs and there is no **mass** leaving the system, so **Eqn 4** can be simplified and solved for **Q** :

$$Q = m_2 \hat{U}_2 - m_1 \hat{U}_1 - m_{in} \hat{H}_{in} \qquad \textbf{Eqn 5}$$

We can determine $\mathbf{U_2}$ and $\mathbf{U_1}$ much as we determined $\mathbf{V_2}$ and $\mathbf{V_1}$ in **part (b)**.

The **NIST Webbook**, using the default reference state, tells that in the initial state :

At **-20°C** :	$U_{sat\ liq}$	**89.095**	**kJ/kg**	$U_{sat\ vap}$	**1299.9**	**kJ/kg**

$$\hat{U}_1 = x_1 \hat{U}_{sat\ vap} + (1 - x_1)\, \hat{U}_{sat\ liq} \qquad \textbf{Eqn 6}$$

U_1	**694.52**	**kJ/kg**

The final state is simpler because the ammonia in the tank is a saturated vapor.

At **600 kPa** :	$U_{sat\ vap}$	**1326.1**	**kJ/kg**	U_2	**1326.1**	**kJ/kg**

Next, we need to determine $\mathbf{H_{in}}$. First, we need to determine the state of the system.

$\mathbf{T_{sat}(P_{in})}$ **17.848** **°C** $\mathbf{T_{in} > T_{sat}}$, therefore we must consult the Superheated Ammonia Tables.

From the **NIST Webbook**, we can obtain : $\mathbf{H_{in}}$ **1521.3** **kJ/kg**

Now, we can plug values back into **Eqn 5** to evaluate **Q** : **Q** **600.83** **kJ**

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

T_2	9.3	°C
m_{in}	0.544	kg

Q	601	kJ

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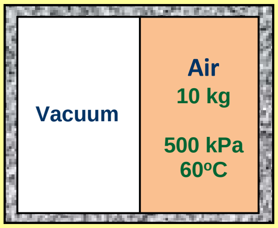
5E-3 Expansion of an Ideal Gas to Fill an Evacuated Chamber 4 pts

The tank, shown below, has two chambers of equal **volume**. The left side holds **10 kg** of **air** at **500 kPa** and **60°C**. The right side is completely evacuated.



When the wall that separates the two chambers within the tank is removed, the **air** expands to fill the right side of the tank. Calculate the final **temperature** and **pressure** in the tank.

Assume **air** behaves as an ideal gas and the process is adiabatic because the tank is well-insulated.

Read : The most important thing to recognize in this problem is that removing the partition is equivalent to allowing the partition to move the to the left (in our diagram) until the ideal gas fills the entire tank. The resisting **force** in this expansion process is **zero** because there is a vacuum in the left chamber of the tank. Since the resisting **force** is **zero**, the **work** done by the expanding gas is also **zero**. If we take the ideal gas to be our system, there is no heat transfer during the expansion either, because the tank is insulated. The 1st Law tells us that $\Delta U = 0$ when no **work** or heat transfer occur during a process on a closed system. Also, because **U** is a function of **T** only for an ideal gas, $T_2 = T_1$. Then, all we need to do is apply the IG EOS to determine P_2.

Diagram: The diagram in the problem statement is adequate.

Given:				**Find:**			
	m	10	kg		P_2	???	kPa
	P_1	500	kPa				
	T_1	60	°C				

Assumptions:

1 - The gas behaves as an ideal gas.

2 - Changes in kinetic and potential energy are negligible.

3 - The tank is perfectly insulated, so the process is adiabatic: **Q = 0**.

Equations / Data / Solve:

The ideal gas is the system that we will analyze. This is a closed system because no **mass** crosses the boundary during the expansion process.

The 1st Law for a closed system is:

$$\cancel{Q} - \cancel{W_b} = \Delta U + \cancel{\Delta E_{pot}} + \cancel{\Delta E_{kin}} \qquad \textbf{Eqn 1}$$

Since the restraining **force** overcome during the expansion is **zero**, the boundary **work** for the expansion is also **zero**. Combining this fact with the 2nd and 3rd assumptions listed above, allows us to simplify **Eqn 1** to **Eqn 2** :

$$\Delta U = 0 \qquad \text{or:} \qquad U_2 = U_1 \qquad \textbf{Eqn 2}$$

For an ideal gas, **internal energy** depends only on the **temperature** of the gas. If the **internal energy** is the same in state 2 as in state 1, then the **temperature** in state 2 must also be the same as the **temperature** in state 1 !

$$T_2 = T_1 \qquad \textbf{Eqn 3}$$

Now that we know $\mathbf{T_2}$, we can apply the Ideal Gas EOS to both states 1 and 2 to determine $\mathbf{P_2}$.

$$P_1 V_1 = n_1 R T_1 \qquad \textbf{Eqn 4} \qquad\qquad P_2 V_2 = n_2 R T_2 \qquad \textbf{Eqn 5}$$

Divide **Eqn 5** by **Eqn 4** and cancel like terms :

$$\frac{P_2 V_2}{P_1 V_1} = \frac{\cancel{n_2} \cancel{R} \cancel{T_2}}{\cancel{n_1} \cancel{R} \cancel{T_1}} \qquad \textbf{Eqn 6}$$

But, we know that $\mathbf{T_1 = T_2}$ and because the system is closed, $\mathbf{n_1 = n_2}$. Also, because the left and right chambers of the tank are equal in size, $\mathbf{V_2 = 2\ V_1}$. Therefore, **Eqn 6** can be simplified as follows:

$$P_2 V_2 = P_1 V_1 \qquad \textbf{Eqn 7}$$

or:

$$P_2 (2 \cancel{V_1}) = P_1 \cancel{V_1} \qquad \textbf{Eqn 8}$$

Solving **Eqn 8** for $\mathbf{P_2}$ yields :

$$P_2 = \frac{P_1}{2} \qquad \textbf{Eqn 9}$$

Putting values into **Eqn 9** gives us the answer : $\mathbf{P_2}$ **250.0** **kPa**

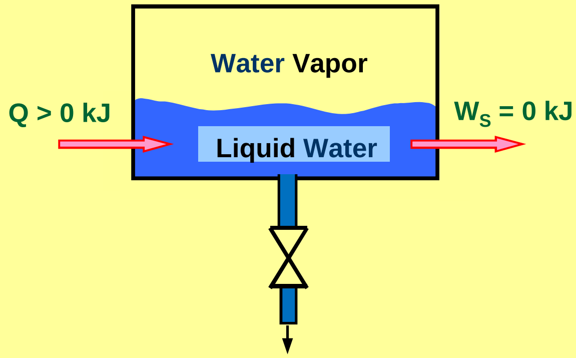
Verify: None of the assumptions can be verified.

Answers :

$\mathbf{P_2}$	**250**	**kPa**

5E-4 Discharging a Tank Containing Water and Steam 6 pts

The rigid tank, shown below, contains 20 L of liquid water and 45 L of water vapor in equilibrium at 200°C.



When the valve in the drain line is opened slightly, liquid water flows slowly out of the tank at a constant rate. Heat transfer into the tank keeps the **temperature** within the tank uniform and constant.

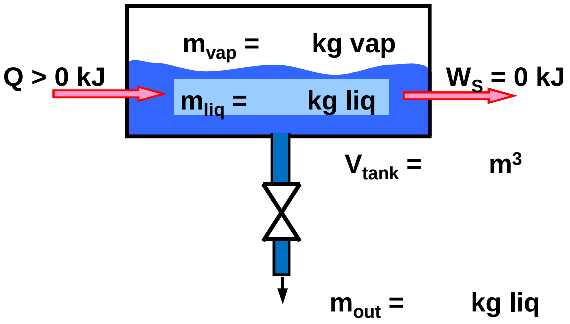
a.) Determine the **mass** of liquid **water** and the **mass** of **water** vapor **initially** in the tank

b.) When the total **mass** of H_2O in the tank is **20%** of what it was initially, determine the **quality** of the vapor-liquid mixture in the tank and the amount of **heat transfer** required up to that point.

Note: this process can be considered a uniform-flow, unsteady process.

Read : The key to this problem is that the process is an isothermal process. As a result, the properties of the liquid inside this system and leaving the system are always the properties of saturated liquid at 200°C. As a result, this is a uniform state process. We can also assume it is a uniform flow process. If we further assume that changes in kinetic and potential energies are negligible and that no **shaft work** occurs, we can use the 1st Law to determine **Q**. **Parts (a)** and **(b)** require use of the Steam Tables or **NIST Webbook** and a working knowledge of the relationship between **mass**, **volume** and **specific volume**, but should not be difficult.

Diagram:



Given:

$V_{1,liq}$	20	L		$T_1 = T_2 =$	200	°C
	0.020	m^3			473.15	K
$V_{1,vap}$	45	L				
	0.045	m^3				
V	0.065	m^3	b.)	f	0.2	kg final/kg init

Find:	**a.)**	$m_{1,vap}$	???	kg
		$m_{1,liq}$	???	kg
		m_1	???	kg
	b.)	x_2	???	kg vap/kg total
		when:	$m_2 = \frac{m_1}{2}$	
	c.)	Q_{12}	???	kJ

Assumptions:

1 - Only saturated liquid water leaves the tank.

2 - The process is isothermal.

3 - Only **flow work** (no **shaft work**) crosses the system boundary.

4 - Changes in kinetic and potential energies are negligible.

5 - Uniform Flow: The properties and **flow rate** of the outlet stream are constant over the **cross-sectional area** of the pipe and with respect to time.

6 - Uniform State: At all times, the properties of the outlet stream are the same as the properties of the contents of the system at that point in time.

Equations / Data / Solve:

Part a.) Because the water liquid and vapor are in equilibrium with each other at all times throughout the process, they are always saturated. Therfore, we can determine the **mass** of liquid and vapor initially in the tank by looking up their **specific volumes** in the Saturated Steam Tables or the **NIST Webbook**.

$$m_{1,liq} = \frac{V_{1,liq}}{\hat{V}_{sat\,liq}} \qquad \textbf{Eqn 1}$$

$$m_{1.vap} = \frac{V_{1,vap}}{\hat{V}_{sat\,vap}} \qquad \textbf{Eqn 2}$$

At 200°C :

$V_{sat\ vap}$	0.12721	m^3/kg	$m_{1,vap}$	0.3537	kg
$V_{sat\ liq}$	0.0011565	m^3/kg	$m_{1,liq}$	17.29	kg

We can determine the total **mass** of water in the system initially from:

$$m_1 = m_{1,vap} + m_{1,liq} \qquad \textbf{Eqn 3}$$

m_1 17.65 kg

Part b.) We know that:

$$m_2 = \frac{m_1}{2} \qquad \textbf{Eqn 4}$$

m_2 3.53 kg

The key here is that we know both the **volume** and the total **mass** in the tank, so we can calculate the **specific volume** and use it to determine the **quality** from :

$$x_2 = \frac{\hat{V}_2 - \hat{V}_{sat\,liq}}{\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}} \qquad \textbf{Eqn 5}$$

The real key to this problem is that the process is isothermal. As a result, the properties of the saturated vapor in the tank remain constant and the properties of the saturated liquid inside the tank and flowing out of the tank also remain constant.

We determine the overall **specific volume** at state 2 from:

$$\hat{V}_2 = \frac{V}{m_2} \qquad \textbf{Eqn 6}$$

Now, we can plug values into **Eqns 6 & 5** :

V_2	0.018417	m^3/kg
x_2	0.1369	kg vap/kg tot

Part c.) To determine **Q**, we need to apply the 1st Law for transient processes and open systems.

Here, we assume that $W_S = 0$, $\Delta E_{kin} = \Delta E_{pot} = 0$. The appropriate form of the 1st Law under these conditions is:

$$\Delta U = m_2\,\hat{U}_2 - m_1\,\hat{U}_1 = Q - W_s + m_{in}\,\hat{H}_{in} - m_{out}\,\hat{H}_{out} \qquad \textbf{Eqn 7}$$

Solving **Eqn 7** for **Q** yields:

$$Q = m_2\,\hat{U}_2 - m_1\,\hat{U}_1 + m_{out}\,\hat{H}_{out} \qquad \textbf{Eqn 8}$$

The **specific enthalpy** of the water leaving the system is the **enthalpy** of saturated liquid water at **200°C**:

H_{out}	852.27	kJ/kg

A mass balance allows us to determine m_{out} :

$$m_{out} = m_1 - m_2 \qquad \textbf{Eqn 9}$$

m_{out}	14.12	kg

Next, we can determine U_1 and U_2 :

$$\hat{U}_1 = x_1\,\hat{U}_{sat\,vap} + (1 - x_1)\,\hat{U}_{sat\,liq} \qquad \textbf{Eqn 10}$$

$$\hat{U}_2 = x_2\,\hat{U}_{sat\,vap} + (1 - x_2)\,\hat{U}_{sat\,liq} \qquad \textbf{Eqn 11}$$

where:

$$x_1 = \frac{m_{1,vap}}{m_1} \qquad \textbf{Eqn 12}$$

Plugging values into **Eqns 10 - 12** yields:

x_1	0.02005	kg vap/kg

At **200°C** :

$U_{sat\,vap}$	2594.20	kJ/kg	U_1	885.43	kJ/kg
$U_{sat\,liq}$	850.47	kJ/kg	U_2	1089.24	kJ/kg

We are finally ready to put numbers into **Eqn 8** to complete this problem.

	$m_2\,U_2$		$m_1\,U_1$		$m_{out}\,H_{out}$	
Q =	3844	-	15625	+	12032	kJ

Q	251.2	kJ

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers :

a.)

$m_{1,vap}$	0.354	kg
$m_{1,liq}$	17.3	kg
m_1	17.6	kg

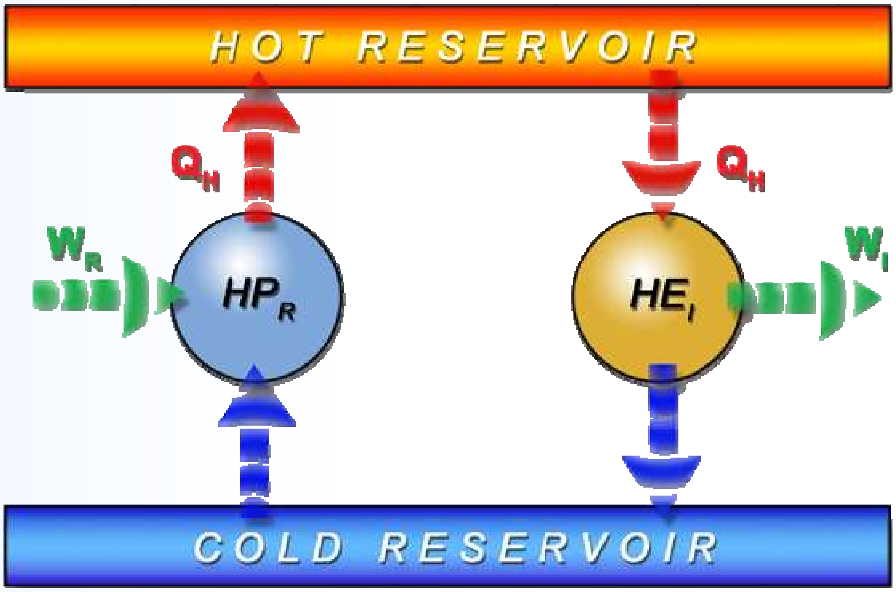
b.)

x_2	0.137	g vap/kg total

c.)

Q_{12}	251	kJ

Chapter 6



The Second Law of Thermodynamics

In this chapter, we begin with the concept of a thermal reservoir and show how they can be used in heat engines, refrigerators and heat pumps.Next, we introduce a simple, intuitive statement of the Second Law of Thermodynamics. In this chapter and the next one, we develop progressively more useful statements of the Second Law. These help us understand and analyze the feasibility and performance of processes and cycles.

Next, we discuss the concept of reversibility and sources of irreversibility. The Carnot Cycle is a reversible cycle that leads to two new and important implications of the Second Law called the Carnot Principles. The Carnot Principles are used with the Kelvin Relationship to establish the fact that the Kelvin temperature scale is a thermodynamic temperature scale. This establishes a relationship between the thermal efficiency of a power cycle and the temperatures of the thermal reservoirs with which it interacts. This leads to the key idea that the usefulness or value of 1 kJ of energy depends on the temperature of the reservoir from which you take it.

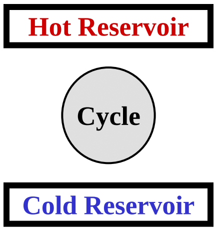
1st Law and Spontaneity

- **1st Law: Energy is neither created nor destroyed**
    - ◊ Places no restriction on the direction that energy flows spontaneously
        - Imagine a cup of water rejecting 100 kJ to the surrounding air and freezing solid.
        - Imagine a cup of water absorbing 100 kJ from the surrounding air and boiling.
    - ◊ We need another law to help us understand why these things do not happen spontaneously.
- **Spontaneity**
    - ◊ Unbalanced forces tend to drive the state of a system towards an equilibrium state
    - ◊ We can harness these unbalanced driving forces to do work for us.
        - The greater the unbalanced driving force, the greater the potential to do work.

- A cup of water left out on a table at room temperature does not spontaneously freeze unless room temperature drops below 0$_o$C.
- A cup of water left out on a table at room temperature does not spontaneously boil unless room temperature climbs above 100$_o$C.
- But if these things did happen, it would NOT VIOLATE the 1st Law !
- As long as 100 kJ leaves the water and 100 kJ enters the surroundings, the 1st Law is satisfied.
- And yet this NEVER happens.
- So, the 1st Law is completely inadequate to explain WHY these things never happen.
- We can take advantage of things that happen spontaneously.
- Don't think of things that are not in equilibrium as being bad or negative.
- We would not be alive if we were in equilibrium with our surroundings !

Thermal Reservoirs and Cycles

- **Thermal Reservoirs**
    - ◊ Bodies than can exchange an infinite amount of heat, but the temperature of the thermal reservoir never changes.
    - ◊ Heat Sink: Reservoir that absorbs heat
    - ◊ Heat Source: Reservoir that puts out heat
- **Types of Thermodynamic Cycles**
    - ◊ Power Cycle
        - Purpose: produce W_S, Input: Q_H, Waste: Q_C
    - ◊ Refrigeration Cycle
        - Purpose: produce Q_C, Input: W_S, Waste: Q_H
    - ◊ Heat Pump Cycle
        - Purpose: produce Q_H, Input: W_S, Waste: Q_C



- Examples of bodies that are nearly perfect thermal reservoirs include a lake, an ocean and the Earth's atmosphere.
- For practical purposes, their temperature does not change because my kitchen refrigerator is operating.
- Another good example of a thermal reservoir is a body in which phase equilibrium exists.
- The temperature of an ice-water bath does not change as heat is transferred into the bath.
- Instead the heat transferred supplies the latent heat of fusion required to melt some of the ice.

Power Cycles Produce Work

- **1st Law: IN = OUT**

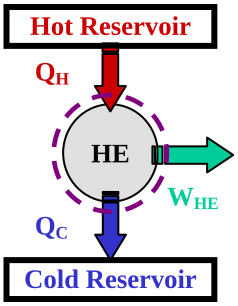
$$Q_H = Q_C + W_{HE}$$

- **Thermal Efficiency of a Power Cycle**

$$\eta = \frac{\text{Desired Result}}{\text{Required Input}} = \frac{W_{HE}}{Q_H}$$

$$\eta = \frac{Q_H - Q_C}{Q_H}$$

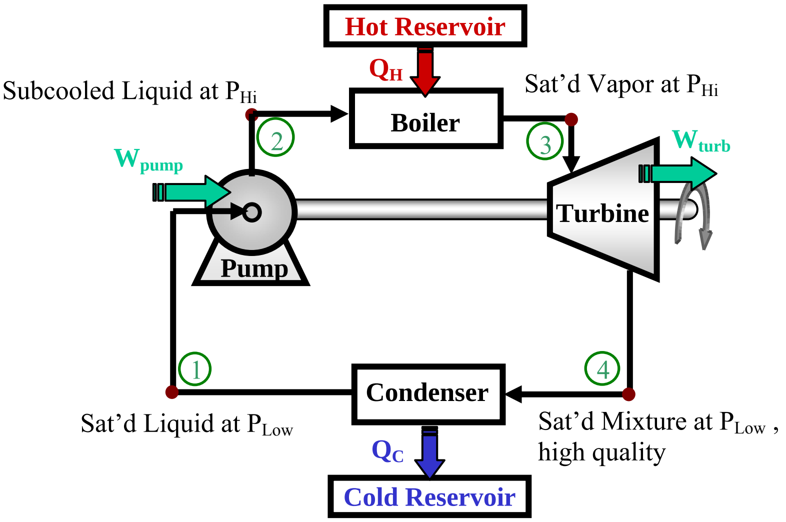
$$\boxed{\eta = 1 - \frac{Q_C}{Q_H}}$$



- Sign Convention Issues
- Do not forget that, in tie-fighter diagrams, all work and heat flows are positive in the directions that the arrows indicate in the diagrams !
- In the 1st Law, I have assumed that the heat engine is operating at steady-state.
- We can calculate the thermal efficiency of any Heat Engine if we know Q_C and Q_H using this simple equation.

Vapor Power Cycle Components

- **Note, in the tie-fighter sign convention :** $W_{HE} = W_{turb} - W_{pump}$



- A vapor power cycle is the type of power cycle used to generate electrical power in coal, oil, natural gas and nuclear power plants.
- This cycle accounts for the vast majority of electrical power production worldwide.
- The sign convention issue pops up again.
- The best way to keep this straight is to always double check the signs of **Q** and **W** to make certain that they make sense.
- Turbines produce work and pumps require work to operate.
- Heat must be added to make the working fluid boil.
- Heat must be removed to make the working fluid condense.

Refrigeration Cycles

- **1st Law:** **IN = OUT**

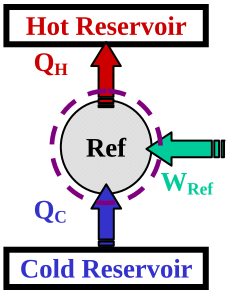
$$Q_H = Q_C + W_{Ref}$$

- **Coefficient of Performance of a Refrigeration Cycle**

$$\beta_{Ref} = COP_R = \frac{\text{Desired Result}}{\text{Required Input}} = \frac{Q_C}{W_{Ref}}$$

$$\beta_{Ref} = COP_R = \frac{Q_C}{Q_H - Q_C}$$

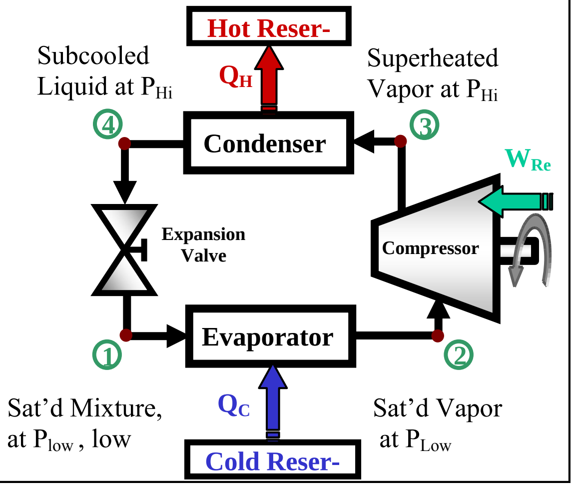
$$\boxed{\beta_{Ref} = COP_R = \frac{1}{\frac{Q_H}{Q_C} - 1}}$$



- In the 1st Law, I have assumed that the heat engine is operating at steady-state.
- Not much new here.
- The value of COPR is usually greater than 1.
  - In this case, that means that $Q_C > W_{ref}$
  - That is what is so “cool” about refrigerators !

Vapor Refrigeration Cycle Components

- The working fluid that flows through the four processes is called a refrigerant.
- The turbine has been replaced by an expansion valve because
  - Expansion valves are less expensive
  - The turbine in a vapor refrigeration cycle produces very little work.



- This is the type of refrigeration cycle used in home refrigerators and in most air conditioners.
- The working fluid flows through four processes in series, but never leaves the system.
- A refrigerator can include a turbine where I have drawn an expansion valve.
  - The problem is that turbines are relatively expensive, require maintenance and, in this type of refrigeration system, produce a small amount of work.
  - Expansion valves are cheap and require little or no maintenance.
  - So, it is hard to find a vapor refrigeration cycle that includes a turbine.

Heat Pump Cycles

- **1st Law: IN = OUT**

  $Q_H = Q_C + W_{HP}$

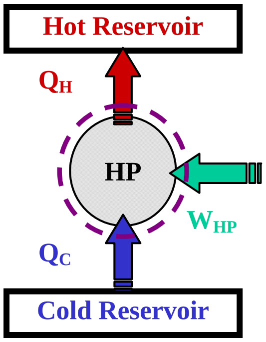
- **Coefficient of Performance of a Refrigeration Cycle**

$$\beta_{HP} = COP_{HP} = \frac{\text{Desired Result}}{\text{Required Input}} = \frac{Q_H}{W_{HP}}$$

$$\beta_{HP} = COP_{HP} = \frac{\text{Desired Result}}{\text{Required Input}} = \frac{Q_H}{W_{HP}}$$

$$\beta_{HP} = COP_{HP} = \frac{Q_H}{Q_H - Q_C}$$

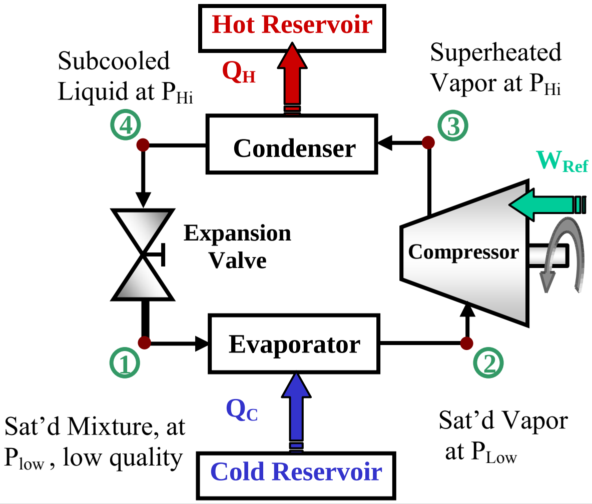
$$\boxed{\beta_{HP} = COP_{HP} = \frac{1}{1 - \frac{Q_C}{Q_H}}}$$



- Very similar to refrigerators.
- The main difference is that the desired result is to add heat to the hot reservoir instead of removing heat from the cold reservoir.

Heat Pump Cycle Components

- The working fluid that flows through the four processes is called a refrigerant.
- The turbine has been replaced by an expansion valve because
  - ◊ Expansion valves are less expensive
  - ◊ The turbine in a vapor heat pump cycle produces very little work.



- The schematic diagram for a HP is also very similar to the schematic for a refrigerator.
- The working fluid is still called a refrigerant.

The 2nd Law of Thermodynamics

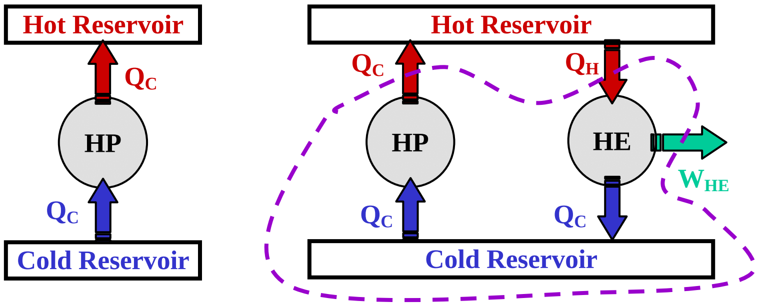
- **Colloquial Statement of the 2nd Law**
  - ◊ Heat flows spontaneously from hot objects to cold objects.
- **The Clausius Statement of the 2nd law**
  - ◊ A thermodynamic cycle cannot transfer heat from a cooler body to a hotter body and produce no other effect on the universe.
  - ◊ The "other effect on the universe" means refrigerators and heat pumps require a work input !

- The colloquial statement of the 2nd Law is concrete and tangible. You should feel very confident about it. All of the experience of your life supports it.
- Over the next few weeks, we will develop several other expressions of the 2nd Law.
- For each new expression of the 2nd Law, we will show that it is equivalent to the one before it.
- In this way, we will develop some very useful and abstract expressions of the 2nd Law.
- The goal is to connect these forms of the 2nd Law in a step-by-step manner back to the colloquial statement of the 2nd Law.
- This should give you the same level of certainty about the validity of these abstract forms of the 2nd Law.

- The simple Clausius Statement has ASTOUNDING implications !
- Let's look at an application of the Clausius Statement of the 2nd Law

Application of the Clausius Statement

- **Can a heat engine have an efficiency of 100% ? That is, can a HE completely convert heat into work ?**



- ◊ This heat pump violates the Clausius Statement of the 2nd Law.

- ◊ The HE complies with both the 1st and 2nd Laws
- ◊ The combined system absorbs heat from a single reservoir and converts it completely into work

- This slide shows that a cycle which violates the Clausius Statement of the 2nd Law could be combined with an ordinary HE (one that does not violate the 1st or 2nd Laws) to make a new system (inside the dashed line) that completely converts heat into work !
- When is it OK to include a reservoir inside the system boundary ?
  - ◊ ONLY when the reservoir has ZERO NET heat exchanged.
  - ◊ So, the key here is that the HP absorbs the same amount of heat from the cold reservoir that the HE rejects back into it.

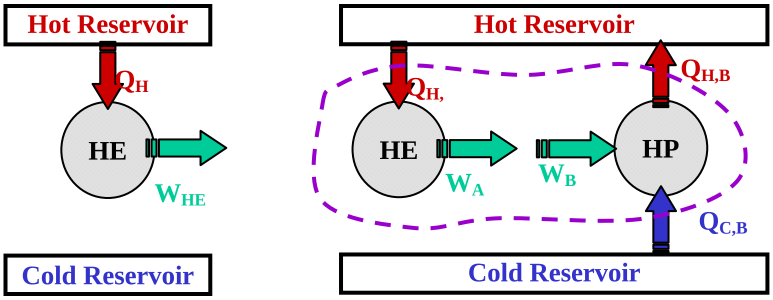
Kelvin-Planck Statement of the 2^{nd} Law

- A thermodynamic cycle cannot produce a net amount of work while exchanging heat with just one thermal reservoir.
- This means that heat engines cannot completely convert heat into work. $\eta < 100\%$
- We showed, on the previous slide, that a cycle which violates the Clauisus Statement also vio-lates the K-P Statement.
- If we can show that the converse is also true (that a cycle which violates the K-P Statement must also violate the Clausius Statement) then we can conclude that the two statements of the 2^{nd} Law are equivalent.

- Here is out first new statement of the 2^{nd} Law.
- It is not obvious how it is related to the Clausius Statement.
- We can show that the K-P and Clausius Statements are equivalent by taking 2 steps.
  - ◊ 1 – We must show that any cycle which violates the Clausius Statement must also violate the K-P Statement.
    - We already did this on the previous slide !
  - ◊ 2 - We must show that any cycle which violates the K-P Statement must also violate the Clausius Statement.
    - This is our mission on the next slide

A Heat Engine That Violates KP Violates Clausius

- **Does a HE that completely converts heat into work violate the Clausius Statement of the 2^{nd} Law ?**



- ◊ This heat engine violates the Kelvin-Planck Statement of the 2^{nd} Law.
- ◊ The HP complies with both the 1^{st} and 2^{nd} Laws
- ◊ The combined system transfers heat from the cold reservoir to the hot reservoir without any work interaction.

- This slide shows that a HE that violates the K-P Statement leads to a combined cycle, a HE plus a HP, that also violates the Clausius Statement.
- Any cycle that violates Clausius must also violate K-P
- Any cycle that violates K-P must also violate Clausius.
- Conclusion: Clausius & K-P are EQUIVALENT statements of the 2^{nd} Law !
- This is a very, very important concept to recognize and accept as fully as you know that heat flows spontaneously from hot things to cold things.
- It is important because …
  - ◊ The K-P Statement of the 2^{nd} Law is much more useful in the analysis of cycles.
  - ◊ The K-P Statement will be the jump-off point for deriving even MORE USEFUL expressions of the 2^{nd} Law.

Perpetual Motion Machines

- **Three kinds of PMMs**
  - ◊ 1st Kind
    - Violate the 1st Law or conservation of mass
    - Create of destroy mass or energy without a nuclear reaction
    - Usually fairly easy to identify
  - ◊ 2nd Kind
    - Violate the 2nd Law of Thermodynamics
    - It is easiest to identify these using the Kelvin-Planck Statement of the 2nd Law
  - ◊ 3rd Kind
    - Produce and consume no work
    - Have no friction
    - Run forever
    - A bit harder to de-bunk

- It is fun to try to identify PMM's and determine which kind they are.
- Let's look at a few example problems of this nature.

Reversible and Irreversible Processes

- **Reversible Processes**
  - ◊ Processes in which the system and the surroundings can both simultaneously return to their initial states after the process is completed.
- **Irreversible Processes**
  - ◊ Processes in which the system and the surroundings cannot both simultaneously return to their initial states after the process is completed.

- A reversible process is the most efficient process.
- No real process is truly reversible.
- In real, irreversible processes, either the system or the surroundings can be returned to its original state, but not BOTH at the same time !
- Some are nearly reversible.
- Let's see what makes a process irreversible so we will know how to make a real process as close to reversible as we can. This means that we will know how to make the efficiency of the process as high as possible !

Sources of Irreversibility

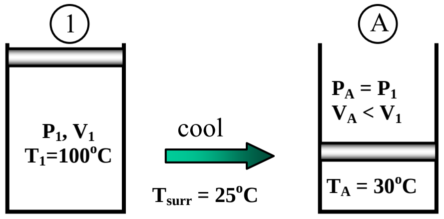
- **Heat Transfer through a finite temperature difference**
- **Friction**
- **Unrestrained, or fast, expansion or compression**

- **Mixing of two different substances**
- **Spontaneous chemical reactions**
- **Electric current flowing through a finite resistance**
- **Inelastic deformation of solids**

- The top 3 sources of irreversibility are the key ones for this course.
- The others are very important, but not so much for the kinds of problems we will consider.
- Now, let's take a closer look at the first 3 sources of irreversibility.
- Minimizing these 3 is the key to designing an efficient process.

Heat Transfer

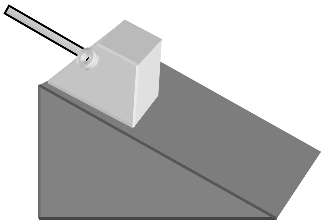
- Heat transfer from the system to the surroundings occurs spontaneously because $\mathbf{T_1} > \mathbf{T_{surr}}$.
- But the system cannot spontaneously return to the initial state because heat cannot spontaneously flow from $\mathbf{T_{surr} = 25^oC}$ to $\mathbf{T_{sys} = 30^oC}$.
- We could use a heat pump to transfer heat from the surroundings into the system to return the system to state 1.
- But the HP would require work from the surroundings to run and this would leave the surroundings in a different state than they started in !
- Any heat transfer through a finite ΔT is therefore irreversible.



- To minimize the irreversibility of a process (and maximize its efficiency) we must keep all the driving forces for heat transfer (**ΔT**'s)as small as practical.
- Think about Newton's law of cooling:
  $Q = h\,A\,\Delta T$
  - ◊ If you need to transfer a lot of heat and you need to keep **ΔT** small, then you must maximize h and A.
  - ◊ The trouble is that you can only make **h** SO big.
  - ◊ A is the area for heat transfer. You can always build BIGGER heat exchangers, that is no problem.
  - ◊ The problem is that BIGGER heat exchangers cost more money, weigh more and take up more space. These can all be big problems.
  - ◊ So, the trade-off becomes efficiency versus HEX size/cost.
  - ◊ The breakeven point usually turns out to be in the neighborhood of **ΔT ~ 10°C**.
    - This yields reasonable efficiency and moderately large HEX's.
    - This is just a rule of thumb. Circumstances can totally change this. Sometimes expense is not a problem (think military, medical or space applications).

Friction

- **Friction converts kinetic energy into internal energy when two bodies in contact with each other are in motion relative to each other.**



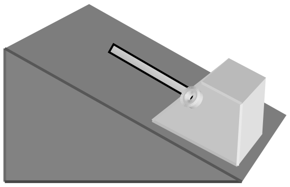
- **Consider what happens when the block spontaneously slides down the inclined plane.**

- The bodies can be solids, liquids or gasses.

- Before we even start, we realize that processes with less friction will be more efficient.
- Why is that ?What is the connection with reversibility and irreversibility ?

Friction

- **As the block slides down the inclined plane, the potential energy of the block decreases.**
- **Friction acts in the direction that <u>opposes</u> motion.**



- **The block does work on the surroundings in order to overcome the frictional force that resists motion.**
- **Friction converts the work into an increase in T and U of the system and surroundings**

- When a resisting force is overcome work is done.
- In this case, the block does work on the surroundings.
- But this work is completely converted into increased U and, therefore, increased T by friction.

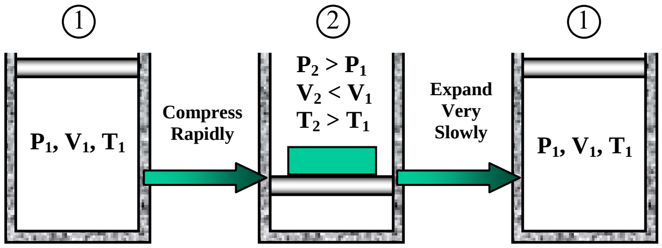
Friction

- **The block never spontaneously slides back up the inclined plane !**
- **Work must be input to increase E_{pot} <u>and</u> to overcome friction.**
- **Friction again converts the work input into another increase in T and U of the system and surroundings.**
- **To return both the block and the surroundings to their original states, we must take heat out of the block and the surroundings and completely convert it to work and return this work to the surroundings.**
- **A device that completely converts heat into work violates the K-P Statement of the 2nd Law and is impossible !**
- **We conclude that any process that includes friction is irreversible !**

- Any process that violates the 2nd Law is impossible.
- An impossible process would be required to return the system and the surroundings to their original states after the process.
- Therefore, the process is irreversible because the "reverse" process is impossible.

Expansion and Compression

- Even without friction, fast expansion and compression are irreversible.



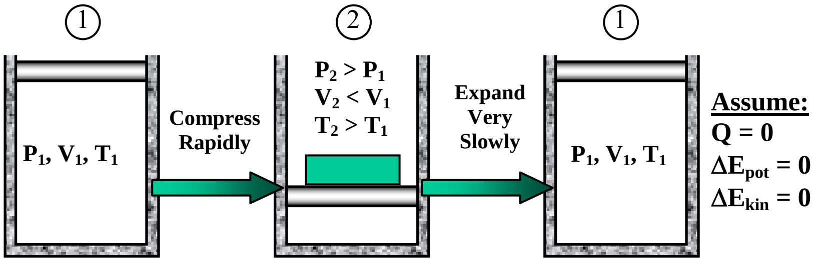
<u>Assume:</u>
Q = 0
$\Delta E_{pot} = 0$
$\Delta E_{kin} = 0$

- Force that must be overcome during the fast compression is <u>greater than</u> $P_1 A_{piston}$.
- Force that is overcome during the slow expansion is <u>equal to</u> $P_1 A_{piston}$.
- We put more work in during the fast compression than we got back out during the slow expansion !

- Recall from our discussion of quasi-equilibrium processes that…
  - ◊ As the compression proceeds rapidly, the molecules don't have time to get out of the way of the advancing piston.
  - ◊ So, the local pressure just inside the face of the piston is GREATER THAN the pressure in the bulk of the fluid inside the piston.
- As a result, the force that must be overcome to compress the gas is always greater than P_1 time A_{piston}.
- We must put in extra work in order to make the process go rapidly.
- We can let the gas expand infinitely slowly back to the original pressure.
  - ◊ In this case, the force which is overcome is exactly equal to the bulk pressure inside the cylinder because the forces acting on the piston are always balanced…the system is always in an equilibrium state.
- The result is that we put more work into the system during the compression than we got out during the expansion.

Expansion and Compression

- If we put in more work than we got out, then U_{sys} and T_{sys} must have increased !



- To return both the system and the surroundings to their original states, we must take heat out of the system and completely convert it to work and return this work to the surroundings.
- A device that completely converts heat into work violates the K-P Statement of the 2nd Law and is impossible !
- We conclude that compression or expansion at a finite rate are irreversible !

- Because the process is adiabatic and changes in E_{kin} and E_{pot} are negligible, this excess work input becomes internal energy.
- This means the temperature of the system increases.
- We must transfer heat out of the system in order to return it to its original temperature.
- We must also return the work to the surroundings in order to return them to their original state.
- The problem is that a device that takes heat from our system and completely converts it into work which can be returned to the surroundings violates the K-P Statement of the 2nd Law and is therefore impossible !
- Since it is impossible to simultaneously return the system and surroundings to their original states, we conclude that compression at a finite rate is irreversible.
- A similar analysis would show that expansion at a finite rate is also irreversible.

Internally Reversible Processes

- **No irreversibilities within the boundary of the system**
  - ◊ No friction inside the system
  - ◊ No rapid expansion or compression inside the system
  - ◊ No mixing of two different substances inside the system
  - ◊ No spontaneous chemical reactions inside the system
  - ◊ No electric current flowing through a finite resistance inside the system
  - ◊ No inelastic deformation of solids inside the system
- **Irreversibilities <u>can</u> exist outside the system**
  - ◊ This usually means that heat can be exchanged between the system and surroundings through a finite temperature difference.

◊

Externally Reversible Processes

- **Irreversibilites do not exist in the surroundings**
  - ◊ This usually means that heat can only be exchanged between the system and surroundings through an infinitely small or infinitessimal temperature difference.
- **Irreversibilites can exist inside the system**

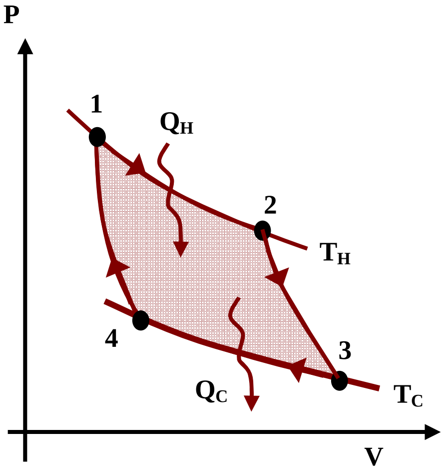
-

Reversible Processes

- **No irreversibilities exist within the system or in the surroundings**
- **No real process is reversible**
- **Some real processes are very nearly reversible**
  - ◊ They have very little friction
  - ◊ They involve only very slow compression or expansion
  - ◊ Heat transfer only takes place between bodies at very similar temperatures.
- **Why bother studying reversible processes if they do not exist ?**
  - ◊ Reversible processes are easier to analyze
  - ◊ Reversible processes provide a best-case to which we can compare the performance of any real process.

Carnot Power Cycle

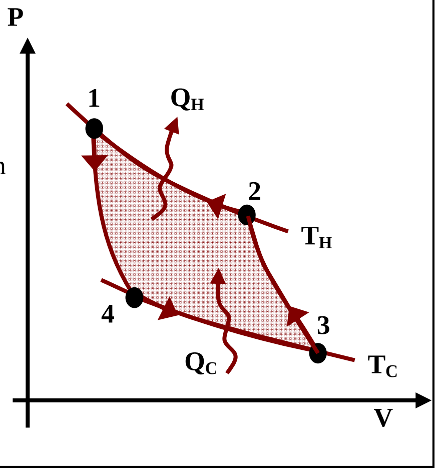
- **Characteristics**
  - Completely Reversible
  - Maximum efficiency between any two thermal reservoirs
- **Steps**
  - 1-2: Isothermal expansion
  - 2-3: Adiabatic expansion
  - 3-4: Isothermal compression
  - 4-1: Adiabatic compression
- **Boundary Work**
  - $W_{12} > 0$ and $W_{23} > 0$
  - $W_{34} < 0$ and $W_{41} < 0$
  - $W_{cycle} > 0$
  - W_{cycle} = enclosed area



- We can imagine a wide variety of reversible cycles, but the Carnot Cycle is the standard against which we will compare all other cycles.
- It is simple and elegant.
- Steps
  - **1-2**: Take some weight off of the back of the piston and let the gas in the cylinder expand. Keep the cylinder submerged in a constant temperature bath at T_H. Heat is transferred from the bath to the gas in the cylinder to keep it at a constant temperature.
  - **2-3**: Take more weights off the back of the piston, but this time keep the cylinder perfectly insulated so that the process is adiabatic. As the gas expands, the temperature drops from T_H to T_C.
  - **3-4**: Put the cylinder into a new constant temperature bath that is at T_C. Add weights to the back of the piston to compress the gas in the cylinder. Heat is transferred from the gas in the cylinder into the bath in order to keep the gas in the cylinder at a constant temperature.
  - **4-1**: Put more weights onto the back of the piston to raise the pressure back to P_1. Keep the cylinder insulated during this compression process so that it occurs adiabatically. As the gas is compressed, its temperature rises from T_C to T_H to complete the cycle.
- Cycles that move clockwise on PV Diagrams are power cycles and the enclosed area is the boundary work output !

Carnot Heat Pump & Refrigeration Cycles

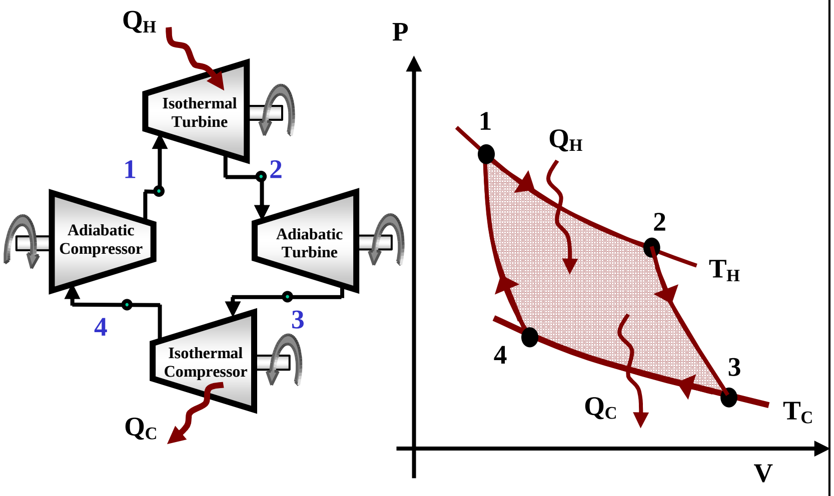
- **If we execute the steps of the Carnot Power Cycle in reverse order, we obtain the Carnot HP / Ref Cycle.**
- **Steps**
  - 1-4: Adiabatic expansion
  - 4-3: Isothermal expansion
  - 3-2: Adiabatic compression
  - 2-1: Isothermal compression
- **Boundary Work**
  - $W_{14} > 0$ and $W_{43} > 0$
  - $W_{32} < 0$ and $W_{21} < 0$
  - $W_{cycle} < 0$
  - $-W_{cycle}$ = enclosed area



- The Carnot Cycle is REVERSIBLE.
- This means that you can operate each step in the reverse order and still come back to the original state. That is, we can consider the reverse cycle !
- The reverse of the Carnot Power Cycle is the Carnot Heat Pump or Refrigeration Cycle.
- The Carnot Refrigeration Cycle absorbs heat from the cold reservoir at T_C and rejects heat to the hot reservoir at T_H.

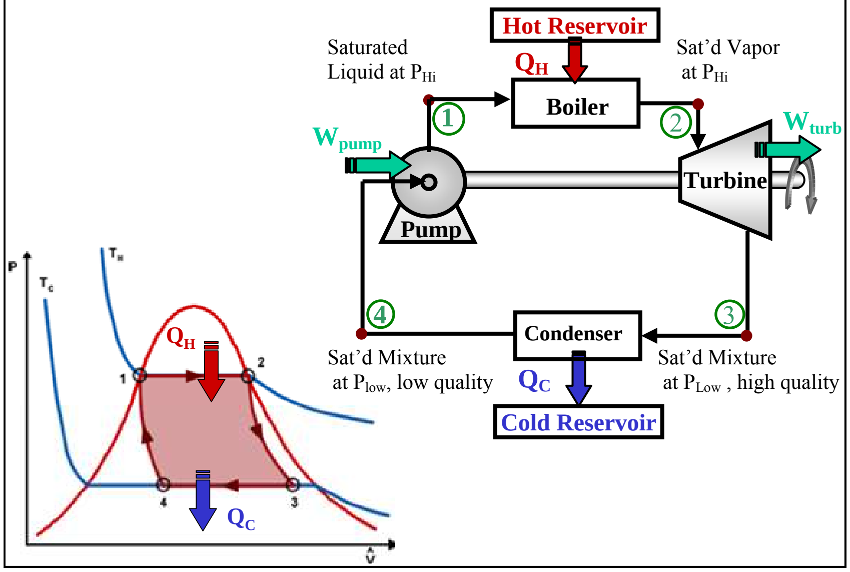
- Cycles that move counter-clockwise on PV Diagrams are refrigeration or heat pump cycles and the enclosed area is the boundary work input !
- Reversing the cycle does not change the magnitudes of Q_H, Q_C and W_{cycle}, just the SIGNS.

Carnot Gas Power Cycle



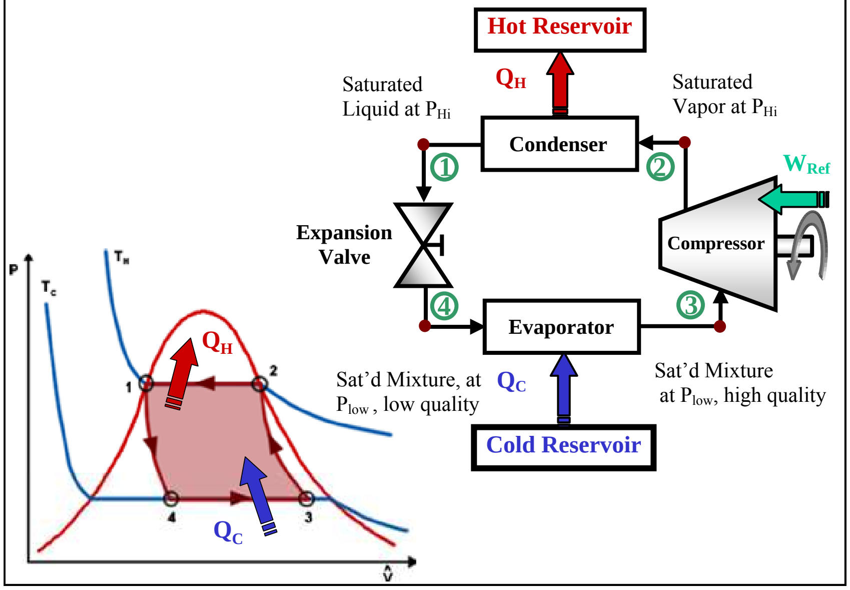
- The Carnot Power Cycle can also be executed with continuous flow equipment as shown here.
- The compression steps take place in compressors and the expansion steps take place in turbines.
- Notice that the inlet to a turbine is at the narrow end of the trapezoid. I remember this because the gas expands in a turbine, so the trapezoid gets wider as the gas flows across it.
- The inlet to a compressor is at the thick end of the trapezoid. I remember this because the gas is compressed to a smaller specific volume in a compressor, so the trapezoid gets narrower as the gas flows across it.
- This is not a very practical cycle because it is difficult to build isothermal turbines and compressors.
  - ◊ It just isn't practical to submerge rotating equipment in constant temperature baths !
  - ◊ Still, in principle, this could be done.
  - ◊ Of course, real processes are NOT reversible, or adiabatic for that matter !

Carnot Vapor Power Cycle



- There are some problems with the Carnot Vapor Power Cycle as well.
- High-efficiency pumps don't work very well with vapor-liquid mixtures.
  - ◊ Pumps are designed to work best on liquids !
- Turbines don't work well if the quality drops below about 90% or even 95%.
  - ◊ Liquid droplets collide with the turbine blades at very high velocity and erode them !
- Again, in principle, the Carnot Vapor Power Cycle could be built.
- Of course, real processes are NOT reversible, or adiabatic for that matter !

Carnot Vapor HP & Ref Cycles

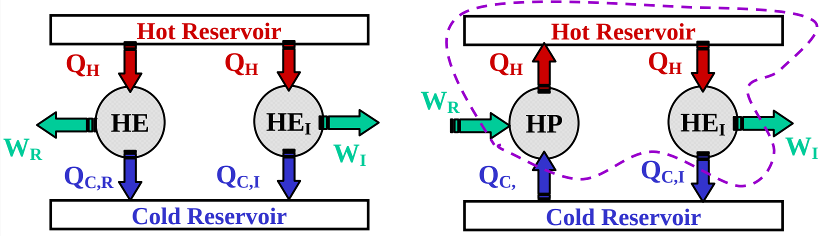


- There are also some problems with the Carnot Vapor HP / Ref Cycle.
- Compressors don't work very well will vapor-liquid mixtures.
  - ◊ Compressors are designed to work best on gases !
- I already removed the turbine in this refrigeration cycle because it just isn't cost effective.
- In principle, the Carnot Vapor HP / Ref Cycle could be built.
- Once again, real processes are NOT reversible, or adiabatic for that matter !

- In order to be a Carnot Cycle, the cycle must include a turbine, not an expansion valve because expansion valves are inherently irreversible !
- In most cases, a turbine in a refrigerator is just not practical.

1^{st} Carnot Principle

- The efficiency of a reversible cycle will always be greater than the efficiency of an irreversible cycle operating between the same two thermal reservoirs.

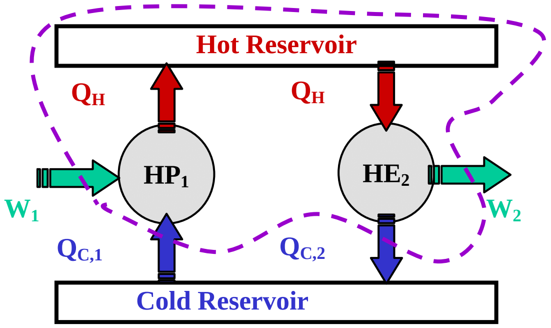


- If: $\eta_R > \eta_I$ Then: $W_I > W_R$ $Q_{C,I} < Q_{C,R}$
  - ◊ The reversible HE could be reversed (right drawing)
  - ◊ System in the dashed line violates the K-P Statement of the 2^{nd} Law !
  - ◊ Conclusion: $\boxed{\eta_I > \eta_R}$

- A reversible HE can be reversed !
- When it is reversed, it becomes a heat pump or a refrigerator.
- When it is reversed, the directions of all the heat and work interactions are also reversed, <u>but they do not change in magnitude</u>.
- So, the HP and the HE in the diagram at right exchange ZERO net heat with the hot reservoir !
- As a result, we can include the reservoir in the system enclosed by the dashed line.
- This system absorbs a net amount of heat from the cold reservoir because $Q_{C,R} > Q_{C,I}$.
- It completely converts this heat into a net amount of work equal to $W_I – W_R$.
- This violates the K-P Statement of the 2nd Law !
- Therefore, it is not possible for an irreversible HE to have a higher efficiency than a reversible HE.
- This confirms what you already knew… reversible heat engines have the maximum efficiency.

2^{nd} Carnot Principle

- All reversible power cycles operating between the same two thermal reservoirs have the same efficiency



- If: $\eta_2 > \eta_1$ Then: $W_2 > W_1$ $Q_{C,2} < Q_{C,1}$
  - ◊ System in the dashed line violates the K-P Statement of the 2^{nd} Law !
  - ◊ Conclusion: $\boxed{\eta_1 = \eta_2}$

- In this diagram, I have already reversed the 1^{st} reversible HE and made it into a HP.
- The Q_H , Q_C and W have the same magnitude and opposite direction as they did when it was a HE.
- Now, the heat pump rejects the same amount of heat to the hot reservoir as the HE takes in.
- So, once again, we can combine the two cycles AND the hot reservoir because there is no NET heat exchange with the hot reservoir.
- The new system, enclosed by the dashed line absorbs a net amount of heat from the cold reservoir equal to $Q_{C,1} - Q_{C,2}$
- The new system completely converts this heat into work equal to $W_2 - W_1$.
- This violates the K-P Statement of the 2^{nd} Law !
- Therefore, it is not possible for one reversible HE to have a higher efficiency than any other reversible HE when they operate between the same two thermal reservoirs.
- All reversible heat engines operating between the same two thermal reservoirs have the same efficiency.

Thermodynamic Temperature Scales

- From Ch 5: $\eta = 1 - \frac{Q_C}{Q_H}$
- 2^{nd} Carnot Principle : $\eta = fxn(T_C, T_H)$
  - ◊ Therefore : $\frac{Q_C}{Q_H} = fxn(T_C, T_H)$
- Thermodynamic Temperature Scales
  - ◊ Operate a HE between two thermal reservoirs
  - ◊ Measure Q_C, Q_H and W
  - ◊ Calculate η
  - ◊ Arbitrarily assign a value to either T_C or T_H
  - ◊ If we know the function: $\eta = fxn(T_C, T_H)$ we can calculate the other, unknown T.
  - ◊ This is a Thermodynamic Temperature Scale

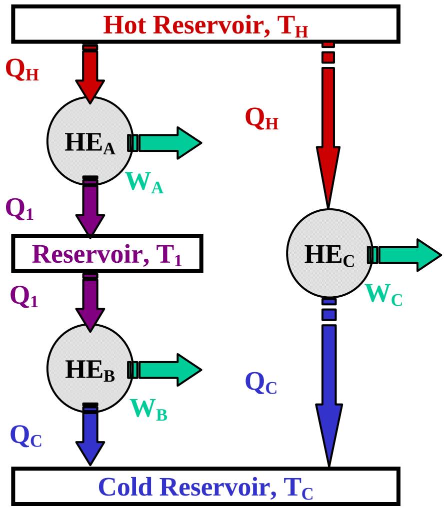
- In Ch 5 we learned that thermal efficiency depends only on the ratio of Q_C to Q_H.
- From the 2^{nd} Carnot Principle, we learned that thermal efficiency depends only on T_C and T_H .
- We can conclude that Q_C / Q_H depends only on T_C and T_H .
- Our goal is to determine this functional relationship !
- This will be a very important result. Why ?
- For starters, it will allow us to define thermodynamic temperature scales.
- Thermodynamic temperature scales are not dependent on the properties of any material !

$Q_C / Q_H = Fxn(T_H , T_C)$

$$\frac{Q_1}{Q_H} = \text{fxn}\left(T_H, T_1\right)$$

$$\frac{Q_C}{Q_1} = \text{fxn}\left(T_1, T_C\right)$$

$$\frac{Q_C}{Q_H} = \text{fxn}\left(T_H, T_C\right)$$



Algebra tells us that :

$$\frac{Q_C}{Q_H} = \frac{Q_C}{Q_1}\,\frac{Q_1}{Q_H}$$

$$\boxed{\text{fxn}\left(T_H, T_C\right) = \text{fxn}\left(T_H, T_1\right)\cdot\text{fxn}\left(T_1, T_C\right)}$$

- Because we are applying the 2nd Carnot Principle to these heat engines, the result of this work will only apply to REVERSIBLE cycles !
- Notice that fxn(T_H, T_1) means plug T_H and T_1 into the function.
  - ◊ It does NOT mean fxn multiplied by T_H or T_1.
  - ◊ Ex: fxn(x,y) = ax + by + c, then fxn (T_H=400, T_1=300) = 400 a + 300 b + c
- The bottom line is:
  - ◊ The function "fxn" must be a very special function in order for the boxed equation to be true
  - ◊ Let's see what kind of function would satisfy the relationship in the box.

The Kelvin Relationship

$$\text{fxn}\left(T_H, T_C\right) = \text{fxn}\left(T_H, T_1\right)\cdot\text{fxn}\left(T_1, T_C\right)$$

- **All of the T_1 terms in f_A and f_B must drop out when they are multiplied together.**
- **Only true if the fxn has the form :** $\text{fxn}(x,y) = \frac{\text{fxn2}(y)}{\text{fxn2}(x)}$

$$\text{fxn}(T_H, T_C) = \frac{\text{fxn2}(T_C)}{\text{fxn2}(T_H)} = \frac{\text{fxn2}(T_1)}{\text{fxn2}(T_H)}\cdot\frac{\text{fxn2}(T_C)}{\text{fxn2}(T_1)}$$

- **Conclusion :** $\frac{Q_C}{Q_H} = \text{fxn}(T_H, T_C) = \frac{\text{fxn2}(T_C)}{\text{fxn2}(T_H)}$
- **Lord Kelvin :** $\text{fxn2}(T) = T$
- **Result :** $\boxed{\frac{Q_C}{Q_H} = \frac{T_C}{T_H}}$ $\boxed{\eta_{Rev} = 1 - \frac{T_C}{T_H}}$

- This is a wonderful result. So, if we know the temperatures of two reservoirs in Kelvin, we can immediately and easily compute the thermal efficiency of a reversible HE operating between those reservoirs.
- What is the thermal efficiency of a reversible HE operating between a human body and the ambient air in the classroom ?
- What is the thermal efficiency of a reversible HE operating between a nuclear reactor at 2000 K and the cooling water at 300 K ?
- This is why we want to have a very hot reservoir to generate electrical power.

- There is only one way for all of the $\mathbf{T_1}$ terms to drop out.
- This leads to such a nice, simple result. It makes life much easier than all those other functions.
- We will use the Kelvin Relationships in most of the problems in the rest of this course !
- Choices other than the one made by Kelvin are possible.
  - ◊ **Fxn2** must be a function that increases monotonically, so **fxn2(T) = e^{T}**
    - This is called a logarithmic temperature scale
    - Temperature on this scale goes from –∞ to +∞ as opposed to **0** to +∞ on the Kelvin scale.
- Now, back to temperature scales…
- Use water at its triple point as one reservoir for a HE and assign its temperature a numerical value of **273.16**.
- Run a reversible HE between this reservoir and another reservoir at an unknown temperature.
- Measure **Q_{triple} and Q_{unk}**.
- **T_{unk} = 273.16 * (Q_{unk}/ Q_{triple})**
- This IS the Kelvin Temperature Scale !
- The Kelvin Temperature Scale IS a THERMODYNAMIC TEMPERATURE SCALE !

The Ideal Gas Temperature Scale

- **Apply the 1^{st} Law to Steps 1-2 and 3-4 carried out in a closed system to determine Q_H and Q_C.**

$$\tilde{Q} - \tilde{W} = \Delta\tilde{U}$$

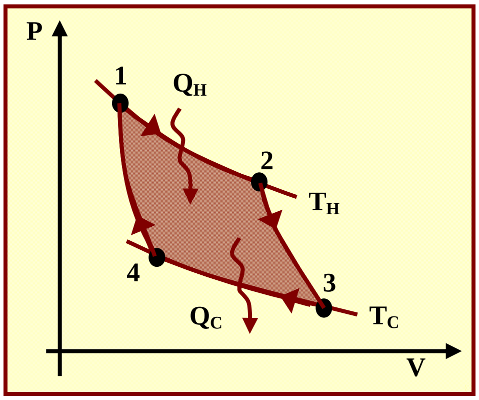
- **Use the definition of boundary work and C_V for ideal gases.**

$$\tilde{Q} = \int \tilde{C}_V^o\, dT + \int \frac{R\,T}{\tilde{V}} d\tilde{V}$$

- **Results :**

$$\tilde{Q}_H = R\,T_H\,\mathrm{Ln}\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

$$\tilde{Q}_C = -R\,T_C\,\mathrm{Ln}\left[\frac{\tilde{V}_4}{\tilde{V}_3}\right]$$



- As usual, assume changes in kinetic and potential energies are negligible.
- The only tricky part here is converting from the 1^{st} Law…which uses our sign convention… to $\mathbf{Q_C}$ and $\mathbf{Q_H}$ which are both positive and DO NOT use the sign convention.
- Here is the key:
  - ◊ $\mathbf{Q_H = Q_{12}}$
  - ◊ $\mathbf{Q_C = -Q_{34}}$
- The results come naturally when you realize that Steps 1-2 and 3-4 are both isothermal.

Adiabatic Steps in a Carnot Cycle

- **Apply the 1^{st} Law to Steps 2-3** $\quad -\tilde{W} = \Delta\tilde{U}$

- **Use the definition of boundary work and C_V for ideal gases.**

$$\int \frac{\tilde{C}_V^o}{T} dT = -\int \frac{R}{\tilde{V}} d\tilde{V}$$

- **Results :**

$$\int_{T_H}^{T_C} \frac{\tilde{C}_V^o}{T} dT = -R\,\mathrm{Ln}\left[\frac{\tilde{V}_3}{\tilde{V}_2}\right]$$

$$\int_{T_C}^{T_H} \frac{\tilde{C}_V^o}{T} dT = -R\,\mathrm{Ln}\left[\frac{\tilde{V}_1}{\tilde{V}_4}\right]$$

- **Conclusion :**

$$\frac{\tilde{V}_4}{\tilde{V}_1} = \frac{\tilde{V}_3}{\tilde{V}_2}$$

$$\boxed{\frac{\tilde{V}_3}{\tilde{V}_4} = \frac{\tilde{V}_2}{\tilde{V}_1}}$$

- ◊ The only tricky part here involves T.
- ◊ In order to understand why this technique is valid, we must write the 1st Law in differential form.
- ◊ **- dW = dU**
- ◊ Then, because we assume only boundary work occurs : **- P dV~ = dU = $\mathbf{C^o_V}$ dT**
- ◊ Ideal Gas EOS : **- (RT/V~) dV = dU = CoV dT**
- ◊ We cannot integrate the left-hand side of this equation at this point because **T** is NOT a constant ! We must get the **T** into the right-hand side of the equation where we will integrate with respect to **T** and take into account the fact that **T** changes during the adiabatic process.
- ◊ Divide by **T** : **- R (dV~ / V~) = ($\mathbf{C^o_V}$ / T) dT**
- ◊ Now, we can integrate !

Ideal Gas & Kelvin Temperature Scales

- **The isothermal steps :**

$$\tilde{Q}_C = -R\,T_C\,\mathrm{Ln}\left[\frac{\tilde{V}_4}{\tilde{V}_3}\right] \qquad \tilde{Q}_H = R\,T_H\,\mathrm{Ln}\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

$$\frac{\tilde{Q}_H}{\tilde{Q}_C} = \frac{R\,T_H\,\mathrm{Ln}\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]}{-R\,T_C\,\mathrm{Ln}\left[\frac{\tilde{V}_4}{\tilde{V}_3}\right]} = \frac{T_H}{T_C}\;\frac{\mathrm{Ln}\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]}{\mathrm{Ln}\left[\frac{\tilde{V}_3}{\tilde{V}_4}\right]}$$

- **The adiabatic steps :** $\frac{\tilde{V}_3}{\tilde{V}_4} = \frac{\tilde{V}_2}{\tilde{V}_1}$
- **Combining yields :** $\boxed{\frac{\tilde{Q}_H}{\tilde{Q}_C} = \frac{T_H}{T_C}}$
- **The Ideal Gas & Kelvin Scales are <u>identical</u> !**

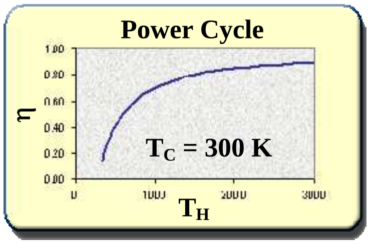
- Most of the algebra here is straightforward.
- The only trick is that **– Ln[A] = Ln[1/A]**.
- The result is very cool.
- It shows in a much clearer way than I could back in Ch 1 why the Ideal Gas and the Kelvin Temperature Scales are identical.
- They are both thermodynamic temperature scales.
- These results enable us to analyze a wide new area of thermodynamic cycle problems.

Carnot Efficiency and COP

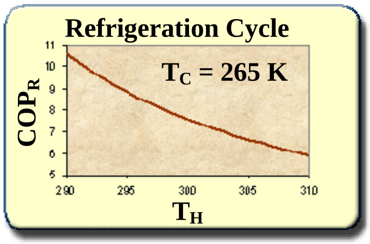
Power Cycles	**Refrigeration Cycles**	**Heat Pump Cycles**
$\eta = 1 - \frac{Q_C}{Q_H}$	$COP_R = \frac{1}{\frac{Q_H}{Q_C} - 1}$	$COP_{HP} = \frac{1}{1 - \frac{Q_C}{Q_H}}$
$\eta_{Rev} = 1 - \frac{T_C}{T_H}$	$COP_{R,rev} = \frac{1}{\frac{T_H}{T_C} - 1}$	$COP_{HP,rev} = \frac{1}{1 - \frac{T_C}{T_H}}$
$\eta_{Irrev} < 1 - \frac{T_C}{T_H}$	$COP_{R,irrev} < \frac{1}{\frac{T_H}{T_C} - 1}$	$COP_{HP,irrev} < \frac{1}{1 - \frac{T_C}{T_H}}$

Performance vs. Reservoir Temperature

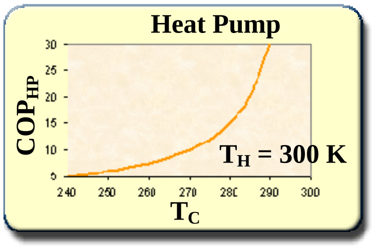
- **All cycles shown here are <u>reversible</u>.**



Power Cycle



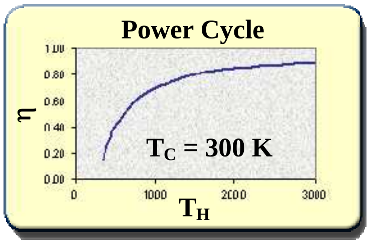
Refrigeration Cycle



Heat Pump

- Power Cycle: $\mathbf{T_C = 300}$ **K**
  - ◊ Efficiency → **100 %** as $\mathbf{T_H} \to \infty$
  - ◊ Efficiency → **0 %** as $\mathbf{T_H} \to \mathbf{T_C}$
- Refrigeration Cycles: $\mathbf{T_C}$ = 265 K
  - ◊ **COP** → ∞ as $\mathbf{T_H} \to \mathbf{T_C}$
  - ◊ **COP** → **0** as $\mathbf{T_H} \to \mathbf{T_H}$
- Heat Pump Cycles: $\mathbf{T_H}$ = 265 K
  - ◊ **COP** → ∞ as $\mathbf{T_C} \to \mathbf{T_H}$
  - ◊ **COP** → **1** as $\mathbf{T_C}$ → 0 K

The Relative Value of Energy



Power Cycle

- **100 J of heat at 1000 K produces more work than 100 J of heat at 300 K**
- **Heat energy available at a higher T is more valuable or has a higher quality.**
- **You can always completely convert work into heat at ANY temperature using friction.**
  - ◊ We can conclude that work is a more valuable form of energy than heat !

- The idea that energy has a certain quality depending on the temperature at which it is available leads to the concept of entropy that we will study in the next chapter !
- No matter what the temperature, if you do some work, the resulting friction yields heat.
- So, work can be completely converted to heat at ANY temperature.
- Heat can NEVER be completely converted into work (K-P statement of the 2nd Law).
- Work is therefore more valuable than heat, regardless of the temperature at which the heat is available.

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6B-1 Home Heat Pump COP 6 pts

A well-insulated house requires **138 MJ/h** to keep the indoor temperature comfortable on a cold day. Under this load, the heat pump compressor uses **7.7 kW** of electrical **power**.

a.) Determine the **COP** of the heat pump

b.) If the heat pump operates **125 hours** in a winter month, what will the homeowner spend on electric heat that month? Residential electricity costs **$0.11/kW-h**.

c.) How much would the homeowner spend that month if she had electrical resistance heating instead of a heat pump?

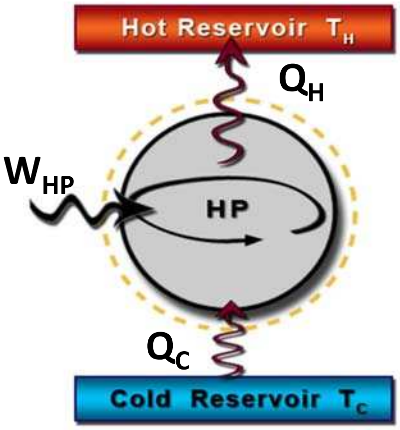
Read : This is a straightforward application of the definition of the **COP** of a heat pump.

Given:

Q_H	138	MJ/h
W_{HP} =	7.7	kW
Price	0.11	$/kW-h
OpTime	125	h/month

Find: COP_{HP} ???

Diagram:



Assumptions:

1 - The heat pump operates at steady-state.

2 - There is no loss of efficiency when the heat pump is started up or shut down by the thermostatic control system.

Equations / Data / Solve:

Begin by writing the definition for the **COP** of a heat pump :

$$COP_{HP} = \frac{Q_H}{W_{HP}} \qquad \textbf{Eqn 1}$$

We can use **Eqn 1** to evaluate the **COP_{HP}**. Watch the units !

Q_H	38.33	kW	COP_{HP}	4.98

First let's see how much it would cost to deliver Q_H to the home using an electrical resistance heater.

An electrical resistance heater, at best, converts all of the electrical work supplied, **W**, into heat released into the home, **Q_H**. Therefore, in order to get **38.33 kW** into your home, you must buy **38.33 kW** of electrical power.

W_{resist} **38.33** **kW**

Now, we can determine how much it would cost to operate the resistance heater for **125 hr/month**.

$$\text{Cost}(\$/\text{month}) = \text{Price}(\$/\cancel{\text{kW}}\cdot\cancel{\text{h}}) \times \text{W}(\cancel{\text{kW}}) \times \text{time}(\cancel{\text{h}}/\text{month}) \qquad \textbf{Eqn 2}$$

$Cost_{resist}$ **527.08** **$/month**

Now, we can apply **Eqn 2** to determine the cost of operating the heat pump for a month.

$Cost_{HP}$ **105.88** **$/month**

The difference between these two costs is the monthly savings: **Savings** **421.21** **$/month**

Verify: We cannot verify the steady-state assumption or the assumption about the thermostatic control system based on the information given in the problem statement.

Answers : **COP_{HP} 5.0** **Savings 421 $/month**

Electrical resistance heaters are not very popular, especially in cold climates.

The **thermal efficiency** of a heat pump drops significantly as the outside **temperature** falls.

When the outside **temperature** drops far enough that the **COP_{HP} ~ 1**, it becomes more practical to use the resistance heater !

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6B-2 Coefficient of Performance of a Heat Pump and a Refrigerator 4 pts

A geothermal heat pump absorbs **15 Btu/s** of heat from the Earth **50 ft** below a house. This heat pump uses a **10 hP** compressor.

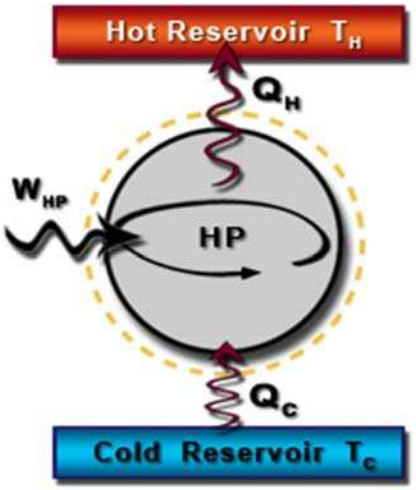
a.) Calculate the **COP** of the heat pump.

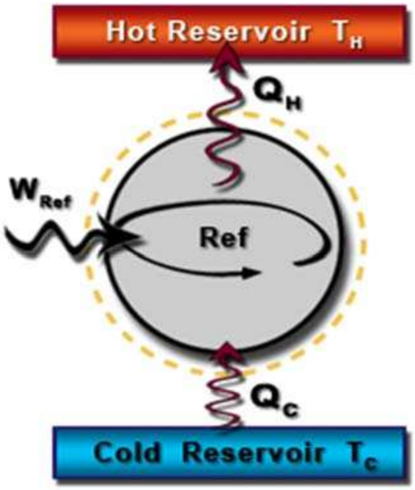
b.) In the summer, the cycle is reversed to cool the house. Calculate the **COP** of the cycle when it is operated as an air-conditioner assuming the working fluid rejects **15 Btu/s** to the Earth.

Read : Here we must apply the definition of **COP** for both refrigerators and heat pumps.

Given:	**W**	**10**	**hP**	**Q_C**	**15**	**Btu/s**
Find:	**a.)**	**COP_{HP}**	**???**	**b.)**	**COP_R**	**???**

Diagram:





Assumptions: 1 - The heat pump and the refrigerator operate at steady-state.

Equations / Data / Solve:

Part a.) If the purpose is to cool the groundwater, then the device is a refrigerator.

So, let's begin with the definition of the **coefficient of performance** for a refrigerator.

$$COP_R = \frac{Q_C}{W_{Ref}} \qquad \text{Eqn 1}$$

We are given the values of both **Q_C** and **W**, so all we need to do is make the units consistent and then plug values into **Eqn 1**.

Conversion Factors :	**1 hP =**	**2545**	**Btu/h**	**W**	**25450**	**Btu/h**
				Q_C	**54000**	**Btu/h**
				COP_R	**2.122**	

Part b.) If the purpose is to heat a building, then the device is a heat pump.
So, let's begin with the definition of the **coefficient of performance** for a heat pump.

$$\mathrm{COP_{HP}} = \frac{Q_H}{W_{HP}}$$ **Eqn 2**

Next, we apply the 1st Law to the heat pump cycle, keeping in mind that we traditionally do not use our sign convention when tie-fighter diagrams are used. The arrows on the diagrams indicate the direction that **heat** and **work** are moving.

$$Q_H = Q_C + W_{HP}$$ **Eqn 3**

Use **Eqn 3** to eliminate $\mathbf{Q_H}$ from **Eqn 2** to get :

$$\mathrm{COP_{HP}} = \frac{Q_C + W_{HP}}{W_{HP}}$$ **Eqn 4**

Now, we can plug in the numbers in consistent units that we used in **part (a)**.

$\mathrm{COP_{HP}}$ **3.122**

Notice that : $$\mathrm{COP_{HP}} = \mathrm{COP_R} + 1$$ **Eqn 5**

This is always true for Carnot Cycles.

Verify: No assumptions to verify that were not given in the problem statement.

Answers :

$\mathrm{COP_R}$	**2.12**
$\mathrm{COP_{HP}}$	**3.12**

6C-1 **Is This a Perpetual Motion Machine ?** **2 pts**

In a day, an electrical resistance heater uses **8.4 kW-h** of electricity. In the same day, the heater delivers **9.7 kW-h** of heat to the house in which it is located. Is this possible or is this a perpetual motion machine?

If so, what kind? Explain your reasoning.

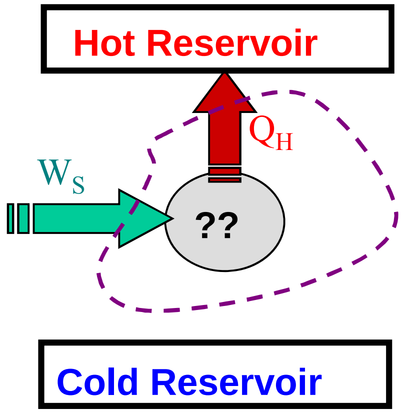
Read : Short and sweet.

Given:

W_s	**8.4**	**kW-h**
Q_H	**9.7**	**kW-h**

Find: **Is this a PMM ? Explain**

Diagram:



Assumptions: None.

Equations / Data / Solve:

Apply the 1st Law to the inventor's process :

$$W_S = Q_H \qquad \text{Eqn 1}$$

But **8.4 kW-h** ≠ **9.7 kW-h** !

Therefore, if the device operates on a true thermodynamic cycle (no change in state after a complete cycle), then it violates the 1st Law.

Verify: No assumptions to verify !

Answers : **It is a PMM of the 1st Kind !**

6C-2 Is This a Perpetual Motion Machine ? 3 pts

Air is compressed by a compressor that is driven by an electric motor. The effluent **air** is warmer than the **air** at the inlet of the compressor. An entrepreneur plans to use this system to heat his home.

He claims that this system is **10%** more efficient than an eletrical resistance heater. Is this possible or is this another perpetual motion machine? If so, what kind? Explain your reasoning.

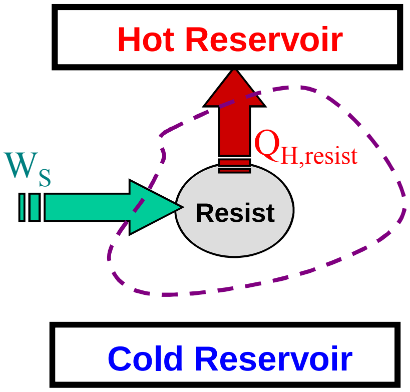
Read : Short and sweet.

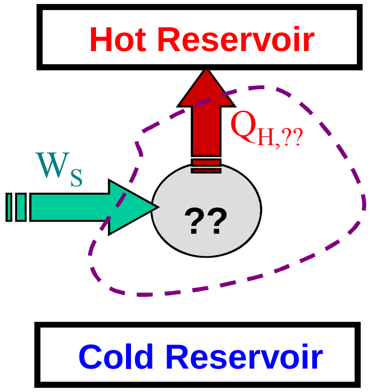
Given: $\eta_{??} = 1.1\ \eta_{resist}$

Find: **Is this a PMM ? Explain**

Assumptions: None.

Diagram:





Equations / Data / Solve:

Efficiency for a heating system is really the **coefficient of performance** and it is defined by :

$$\mathrm{COP} = \frac{Q_H}{W_S} \qquad \text{Eqn 1}$$

The **COP** for the resistance heater is **1** because it converts <u>all</u> of the electrical energy input into **heat** rejected into the hot reservoir…your home.

If the **COP** of the compressor/heater device is **10%** <u>greater</u>, then : **COP =** **1.1**

And:

$$Q_H = 1.1 \cdot W_S \qquad \text{Eqn 2}$$

But if you consider a system in which the air is <u>recycled</u> as the working fluid, then the 1st Law for the compressor/heater system is:

$$W_S = Q_H \qquad \text{Eqn 3}$$

Eqns 2 and **3** cannot both be true.

Therefore, the inventor's device violates the 1st Law !

Verify: No assumptions to verify !

Answers : **It is a PMM of the 1st Kind !**

6D-1 Determine Whether Water Condensing is a Reversible Process 2 pts

A piston-and-cylinder device with a free-floating piston contains saturated **water** vapor at **225°C**. Heat is lost from the **water** to the surroundings which are at **25°C**.
Eventually, all of the **water** has condensed and the cylinder contains saturated liquid **water** at **225°C**. Is this process internally reversible? Reversible? Irreversible? Explain your answer.

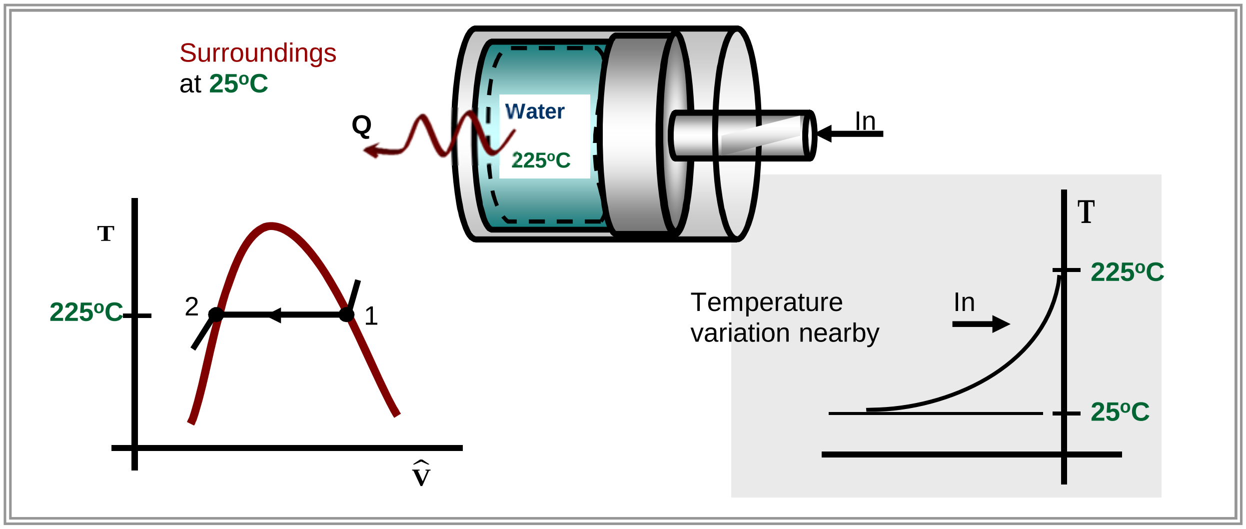
Read : What can be said about this case of heat transfer process for **(a)** isothermal cooling of the water and **(b)** contact between **225°C** and **25°C** surroundings?

Given:

T_w	225	°C
T_{surr}	25	°C

Find: **Internally Reversible ? Reversible ? Explain.**

Diagram:



Assumptions:

1 - The system is the water in the piston-and-cylinder assembly.
2 - The system undergoes a constant-temperature process from saturated vapor to saturated liquid.

Equations / Data / Solve:

No equations are needed to resolve this problem.

There are no equations to solve.

Verify: None of the assumptions made can be verified with the given information.

Answers : Since the **temperature** is constant during the process, the **pressure** also remains constant. As shown by the **T-V Diagram**, the process is a sequence of equilibrium states, and thus is internally reversible.

The process is not reversible because there is a significant irreversibility in the surroundings. The spontaneous heat transfer taking place between the water at **225°C** and the surroundings at **25°C** is irreversible.

6E-1 Heat, Work and Efficiency of a Water Vapor Power Cycle 8 pts

A piston-and-cylinder device containing 12 kg of **water** carries out a Carnot power cycle. The maximum **pressure** is 2.0 MPa and the minimum **pressure** is 110 kPa.

During the isothermal expansion, the **water** is heated from a **quality** of 14% until it is a saturated vapor. The cycle produces 500 kJkg of work during the adiabatic expansion.

a.) Sketch the process path for the cycle on a **PV Diagram**
b.) Calculate **Q** and **W**, in **kJ**, for each process in the cycle
c.) Calculate the **thermal efficiency** of the cycle.

Read : Apply the 1st Law (for a closed system) to get **Q** and **W**.
Use the 1st Law applied to **step 2-3** to determine U_3 and x_3.
The trick is to get Q_{34}. Use T_C, T_H, Q_{12} and the Carnot Efficiency of this reversible cycle to determine Q_{34}.

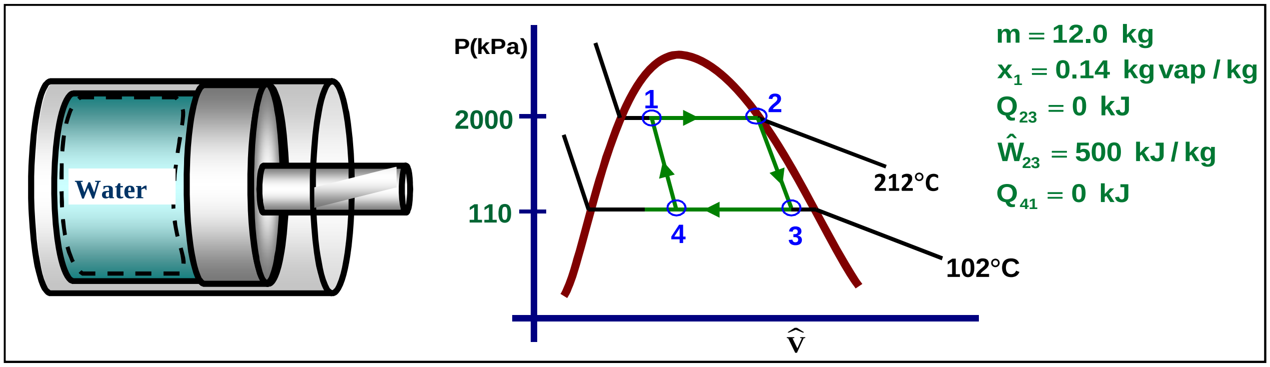
Given:

P_1	2	MPa	$P_3 = P_4$	110	kPa
x_1	0.14		$Q_{23} = Q_{41}$	0	kJ/kg
P_2	2	MPa	W_{23}	500	kJ/kg
x_2	1		m	12	kg

Find:

Part (a)	PV Diagram		
Part (b)	$Q_{12}, Q_{23}, Q_{34}, Q_{41}$	?	kJ
	$W_{12}, W_{23}, W_{34}, W_{41}$	?	kJ
Part (c)	η	?	

Diagram:
Part a.)



Assumptions:

1 - The system undergoes a Carnot Cycle.
  - **Steps 1-2** and **3-4** are isothermal.
  - **Steps 2-3** and **4-1** are adiabatic.
  - All steps are reversible.

2 - The water inside the cylinder is the system and it is a closed system.
3 - Changes in kinetic and potential energies are negligible.
4 - Boundary work is the only form of **work** interaction during the cycle.

Equations / Data / Solve:

Part b.) Begin by applying the 1st law for closed systems to each step in the Carnot Cycle. Assume that changes in kinetic and potential energies are negligible.

$$\Delta U = Q - W \qquad \text{Eqn 1}$$

Step 1 - 2

Apply the 1st Law, **Eqn 1**, to **step 1-2** : $\Delta U_{12} = Q_{12} - W_{12}$ **Eqn 2**

Boundary work at for a constant **pressure** process, like **step 1-2**, can be determined from :

$$W_{12} = \int_1^2 P\,dV = P\Delta V = mP\left(\hat{V}_2 - \hat{V}_1\right) \qquad \text{Eqn 3}$$

Now, we can substitute **Eqn 3** into **Eqn 1** to get : $\Delta U_{12} = Q_{12} - P\Delta V_{12}$ **Eqn 4**

The definition of **enthalpy** is: $\Delta H = \Delta U + \Delta (PV)$ **Eqn 5**

For isobaric processes, **Eqn 5** becomes : $\Delta H = \Delta U + P\Delta V$ **Eqn 6**

Now, combine **Eqns 4** and **6** to get : $Q_{12} = \Delta H_{12} = m\Delta\hat{H}_{12}$ **Eqn 7**

We know the **pressure** and the **quality** of **states 1** and **2**, so we can use the Saturation Table in the Steam Tables to evaluate **V** and **H** for **states 1** and **2** so we can use **Eqns 3** and **7** to evaluate Q_{12} and W_{12}.

Properties are determined from **NIST WebBook**:

$$\hat{V} = \hat{V}_{sat\,liq} + x\,(\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}) \qquad \text{Eqn 8}$$

At P_1 and x_1:

$V_{sat\ liq,\ 1}$	0.0011767	m^3/kg			
$V_{sat\ vap,\ 1}$	0.099585	m^3/kg	V_1	0.014954	m^3/kg

$$\hat{U} = \hat{U}_{sat\,liq} + x\,(\hat{U}_{sat\,vap} - \hat{U}_{sat\,liq}) \qquad \text{Eqn 9}$$

$U_{sat\ liq,\ 1}$	906.14	kJ/kg			
$U_{sat\ vap,\ 1}$	2599.1	kJ/kg	U_1	1143.2	kJ/kg

$$\hat{H} = \hat{H}_{sat\,liq} + x\,(\hat{H}_{sat\,vap} - \hat{H}_{sat\,liq})$$

$H_{sat\ liq,\ 1}$	908.50	kJ/kg			
$H_{sat\ vap,\ 1}$	2798.3	kJ/kg	H_1	1173.1	kJ/kg

Saturated vapor at P_2:

V_2	0.099585	m^3/kg			
U_2	2599.1	kJ/kg	W_{12}	2031.147	kJ
H_2	2798.3	kJ/kg	Q_{12}	19502.68	kJ

Step 2 - 3

Apply the 1st Law, **Eqn 1**, to **step 2-3** :

$$\Delta U_{23} = Q_{23} - W_{23} \qquad \text{Eqn 10}$$

The **specific heat transferred** and **specific work** for **step 2-3** are given in the problem statement.

Q_{23}	0	kJ	W_{23}	6000	kJ

We can plug these values into **Eqn 8** to determine ΔU_{23} :

ΔU_{23}	-6000	kJ

We already determined U_2, so we can now determine U_3 :

$$\hat{U}_3 = \frac{\Delta \hat{U}_{23}}{m} + \hat{U}_2 \qquad \text{Eqn 11}$$

U_3	2099.1	kJ/kg

We can use this value of U_3 to determine the unknown quality, x_3 , using :

$$x_3 = \frac{\hat{U}_3 - \hat{U}_{sat\,liq,3}}{\hat{U}_{sat\,vap,3} - \hat{U}_{sat\,liq,3}} \qquad \text{Eqn 12}$$

Properties are determined from **NIST WebBook**:

At P_3:

$U_{sat\ liq,\ 3}$	428.72	kJ/kg			
$U_{sat\ vap,\ 3}$	2508.7	kJ/kg	x_3	0.8031	kg vap/kg

$$\hat{V} = \hat{V}_{sat\,liq} + x\,(\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}) \qquad \text{Eqn 13}$$

At P_3 and x_3:

$V_{sat\ liq,\ 3}$	0.0010453	m^3/kg			
$V_{sat\ vap,\ 3}$	1.54946	m^3/kg	V_3	1.2445	m^3/kg

$$\hat{H} = \hat{H}_{sat\,liq} + x\,(\hat{H}_{sat\,vap} - \hat{H}_{sat\,liq}) \qquad \text{Eqn 14}$$

$H_{sat\ liq,\ 3}$	428.84	kJ/kg			
$H_{sat\ vap,\ 3}$	2679.2	kJ/kg	H_3	2236.0	kJ/kg

Step 3 - 4

Apply the 1st Law, **Eqn 1**, to **step 3-4** :

$$\Delta U_{34} = Q_{34} - W_{34} \qquad \text{Eqn 15}$$

Because **step 3-4** is isobaric, just like **step 1-2**, **Eqn 7** is the simplified form of the 1st Law :

$$Q_{34} = \Delta H_{34} = m\,\Delta \hat{H}_{34} \qquad \text{Eqn 16}$$

To determine the properties at **state 4**, we make us of the relationship between the absolute Kelvin temperature scale and **heat transferred** in a Carnot Cycle.

$$\frac{|Q_{34}|}{|Q_{12}|} = \frac{T_C}{T_H} \qquad \text{Eqn 17}$$

Solve **Eqn 13** for Q_{34} :

$$|Q_{34}| = \left(\frac{T_C}{T_H}\right)|Q_{12}| \qquad \text{Eqn 18}$$

$T_H = T_{sat}(P_1)$:	T_H	485.53	K	Q_{34}	-15080.8	kJ
$T_C = T_{sat}(P_3)$:	T_C	375.44	K	Q_{34}	-1256.7	kJ/kg

Now, we can use **Q_{34}** and **Eqn 12** to determine **H_4** as follows:

$$Q_{34} = m\left(\hat{H}_4 - \hat{H}_3\right) \qquad \textbf{Eqn 19} \qquad \text{or :} \qquad \hat{H}_4 = \hat{H}_3 + \frac{Q_{34}}{m} \qquad \textbf{Eqn 20}$$

H_4	979.29	kJ/kg

Now that we know the values of <u>two</u> intensive properties at **state 4**, **T_4** and **H_4**, we can evaluate <u>all</u> the other properties using the Saturation Tables in the Steam Tables.

Properties are determined from **NIST WebBook**:

$$x_4 = \frac{\hat{H}_4 - \hat{H}_{sat\,liq,4}}{\hat{H}_{sat\,vap,4} - \hat{H}_{sat\,liq,4}} \qquad \textbf{Eqn 21}$$

At **P_4**:

$H_{sat\ liq,\ 4}$	428.84	kJ/kg			
$H_{sat\ vap,\ 4}$	2679.2	kJ/kg	x_4	0.24461	kg vap/kg

$$\hat{V} = \hat{V}_{sat\,liq} + x\,(\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}) \qquad \textbf{Eqn 22}$$

At **P_4** and **x_4**:

$V_{sat\ liq,\ 4}$	0.0010453	m^3/kg			
$V_{sat\ vap,\ 4}$	1.54946	m^3/kg	V_4	0.37980	m^3/kg

$$\hat{U} = \hat{U}_{sat\,liq} + x\,(\hat{U}_{sat\,vap} - \hat{U}_{sat\,liq}) \qquad \textbf{Eqn 23}$$

$U_{sat\ liq,\ 4}$	428.72	kJ/kg			
$U_{sat\ vap,\ 4}$	2508.7	kJ/kg	U_4	937.51	kJ/kg

At last we have **U_4** and we can plug it into **Eqn 11** to evaluate **W_{34}** :

$$W_{34} = Q_{34} - m\left(\hat{U}_4 - \hat{U}_3\right) \qquad \textbf{Eqn 24}$$

W_{34}	-1141.45	kJ/kg

Step 4 - 1

The **heat transferred** for **step 4-1** is given in the problem statement.

Apply the 1st Law, **Eqn 1**, to **step 4-1** :

$$\Delta U_{41} = Q_{41} - W_{41} \qquad \textbf{Eqn 25}$$

Solve **Eqn 25** for **W_{41}** :

$$W_{41} = Q_{41} - \Delta U_{41} = Q_{41} - m\left(\hat{U}_1 - \hat{U}_4\right) \qquad \textbf{Eqn 26}$$

W_{41}	-2467.79	kJ

Part c.) The efficiency of a Carnot Cycle is defined by:

$$\eta = \frac{W_{cycle}}{Q_{in}} \qquad \text{Eqn 27}$$

Where :

$$W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41} \qquad \text{Eqn 28}$$

And :

$$Q_{in} = Q_{12} \qquad \text{Eqn 29}$$

Q_{in}	19502.7	kJ
W_{cycle}	4421.9	kJ

η 0.2267

Or the **efficiency** can be determined in terms of reservoir **temperatures**:

$$\eta = 1 - \frac{T_C}{T_H} \qquad \text{Eqn 30}$$

η 0.2267

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

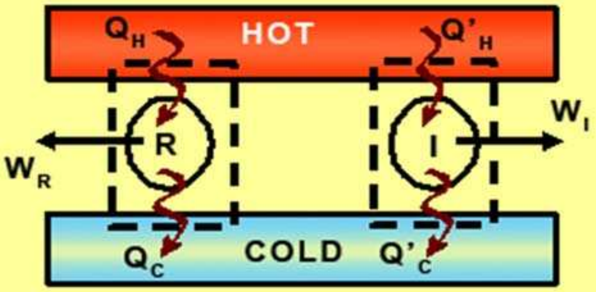
Answers :

Process	Q	W
1-2	19502.7	2031.1
2-3	0	6000.0
3-4	-15080.8	-1141.5
4-1	0.0	-2467.8
Cycle	4421.9	4421.9

The thermal efficiency of the process is : 22.7%

6E-2 Performance of Reversible and Irreversible Power Cycles 4 pts

Two power cycles are available for your use and analysis. One is reversible (**R**) and one is irreversible (**I**). You also have two thermal reservoirs at your disposal. You connect both cycles to both reservoirs, as shown below.



a.) If each cycle receives the same amount of heat from the hot reservoir, show that the irreversible cycle rejects more heat to the cold reservoir than the reversible cycle does.

b.) If each cycle produces the same net amount of work, show that the irreversible cycle must absorb more heat from the hot reservoir than the reversible cycle does.

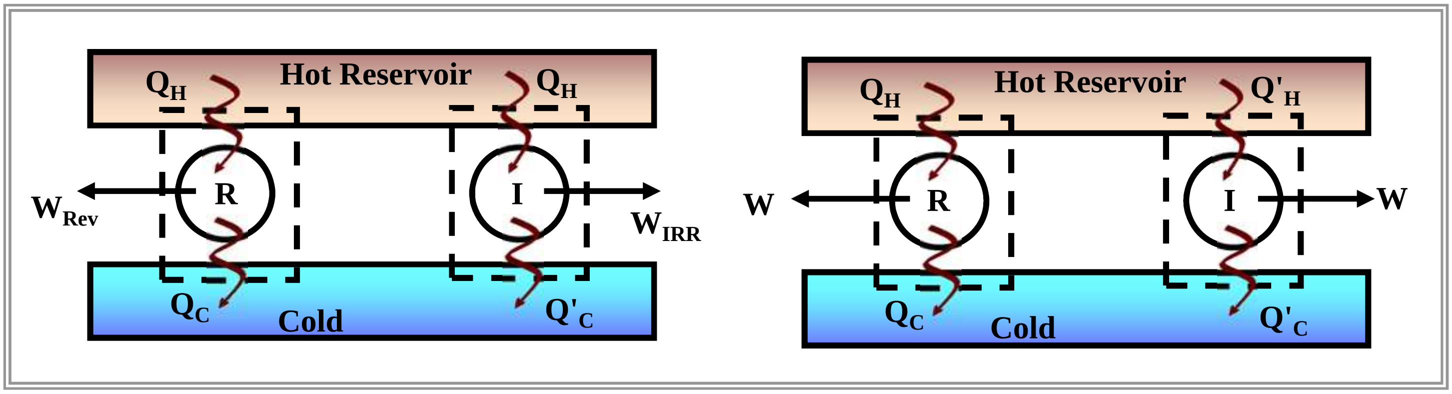
Read : Between reversible and irreversible cycles, the Carnot Corollaries indicate that $\eta_{Rev} > \eta_{IRR}$.
Use the 1st Law and the Carnot Corollaries to demonstrate these two points.

Given: A reversible power cycle **R** and an irreversible power cycle **I** operate between the same two thermal reservoirs.

Find: Show that:

Part (a)	for $Q_H = Q'_H$	$Q'_C > Q_C$
Part (b)	for $Q_R = W_I$	$Q'_H > Q_H$

Diagram:



Assumptions: 1 - The system **R** undergoes a reversible power cycle while system **I** undergoes an irreversible power cycle.

Equations / Data / Solve:

No equations are needed to answer this problem.

This problem is a proof. Therefore, the equations needed will be determined in the answer questions section.

Verify: The assumptions in this problem cannot be verified with the given information.

Answers :

Part (a) By the first Carnot Corollary, $\eta_{\mathbf{Rev}}$ **>** $\eta_{\mathbf{IRR}}$.

Since <u>both</u> cycles receive the <u>same</u> amount of energy, $\mathbf{Q_H}$, it follows that:

$$\mathbf{W_R > W_I} \qquad \textbf{Eqn 1}$$

An energy balance on cycle **R** is:

$$\mathbf{W_R = Q_H - Q_C} \qquad \textbf{Eqn 2}$$

An energy balance on cycle **I** is:

$$\mathbf{W_I = Q_H - Q'_C} \qquad \textbf{Eqn 3}$$

Combining **Eqns 1**, **2** and **3** yields :

$$\mathbf{Q_H - Q_C > Q_H - Q'_C} \qquad \textbf{Eqn 4}$$

The $\mathbf{Q_H}$ terms in **Eqn 4** cancel out and we obtain:

$$\mathbf{Q'_C > Q_C} \qquad \textbf{Eqn 5}$$

Thus, not only do <u>actual</u> cycles develop <u>less</u> **work** they <u>also</u> <u>discharge</u> <u>more</u> energy by heat transfer to their surroundings, thereby <u>increasing</u> the effect of thermal polution.

Part (b) By the first Carnot Corollary, $\eta_{\mathbf{Rev}}$ **>** $\eta_{\mathbf{IRR}}$ and from the problem statement we know:

$$\mathbf{W_R = W_I = W} \qquad \textbf{Eqn 6}$$

Efficiency is defined by:

$$\eta = \frac{\mathbf{W}}{\mathbf{Q_H}} \qquad \textbf{Eqn 7}$$

Therefore, because $\eta_{\mathbf{Rev}}$ **>** $\eta_{\mathbf{IRR}}$:

$$\frac{\mathbf{W}}{\mathbf{Q_H}} > \frac{\mathbf{W}}{\mathbf{Q'_H}} \qquad \textbf{Eqn 8}$$

The **work** terms <u>cancel</u> because <u>both</u> cycles produce the <u>same</u> amount of **work**. Therefore, **Eqn 8** becomes :

$$\boxed{\mathbf{Q'_H > Q_H}} \qquad \textbf{Eqn 9}$$

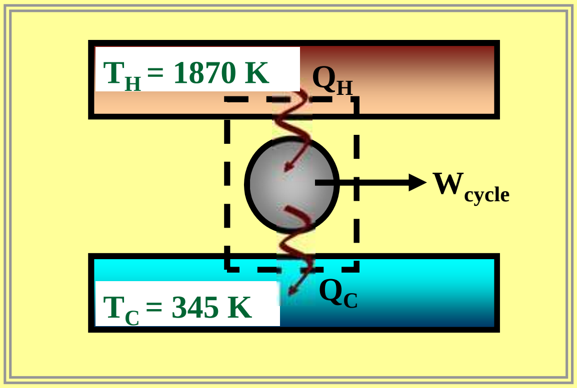
Notice that:

$$\mathbf{Q'_C > Q_C} \qquad \textbf{Eqn 10}$$

If the hot reservoir were <u>maintained</u> by, say, energy from the combustion of a fossil fuel, the irreversible cycle would have the <u>greater</u> fuel requirement. Also, note the irreversible cycle would also have the <u>greater</u> energy <u>discharge</u> to the cold reservoir, <u>increasing</u> the magnitude of thermal polution.

6E-3 Determining Whether a Power Cycle is Reversible, Irreversible or Impossible 4 pts

Consider the power cycle described by the diagram, below. Consider each of the following cases and determine whether the power cycle in each case is **reversible**, **irreversible** or **impossible**.



a.) Q_H = 1150 kJ, W_{cycle} = 988 kJ
b.) Q_H = 1122 kW, Q_C = 207 kW
c.) W_{cycle} = 1660 kJ, Q_C = 499 kJ
d.) η = 74%

Read : What is the relationship between Q_C/Q_H and T_C/T_H for a reversible cycle?

Read about the Kelvin temperature scale. How does this allow you to determine the maximum theoretical efficiency of a thermodynamic cycle from T_C and T_H?

Note that the maximum efficiency by definition is associated with a reversible cycle.

Any real cycle always possesses some losses or friction and these irreversibilities reduce η below η_{max}.

Diagram: The diagram in the problem statement is adequate.

Given:

	T_H	1870	K				
	T_C	345	K				
a.)	Q_H	1150	kJ	c.)	W_{cycle}	1660	kJ
	W_{cycle}	988	kJ		Q_C	499	kJ
b.)	Q_H	1122	kJ	d.)	η	74%	
	Q_C	207	kJ				

Find: **Reversible ?** **Irreversible ?** **Impossible ?**

Assumptions: **1 -** The system shown undergoes a power cycle.

Equations / Data / Solve:

To determine if each case is reversible, irreversible, or impossible we need to compare the actual efficiency of the case to the maximum efficiency. There are **3** possibilities :

If the efficiency of the process equals the maximum efficiency, then the process is **reversible**.
If the efficiency of the process is less than the maximum efficiency, then the process is **irreversible**.
If the efficiency of the process is greater than the maximum efficiency, then the process is **impossible**.

Since the maximum efficiency, by definition, is associated with a 'reversible' cycle, the maximum thermal efficiency for any power cycle operating between thermal resevoirs T_H and T_C is:

$$\eta_{max} = 1 - \frac{T_C}{T_H}$$

Eqn 1

The efficiency for each case is determined by:

$$\eta = \frac{W_{cycle}}{Q_H}$$

Eqn 2

Only in **part (a)** do we know W_{cycle} and Q_H but realizing:

$$W_{cycle} = Q_H - Q_C$$

Eqn 3

or :

$$Q_H = W_{cycle} + Q_C$$

Eqn 4

Combining **Eqns 2** and **4** gives an equation we can use to resolve **parts (b)** and **(c)** :

$$\eta = \frac{W_{cycle}}{W_{cycle} + Q_C} = 1 - \frac{Q_C}{Q_H}$$

Eqn 5

	Plug values into **Eqn 1** to determine the maximum thermal efficiency :	η_{max}	**0.816**
a.)	Plug values into **Eqn 2** to determine η :	η	**0.859**
b.)	Plug values into **Eqn 5** to determine η :	η	**0.816**
c.)	Plug values into **Eqn 5** to determine η :	η	**0.769**
d.)	Given :	η	**0.740**

Verify: The assumptions made in this problem cannot be verified with the given information.

Answers :

Part (a) The process is **impossible** because the efficiency of the process is greater than the maximum efficiency for any power cycle operating between thermal resevoirs at T_H and T_C.

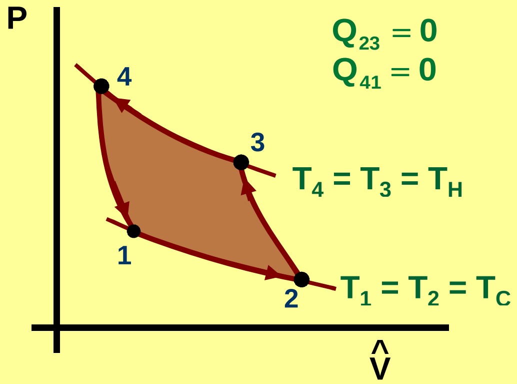
Part (b) The process is **reversible** because the efficiency of the process equals the maximum efficiency for any power cycle operating between thermal resevoirs at T_H and T_C.

Part (c) The process is **irreversible** because the efficiency of the process is less than the maximum efficiency for any power cycle operating between thermal resevoirs at T_H and T_C.

Part (d) The process is **irreversible** because the efficiency of the process is less than the maximum efficiency for any power cycle operating between thermal resevoirs at T_H and T_C.

6E-4 Pressure, Work and COP for a Carnot Gas Refrigeration Cycle 8 pts

The **PV Diagram**, below, shows the process path for a Carnot refrigeration cycle carried out in a piston-and-cylinder device with **8.4 lb_m** of **air** as the working fluid.



The maximum **temperature** in the cycle is **80°F** and the minimum **temperature** is **-10°F**. The isothermal expansion requires **126 Btu** of **heat transfer** and the **volume** of **air** in the cylinder at the end of the isothermal compression is **1.74 ft^3**. Assume **air** behaves as an ideal gas with a constant **heat capacity ratio** of **γ = 1.4**. The following relationship is valid for the adiabatic steps in the Carnot cycle:

$$T P^{1-\gamma/\gamma} = C$$

Calculate…

a.) The **pressure** at **states 1 to 4**

b.) The **work** for each of the four processes

c.) The **COP** for the refrigeration cycle.

Read :

Part (a)

Note the direction (**CW** or **CCW**) of the cycle. It will be the opposite direction to that of a power cycle. With the final **volume**, **V_4** given, use the ideal gas EOS to determine **P_4**. Use the equation given in the problem statement to determine **P_1**. You can then calaculate **V_1** from the ideal gas EOS. Use an energy balance to determine **V_2** and then the ideal gas EOS to find **P_2**. Finally, use the equation in the problem statement again to determine **P_3**.

Part (b)

Notice that **$(U_2 - U_1) = 0$** for an ideal gas undergoing an isothermal process because **U** is a function of **T** only. Carefully apply an energy balance to **step 2-3** (watch the sign of each term). Here **$dU = C_V\, dT = R/(\gamma-1)\, dT$** for the case of constant γ. Note that **MW_{air} = 28.97 lb_m/lbmol**.

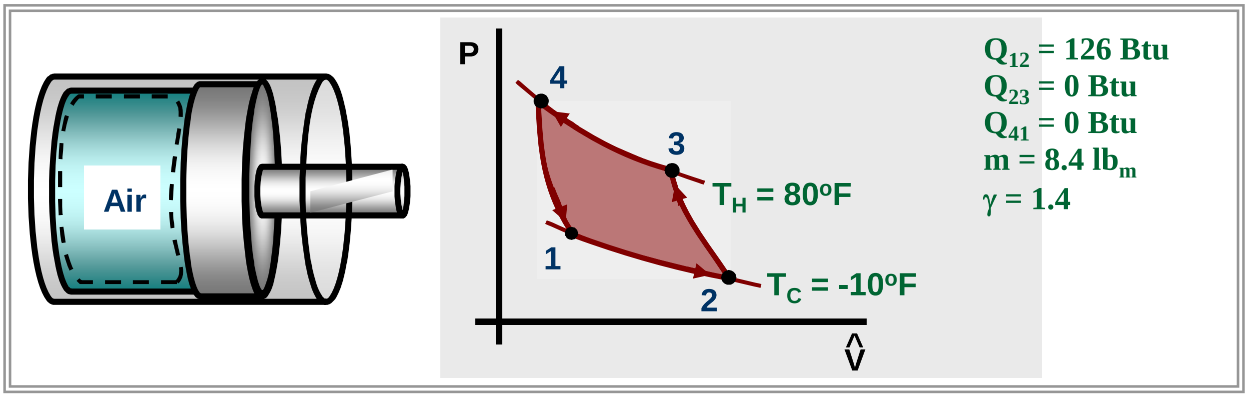
Given:

m	8.4	lb_m	γ	1.4	
T_C	-10	°F	T_H	80	°F
	449.67	°R	T_H	539.67	°R
Q_{12}	126	Btu	V_4	1.74	ft^3

Find:

a.)	P_1, P_2, P_3, P_4	???	lb_f/in^2
b.)	$W_{12}, W_{23}, W_{34}, W_{41}$	???	Btu
c.)	COP_R	???	

Diagram:



Q_{12} = 126 Btu
Q_{23} = 0 Btu
Q_{41} = 0 Btu
m = 8.4 lb_m
γ = 1.4

Assumptions:

1 - The system consists of air modeled as an ideal gas with γ **= 1.4**.
2 - Boundary work is the only form of **work** interaction.
3 - Changes in kinetic and potential energies are negligible.
4 - Because the cycle is a Carnot Cycle, the cycle is reversible and therefore each step in the cycle is a reversible process.
5 - The **heat capacities** of the ideal gas are constant.

Equations / Data / Solve:

R	1545.35	(ft * lbf)/(lbmol * °R)	Conversion Factors: 1 ft² =	144	in²
R	1.986	Btu/lbmol-°R	1 Btu =	778	ft-lb_f
MW_{air}	29	lb_m/lbmol	1 L =	0.0353	ft³
T_{ref}	459.67	°R			

Part a.) Since air is modeled as an ideal gas, we can determine the **pressure** at **state 4** from the ideal gas EOS:

$$P_4 = \frac{m}{MW}\frac{RT_4}{V_4} \qquad \text{Eqn 1}$$

P_4 964 lb_f/in^2

Now, let's apply the equation given in the problem statement to **step 1-4** :

$$T_1 P_1^{1-\gamma/\gamma} = T_4 P_4^{1-\gamma/\gamma} \qquad \text{Eqn 2}$$

Solving for P_1 :

$$P_1 = P_4 \left(\frac{T_4}{T_1}\right)^{\gamma/(1-\gamma)} \qquad \text{Eqn 3}$$

P_1 509.1 lb_f/in^2

P_2 can also be determined from the ideal gas EOS:

$$P_2 = \frac{m}{MW}\frac{RT_2}{V_2} \qquad \text{Eqn 4}$$

The problem is that we don't know V_2. We could determine V_2 if we knew the value of W_{12} because :

The **work** is:

$$W_{12} = \int_1^2 P\,dV \qquad \text{Eqn 5}$$

We can then use the IG EOS for isothermal **step 1-2** :

$$P = \frac{nRT_1}{V} \qquad \text{Eqn 6}$$

Substituting **Eqn 6** into **Eqn 5** yields :

$$W_{12} = \int_1^2 \frac{n R T_1}{V} dV \qquad \textbf{Eqn 7}$$

Integrating **Eqn 7** yields :

$$W_{12} = n R T_C \, Ln\left(V_2 / V_1\right) \qquad \textbf{Eqn 8}$$

Solving for **Eqn 8** for V_2 :

$$V_2 = \exp\left[\frac{W_{12}}{n R T_C}\right] V_1 \qquad \textbf{Eqn 9}$$

Now, the issue is that we still don't know W_{12}. Let's write the 1st Law for **step 1-2**, assuming changes in kinetic and potential energies are negligible.

$$\Delta U_{12} = Q_{12} - W_{12} \qquad \textbf{Eqn 10}$$

Since the internal energy of an ideal gas depends on **temperature** only and the **temperature** is constant along **Process 1-2**, $U_2 = U_1$ and **Eqn 10** reduces to:

$$Q_{12} = W_{12} \qquad \textbf{Eqn 11}$$

W_{12} **126 Btu**

We can determine the number of **moles** in the system from :

$$n = \frac{m}{MW} \qquad \textbf{Eqn 12}$$

n **0.28966 lbmoles**

Then, we can evaluate V_1 from the Ideal Gas EOS :

$$V_1 = \frac{n R T_1}{P_1} \qquad \textbf{Eqn 13}$$

V_1 **2.7456** ft^3

Now, we can substitute W_{12} and V_1 into **Eqn 9** to evaluate V_2 : V_2 **4.4683** ft^3

At last we can use V_2 in **Eqn 4** to evaluate P_2 : P_2 **312.8** lb_f/in^2

That leaves us P_3 yet to be determined for **part (a)**.

P_3 can be determined by applying **Eqn 3** to **step 2-3** :

$$P_3 = P_2 \left(\frac{T_2}{T_3}\right)^{\gamma/(1-\gamma)} \qquad \textbf{Eqn 14}$$

P_3 **592.4** lb_f/in^2

Verify : The ideal gas assumption needs to be verified.

We need to determine the **specific volume** at each state and check **if** :

$$\tilde{V} > 80 \ \ ft^3 / lbmol \qquad \textbf{Eqn 15}$$

From the Ideal Gas EOS, we obtain :

$$\tilde{V} = \frac{R T}{P} \qquad \textbf{Eqn 16}$$

V_1	**11.38**	$ft^3/lbmol$	V_3	**9.78**	$ft^3/lbmol$
V_2	**18.51**	$ft^3/lbmol$	V_4	**6.01**	$ft^3/lbmol$

It is **NOT** accurate to treat the air in this process as an ideal gas !
We were instructed to do so and we did, but we need to keep in mind that the results may not be accurate to 2 significant figures.

P_1	**509**	**lb_f/in^2**
P_2	**313**	**lb_f/in^2**

P_3	**592**	**lb_f/in^2**
P_4	**964**	**lb_f/in^2**

Part b.) Let's begin by writing the 1st Law, open systems, steady-state with changes in kinetic and potentail energies negligible :

$$\Delta U = Q - W$$ **Eqn 16**

Apply **Eqn 16** to each step in the cycle :

$$\cancel{\Delta U_{12}} = Q_{12} - W_{12}$$ or : $$Q_{12} = W_{12}$$ **Eqn 17**

$$\Delta U_{23} = \cancel{Q_{23}} - W_{23}$$ or : $$W_{23} = -\Delta U_{23}$$ **Eqn 18**

$$\cancel{\Delta U_{34}} = Q_{34} - W_{34}$$ or : $$Q_{34} = W_{34}$$ **Eqn 19**

$$\Delta U_{41} = \cancel{Q_{41}} - W_{41}$$ or : $$W_{41} = -\Delta U_{41}$$ **Eqn 20**

$\Delta U_{12} = \Delta U_{34} = 0$ because these steps are isothermal processes and the system contains an ideal gas.
$Q_{23} = Q_{41} = 0$ because these steps are adiabatic.

Integrating the definition of the **constant volume heat capacity** we obtain :

$$\Delta U_{AB} = \int_{T_A}^{T_B} C_V \, dT$$ **Eqn 21**

For an ideal gas with constant **heat capacities**, **Eqn 21** becomes :

$$\Delta U_{AB} = n \tilde{C}^o_V \left(T_B - T_A \right)$$ **Eqn 22**

The **heat capacities** of ideal gases are related by the following equations :

$$\tilde{C}^o_P = \tilde{C}^o_V + R$$ **Eqn 23**

$$\frac{\tilde{C}^o_P}{\tilde{C}^o_V} = \gamma = 1 + \frac{R}{\tilde{C}^o_V}$$ **Eqn 24**

$$\tilde{C}^o_V = \frac{R}{\gamma - 1}$$ **Eqn 25**

C_V	**5.0**	**Btu/lbmole-^{o}R**

Now, we can combine **Eqn 22** with **Eqns 18** & **20** for **steps 2-3** and **4-1**:

W_{23}	**-129.432**	**Btu**
W_{41}	**129.432**	**Btu**

We already know, from **part (a)**, that : **W_{12}** **126** **Btu**

So, now we need to evaluate **W_{34}**. Because **step 3-4** is adiabatic (like **step 1-2**) we can apply **Eqn 8** to **step 3-4** as follows :

$$W_{34} = n R \, T_H \, Ln\left(V_4 / V_3 \right)$$ **Eqn 26**

W_{34} **-151.195** **Btu**

The assumptions made in this part of the problem cannot be verified with the given information.

W_{12}	126	Btu
W_{23}	-129.43	Btu

W_{34}	-151.19	Btu
W_{41}	129.43	Btu

Part c.) The **coefficient of performance** of a Carnot Cycle is:

$$COP_{R,max} = \frac{T_C}{T_H - T_C}$$ **Eqn 27**

COP_R **5.0**

Verify: The assumptions made in this part of the problem cannot be verified with the given information.

Answers : **a.)**

P_1	510	lb_f/in^2
P_2	310	lb_f/in^2

P_3	590.0	lb_f/in^2
P_4	960.0	lb_f/in^2

b.)

W_{12}	126	Btu
W_{23}	-129	Btu

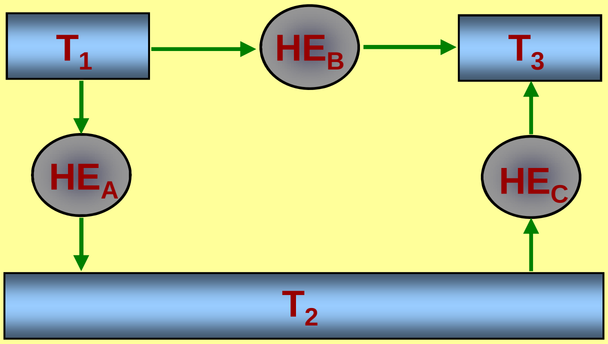
W_{34}	-151	Btu
W_{41}	129	Btu

c.)

COP_R	5.0

6F-1 Relationship Between Carnot Cycle Efficiencies 5 pts

Three Carnot heat engines operate between three thermal reservoirs, as shown in the diagram, below.



Derive an **equation** for the thermal efficiency of $\mathbf{HE_C}$ (η_C), in terms of the thermal efficiency of $\mathbf{HE_A}$ (η_A) and the thermal efficiency of $\mathbf{HE_B}$ (η_B).

Read : The key to this problem is the fact that all **three** heat engines are Carnot Engines and their efficiencies are completely determined by the **temperatures** of the **three** reservoirs. Our goal is to algebraically manipulate the **three** equations for the **three** efficiencies in order to eliminate all **three temperatures**.

Given:
$\mathbf{HE_1}$ absorbs heat from a reservoir at $\mathbf{T_1}$ and rejects heat to a reservoir at $\mathbf{T_2}$.
$\mathbf{HE_2}$ absorbs heat from a reservoir at $\mathbf{T_2}$ and rejects heat to a reservoir at $\mathbf{T_3}$.
$\mathbf{HE_3}$ absorbs heat from a reservoir at $\mathbf{T_1}$ and rejects heat to a reservoir at $\mathbf{T_3}$.

Find: $\eta_3 = \mathbf{fnx}(\eta_1, \eta_2)$

Diagram: The diagram in the problem statement is adequate.

Assumptions: **1 -** All **three** heat engines are **Carnot Heat Engines**.

Equations / Data / Solve:

Begin by applying the equation relating the **thermal efficiency** of a **Carnot Engine** to the **temperatures** of the reservoirs to all **three** heat engines.

$$\eta_A = \mathbf{1} - \frac{\mathbf{T_2}}{\mathbf{T_1}} \qquad \textbf{Eqn 1}$$

$$\eta_B = \mathbf{1} - \frac{\mathbf{T_3}}{\mathbf{T_2}} \qquad \textbf{Eqn 2}$$

$$\eta_C = \mathbf{1} - \frac{\mathbf{T_3}}{\mathbf{T_1}} \qquad \textbf{Eqn 3}$$

Rearrange **Eqns 1** & **2** as follows :

$$\frac{T_2}{T_1} = 1 - \eta_A$$ **Eqn 4**

$$\frac{T_3}{T_2} = 1 - \eta_B$$ **Eqn 5**

Multiply **Eqn 4** by **Eqn 5** to get :

$$\frac{T_2}{T_1}\,\frac{T_3}{T_2} = \frac{T_3}{T_1} = (1 - \eta_B)\,(1 - \eta_A)$$ **Eqn 6**

Now, substitute **Eqn 6** back into **Eqn 3** :

$$\eta_C = 1 - (1 - \eta_B)\,(1 - \eta_A)$$ **Eqn 7**

Simplify **Eqn 7** :

$$\eta_C = \cancel{1} - (\cancel{1} - \eta_B - \eta_A + \eta_A\,\eta_B)$$ **Eqn 8**

Finally :

$$\eta_C = \eta_B + \eta_A - \eta_A\,\eta_B$$ **Eqn 9**

Verify: We cannot verify that the heat engines are **Carnot Engines**, but the problem statement instructed us to make this assumption.

Answers : $\boxed{\eta_C = \eta_B + \eta_A - \eta_A\,\eta_B}$

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6G-1 Efficiency and Coefficient of Performance of Carnot Cycles 4 pts

A Carnot Cycle operates between thermal reservoirs at **55°C** and **560°C**. Calculate…

a.) The **thermal efficiency**, η, if it is a power cycle

b.) The **COP** if it is a refrigerator

c.) The **COP** if it is a heat pump

Read : This is a straightforward application of the definitions of **efficiency** and **coefficient of performance**.

Given:						
	T_H	560	°C	T_C	55	°C
	T_H	833.15	K	T_C	328.15	K

Find:						
	η	???	COP_R	???	COP_{HP}	???

Diagram: Not necessary for this problem.

Assumptions: None.

Equations / Data / Solve:

Part a.) The **thermal efficiency** of a **Carnot Cycle** depends only on the **temperatures** of the thermal reservoirs with which it interacts. The equation that defines this relationship is :

$$\eta = 1 - \frac{T_C}{T_H} \qquad \text{Eqn 1}$$

Just be sure to use absolute **temperature** in **Eqn 1** ! In this case, convert to Kelvin. **Temperatures** in Rankine will work also.

η **60.6%**

Part b.) The **coefficient of performance** of a **Carnot Refrigeration Cycle** also depends only on the **temperatures** of the thermal reservoirs with which it interacts. The equation that defines this relationship is :

$$COP_R = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{T_C}{T_H - T_C} \qquad \text{Eqn 2}$$

Using **T** in Kelvin yields : COP_R **0.6498**

This is an exceptionally BAD COP_R because it is less than **1.** This isn't terribly surprising when you consider that the refrigerator must reject heat to a thermal reservoir at **560°C** !!

Part c.) The **coefficient of performance** of a **Carnot Heat Pump Cycle** also depends only on the **temperatures** of the thermal reservoirs with which it interacts. The equation that defines this relationship is :

$$COP_{HP} = \frac{1}{1 - \frac{T_C}{T_H}} = \frac{T_H}{T_H - T_C} \qquad \text{Eqn 3}$$

Using T in Kelvin yields : **COP_{HP}** **1.6498**

This is a BAD **COP_{HP}** because it is just barely greater than **1**. This isn't terribly surprising when you consider that the heat pump must put out heat to a reservoir at **560°C** !!

Notice also that : $$COP_{HP} = COP_R + 1 \qquad \text{Eqn 4}$$

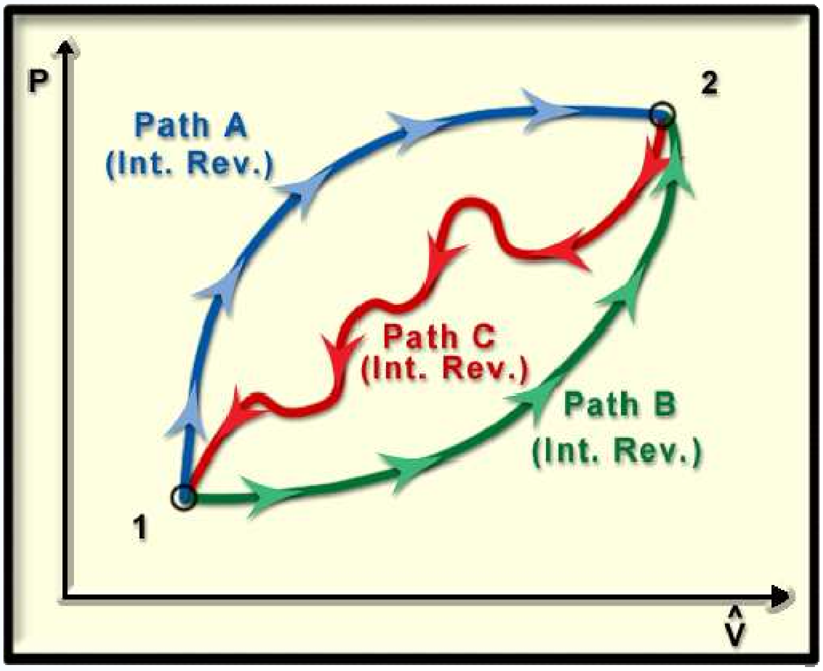
This is always true for **Carnot Cycles**.

Verify: No assumptions to verify that were not given in the problem statement.

Answers :

η	**60.6%**	COP_R	**0.650**	COP_{HP}	**1.65**

Chapter 7



Entropy

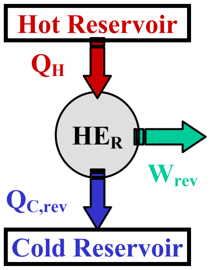
In this chapter, we use the Kelvin Relationship and the Carnot Pricniples to show that the Clausius Inequality is true. This leads to the definition of entropy. The TS Diagram will be used frequently in the remainder of this course because it provides a great deal of insight into the performance of processes and cycles. The Principle of Increasing Entropy leads to the concept of entropy generation.

The 1st and 2nd Gibbs Equations are introduced to facilitate the evaluation of changes in entropy associated with processes. We apply the Gibbs equations to incompressible liquids and ideal gases. This analysis leads to the Ideal Gas Entropy Function and to relative properties. These are tabulated in the Appendix.

The chapter concludes with a discussion of polytropic, isentropic and other special processes and their representation on PV and TS Diagrams.

The Clausius Inequality

$$\boxed{\oint \frac{\delta Q}{T} \le 0}$$



- **Cyclic Integrals**
  - ◊ Integrate through all the steps in a cycle and return to the initial state.
- **Inexact Differentials: δQ & δW**
  - ◊ Used for path variables, Q and W
- **Evaluating Cyclic Integrals**
  - ◊ Example 1: Carnot HE

$$\oint \delta Q = \int_1^2 \delta Q + \int_3^4 \delta Q = Q_{12} + Q_{34} = Q_H - Q_C > 0$$

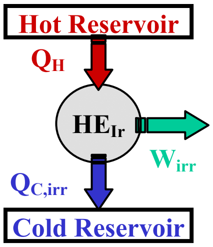
  - ◊ Example 2: Carnot HE

$$\oint \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + \int_3^4 \frac{\delta Q}{T} = \frac{\int_1^2 \delta Q}{T_H} + \frac{\int_3^4 \delta Q}{T_C} = \frac{Q_{12}}{T_H} + \frac{Q_{34}}{T_C} = \frac{Q_H}{T_H} - \frac{Q_{C,rev}}{T_C}$$

- Example 2
  - ◊ Because the Carnot Cycle is completely reversible, all heat transfer must occur through an infinitesimal temperature difference.
  - ◊ Therefore, the temperature of the hot reservoir must be equal to the temperature of the working fluid in the system to which it transfers heat.
  - ◊ And the temperature of the cold reservoir must be equal to the temperature of the working fluid in the system from which it receives heat.
  - ◊ Since the reservoir temperatures are constant, the temperatures within the system where the heat exchange occurs must also be constant.

- Cyclic integrals are new, but they are not scary or terribly difficult when applied to thermodynamic cycles.
- All you need to do is integrate through all the steps of the cycle, so that you begin and end in the same state.
- The funky "δ" is a common way of indicating that the differential is not an exact differential, "δ", nor is it a partial differential, "∂".
- It is absolutely crucial that you understand these two examples of how to evaluate a cyclic integral.
- Example 1
  - ◊ In the cyclic integral, our sign convention applies, but in the tie-fighter diagram, both $\mathbf{Q_H}$ and $\mathbf{Q_C}$ are positive quantities.
  - ◊ When we evaluate the cyclic integral, we get something like $\mathbf{Q_{12} + Q_{23} + Q_{34} + Q_{41}}$ for a Carnot cycle.
  - ◊ In a Carnot Cycle, Steps 2-3 and 4-1 are adiabatic, so $\mathbf{Q_{23} = Q_{41} = 0}$.
  - ◊ So, the cyclic integral is just $\mathbf{Q_{12} + Q_{34}}$.
  - ◊ Because of the sign convention conflict, $\mathbf{QH = Q_{12}}$ and $\mathbf{Q_C = - Q_{34}}$.
  - ◊ Therefore, the cyclic integral is $\mathbf{Q_H - Q_C}$.
  - ◊ The 1st Law tells us that $\mathbf{Q_H = Q_C + W_{HE}}$
  - ◊ Since $\mathbf{W_{HE} > 0}$, $\mathbf{Q_H > Q_C}$ and finally $\mathbf{Q_H - Q_C > 0}$

Clausius: Int. Rev. and Irrev. Cycles



- Reversible Cycle, such as Carnot: $\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_{C,rev}}{T_C}$
  - ◊ Kelvin: $\frac{Q_{C,rev}}{Q_H} = \frac{T_C}{T_H}$ or: $\frac{Q_{C,rev}}{T_C} = \frac{Q_H}{T_H}$
  - ◊ Therefore: $\oint \frac{\delta Q}{T} = 0$
- Irreversible Cycles: $\eta_{rev} > \eta_{irr}$
  - ◊ Definition of efficiency : $W_{rev} > W_{irr}$
  - ◊ 1^{st} Law : $Q_H - Q_{C,rev} > Q_H - Q_{C,irr}$

$$Q_{C,rev} < Q_{C,irr}$$

$$\oint \frac{\delta Q_{irr}}{T} = \frac{Q_H}{T_H} - \frac{Q_{C,irr}}{T_C} = \frac{Q_{C,rev}}{T_C} - \frac{Q_{C,irr}}{T_C} < 0$$

- **All Cycles :** $\boxed{\oint \frac{\delta Q}{T} \le 0}$

- Conclusions
- ◊ The Clausius Inequality is TRUE ! The equality applies for int. rev. cycles and the "less than" part applies for irreversible cycles.
- ◊ Why relax the rule from reversible to just internally reversible ? If the temperatures inside the system are NOT equal to the temperatures of the hot and cold reservoirs, then the heat exchange is NOT reversible.
- ◊ But, the Kelvin Relationship applies if the cycle is internally reversible.
- ◊ As long as the cycle is internally reversible, the equality in the Clausius Inequality applies.
- ◊ The temperature within the system at which heat exchange occurs does not need to be constant for the equality part of Clausius to hold true. It just makes it a WHOLE LOT easier to evaluate the cyclic integral.

- Reversible Cycles
  - ◊ Kelvin Relationship applies to reversible cycles like Carnot
    - Depends on the fact that the temperature scale we use, the Kelvin Scale, is a thermodynamic temperature scale
- Irreversible Cycles:
  - ◊ Consider an irreversible HE that operates between the same hot and cold thermal reservoirs and receives the same amount of heat from the hot reservoir, $\mathbf{Q_H}$.
  - ◊ 1st Carnot Principle : the reversible HE is more efficient than the irreversible HE
  - ◊ Thermal efficiency is defined as the ratio of the work output to $\mathbf{Q_H}$.
    - Since $\mathbf{Q_H}$ is the same for both HE's and the efficiency of the reversible HE is greater, we conclude that the work output of the irreversible HE must be less than the work output of the reversible HE.
  - ◊ The 1st Law allows us to replace the work output with $\mathbf{Q_H - Q_C}$ for each HE.
  - ◊ The $\mathbf{Q_H}$term on each side of the inequality cancels leaving us with the fact that the irreversible HE must reject more heat to the cold reservoir than the reversible HE does.
  - ◊ The Cyclic Integral
    - We can now apply the cyclic integral of **dQ/T** to the irreversible process.
    - The key here is that we can use the Kelvin Relationship (applied to the reversible HE, not this irreversible one) to replace $\mathbf{Q_H / T_H}$ with $\mathbf{Q_{C,rev} / T_C}$.
    - We just showed that $\mathbf{Q_{C,rev} < Q_{C,irr}}$, so we conclude that the cyclic integral <u>must</u> be negative for all irreversible cycles !

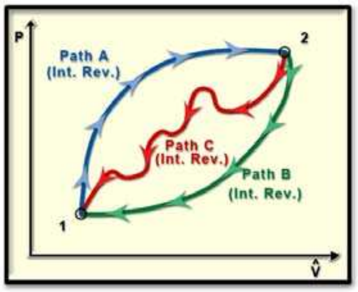
Entropy

- **Cycle 1-A-2-B-1:**

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$$

- **Cycle 1-A-2-C-1:**

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_C = 0$$



- **Subtract Eqns:** $\int_2^1 \left(\frac{\delta Q}{T}\right)_C = \int_2^1 \left(\frac{\delta Q}{T}\right)_B$

$$\int \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

- Does not depend on path !
- It is a state variable or property !

- **Definition of Entropy:** $\boxed{dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}}}$

- Cycles 1-A-B-2 and 1-A-C-2 are internally reversible, so the cyclic integral of **dQ / T = 0**.
- This means that the integral of **dQ / T** from state 2 to state 1 must be the same along paths B and C.
- Since we didn't specify anything special about paths B and C except that they are internally reversible.
- We didn't specify anything special at all about states 1 and 2.
- We can conclude that the integral of d **Q / T** must be the same for ALL INTERNALLY REVERSIBLE paths between ANY two states.
- When the change in a quantity between two states does not depend on the path, we call it a state variable or property.
- This new property is called entropy and it is defined by the equation in the box.
- This equation really only tells how to calculate CHANGES in entropy.
  - ◊ So, like enthalpy and elevation above sea level, we must choose a reference state for entropy.
  - ◊ In the Fluid and Thermal Property Tables of the NIST Webbook, they always list the reference state and it always includes the reference state for entropy. Check it out for yourself.

ΔS: Int. Rev. and Irrev. Processes

- **Change in Entropy:** $\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}}$ [=] kJ / K

$$\left(S_2 - S_1\right)_{\text{int rev}} = \left(S_2 - S_1\right)_{\text{irrev}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}}$$

- **Problem:** Since Int. Rev. processes do not exist, how do we evaluate ΔS ?
- **Special Case:** Internally Reversible, Isothermal Processes

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = \frac{1}{T}\int_1^2 \delta Q_{\substack{\text{int}\\\text{rev}}} = \frac{Q_{\substack{\text{int}\\\text{rev}}}}{T}$$

Especially useful for evaluating $\Delta S_{\text{reservoir}}$ because $T_{\text{reservoir}}$ **= constant**

- Although we have to follow an internally reversible path to evaluate ΔS_{12}, $S_2 - S_1$ has the same value whether the process follows an internally reversible path or a IRREVERSIBLE path.
- This result is enormously important !
  - ◊ In the real world, we won't analyze any internally reversible processes because they don't really exist.
  - ◊ But, because entropy is a state variable or property, we can use it to analyze real, IRREVERSIBLE processes anyway !
- So, how do we evaluate **ΔS** for processes OTHER THAN reservoirs ?
  - ◊ We cannot use reversible processes because they don't exist !
  - ◊ It turns out that **ΔS** is related to changes in other properties that are easier to measure.
  - ◊ In Lesson D, we will derive the Gibbs Equations and we will see how **ΔS** can be calculated using the heat capacity and equations of state.

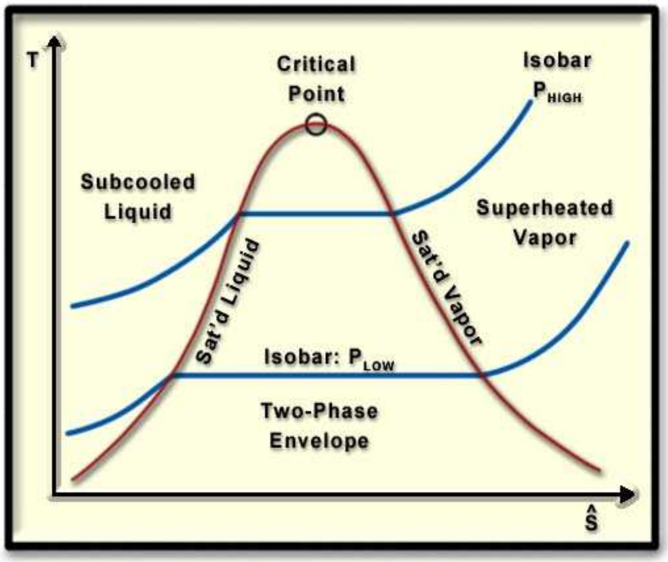
Entropy of a Pure Substance

- S = fxn(T, P, phase)
- **NIST Webbook**
  - Thermophysical Properties of Fluid Systems
  - Specific entropy is listed in the thermodynamic data tables
  - Observations: $\hat{S}_{sat\,vap} > \hat{S}_{sat\,liq}$
  - All substances at all T : $\hat{S}\uparrow$ **with** $\uparrow T$
  - IG and many real gases : $\hat{S}\downarrow$ **with** $\uparrow P$
  - Sat'd Mixtures: $\hat{S}_{sat\,mix} = x\ \hat{S}_{sat\,vap} + (1 - x)\ \hat{S}_{sat\,liq}$
  - Subcooled Liquids: $\hat{S}_{subcooled}(T, P) \approx \hat{S}_{sat\,liq}(T, P^*)$

Where P* = vapor pressure = P_{sat}

- Specific entropy is just another property that we can look up in tables of thermodynamic data.
- It is available in the NIST Webbook, in the back of the Schaum's Outline and in the tables you downloaded from Thermo-CD (most notably the tables for R-134a).
- A quick look at the thermodynamic tables should convince you that specific entropy increases with increasing **T** as long as a phase change does not occur.
- For ideal gases and most real gases at most values of **T** & **P**, specific entropy decreases as pressure increases.
- You can use quality to calculate the specific entropy of a saturated mixture (just as you can for any other specific property).
- Subcooled Liquids
  - If data is available for subcooled liquids, then you should definitely use it.
  - If data is not available, then you can approximate the specific entropy using the specific entropy of the saturated liquid AT THE SAME TEMPERATURE.
  - This is a reasonable approximation as long as the liquid is incompressible (**V^ = constant** and $C_P = C_V$)
  - This approximation is most accurate if the actual pressure, **P**, is not a great deal larger than the vapor pressure, **P*.**

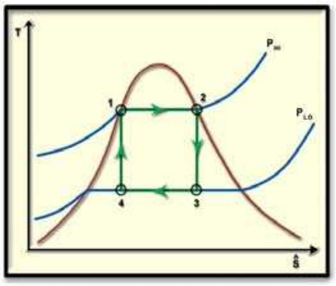
T-S Diagram



- The T-S Diagram will be the most useful diagram in the remainder of this course.
- The good news and the bad news is that it looks very similar to the PV Diagram !
  - Good news because you will learn how to use the diagram quickly.
  - Bad news because it is easy to get the two confused !
  - TS Diagrams look even more similar to TV Diagrams, but we have not used TV Diagrams much.
- The TS Diagram has isobars.
  - Isobars go from lower left to upper right because specific entropy increases as T increases.
  - Isobars are horizontal in the two-phase envelope because the phase transition entails a large change in the specific entropy, but occurs at a constant temperature.
- You are probably wondering WHY the TS Diagram is going to be so useful.
- Let's see what a Carnot Cycle looks like on a TS Diagram.

Carnot Cycle

- **Steps**
  - 1-2: Isothermal expansion
  - 2-3: Adiabatic expansion
  - 3-4: Isothermal compression
  - 4-1: Adiabatic compression



- **Step 1-2:** $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$

$$= \frac{1}{T}\int_1^2 \delta Q_{rev} = \frac{Q_H}{T_H}$$

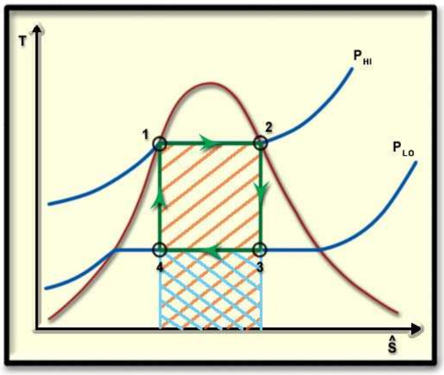
- **Step 2-3:** $S_3 - S_2 = \int_2^3 \left(\frac{\cancel{\delta Q}}{T}\right)_{rev} = 0$ **Isentropic !**
- **Step 3-4:** $S_4 - S_3 = \int_3^4 \left(\frac{\delta Q}{T}\right)_{rev} = \frac{1}{T}\int_3^4 \delta Q_{rev} = \frac{Q_{34}}{T_C} = -\frac{Q_C}{T_C}$
- **Step 4-1:** $S_2 - S_1 = \int_1^2 \left(\frac{\cancel{\delta Q}}{T}\right)_{rev} = 0$ **Isentropic !**

- We use the definition of entropy to evaluate changes in entropy for each step in the Carnot Cycle.
- Steps 2-3 and 4-1 are both adiabatic and reversible. Therefore there is NO CHANGE in the entropy of the system during these steps.
  - Steps 2-3 and 4-1 are ISENTROPIC !
- Steps 1-2 and 3-4 occur at constant temperature.
  - So, the definition of entropy simplifies (just as it did for a reservoir).
  - **ΔS** is just **Q / T**.
- So, what about work ?

Heat, Work and TS Diagrams

$$\hat{Q}_H = T_H\left(\hat{S}_2 - \hat{S}_1\right)$$

$$\hat{Q}_C = T_C\left(\hat{S}_3 - \hat{S}_4\right)$$



- **1st Law Cycle**

$$\hat{Q}_H = \hat{W} + \hat{Q}_C$$

$$\hat{W} = \hat{Q}_H - \hat{Q}_C$$

- **Q_H = area under path for step 1-2**
- **Q_C = positive area under path for step 3-4**
- **W = area enclosed by the cycle !**

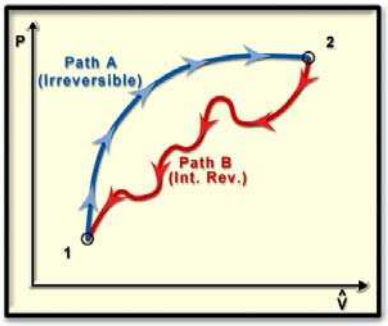
- Areas under process paths on TS Diagrams represent heat transferred in reversible processes.
- Application of the 1st Law lets us also associate work with the area enclosed by the cycle.
- This simple, tangible interpretation of area makes it much easier to understand and compare the performance of complex cycles that we will study later.
- For example, if we decrease $\mathbf{P_{LO}}$, does the efficiency of this power cycle increase or decrease ?

Principle of Increasing Entropy

- **Clausius:** $\oint \frac{\delta Q}{T} \le 0$
- **Apply to Cycle 1-A-2-B**

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B < 0$$

- **Introduce Entropy:** $S_1 - S_2 = \int_2^1 \left(\frac{\delta Q}{T}\right)_{int\,rev}$



- **Substitute into Clausius:** $\int_1^2 \left(\frac{\delta Q}{T}\right)_A + S_1 - S_2 < 0$
- **Rearrange:** $S_2 - S_1 > \int_1^2 \left(\frac{\delta Q}{T}\right)_A$
- **Differential Form:** $\boxed{dS \ge \frac{\delta Q}{T}}$

- The entropy sneaks into the equation because path B is reversible.
- The result is that the entropy change from 1 to 2 is GREATER THAN (or equal to if process A were reversible) the integral of **dQ / T** from state 1 to state 2.
- This is the principle of increasing entropy.
  - ◊ It does NOT mean that entropy cannot decrease !
  - ◊ All you need to do to decrease the entropy of a system is to transfer some heat out of it !
- So, what does this principle tell us ?

Entropy Generation

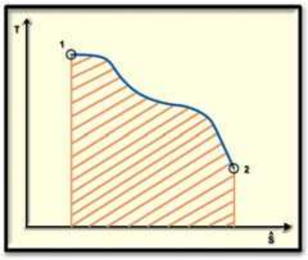
- **Newest Statement of the 2nd Law :** $dS \ge \frac{\delta Q}{T}$
- **Make Clausius into an equality:** $dS = \frac{\delta Q}{T} + dS_{gen}$
- **Entropy Generation,** S_{gen}
  - ◊ Internally reversible processes: $S_{gen} = 0$
  - ◊ Irreversible processes: $S_{gen} > 0$
  - ◊ Impossible processes: $S_{gen} < 0$

- We define entropy generation by making the Clausius Inequality into an equality !
- One of the uses for entropy generation will be to allow us to quickly determine if a process is reversible, irreversible or impossible !
- This is the important implication of the Principle of Increasing Entropy.

TS Diagram

- **Area under the curve:** $\text{Area} = \int_1^2 T\, d\hat{S}$
- **But:** $\delta \hat{Q}_{\text{Int Rev}} = T\, d\hat{S}$

$$\text{Area} = \int_1^2 \delta \hat{Q}_{\text{Int Rev}} = \hat{Q}_{\text{Int Rev}}$$



- **Irreversible Processes:** $d\hat{S} = \frac{\delta \hat{Q}_{\text{Irrev}}}{T} + d\hat{S}_{\text{gen}}$

$$\delta \hat{Q}_{\text{Irrev}} = T\, d\hat{S} - T\, d\hat{S}_{\text{gen}}$$

$$\hat{Q}_{\text{Irrev}} = \text{Area} - \int_1^2 T\, d\hat{S}_{\text{gen}} \qquad \hat{Q}_{\text{Irrev}} < \text{Area}$$

- **S_{gen} is a path variable ! We must use $\delta \hat{S}_{gen}$:** $\boxed{dS = \frac{\delta Q}{T} + \delta S_{\text{gen}}}$

- Any 2 states can be connected by both reversible and irreversible process paths.
- Reversible and irreversible paths result in different values for S_{gen}.
- So, S_{gen} depends on the process path.
- S_{gen} is a path variable and therefore requires an <u>inexact</u> differential.

Isolated Systems

- **Entropy Statement of the 2nd Law:** $\Delta S = \int \frac{\delta Q}{T} + S_{\text{gen}}$
- **Isolated Systems, $\delta Q = 0$, there-** $\Delta S_{\text{isolated}} = S_{\text{gen}} \geq 0$
- **The universe is an isolated system:** $\boxed{\Delta S_{\text{univ}} = S_{\text{gen}} \geq 0}$

The Principle of Increasing Entropy
(a consequence of the 2nd Law)

- **Divide the universe into two regions**
  - ◊ The system and the surroundings:

$$\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = S_{\text{gen}} \geq 0$$

- The universe is an isolated system
  - ◊ No mass enters or leaves
  - ◊ There is nothing else with which to exchange heat !
  - ◊ Any change in the entropy of the universe can only be attributed to entropy generation !
  - ◊ Since the total $S_{gen} \geq 0$ for any process, the entropy of the universe can only increase !
  - ◊ Keep in mind that universe is just the combination of the system and all of its surroundings (including any reservoirs).

The Gibbs Equations

- **Definition of entropy:** $\delta Q_{Int\,Rev} = T\,dS$
- **1^{st} Law, closed system, W_b only:** $\delta Q - \delta W = dU$
- **Boundary work for an internally reversible process:** $\delta W_{Int\,Rev} = P\,dV$
- **1^{st} Gibbs Equation:** $\boxed{dU = T\,dS - P\,dV}$
- **Definition of enthalpy:** $dH = dU + d(PV)$

  $= dU + P\,dV + V\,dP$
- **Substitute in the 1^{st} Gibbs Eqn:** $dH = T\,dS \boxed{- P\,dV + P\,dV} + V\,dP$
- **2^{nd} Gibbs Eqn:** $\boxed{dH = T\,dS + V\,dP}$

- BIG result !
  - ◊ If we can determine **P**, **V**, **T** and **U** or **H**, the Gibbs Eqns can be used to calculate **ΔS** !
  - ◊ As a result, we can look up specific entropy in the NIST Webbook or in any thermodynamic table.
  - ◊ We must still choose a reference state (**T**, **P** and phase) at which **S = 0**.
- So, HOW do we use the Gibbs Eqns to evaluate changes in entropy ?
  - ◊ We need an EOS to do this.
  - ◊ Incompressible Liquids are the simplest, so we will consider them first.
  - ◊ Ideal Gas EOS is the simplest EOS, so we will consider that next.
  - ◊ More accurate, sophisticated EOS's are tougher to use to evaluate **ΔS**.
    - Good news. This is beyond the scope of this course !
    - If it is not an incompressible liquid or an ideal gas, we will always use the thermodynamic tables.

Incompressible Liquids

- **Assumptions:** $\tilde{V} \approx 0$ $\quad d\tilde{V} = 0$ $\quad \tilde{C}_P = \tilde{C}_V = \tilde{C}$
- **Gibbs Equations:** $d\tilde{U} = T\,d\tilde{S} - P\,\cancel{d\tilde{V}}$

  $d\tilde{H} = T\,d\tilde{S} + \cancel{\tilde{V}}\,dP$
- **Enthalpy, internal energy and heat capacity:** $d\tilde{U} = \tilde{C}_V\,dT = \tilde{C}\,dT$

  $d\tilde{H} = \tilde{C}_P\,dT = \tilde{C}\,dT$

  $$d\tilde{S} = \frac{dH}{T} = \frac{dU}{T} = \frac{\tilde{C}}{T}\,dT$$
- **Result:** $\boxed{d\tilde{S} = \frac{\tilde{C}}{T}\,dT}$

- The molar volume of an incompressible liquid is constant.
- If we further assume that the molar volume is very, very small, The Gibbs Equations simplify even further.
- Remember that for a perfectly incompressible liquid, $\mathbf{C_P = C_V}$.
- As a result, **ΔH = ΔU.**
- The result is that both Gibbs Equations reduce to the same equation…the one in the box.
- This makes it easy to evaluate changes in entropy of an incompressible liquid.
- It also means that the molar entropy of a subcooled liquid is the same as the molar entropy of a saturated liquid at the same temperature.
- The best part is that our best estimate of the heat capacity of a liquid is often just a constant.
- In which case, **ΔS = C Ln(T2/T1)**
- It doesn't get much better than that !

Ideal Gases

- **Gibbs Equations:**

$$d\tilde{U} = T\,d\tilde{S} - P\,d\tilde{V} \qquad \Delta\tilde{S} = \int_1^2 d\tilde{S} = \int_1^2 \frac{d\tilde{U}}{T} + \int_1^2 \frac{P}{T}\,d\tilde{V}$$

$$d\tilde{H} = T\,d\tilde{S} + \tilde{V}\,dP \qquad \Delta\tilde{S} = \int_1^2 d\tilde{S} = \int_1^2 \frac{d\tilde{H}}{T} - \int_1^2 \frac{\tilde{V}}{T}\,dP$$

- **Ideal Gas Heat Capacities and EOS:**

$$\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}_V^o}{T}\,dT + \int_1^2 \frac{P}{T}\,d\tilde{V} = \int_1^2 \frac{\tilde{C}_V^o}{T}\,dT + \int_1^2 \frac{R}{\tilde{V}}\,d\tilde{V}$$

$$\Delta\tilde{S} = \int_1^2 \frac{d\tilde{H}}{T} - \int_1^2 \frac{\tilde{V}}{T}\,dP = \int_1^2 \frac{\tilde{C}_P^o}{T}\,dT - \int_1^2 \frac{R}{P}\,dP$$

- **Integration yields:**

$$\boxed{\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}_V^o}{T}\,dT + R\,Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]}$$

$$\boxed{\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}_P^o}{T}\,dT - R\,Ln\left[\frac{P_2}{P_1}\right]}$$

- There are three ways to evaluate **ΔS** using these ideal gas forms of the Gibbs Equations.
  - ◊ Use the Shomate Equation from the NIST Webbook.
  - ◊ Use the Ideal Gas Property Tables in Thermo-CD.
    - Do not mix and match data from different sources without verifying that they use the same reference state !
  - ◊ Use relative Pressure, also provided in the Ideal Gas Property Tables in Thermo-CD.
    - We will cover this in the next class

Using the Shomate Equation

- **This method is accurate for ideal gases, but it is tedious.**

$$\int_1^2 \frac{\tilde{C}_P^o}{T}\,dT = \int_{T_1}^{T_2} \frac{1}{T}\left[A + \frac{B}{1000}T + \frac{C}{1000^2}T^2 + \frac{D}{1000^3}T^3 + 1000^2\frac{E}{T^2}\right]dT$$

$$= A\,Ln\frac{T_2}{T_1} + \frac{B}{1000}\left(T_2 - T_1\right) + \frac{C/2}{1000^2}\left(T_2^2 - T_1^2\right) + \ldots$$

$$+ \frac{D/3}{1000^3}\left(T_2^3 - T_1^3\right) - 1000^2\,\frac{E}{2}\left(\frac{1}{T_2^2} - \frac{1}{T_1^2}\right)$$

$$\int_{T_1}^{T_2} \frac{\tilde{C}_V^o}{T}\,dT = \int_{T_1}^{T_2} \frac{\tilde{C}_P^o - R}{T}\,dT = \int_{T_1}^{T_2} \frac{\tilde{C}_P^o}{T}\,dT - R\,Ln\left[\frac{T_2}{T_1}\right]$$

- Using the Shomate Equation is a tiny bit different than when we used it before because we are evaluating the integral of $\mathbf{C_P}$ over **T** with respect to **T** and NOT just the integral of $\mathbf{C_P}$ **dT**.
  - ◊ Don't make this silly mistake !
- The calculations here are not hard, they are just tedious.
- We can use the fact that, for an ideal gas, $\mathbf{C_P} = \mathbf{C_V} + \mathbf{R}$ to help us evaluate the integral of $\mathbf{C_V}$ **dT** using the Shomate Equation, as shown here.
- The good news is that the Ideal Gas Properties Table essentially eliminates our reliance on the Shomate Equation for some common gases.

Ideal Gas Entropy Function

- **Definition:** $\tilde{S}^o_T = \int_{T_{ref}}^{T} \frac{\tilde{C}^o_P}{T}\, dT$ Where T_{ref} is the reference temperature and: $\tilde{S}^o_{Tref} = 0$
- **Relationship with Shomate Eqn :** $\int_{T_1}^{T_2} \frac{\tilde{C}^o_P}{T}\, dT = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1}$
- **1st Gibbs Eqns for ideal gases :**

$$\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}^o_V}{T}\, dT + R\, Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

$$= \int_1^2 \frac{\tilde{C}^o_P}{T}\, dT - R\, Ln\left[\frac{T_2}{T_1}\right] + R\, Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

- **Substitute the Ideal Gas Entropy Function :**

$$\boxed{\Delta\tilde{S} = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1} - R\, Ln\left[\frac{T_2}{T_1}\right] + R\, Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]}$$

- The ideal gas entropy function, $\mathbf{S^o}$ is tabulated for some gases in Thermo-CD.
- The table contains the results of many, many integrals of the Shomate Equation.
- Instead of integrating Shomate, you can simply interpolate to determine values of $\mathbf{S^o}$.
- Then, we can re-write the ideal gas form of the 1st Gibbs Equation in terms of the ideal gas entropy function.
- The resulting equation in the box allows you to calculate **ΔS** for an ideal gas as long as you know the initial and final **T** and molar volume.
- What if you only know **T** and P ?
  - ◊ You could solve for **V** using the ideal gas EOS, or you could use the 2nd Gibbs Equation.
    - See the next slide.

Ideal Gas Entropy Function

- **Definition:** $\tilde{S}^o_T = \int_{T_{ref}}^{T} \frac{\tilde{C}^o_P}{T}\, dT$ Where T_{ref} is the reference temperature and: $\tilde{S}^o_{Tref} = 0$
- **Relationship with Shomate Eqn :** $\int_{T_1}^{T_2} \frac{\tilde{C}^o_P}{T}\, dT = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1}$
- **1st Gibbs Eqns for ideal gases :**

$$\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}^o_P}{T}\, dT - R\, Ln\left[\frac{P_2}{P_1}\right]$$

- **Substitute the Ideal Gas Entropy Function :**

$$\boxed{\Delta\tilde{S} = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1} - R\, Ln\left[\frac{P_2}{P_1}\right]}$$

- We can also write Gibbs 2nd Equation in terms of the Ideal Gas Entropy Function, $\mathbf{S^o}$.
- This gives us nice, neat, algebraic equation (no integrals) that allows us to compute **ΔS** for an ideal gas if we know the initial and final pressures.
- The combination of this result and the result on the previous page give us a complete toolkit of equations for calculating **ΔS** for ideal gases !
- The OTHER columns in the Ideal Gas Property Tables are pretty useful as well.
  - ◊ $\mathbf{H^o}$ is the integral from $\mathbf{T_{ref}}$ to **T** of $\mathbf{C^o_P}$.
  - ◊ $\mathbf{U^o}$ is the integral from $\mathbf{T_{ref}}$ to **T** of $\mathbf{C^o_V}$.
  - ◊ There is no longer any reason to integrate the Shomate Equation for gases that appear in the Ideal Gas Property Tables !!
- Real gases require more sophisticated EOS's like the ones used to generated the steam tables and other thermodynamic tables.
  - ◊ These complicated EOS's are used with the Gibbs Equations to generated the values of specific entropy that we find in the thermodynamic tables.
  - ◊ The computations are beyond the scope of this course.
  - ◊ We will just trust NIST and use the data that they provide for specific entropy.

Relative Thermodynamic Properties

- **Gibbs Equations for ideal gases:**

$$\Delta\tilde{S} = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1} - R\ \mathrm{Ln}\left[\frac{T_2}{T_1}\right] + R\ \mathrm{Ln}\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right] \qquad \Delta\tilde{S} = \tilde{S}^o_{T_2} - \tilde{S}^o_{T_1} - R\ \mathrm{Ln}\left[\frac{P_2}{P_1}\right]$$

- **Consider an isentropic process from a reference state (T_{ref}, P_{ref}, V_{ref}) to any other state (T, P, V).**
  - Note, $\tilde{S}^o_{Tref} = 0$. Gibbs Eqns become:

$$\mathrm{Ln}\left[\frac{\tilde{V}}{\tilde{V}_{ref}}\right] = \mathrm{Ln}\left[\frac{T}{T_{ref}}\right] - \frac{\tilde{S}^o_T}{R} \qquad \frac{\tilde{S}^o_T}{R} = \mathrm{Ln}\left[\frac{P}{P_{ref}}\right]$$

- **Define Relative Properties:**
  (IG Properties Tables, functions of T only)

$$\boxed{V_R = \frac{\tilde{V}}{\tilde{V}_{ref}}} \qquad \boxed{P_R = \frac{P}{P_{ref}}}$$

- **Advantage of using Relative Properties:**

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{V_R(T_1)}{V_R(T_2)} \qquad \frac{P_1}{P_2} = \frac{P_R(T_1)}{P_R(T_2)}$$

- Relative Thermodynamic Properties provide another way to analyze isentropic processes on ideal gases that lets us avoid the tedium of integrating the Shomate Equation.
- Limitations:
  - Ideal Gases only
  - Isentropic processes only
  - Tables are only available for seven common gases: air, $\mathbf{N_2}$, $\mathbf{O_2}$, $\mathbf{CO_2}$, **CO**, $\mathbf{H_2}$, $\mathbf{H_2O}$.
  - For any other ideal gases, we must resort to integrating the Shomate Equation.

Using Relative properties

- **Advantage of using Relative Properties:**

$$\boxed{V_R = \frac{\tilde{V}}{\tilde{V}_{ref}}} \qquad \boxed{P_R = \frac{P}{P_{ref}}}$$

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{V_R(T_1)}{V_R(T_2)} \qquad \frac{P_1}{P_2} = \frac{P_R(T_1)}{P_R(T_2)}$$

- **Example:** Air expands in a turbine from 1000 K and 1 MPa to the surroundings at 100 kPa. If the air is an ideal gas and the turbine is isentropic, what is the temperature of the turbine exhaust ?

$$P_R(T_2) = P_R(T_1)\,\frac{P_2}{P_1} = 86.792\ \frac{100\,\mathrm{kPa}}{1000\,\mathrm{kPa}} = 8.6792$$

T (K)	P_R
540	8.3101
T_2	8.6792
550	8.8893

By interpolation :

T_2 = **546.4 K**

- The advantage of using them is most evident in the last two equations in this slide.
  - In any isentropic process from ideal gas state 1 to ideal gas state 2, there are 6 variables of interest: **P**, **V** and **T** for each state.
  - We can also write the IG EOS for each state.
  - Given $\mathbf{P_1}$, $\mathbf{T_1}$ and $\mathbf{P_2}$ we can determine $\mathbf{PR_2}$ and get $\mathbf{T_2}$ by interpolation on the IG Property Tables.
  - A similar analysis can be used if volumes are known.
  - This method is pretty quick and straightforward.

Isentropic Processes

- **Assumptions:** **Ideal Gas with constant $\tilde{C}_P^o$ and $\tilde{C}_V^o$**
- **Key equations:** $\tilde{C}_P^o = \tilde{C}_V^o + R$ **and:** $\gamma = \dfrac{\tilde{C}_P^o}{\tilde{C}_V^o} > 1$
- **Results:** $T_1\ \tilde{V}_1^{\gamma-1} = T_2\ \tilde{V}_2^{\gamma-1} = \text{constant}$

$$T_1\ P_1^{\frac{1-\gamma}{\gamma}} = T_2\ P_2^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$P_1\ \tilde{V}_1^{\gamma} = P_2\ \tilde{V}_2^{\gamma} = \text{constant}$$

- The derivation of the equations presented here is given in Thermo-CD.

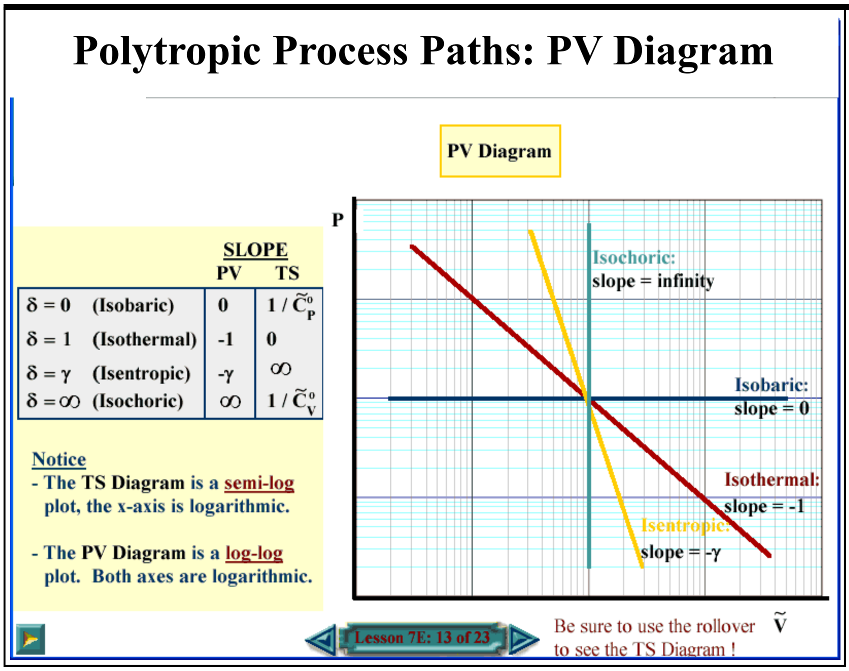
Polytropic Processes

- **Definition:** $P_1\ \tilde{V}_1^{\delta} = P_2\ \tilde{V}_2^{\delta} = \text{constant}$
- **Other relationships:** $T_1\ \tilde{V}_1^{\delta-1} = T_2\ \tilde{V}_2^{\delta-1} = \text{constant}$

$$T_1\ P_1^{\frac{1-\delta}{\delta}} = T_2\ P_2^{\frac{1-\delta}{\delta}} = \text{constant}$$

- **An isentropic process is a polytropic process where:** $\delta = \gamma$ $P_1\ \tilde{V}_1^{\gamma} = P_2\ \tilde{V}_2^{\gamma}$
- **Isobaric process:** $\delta = 0$ $P_1 = P_2$
- **Isothermal process:** $\delta = 1$ $P_1\ \tilde{V}_1 = P_2\ \tilde{V}_2$
- **Isochoric process:** $\delta = \infty$ $\tilde{V}_1 = \tilde{V}_2$

- All of the equations on the previous slide also apply to any polytropic process.
  - ◊ All we need to do is replace γ with δ.
- Isentropic processes for ideal gases with constant heat capacities are a special case or subset of the more general polytropic process.
- It also turns out that isobaric processes on ideal gases are just polytropic processes in which $\delta = \mathbf{0}$.
- Isothermal processes on ideal gases are just polytropic processes in which $\delta = \mathbf{1}$.
- Isochoric processes on ideal gases are just polytropic processes in which $\delta = \infty$.



Polytropic Process Paths: PV Diagram

PV Diagram

		SLOPE	
		PV	**TS**
$\delta = 0$	**(Isobaric)**	**0**	**$1 / \tilde{C}^{\circ}_P$**
$\delta = 1$	**(Isothermal)**	**-1**	**0**
$\delta = \gamma$	**(Isentropic)**	**$-\gamma$**	**∞**
$\delta = \infty$	**(Isochoric)**	**∞**	**$1 / \tilde{C}^{\circ}_V$**

Notice

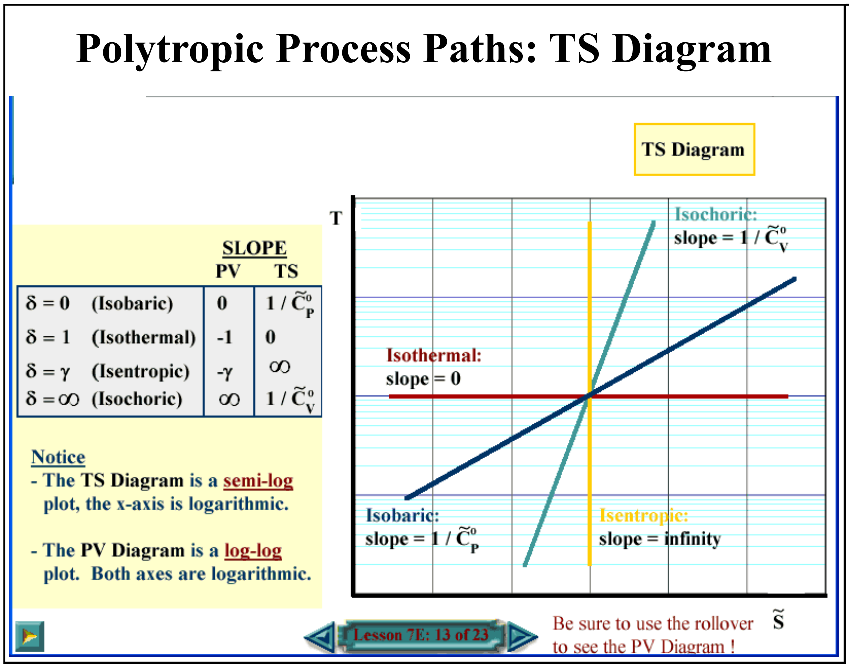
- The TS Diagram is a semi-log plot, the x-axis is logarithmic.

- The PV Diagram is a log-log plot. Both axes are logarithmic.

Be sure to use the rollover to see the TS Diagram !

Lesson 7E: 13 of 23

- It is critical that you notice that this is a semi-logarithmic plot.
- Both the P axis and the V axis are logarithmic.
- The logarithmic axis has the same effect on the shape of the data plotted as taking the logarithm of your P data, but has the BIG advantage of not messing up the numbers.
  - ◊ You should always consider using a logarithmic axis instead of plotting the log of your data on an ordinary, linear axis.
  - ◊ BUT areas on these logarithmic coordinates are not equal to WORK !
    - Don't get caught in this trap.
- The cool part is that polytropic processes for ideal gases with constant heat capacities are straight lines on this plot.
  - ◊ The **slope** = - δ
  - ◊ Note, a line with a slope of ∞ is the same as a line with a slope of $-\infty$.



Polytropic Process Paths: TS Diagram

TS Diagram

		SLOPE	
		PV	**TS**
$\delta = 0$	**(Isobaric)**	**0**	**$1 / \tilde{C}^{\circ}_P$**
$\delta = 1$	**(Isothermal)**	**-1**	**0**
$\delta = \gamma$	**(Isentropic)**	**$-\gamma$**	**∞**
$\delta = \infty$	**(Isochoric)**	**∞**	**$1 / \tilde{C}^{\circ}_V$**

Notice

- The TS Diagram is a semi-log plot, the x-axis is logarithmic.

- The PV Diagram is a log-log plot. Both axes are logarithmic.

Be sure to use the rollover to see the PV Diagram !

Lesson 7E: 13 of 23

- This plot is different. It is a semi-logarithmic plot.
  - ◊ The T axis is logarithmic and the S axis is an ordinary, linear axis.
- This plot also produces straight lines for all polytropic processes !
  - ◊ But the slopes are different.
  - ◊ On the next slide, we will see why these paths are linear and how to determine their slopes.

Polytropic Process Paths: TS Diagram

- **Gibbs Eqns, IG, Constant Heat Capacities:**

$$\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}_V^o}{T}\,dT + R\,Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right] = \tilde{C}_V^o\,Ln\left[\frac{T_2}{T_1}\right] + R\,Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

$$\Delta\tilde{S} = \int_1^2 \frac{\tilde{C}_P^o}{T}\,dT - R\,Ln\left[\frac{P_2}{P_1}\right] = \tilde{C}_P^o\,Ln\left[\frac{T_2}{T_1}\right] - R\,Ln\left[\frac{P_2}{P_1}\right]$$

$$\text{Slope} = \frac{Ln\left[{T_2}/{T_1}\right]}{\Delta\tilde{S}}$$

- **Isentropic process:** $\delta = \gamma$ $\text{Slope} = \pm\infty$
- **Isobaric process:** $\delta = 0$ $\text{Slope} = \frac{1}{\tilde{C}_P^o}$
- **Isothermal process:** $\delta = 1$ $\text{Slope} = 0$
- **Isochoric process:** $\delta = \infty$ $\text{Slope} = \frac{1}{\tilde{C}_V^o}$

- The key here is that our TS Diagram is semi-logarithmic.
  - ◊ The T axis is logarithmic.
  - ◊ This is equivalent to plotting **Ln[T] vs. S**.
  - ◊ The slope of a line on such a plot would be: **$\{Ln[T_2]-Ln[T_1]\} / \{S_2-S_1\}$**
  - ◊ Or, by using properties of logarithms… **slope = $\{Ln[T_2/T_1]\} / \{S_2-S_1\}$**
- Now, let's use the two Gibbs Eqns to determine the slope of each of the special types of polytropic processes on a semi-logarithmic TS Diagram.
  - ◊ Isentropic: **$\Delta S = 0$** and **ΔS** is in the denominator of the slope, so the slope is ∞ (or **-** ∞)
  - ◊ Isobaric: **$P_2 = P_1$**, so the **$R\,Ln[P_2/P_1]$** term in the 2nd Gibbs Eqn is zero.
    - When we then solve the 2nd Gibbs Eqn for the slope, we get **$1 / C^o_P$**.
  - ◊ Isothermal: **$T_2 = T_1$**, so the numerator of the slope is zero and the slope is therefore zero as well.
  - ◊ Isochoric: **$V_2 = V_1$**, so the **$R\,Ln(V_2/V_1)$** term in the 1st Gibbs Eqn is zero.
    - When we then solve the 1st Gibbs Eqn for the slope, we get **$1 / C^o_V$**.

Boundary Work in Polytropic Processes

- **Internally Reversible Processes:**

$$\tilde{W}_b = \int_1^2 P\,d\tilde{V} = \int_1^2 \frac{C}{\tilde{V}^\delta}\,d\tilde{V} = P_1\,\tilde{V}_1^\delta \int_1^2 \frac{d\tilde{V}}{\tilde{V}^\delta}$$

- **$\delta = 1$**

$$\tilde{W}_b = P_1\,\tilde{V}_1 \int_1^2 \frac{d\tilde{V}}{\tilde{V}} = P_1\,\tilde{V}_1\,Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

  - ◊ **If also an IG then Isothermal:**

$$\tilde{W}_{b,IG} = R\,T\,Ln\left[\frac{\tilde{V}_2}{\tilde{V}_1}\right]$$

- **$\delta \neq 1$:**

$$\tilde{W}_b = P_1\,\tilde{V}_1^\delta \left[\frac{\tilde{V}^{(-\delta+1)}}{-\delta+1}\right]\Bigg|_1^2 = \frac{P_2\,\tilde{V}_2 - P_1\,\tilde{V}_1}{1-\delta}$$

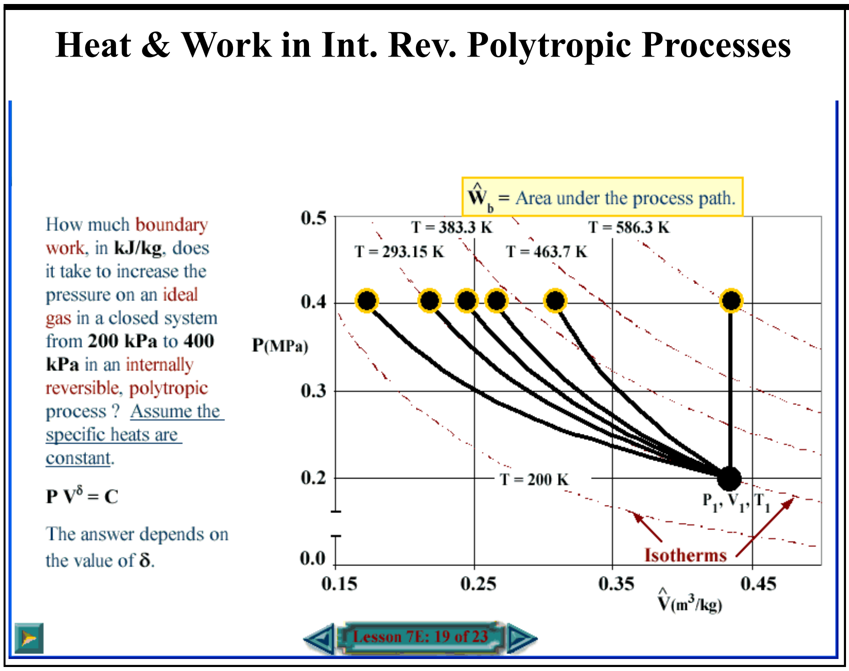
$$\tilde{W}_{b,IG} = \frac{R}{1-\delta}\left(T_2 - T_1\right)$$

- **$\delta = \gamma$: Isentropic:**

$$\tilde{W}_b = -\Delta\tilde{U} = -\tilde{C}_V^o\left(T_2 - T_1\right)$$

$$\tilde{W}_{b,IG} = \frac{R}{1-\gamma}\left(T_2 - T_1\right)$$

- It is pretty straightforward to integrate **P dV** to obtain an equation for boundary work in an internally reversible, isothermal process.
- It is a bit more messy for polytropic processes that are not isothermal.
  - ◊ Fortunately, the result is fairly simple.
- A process that is both internally reversible AND isentropic, must be ADIABATIC.
  - ◊ So, we can use the 1st Law to evaluate the boundary work.



Heat & Work in Int. Rev. Polytropic Processes

How much boundary work, in **kJ/kg**, does it take to increase the pressure on an ideal gas in a closed system from **200 kPa** to **400 kPa** in an internally reversible, polytropic process ? Assume the specific heats are constant.

$P V^{\delta} = C$

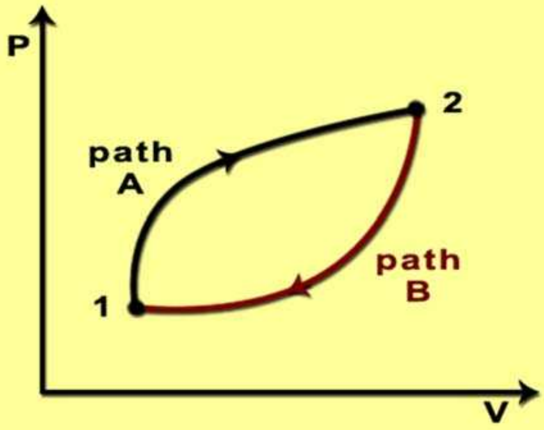
The answer depends on the value of δ.

$\hat{W}_b$ = Area under the process path.

- Working from right to left, each of these process paths requires more work to raise the pressure from **200 kPa** to **400 kPa**.
  - ◊ The 1st path is isochoric ($\delta = \infty$).The pressure rises because heat is transferred into the system.
  - ◊ The 2nd path is polytropic with $\delta = \mathbf{2}$. The boundary work for the process increased, but the heat transfer requirement decreased.
  - ◊ The 3rd path from the right is polytropic with $\delta = \gamma = \mathbf{1.3985}$. The boundary work for the process increased, but the heat transfer requirement is ZERO. This is the first process path that is genuinely a pure compression process. We are not using heat to raise the pressure.
  - ◊ The remaining paths are all polytropic with $\delta < \gamma$, including an isothermal path with $\delta = \mathbf{1}$. These paths can be considered to be pure compressions. They all require more work than the isentropic path and they must reject some heat.
  - ◊ We conclude that the isentropic path requires the least work of all of the “pure” compression processes.
  - ◊ This represents an opportunity for us to define some sort of efficiency.
  - ◊ We will do that in the next chapter !

7A-1 Process Paths and Cyclic Integrals 4 pts

Consider the cycle made of **path A** followed by **path B** shown below.



The following **integrals** have been evaluated.

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A = -24.7 \text{ kJ / K}$$

$$\int_2^1 \left(\frac{\delta Q}{T}\right)_B = 41.3 \text{ kJ / K}$$

Is this cycle **reversible**, **irreversible** or **impossible**?

Read : This is a straightforward application of the Clausius Inequality to a thermodynamic cycle interacting with two thermal reservoirs.

Diagram: See the problem statement.

Given: $\int_1^2 \left(\frac{\delta Q}{T}\right)_A =$ -24.7 kJ/K $\int_2^1 \left(\frac{\delta Q}{T}\right)_B =$ 41.3 kJ/K

Find: Is this cycle **reversible**, **irreversible** or **impossible**?

Assumptions: None.

Equations / Data / Solve:

The Clausius Inequality is: $$\oint \frac{\delta Q}{T} \leq 0$$ **Eqn 1**

If the cyclic integral is … ... positive, the cycle is impossible
... negative, the cycle is irreversible
... zero, the cycle is reversible

Since the cycle is made up of two **steps**, **A** & **B**, we can expand **Eqn 1** as follows:

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B$$ **Eqn 2**

Now, we can plug in values from the problem statement. $\oint \frac{\delta Q}{T} =$ **16.6** **kJ/K**

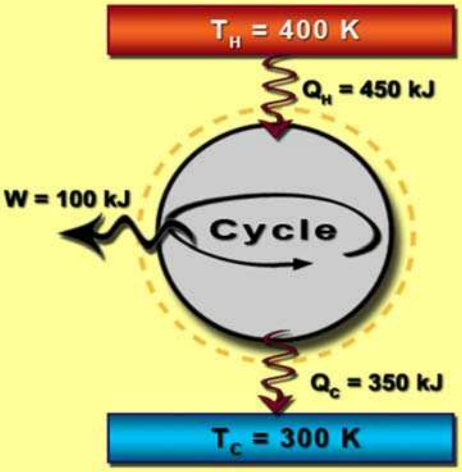
Because the cyclic integral is greater than zero, the cycle is impossible.

Verify: None.

Answers : **Because the cyclic integral is greater than zero, the cycle is impossible.**

7A-2 Properties of Cyclic Integrals 3 pts

Consider the heat engine shown below. The cyclic integral of δQ is greater than **zero**. Does this violate the Clausius Inequality? Explain.



Read : This problem is designed to make you think very carefully about how to use a cyclic integral. The key is that the **temperature** is not the same for both of the heat transfer interactions in this cycle.

Diagram: See the problem statement.

Given:

T_H	400	K	Q_H	450	kJ
T_C	300	K	Q_C	-350	kJ
			W	-100	kJ

$$\oint \delta Q > 0$$

Find: Does the cycle violate the Clausius Inequality?

Assumptions: None.

Equations / Data / Solve:

The Clausius Inequality is:

$$\oint \frac{\delta Q}{T} \le 0 \qquad \text{Eqn 1}$$

Because the cycle only exchanges **heat** with the hot and cold thermal reservoirs, the integrals can be simplified:

$$\oint \delta Q = Q_H + Q_C \qquad \text{Eqn 2}$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} + \frac{Q_C}{T_C} \qquad \text{Eqn 3}$$

$\oint \delta \mathbf{Q} =$ **100** **kJ**

$\oint \frac{\delta \mathbf{Q}}{\mathbf{T}} =$ **-0.04167** **kJ/K**

This cycle is irreversible because the cyclic integral in the Clausius Inequality is less than **zero**.

It is true that the cyclic integral of δ**Q > 0**. But the Clausius Inequality is still satisfied.

Confusion about the cyclic integrals sometimes arises if you mistakenly pull **T** out of the cyclic integral.

$$\oint \frac{\delta \mathbf{Q}}{\mathbf{T}} \neq \frac{\oint \delta \mathbf{Q}}{\mathbf{T}}$$ **Eqn 4**

You could only pull **T** out of the cyclic integral if ALL of the **heat** exchange across the system boundary went to or from reservoirs that were ALL at the SAME **temperature**.

That is almost never going to happen. It is definitely not the case in this problem as the hot and cold reservoirs are at **400 K** and **300 K**, respectively.

Verify: None.

Answers : Yes, the cycle does indeed satisfy the Clausius Inequality.

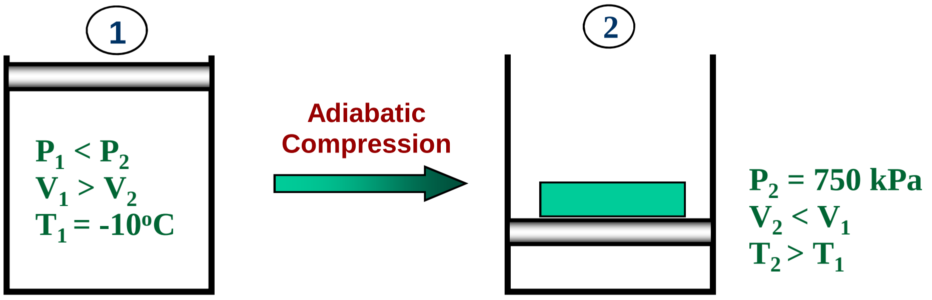
7B-1 Reversible Adiabatic Compression of R-134a 5 pts

Saturated **ammonia** vapor at **-10°C** is compressed in an **insulated** piston-and-cylinder device until the **pressure** reaches **750 kPa**. Assuming the process is internally reversible, calculate the **work** for this process in kJ/kg.

Read : The key to solving this problem is to recognize that any process that is both adiabatic and reversible is **ISENTROPIC**. This means that $S_2 = S_1$ and this allows you to fix state 2 and evaluate U_2. Use U_2 in the 1st Law to evaluate **W**.

Given:				**Find:**			
	T_1	-10	°C		W	???	kJ/kg
	P_2	750	kPa				

Diagram:



Assumptions:

1 - Process is internally reversible.
2 - Changes in kinetic and potential energies are negligible.
3 - **Boundary work** is the only form of **work** that crosses the system boundary.

Equations / Data / Solve:

Begin by applying the 1st Law to the process, assuming changes in kinetic and potential energies are negligible:

$$\hat{Q} - \hat{W} = \Delta\hat{U} \qquad \textbf{Eqn 1}$$

The process is adiabatic so **Eqn 1** can be simplified to :

$$\hat{W} = -\Delta\hat{U} = \hat{U}_1 - \hat{U}_2 \qquad \textbf{Eqn 2}$$

Use the **NIST Webbook** to obtain properties for state 1, saturated vapor at **-10°C** :

P_1	290.71	kPa	U_1	1309.9	kJ/kg
			S_1	5.4701	kJ/kg-K

Because the process is both reversible and adiabatic, it is isentropic.

Therefore : S_2 5.4701 kJ/kg-K

At this point we know values of two intensive variables for state 2, so we can use the **NIST Webbook** to determine the value of any other property. In this case, we need $\mathbf{U_2}$. First we need to determine the phases that exist at state 2.

At $\mathbf{P_2}$: $\mathbf{T_{sat}}$ **54.05** $\mathbf{^oC}$

$\mathbf{S_{sat\ vap}}$	**4.7209**	**kJ/kg-K**
$\mathbf{S_{sat\ liq}}$	**1.5744**	**kJ/kg-K**

Because $\mathbf{S_2 > S_{sat\ vap}}$ at $\mathbf{P_2}$, we can conclude that state 2 is a superheated vapor. We could have reached the same conclusion after careful consideration of a **TS Diagram**.

We can get the following data from the superheated Ammonia Tables or from the **NIST Webbook** :

At **750 kPa** :

$\mathbf{T\ (^oC)}$	**S (kJ/kg-K)**	**U (kJ/kg)**
50	**5.4388**	**1401.6**
$\mathbf{T_2}$	**5.4701**	$\mathbf{U_2}$
75	**5.6233**	**1449.4**

Interpolation yields :

$\mathbf{T_2}$	**54.25**	$\mathbf{^oC}$
$\mathbf{U_2}$	**1409.73**	**kJ/kg**

Now, we can plug values back into **Eqn 2** : **W** **-99.80** **kJ/kg**

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

W	**-99.8**	**kJ/kg**

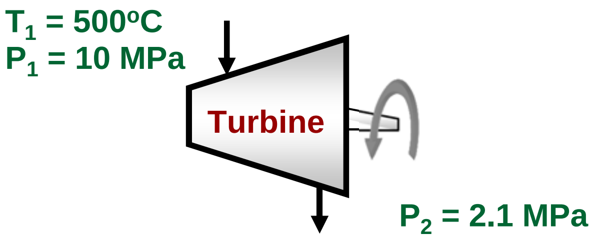
7B-2 Work Output of an Adiabatic, Reversible Turbine 5 pts

An adiabatic turbine lets **10 Mpa steam** down to **2.1 Mpa**. Determine the maximum **work** output if the inlet **temperature** is **500°C** and changes in kinetic and potential energies are negligible.

Read : The key to solving this problem is to recognize that any process that is both adiabatic and reversible is **ISENTROPIC**. This means that $S_2 = S_1$ and this allows you to fix state 2 and evaluate H_2. Use H_2 in the 1st Law to evaluate W_S.

Given:				**Find:**			
	T_1	500	°C		W_S	???	kJ/kg
	P_1	10000	kPa				
	P_2	2100	kPa				

Diagram:



Assumptions:

1 - The turbine is both adiabatic and reversible.
2 - Changes in kinetic and potential energies are negligible.
3 - **Shaft work** is the only form of **work** that crosses the system boundary.

Equations / Data / Solve:

Begin by applying the 1st Law to the process, assuming changes in kinetic and potential energies are negligible:

$$\hat{Q} - \hat{W}_S = \Delta\hat{H} \qquad \textbf{Eqn 1}$$

The process is adiabatic so **Eqn 1** can be simplified to :

$$\hat{W}_S = -\Delta\hat{H} = \hat{H}_1 - \hat{H}_2 \qquad \textbf{Eqn 2}$$

Use the **NIST Webbook** to obtain properties for **state 1**. First we have to determine the phases present.

At P_1 : T_{sat} 311.00 °C

Since $T_1 > T_{sat}$, **state 1** is a superheated vapor.
The superheated Steam Tables and the **NIST Webbook** yield :

H_1	3375.1	kJ/kg	S_1	6.5995	kJ/kg-K

Because the process is both reversible and adiabatic, it is isentropic.

Therefore, $S_2 = S_1$: S_2 6.5995 kJ/kg-K

At this point we know values of two intensive variables for **state 2**, so we can use the **NIST Webbook** to determine the value of any other property. In this case, we need H_2. First, we need to determine the phases that exist at **state 2**.

At P_2 : T_{sat} 214.86 °C

$S_{sat\ vap}$ 6.3210 kJ/kg-K

$S_{sat\ liq}$ 2.4699 kJ/kg-K

Because $S_2 > S_{sat\ vap}$ at P_2, we can conclude that **state 2** is a superheated vapor. We could have reached the same conclusion after careful consideration of a **TS Diagram**.

We can get the following data from the superheated Steam Tables or from the **NIST Webbook** :

At **2.1 MPa** :

T (°C)	S (kJ/kg-K)	H (kJ/kg)
265	6.5903	2937.1
T_2	6.5995	H_2
270	6.6133	2949.4

Interpolation yields :

T_2 266.99 °C

H_2 2941.99 kJ/kg

Now, we can plug values back into **Eqn 2** : W_S 433.13 kJ/kg

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

W	433	kJ/kg

7B-3 Entropy Change of an Isobaric Process 6 pts

A piston-and-cylinder device with a free-floating piston contains **2.6 kg** of saturated **water** vapor at **150°C**. The **water** loses heat to the surroundings until the cylinder contains saturated liquid **water**.

The surroundings are at **20°C**. Calculate… **a.) ΔS_{water}, b.) $\Delta S_{surroundings}$, c.) $\Delta S_{universe}$**

Read : Calculating ΔS for the water in the cylinder is straightforward. The key to calculating ΔS_{surr} is the fact that the surroundings behave as a thermal reservoir. The **temperature** of the surroundings does not change and there are no irreversibilities in the surroundings that are associated with the process. The key to calculating ΔS_{univ} is the fact that the universe is made up of the combination of the system and the surroundings. Consequently, $\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr}$.

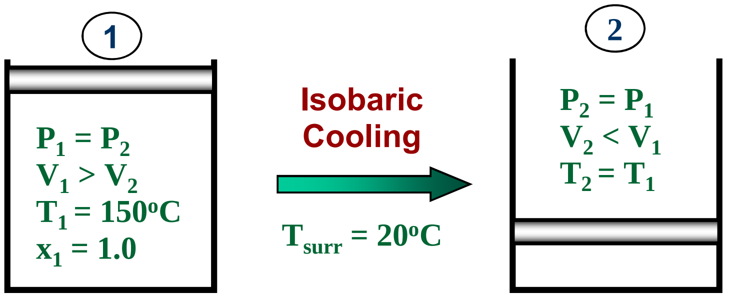
Given:

m	2.60	kg
T_1	150	°C
x_1	1	kg vap/kg
T_{surr}	20	°C
$P_1 = P_2$		
x_2	0	kg vap/kg

Find:

ΔS_{sys}	???	kJ/K
ΔS_{surr}	???	kJ/K
ΔS_{univ}	???	kJ/K

Diagram:



Assumptions:

1 - Changes in kinetic and potential energies are negligible.
2 - **Boundary work** is the only form of **work** that crosses the system boundary.
3 - The surroundings behave as a thermal reservoir.

Equations / Data / Solve:

Part a.) Because both states are saturated we can obtain the **specific entropies** directly from the Steam Tables or the **NIST Webbook**.

At $T_1 = 150°C$:

S_1	6.8371	kJ/kg-K
S_2	1.8418	kJ/kg-K

Therefore :

$$\Delta S_{sys} = S_2 - S_1 \quad \text{Eqn 1}$$

ΔS_{sys}	-4.9953	kJ/kg-K
	-12.988	kJ/K

Part b.) The surroundings behave as a thermal reservoir.
We can calculate ΔS_{surr} from:

$$\Delta \hat{S}_{surr} = \int_1^2 \left(\frac{\delta \hat{Q}}{T} \right)_{rev} = \frac{1}{T} \int_1^2 \delta \hat{Q}_{rev} = \frac{\hat{Q}_{surr}}{T_{surr}} = -\frac{\hat{Q}_{sys}}{T_{surr}} \qquad \text{Eqn 2}$$

We can determine Q_{sys} by applying the 1st Law using the water within the cylinder as the system.

$$\hat{Q}_{sys} - \hat{W} = \Delta \hat{U} \qquad \text{Eqn 3}$$

$$\hat{Q}_{sys} - \hat{W}_b + \cancel{\hat{W}_S} = \Delta \hat{U} \qquad \text{Eqn 4}$$

Because the process is isobaric, the **boundary work** is :

$$\hat{W}_b = P \Delta \hat{V} \qquad \text{Eqn 5}$$

Now, substitute **Eqn 5** into **Eqn 4** to get :

$$\hat{Q}_{sys} = P \Delta \hat{V} + \Delta \hat{U} = \Delta \hat{H} \qquad \text{Eqn 6}$$

We can look up **enthalpy** values for states 1 & 2 in the Saturated Steam Table or in the **NIST Webbook**.

At **T_1 = 150°C** :

H_1	2745.9	kJ/kg
H_2	632.18	kJ/kg

Next plug H_1 and H_2 into **Eqn 6**. Q_{surr} is equal in magnitude, but opposite in sign to Q_{sys} because the **heat** leaving the system enters the surroundings.

Q_{sys}	-2113.7	kJ/kg
Q_{surr}	2113.7	kJ/kg

Now, we can plug numbers into **Eqn 2** to calculate ΔS_{surr}.

ΔS_{surr}	7.2105	kJ/kg-K
	18.747	kJ/K

Part c.) The universe is made up of the combination of the system and the surroundings. Therefore :

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr} \qquad \text{Eqn 7}$$

So, all we need to do is plug values into **Eqn 7** that we determined in **parts (a)** and **(b)**.

ΔS_{univ}	5.7595	kJ/K

ΔS_{univ} > 0 because the **heat transfer** to the surroundings was not reversible.

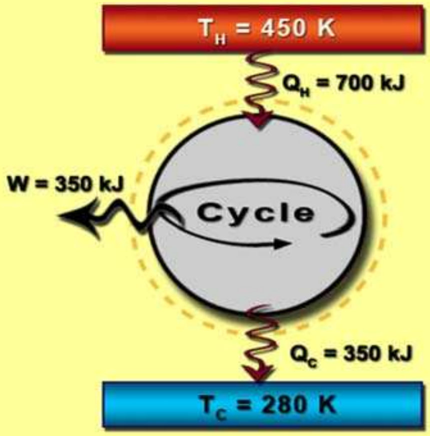
Verify: None of the assumptions made in this problem solution can be verified.

Answers :

ΔS_{sys}	-12.988	kJ/K
ΔS_{surr}	18.747	kJ/K
ΔS_{univ}	5.7595	kJ/K

7C-1 Entropy Change of the Universe for a Cycle 3 pts

Calculate $\Delta S_{universe}$ for the power cycle shown below. Is this cycle reversible, irreversible or impossible?



Read : The key to this problem is that the sign of **ΔS_{univ}** determines whether a process is impossible, reversible or irreversible. Use the definition of **entropy** to evaluate **ΔS** for each reservoir and for the cycle and add them up to get **ΔS_{univ}**.

Diagram: See the problem statement.

Given:

T_H	450	K	Q_H	700	kJ
T_C	280	K	Q_C	-350	kJ

Find: Is this cycle **reversible**, **irreversible** or **impossible**?

Assumptions: 1 - The cycle only exchanges **heat** with the hot and cold reservoirs shown.

Equations / Data / Solve:

In this problem, the universe consists of the cycle, the hot reservoir and the cold reservoir. We can calculate **ΔS_{univ}** from:

$$\Delta S_{univ} = \Delta S_{cycle} + \Delta S_{hot} + \Delta S_{cold} \qquad \text{Eqn 1}$$

Because the cycle begins and ends in the same **state**, **$S_{init} = S_{final}$** and **$\Delta S_{cycle} = 0$**.

By definition, the **temperatures** of the thermal reservoirs remain constant and there are no irreversibilities within the reservoirs because no process takes place in either reservoir. As a result, it is relatively simple to calculate ΔS_{hot} and ΔS_{cold} using the following simplifications of the definition of **entropy**.

$$\Delta S_{hot} = \oint \frac{\delta Q_H}{T_H} = \frac{-Q_H}{T_H} \qquad \textbf{Eqn 2}$$

$$\Delta S_{cold} = \oint \frac{\delta Q_C}{T_C} = \frac{-Q_C}{T_C} \qquad \textbf{Eqn 3}$$

ΔS_{hot}	**-1.556**	**kJ/K**
ΔS_{cold}	**1.250**	**kJ/K**

Now, we can plug values back into **Eqn 1** to complete this problem.

ΔS_{univ}	**-0.306**	**kJ/K**

If the ΔS_{univ} is …

... negative, the cycle is impossible
... **zero**, the cycle is reversible
... positive, the cycle is irreversible

This cycle is impossible because $\Delta S_{univ} < 0$.

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **This cycle is impossible because $\Delta S_{univ} < 0$.**

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7D-1 ΔS of H2 in a Compression Process 6 pts

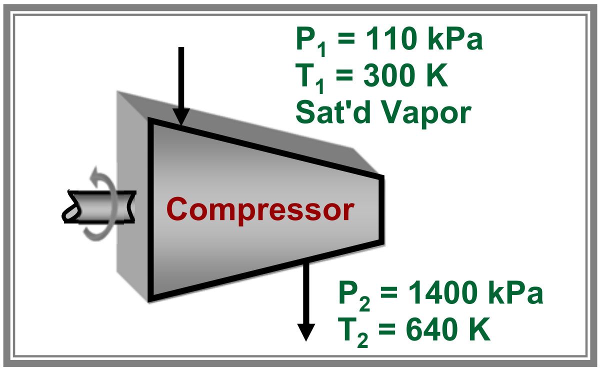
In a piston-and-cylinder device, **carbon dioxide** (**CO_2**) gas is compressed from **110 kPa** and **300K** to **1.4 MPa** and **640K**. Determine the change in the **specific entropy** of the **CO_2** assuming it behaves as an ideal gas. Use…

a.) The Shomate Heat Capacity Equation
b.) The Ideal Gas Entropy Function
c.) The **NIST Webbook**
d.) Compare your answers from **parts (a)** and **(b)** to your answer in **part (c)**.

Read : This problem is an application of the 2nd Gibbs Equation. In **part (a)** we must evaluate the integal of **C_P / T dT** ourselves using the Shomate Equation. The results we obtain should very closely match the results we get in **part (b)** when we use the the **Ideal Gas Entropy Function** (values obtained form the Ideal Gas Property Tables). Finally, in **part (c)** we can determine how much error was introduced by our ideal gas assumption using the **NIST Webbook** to evaluate the **specific entropy** change.

Given:	**T_1**	**300**	**K**	**T_2**	**640**	**K**
	P_1	**110**	**kPa**	**P_2**	**1400**	**kPa**
Find:	**ΔS**	**???**	**kJ/kg-K**			

Diagram:



Assumptions: **1 -** The carbon dioxide behaves as an ideal gas.

Equations / Data / Solve:

Part a.) The 2nd Gibbs Equation is the one best suited to this problem because we know the inlet and outlet **pressures**. The 2nd Gibbs Equation for ideal gases is:

$$\Delta\hat{S} = \frac{1}{MW}\left[\int_1^2 \tilde{C}_P \frac{dT}{T} - R\,Ln\left(\frac{P_2}{P_1}\right)\right] \qquad \text{Eqn 1}$$

The **heat capacity** is determined from the Shomate Equation.

$$\tilde{C}_P^o = A + BT + CT^2 + DT^3 + E/T^2$$ **Eqn 2**

The values of the constants in the Shomate Equation for carbon dioxide are obtained from the **NIST WebBook**:

T (K)	298. - 1200.
A	24.99735
B	55.18696
C	-33.69137
D	7.948387
E	-0.136638

Substituting the **Eqn 2** into **Eqn 1** and integrating yields:

$$\Delta\tilde{S} = A \cdot Ln\frac{T_2}{T_1} + \left(\frac{B}{1000}\right)(T_2 - T_1) + \left(\frac{(1/2)C}{1000^2}\right)(T_2^2 - T_1^2) + \left(\frac{(1/3)D}{1000^3}\right)(T_2^3 - T_1^3) - \left(\frac{(1/2)E}{1000^{-2}}\right)(T_2^{-2} - T_2^{-2}) - R \cdot Ln\frac{P_2}{P_1}$$ **Eqn 3**

We will need the value of the **Universal Gas Constant** and the **molecular weight** to determine the change in the **specific entropy**.

R **8.314** **J/mole-K** **MW** **44.010** **g/mol**

Now, we can substitute values into **Eqn 3** to complete **part (a)** :

$$\int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} =$$ **32.351** **J/mole-K**

$$R\,Ln\frac{P_2}{P_1} =$$ **21.149** **J/mole-K**

Now, we can plug values into **Eqn 3** :

ΔS **11.202** **J/mole-K**
ΔS **0.2545** **kJ/kg-K**

Part b.) In this part of the problem, we use the 2nd Gibbs Equation in terms of the Ideal Gas Entropy Function:

$$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW} \cdot Ln\frac{P_2}{P_1}$$ **Eqn 4**

Properties are determined from Ideal Gas Property Tables:

At **T_1**: **S^o_{T1}** **0.0052249** **kJ/kg-K**
At **T_2**: **S^o_{T2}** **0.74030** **kJ/kg-K**

Now, we can plug values into Eqn 4 : **ΔS** **0.2545** **kJ/kg-K**

Part c.) The **NIST Webbook** yields the following values for the **specific entropy** of carbon dioxide:

S_1 **2.7264** **kJ/kg-K**
S_2 **2.9796** **kJ/kg-K**

Therefore : **ΔS** **0.2532** **kJ/kg-K**

Verify: The ideal gas assumption needs to be verified.
We need to determine the **specific volume** at each state and check **if** :

$$\tilde{V} > 20 \ \mathrm{L/mol}$$

Solving the Ideal Gas EOS for **molar volume** yields :

$$\tilde{V} = \frac{RT}{P}$$

Plugging in values gives us :

V_1	22.7	L/mol
V_2	3.80	L/mol

The **specific volume** at **state 2** is much less than **20 L/mol**, so the ideal gas assumption is questionable at

Answers :

a.)	ΔS	0.255	kJ/kg-K
b.)	ΔS	0.255	kJ/kg-K
c.)	ΔS	0.253	kJ/kg-K

Comparison:

The results in parts (a) and (b) are identical. This is not a surprise, assuming we integrated the Shomate Equation correctly !

The error in ΔS associated with the ideal gas assumption in this problem is: 0.53%

We expected the error to be greater than 1% since the molar volume is much less than 20 L/mole.

7D-2 **Calculating ΔS from Ideal Gas Tables and from Ideal Gas Heat Capacities** **5 pts**

Hydrogen (H_2) gas is compressed from **4.8 bar** and **320K** to **15.4 bar** and **1300K**. Determine the change in the **specific entropy** of the H_2, in kJ/kg, assuming the H_2 behaves as an ideal gas. Use…

a.) The Shomate Heat Capacity Equation

$$\Delta\tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} - R\,Ln\frac{P_2}{P_1}$$

b.) The Ideal Gas Entropy Function

$$\Delta\tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \tilde{S}_{T2}^o - \tilde{S}_{T1}^o - R\,Ln\frac{P_2}{P_1}$$

c.) with constant **Heat Capacity**, C_P, determined at **810K** and **10.1 bar**.

$$\Delta\tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} - R\,Ln\frac{P_2}{P_1}$$

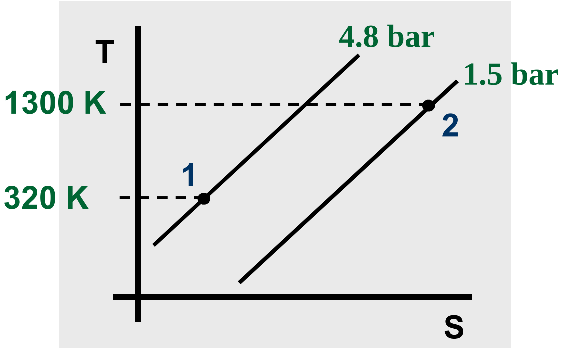
Read : All equations given are for the molar change in **entropy**. Make sure and divide your final answer by the **molecular weight** of hydrogen to obtain a final answer as the change in **specific entropy**, in **kJ/kg-K**.

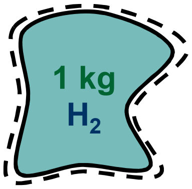
Given:

	m	1	kg	**Hydrogen**		
	T_1	320	K	T_2	1300	K
	P_1	4.8	bar	P_2	15.4	bar
c.)	T_C	810	K	P_C	10.1	bar

Find: Part (a) - (c) ΔS ??? kJ/(kg K)

Diagram:





Assumptions: 1 - The system consists of **one kg** of hydrogen, which behaves as an ideal gas.

Equations / Data / Solve:

Part a.) Here, we use the equation given in the problem statement:

$$\Delta \tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} - R\,Ln\frac{P_2}{P_1} \qquad \textbf{Eqn 1}$$

The heat capacity is determined from the Shomate Equation.

$$\tilde{C}_P^o = A + BT + CT^2 + DT^3 + E/T^2 \qquad \textbf{Eqn 2}$$

The values of the constants in the Shomate Equation for hydrogen are obtained from the **NIST WebBook**:

T (K)	298 - 1500
A	33.1078
B	-11.508
C	11.6093
D	-2.8444
E	-0.15967

Substituting **Eqn 2** into **Eqn 1** and integrating yields:

$$\Delta\tilde{S} = A \cdot Ln\frac{T_2}{T_1} + \left(\frac{B}{1000}\right)(T_2 - T_1) + \left(\frac{(1/2)C}{1000^2}\right)(T_2^2 - T_1^2) + \left(\frac{(1/3)D}{1000^3}\right)(T_2^3 - T_1^3) - \left(\frac{(1/2)E}{1000^{-2}}\right)(T_2^{-2} - T_2^{-2}) - R \cdot Ln\frac{P_2}{P_1} \qquad \textbf{Eqn 3}$$

We will need the value of the **Universal Gas Constant** and the **molecular weight** to determine the change in the **specific entropy**.

R 8.314 **J/mole-K** **MW** 2.016 **g/mol**

Now, we can substitute values into **Eqn 3** to complete **part (a)** :

$$\int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} = \quad 41.564 \ \text{J/mole-K}$$

$$R \cdot Ln\frac{P_2}{P_1} = \quad 9.6921 \ \text{J/mole-K}$$

Now, we can plug values into **Eqn 3** :

ΔS **31.872** **J/mole-K**

ΔS **15.810** **kJ/kg-K**

Part b.) In this part of the problem, we use the equation given in the problem statement:

$$\Delta\tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \tilde{S}_{T2}^o - \tilde{S}_{T1}^o - R\,Ln\frac{P_2}{P_1} \qquad \textbf{Eqn 4}$$

Properties are determined from Ideal Gas Entropy Tables:

At T_1: S^o_{T1} **1.0139** **kJ/kg-K**

At T_2: S^o_{T2} **21.631** **kJ/kg-K**

Now, we can plug values into **Eqn 4** :

ΔS **15.809** **kJ/kg-K**

Part c.) Once again, we will use the equation given in the problem statement:

$$\Delta\tilde{S} = \tilde{S}_2 - \tilde{S}_1 = \tilde{C}_P^o \, \mathrm{Ln}\frac{T_2}{T_1} - R\,\mathrm{Ln}\frac{P_2}{P_1} \qquad \text{Eqn 5}$$

Heat capacity is determined from **NIST WebBook**: At **810 K**: C^o_P **29.679** **J/(mol K)**

$$\int_{T_1}^{T_2} \tilde{C}_P^o \frac{dT}{T} = \tilde{C}_P^o \cdot \mathrm{Ln}\left(\frac{T_2}{T_1}\right) =$$ **41.604** **J/mole-K**

Now, we can plug values into **Eqn 5** :

ΔS	**31.912**	**J/mole-K**
ΔS	**15.830**	**kJ/kg-K**

Verify: The ideal gas assumption needs to be verified.

We need to determine the **specific volume** at each **state** and check **if** : $\tilde{V} > 5\,L/mol$

(hydrogen is a diatomic gas).

Solving the Ideal Gas EOS for **molar volume** yields : $$\tilde{V} = \frac{RT}{P}$$

Conversion Factors:

1 L =	**0.001**	**m^3**
1 bar =	**100000**	**N/m^2**
1 J =	**1**	**N-m**

Plugging in values gives us :

V_1	**5.54**	**L/mol**
V_2	**7.02**	**L/mol**

The **specific volume** at each **state** is greater than **5 L/mol** and therefore the ideal gas assumption is reasonable.

Answers :

a.)	**ΔS**	**15.81**	**kJ/kg-K**
b.)	**ΔS**	**15.81**	**kJ/kg-K**
c.)	**ΔS**	**15.83**	**kJ/kg-K**

Comparison:

The results in parts (a) and (b) are identical. This is not a surprise, assuming we integrated the Shomate Equation correctly !

The error in ΔS associated with the ideal gas assumption in this problem is: 0.13%

We expected the error to be less than 1% since the molar volumes are greater than 5 L/mole.

7D-3 Work, Efficiency and the T-S Diagram for an Ideal Gas Power Cycle 8 pts

Air contained in a piston-and-cylinder device undergoes a power cycle made up of **three** internally reversible processes.

Step 1-2: Adiabatic compression from **20 psia** and **570°R** to **125 psia**
Step 2-3: Isothermal expansion to **20 psia**
Step 3-1: Isobaric compression

a.) Sketch the process path for this power cycle on both **PV** and **TS diagrams**
b.) Calculate T_3 in **°F**
c.) Calculate the boundary work in **Btu/lb$_m$**
d.) Calculate the **thermal efficiency** of the power cycle

Read : For **part (a)** sketch the cycle first to get a better understanding of the processes.
For **part (b)** recall that the **Process 2-3** is isothermal and therefore $T_3 = T_2$. Determine $S^o(T_2)$ and look it up in the Ideal Gas Entropy Table for air to determine T_2.
For **part (c)** determine the net **work** by determining the **work** for each process and then adding them together.
For **part (d)** determine the **thermal efficiency** as the ratio of the net **work** to the **heat** going into the system.

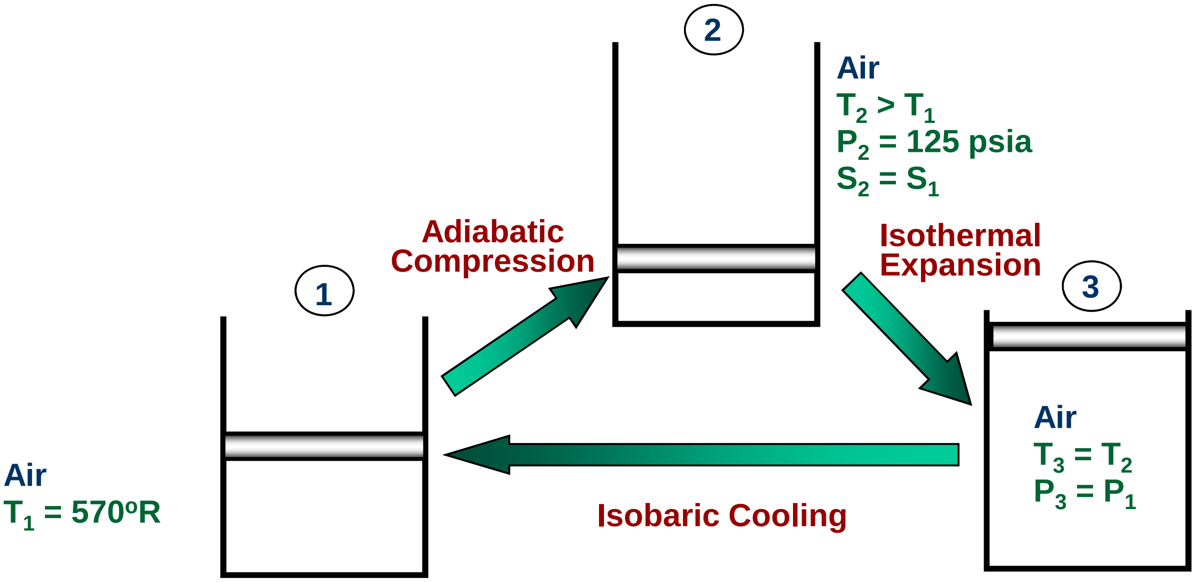
Given:

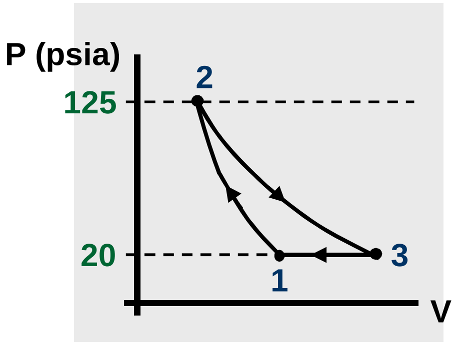
T_1	570	°R	P_2	125	psia
P_1	20	psia	P_3	20	psia

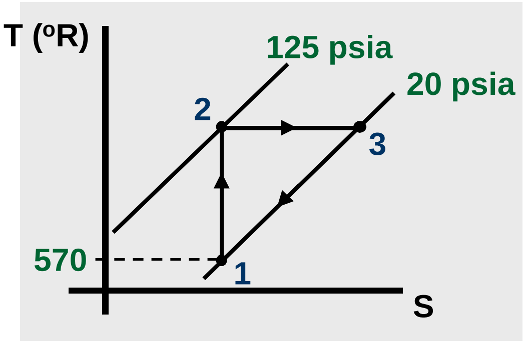
Find:

Part (a)	Sketch PV and TS diagrams			Part (c)	W_{cycle}	?	Btu/lb$_m$
Part (b)	T_3	?	°R	Part (d)	η	?	

Diagram:







Assumptions:

1 - The system is the air inside the cylinder.
2 - The air is modeled as an ideal gas.
3 - Each process is internally reversible.
4 - **Boundary work** is the only form of **work** that crosses the system boundary.
5 - There is no change in kinetic or potential energy for either of the two processes.

Equations / Data / Solve:

Part b.) Since **Process 2-3** is isothermal:

$$T_3 = T_2 \qquad \text{Eqn 1}$$

So we should work on determining T_2.

Let's apply the 2nd Gibbs Equation for ideal gases to **Process 1-2** :

$$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\cdot \text{Ln}\frac{P_2}{P_1} \qquad \text{Eqn 2}$$

We can determine T_2 and thus T_3 from:

$$\hat{S}^o(T_2) = \hat{S}^o(T_1) + \frac{R}{MW}\cdot \text{Ln}\left(\frac{P_2}{P_1}\right) \qquad \text{Eqn 3}$$

Lookup $S^o(T_1)$ in the Ideal Gas Entropy Tables: $S^o(T_1)$ 0.014381 Btu/lb$_m$-°R

Now, plug values into **Eqn 3** to determine $S^o(T_2)$:

R	1.987	Btu/lbmol-°R
MW	28.97	lb$_m$/lbmol

$S^o(T_2)$ 0.14007 Btu/lb$_m$-°R

Now that we know the value of S^o at T_2, we can interpolate on the air Ideal Gas Property Table to determine T_2.

T (°R)	S^o (Btu/lb$_m$-°R)
950	0.13939
T_2	0.14007
960	0.14202

Interpolation yields : $T_2 = T_3 =$ 952.60 °R
492.93 °F

Part c.) The net **work** is the sum of the **work** done during each process:

$$\hat{W}_{cycle} = \hat{W}_{12} + \hat{W}_{23} + \hat{W}_{31} \qquad \text{Eqn 4}$$

We need to determine the **work** involved in each process. Begin with **Process 1-2**.

Since **Process 1-2** is adiabatic, and changes in internal and kinetic energies are negligible, the appropriate form of the 1st Law is :

$$\cancel{\hat{Q}_{12}} - \hat{W}_{12} = \Delta\hat{U} + \cancel{\Delta\hat{E}_{kin}} + \cancel{\Delta\hat{E}_{pot}} \qquad \text{Eqn 5}$$

$$\hat{W}_{12} = -\left(\hat{U}_2 - \hat{U}_1\right) \qquad \text{Eqn 6}$$

Because we know both T_1 and T_2, we can look up the **U**'s in the Ideal Gas Property Table:

At T_1 = 570 °R, no interpolation is required: U_1 5.6705 Btu/lb$_m$

T (^oR)	U^o (Btu/lb$_m$)
950	72.806
952.60	U_2
960	74.631

Interpolation yields : U_2 73.281 Btu/lb$_m$

Now, we can plug values back into **Eqn 6** to determine W_{12} : W_{12} -67.610 Btu / lb$_m$

Boundary work done by the system during **Process 2-3** can be calculated from the definition of **boundary work** :

$$\hat{W}_{23} = \int_2^3 P\,dV \qquad \text{Eqn 7}$$

For an ideal gas substitute **P = nRT/V** into **Eqn 7** :

$$\hat{W}_{23} = \int_2^3 \frac{R}{MW}\frac{T}{V}dV \qquad \text{Eqn 8}$$

Integrate **Eqn 2**
(the process is isothermal, $T_2 = T_3$) :

$$\hat{W}_{23} = \frac{R}{MW}T_2\,Ln\frac{V_3}{V_2} \qquad \text{Eqn 9}$$

Since $P_2V_2 = nRT_2$ and $P_3V_3 = nRT_3$ and
$T_2 = T_3$, we conclude that $P_2\,V_2 = P_3\,V_3$, or :

$$\frac{V_3}{V_2} = \frac{P_2}{P_3} \qquad \text{Eqn 10}$$

Combining **Eqn 10** and **Eqn 9** yields :

$$\hat{W}_{23} = \frac{R}{MW}T_2\,Ln\frac{P_2}{P_3} \qquad \text{Eqn 11}$$

We can plug numbers into **Eqn 11** to evaluate W_{23} : W_{23} 119.74 Btu / lb$_m$

Boundary work done by the system during **Process 3-1** can be calculated from the definition of **boundary work** :

$$\hat{W}_{31} = \int_2^3 P\,d\hat{V} \qquad \text{Eqn 12}$$

Since **Process 3-1** is isobaric
(**P=constant**), **Eqn 12** simplifies to :

$$\hat{W}_{31} = P\left(\hat{V}_1 - \hat{V}_3\right) \qquad \text{Eqn 13}$$

Since :

$$P_1\,\hat{V}_1 = \frac{R\,T_1}{MW} \qquad \text{Eqn 14}$$

$$P_3\,\hat{V}_3 = \frac{R\,T_3}{MW} \qquad \text{Eqn 15}$$

And since $P_1 = P_3$, **Eqns 13**, **14** & **15** can be combined to obtain :

$$\hat{W}_{31} = \frac{R}{MW}\left(T_1 - T_3\right) \qquad \text{Eqn 16}$$

Now, we can plug values into **Eqn 16** to evaluate W_{31} : W_{31} -26.24 Btu / lb$_m$

Now, we can calculate W_{cycle} from the sum of the **work** terms for each step, using **Eqn 4** :

W_{cycle} 25.88 Btu / lb$_m$

Part d.) The **thermal efficiency** of the cycle is defined by :

$$\eta = \frac{\hat{W}_{cycle}}{\hat{Q}_{in}}$$ Eqn 17

We know $\mathbf{W_{cycle}}$, so we need to determine $\mathbf{Q_{in}}$. We also know that $\mathbf{Q_{12} = 0}$ (adiabatic process) and from the **TS Diagram** it can be concluded that $\mathbf{Q_{23} > 0}$ and $\mathbf{Q_{31} < 0}$. Therefore, $\mathbf{Q_{in} = Q_{23}}$.

$$\eta = \frac{\hat{W}_{cycle}}{\hat{Q}_{23}}$$ Eqn 18

Now we need to determine the **heat transferred** into the cycle during **Process 2-3**. Start from the definition of **entropy** :

$$\delta \hat{Q}_{int\,rev} = T\,d\hat{S}$$ Eqn 19

Because **Process 2-3** is internally reversible, we can integrate **Eqn 19** to get:

$$\hat{Q}_{23} = \int_2^3 T\,d\hat{S}$$ Eqn 20

Now, because **Process 2-3** is isothermal, the **T** pops out of the integral and **Eqn 20** is easy to integrate:

$$\hat{Q}_{23} = T(\hat{S}_3 - \hat{S}_2)$$ Eqn 21

Now, we can again apply the 2nd Gibbs Equation for ideal gases to **Process 2-3** to evaluate Δ**S**:

$$\Delta\hat{S} = \hat{S}_3 - \hat{S}_2 = \hat{S}^o_{T3} - \hat{S}^o_{T2} - \frac{R}{MW}\cdot \mathrm{Ln}\frac{P_3}{P_2}$$ Eqn 22

Since the process is isothermal: $\mathbf{S^o(T_2) = S^o(T_3)}$
and Eqn 22 simplifies to:

$$\Delta\hat{S}_{23} = -\frac{R}{MW}\cdot \mathrm{Ln}\frac{P_3}{P_2}$$ Eqn 23

When we substitute **Eqn 23** into **Eqn 21** we get :

$$\hat{Q}_{23} = T_2\left(-\frac{R}{MW}\cdot \mathrm{Ln}\frac{P_3}{P_2}\right)$$ Eqn 24

ΔS_{23} **0.12569** $\mathbf{Btu/lb_m\text{-}^oR}$ Q_{23} **119.74** $\mathbf{Btu / lb_m}$

Finally, we plug values back into **Eqn 18** to evaluate the **thermal efficiency** of the cycle :

η **21.62%**

Verify: The ideal gas assumption needs to be verified.
We need to determine the **specific volume** at each **state** and check **if**:

$$\tilde{V} > 80\ \ \mathrm{ft^3 / lbmol}$$

Air can be considered a diatomic gas.

Solving the Ideal Gas EOS for **molar volume** yields :

$$\tilde{V} = \frac{RT}{P}$$

Use :			
	R	10.7316	$\mathbf{psia\text{-}ft^3 / lbmol\text{-}^oR}$
	V_1	305.85	$\mathbf{ft^3/lbmol}$
	V_2	81.78	$\mathbf{ft^3/lbmol}$
	V_3	511.15	$\mathbf{ft^3/lbmol}$

The **specific volume** is greater than **80** $\mathbf{ft^3/lbmol}$ for all **states** so the ideal gas assumption is valid.

Answers :						
a.)	**See diagrams above.**			**c.)**	$\mathbf{W_{cycle}}$	**25.9** $\mathbf{Btu/lb_m}$
b.)	$\mathbf{T_3}$	**953**	$\mathbf{^oR}$	**d.)**	η	**21.6%**

7D-4 ΔS and the T-S Diagram for Ideal Gas Processes 8 pts

An ideal gas is contained in a piston-and-cylinder device in which the system moves from **state 1** to **state 2**.

a.) If T_2 is greater than T_1, show that the ΔS_{12} is **greater** if the process is isobaric than if it is isochoric. Sketch the isobaric and isochoric process paths on **PV** and **TS diagrams**.

b.) Use your **TS Diagram** from **part (a)** to show that an isochoric path passing through a state has a **greater slope** than an isobaric path passing through the same state.

c.) If P_2 is greater than P_1, show that the **ratio** of ΔS_{12} for an isothermal process to ΔS_{12} for an isochoric process is $(1 - \gamma)$. Sketch the isothermal and isochoric proces paths on **PV** and **TS diagrams**.

Read : Sketch the process in **parts (a)**, **(b)** and **(c)** first to get a better understanding of the processes.
For **part (a)** use equations relating **entropy** to C_p and C_V.
For **part (b)** recall that the slope on a **TS Diagram** is **(dT/dS)**.
For **part (c)** determine **ΔS** for each process and determine the **ratio**.

Given: A closed system consisting of an ideal gas with constant **specific heat ratio** γ.

Find:

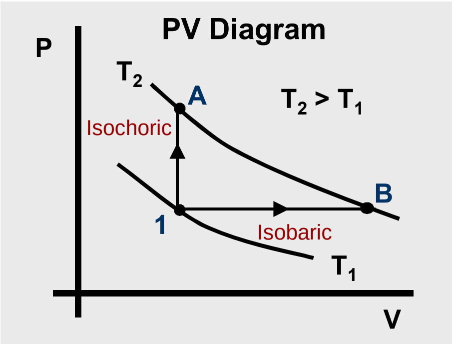
Part (a) For the process where the **T** increases from T_1 to T_2: show that **ΔS** is greater if the change in state occurs at constant **P** than if it occurs at constant **V**.
Sketch **PV** and **TS Diagrams** for the process.

Part (b) Show on a **TS Diagram** that a line of constant **specific volume** passing through a state has a greater slope than a line of constant **P**.

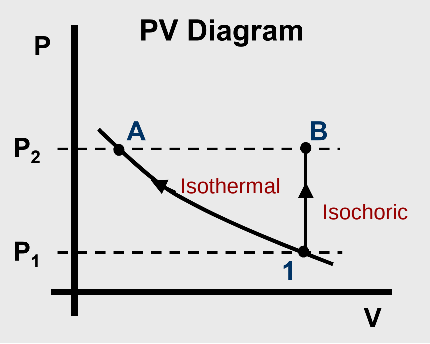
Part (c) For the process where the **P** increases from P_1 to P_2: show that the ratio of **ΔS** for an isothermal process to **ΔS** for a constant **specific volume** process is $(1 - \gamma)$.
Sketch **PV** and **TS Diagrams** for the process.

Diagram:

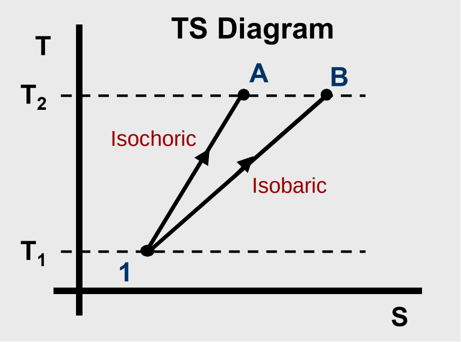
Part (a) and **(b):**



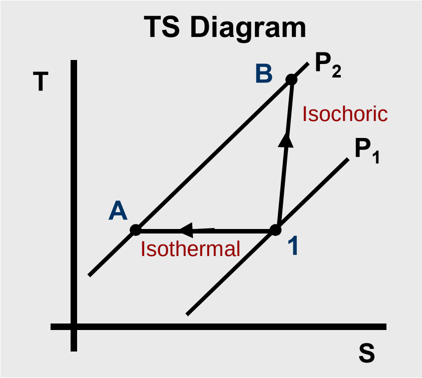
Part (c):



Part (a) and **(b):**



Part (c):



Assumptions: **1 -** The system consists of an ideal gas with constant **specific heats**.

Equations / Data / Solve:

Part a.) There are two key equations for calculating the **entropy change** of an ideal gas.

$$\Delta \widetilde{S} = \int \widetilde{C}_V^{o} \frac{dT}{T} + R \ln\left(\frac{\widetilde{V}_2}{\widetilde{V}_1}\right) \qquad \text{Eqn 1}$$

$$\Delta \widetilde{S} = \int \widetilde{C}_P^{o} \frac{dT}{T} - R \ln\left(\frac{P_2}{P_1}\right) \qquad \text{Eqn 2}$$

For **Process 1-A**, **specific volume** is **constant**. For **Process 1-B**, **pressure** is constant.
We can apply **Eqn 1** to **Process 1-A** and **Eqn 2** to **Process 1-B**.

$$\Delta \widetilde{S} = \int_{T_1}^{T_A} \widetilde{C}_V^{o} \frac{dT}{T} \qquad \text{Eqn 3}$$

$$\Delta \widetilde{S} = \int_{T_1}^{T_B} \widetilde{C}_P^{o} \frac{dT}{T} \qquad \text{Eqn 4}$$

Because the **specific heats** are constant, **Eqns 3 & 4** can be integrated to obtain :

$$S_A - S_1 = \widetilde{C}_V^{o} \, Ln \frac{T_2}{T_1} \qquad \text{Eqn 5}$$

$$S_B - S_1 = \widetilde{C}_P^{o} \, Ln \frac{T_2}{T_1} \qquad \text{Eqn 6}$$

Notice that both the intial and final **temperatures** are the same: $T_A = T_B = T_2$.

Next, we can take the ratio of **Eqn 2** to **Eqn 1** :

$$\frac{S_B - S_1}{S_A - S_1} = \frac{\widetilde{C}_P^{o} \cancel{Ln(T_2 / T_1)}}{\widetilde{C}_V^{o} \cancel{Ln(T_2 / T_1)}} \qquad \text{Eqn 7}$$

Cancelling terms leaves us with :

$$\frac{S_B - S_1}{S_A - S_1} = \frac{\widetilde{C}_P^{o}}{\widetilde{C}_V^{o}} \qquad \text{Eqn 8}$$

For ideal gases :

$$\widetilde{C}_P^{o} = \widetilde{C}_V^{o} + R \qquad \text{Eqn 9}$$

Use **Eqn 9** to eliminate C_P from **Eqn 8** :

$$\frac{S_B - S_1}{S_A - S_1} = \frac{\widetilde{C}_V^{o} + R}{\widetilde{C}_V^{o}} = 1 + \frac{R}{\widetilde{C}_V^{o}} \qquad \text{Eqn 10}$$

Because **R** and C_V are both positive numbers, we can conclude that :

$$(S_B - S_1) > (S_A - S_1) \qquad \text{Eqn 11}$$

Part b.) Here, we compare, at **state 1** **(dT/dS)$_V$** to **(dT/dS)$_P$**.

Since **(dT/dS)** at fixed **V** (or fixed **P**) is :

$$\frac{dT}{dS} = \lim_{\Delta S \to 0} \frac{\Delta T}{\Delta S} \qquad \textbf{Eqn 12}$$

In **part (a)**, we showed that, for the same ΔT, **ΔS** at constant **P** is greater than **ΔS** at constant **V**.

Consequently :

$$\boxed{\left(\frac{\partial T}{\partial S}\right)_V > \left(\frac{\partial T}{\partial S}\right)_P} \qquad \textbf{Eqn 13}$$

On a **TS Diagram**, a constant **specific volume** line passing through **State 1** has a greater slope than a constant **pressure** line passing through the same **state**.

Part c.) For **Process 1-A**, **temperature** is constant. For **Process 1-B**, **volume** is constant.
Apply **Eqn 2** to **Process 1-A** and **Eqn 1** to **Process 1-B**.

$$\tilde{S}_A - \tilde{S}_1 = -R\ln\left(\frac{P_2}{P_1}\right) \qquad \textbf{Eqn 14}$$

$$\tilde{S}_B - \tilde{S}_1 = \int_{T_1}^{T_B} \tilde{C}_V^o \frac{dT}{T} = \tilde{C}_V^o \, Ln\left[\frac{T_B}{T_1}\right] \qquad \textbf{Eqn 15}$$

We need to consider the ratio of **Eqn 14** to Eqn 15 and compare its value to **1** to determine which is greater, **ΔS$_{1-A}$** or **ΔS$_{1-B}$**.

$$\frac{\tilde{S}_A - \tilde{S}_1}{\tilde{S}_B - \tilde{S}_1} = \frac{-R\,Ln\left(\frac{P_2}{P_1}\right)}{\tilde{C}_V^o \, Ln\left(\frac{T_B}{T_1}\right)} \qquad \textbf{Eqn 16}$$

But, for ideal gases undergoing a constant **volume** such as **Process 1-B** :

$$P_1 V = nRT_1 \qquad \textbf{Eqn 17}$$

and :

$$P_2 V = nRT_B \qquad \textbf{Eqn 18}$$

Therefore :

$$\frac{P_2}{P_2} = \frac{T_B}{T_1} \qquad \textbf{Eqn 19}$$

Now, we can use **Eqns 9** & **19** to simplify **Eqn 16** :

$$\frac{\tilde{S}_A - \tilde{S}_1}{\tilde{S}_B - \tilde{S}_1} = \frac{-\left(\tilde{C}_P^o - \tilde{C}_V^o\right)}{\tilde{C}_V^o} = 1 - \frac{\tilde{C}_V^o}{\tilde{C}_V^o} = 1 - \gamma \qquad \textbf{Eqn 20}$$

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers : **Part a.)** $\boxed{(S_B - S_1) > (S_A - S_1)}$

Part b.)

$$\boxed{\left(\frac{\partial T}{\partial S}\right)_V > \left(\frac{\partial T}{\partial S}\right)_P}$$

Part c.)

$$\boxed{\frac{\tilde{S}_A - \tilde{S}_1}{\tilde{S}_B - \tilde{S}_1} = 1 - \gamma}$$

7D-5 Compressed-Air-Driven Turbine 8 pts

A small cylinder of compressed **air** stores energy, just like a battery. When you want to recover the energy from the compressed **air**, release the air through a turbine and vent the **air** to the surroundings.

Use the shaft work to generate electricity.

Consider a cylinder that contains **air** at **400 psia** and **$1000^{o}F$**. When the **air** in the cylinder flows out through the turbine, it produces **250 Btu** of shaft work by the time the **pressure**

in the cylinder reaches **75 psia**. The turbine exhausts to ambient pressure, **14.7 psia**. Determine the **volume** of the cylinder in **ft^3**.

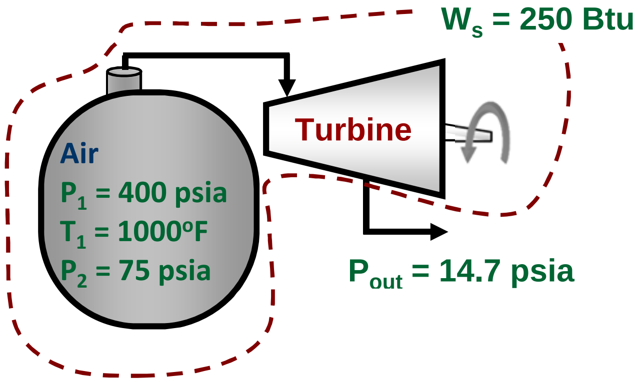
Assume the **air** behaves as an ideal gas, the turbine and the cylinder are internally reversible, the entire process is adiabatic and changes in kinetic and potential energies are negligible.

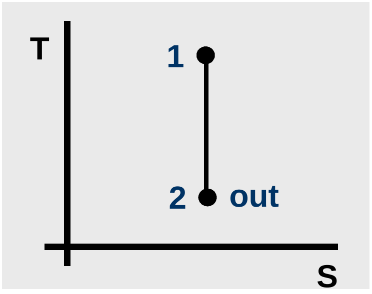
Read : The key to this process is that it is entirely isentropic. This will let us determine the initial and final properties of the air in the tank, as well as the properties of the turbine exhaust. The best part is that the properties of the turbine exhaust do not change during the process.

Given:	P_1	400	psia	P_2	75	psia
	T_1	1000	^{o}F	P_{out}	14.7	psia
	W_s	250	Btu			

Find: V ? ft^3

Diagram:





Assumptions:

1 - The system is shown in the diagram.
2 - For the system, heat exchange with the surroundings is negligible.
3 - Changes in kinetic and potential energies are negligible.
4 - The process is reversible.
5 - The air behaves as an ideal gas. This is a very questionable assumption at these **pressures**, but the problem statement instructed us to make it !

Equations / Data / Solve:

We want to evaluate the **volume** of the tank in the absence of irreversibilities.

We can begin by applying the 1st Law to this system.

$$\Delta U = \cancel{Q} - W_s + \cancel{n}_{in}\,\tilde{H}_{in} - n_{out}\,\tilde{H}_{out} \qquad \textbf{Eqn 1}$$

We can simplify **Eqn 1** because the process is adiabatic and we have assumed that changes in kinetic and potential energies are negligible and because there is no **mass** flow into the system.

$$\Delta U = -W_s - n_{out}\,\tilde{H}_{out} \qquad \textbf{Eqn 2}$$

The mass conservation equation for this process is :

$$-\Delta n = n_1 - n_2 = n_{out} \qquad \textbf{Eqn 3}$$

Combining **Eqns 1** & **2** yields :

$$W_s = -\Delta U - \Delta n \cdot \tilde{H}_{out} \qquad \textbf{Eqn 4}$$

Because the entire process is reversible and adiabatic, the process is **isentropic**. Therefore, S_{out} can be determined and does not change during the process. Because **two** intensive properties, S_{out} and P_{out}, are constant, we can conclude that the **state** of the turbine exhaust is constant and, therefore, T_{out} and H_{out} are constant as well. Therefore, **Eqn 4** becomes :

$$W_s = \tilde{U}_1 n_1 - \tilde{U}_2 n_2 + \tilde{H}_{out}\,(n_2 - n_1) \qquad \textbf{Eqn 5}$$

The initial and final **moles** of **air** in the tank can be determined from the ideal gas EOS:

$$PV = nRT \qquad \textbf{Eqn 6}$$

$$n = \frac{PV}{RT} \qquad \textbf{Eqn 7}$$

Apply **Eqn 7** to both the initial and final **states** of the tank contents and combine these with **Eqn 5** to get:

$$W_s = \tilde{U}_1\left(\frac{P_1 V}{R T_1}\right) - \tilde{U}_2\left(\frac{P_2 V}{R T_2}\right) + \tilde{H}_{out}\left[\left(\frac{P_2 V}{R T_2}\right) - \left(\frac{P_1 V}{R T_1}\right)\right] \qquad \textbf{Eqn 8}$$

Now, we can solve **Eqn 8** for the unknown **volume** of the tank :

$$V = \frac{W_s\ R}{\left[\tilde{U}_1 \dfrac{P_1}{T_1} - \tilde{U}_2 \dfrac{P_2}{T_2} + \tilde{H}_{out}\left[\dfrac{P_2}{T_2} - \dfrac{P_1}{T_1}\right]\right]} \qquad \textbf{Eqn 9}$$

An alternate way to express **Eqn 9**:

$$V_{tank} = \frac{W_S\,(R\,/\,MW)}{\dfrac{P_1}{T_1}\left(\hat{U}_1 - \hat{H}_{out}\right) - \dfrac{P_2}{T_2}\left(\hat{U}_2 - \hat{H}_{out}\right)} \qquad \textbf{Eqn 9a}$$

The air remaining in the tank undergoes an isentropic expansion from P_1, T_1 to P_2, T_2.

At this point, we can solve this problem by either of two methods. We can apply the 2nd Gibbs Equation for ideal gases and the Ideal Gas Entropy Function or we can use the Ideal Gas Relative Pressure, P_r.

Method 1: Use the Ideal Gas Entropy Function.

The 2nd Gibbs Equation for ideal gases in terms of the Ideal Gas Entropy Function is :

$$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\,Ln\frac{P_2}{P_1} = 0 \qquad \text{Eqn 10}$$

We can apply **Eqn 10** to the process that the air inside the tank undergoes AND to the process that the air undergoes as it passes through the turbine:

$$\Delta\hat{S} = \hat{S}_{out} - \hat{S}_1 = \hat{S}^o_{Tout} - \hat{S}^o_{T1} - \frac{R}{MW}\,Ln\frac{P_{out}}{P_1} = 0 \qquad \text{Eqn 11}$$

We can solve **Eqns 10** & **11** for the unknowns S^o_{T2} and S^o_{Tout} :

$$\hat{S}^o_{T2} = \hat{S}^o_{T1} + \frac{R}{MW}\,Ln\frac{P_2}{P_1} \qquad \text{Eqn 12}$$

$$\hat{S}^o_{Tout} = \hat{S}^o_{T1} + \frac{R}{MW}\,Ln\frac{P_{out}}{P_1} \qquad \text{Eqn 13}$$

We can look up S^o_{T1} in the Ideal Gas Property Tables and use it with the known pressures in **Eqn 13** to determine S^o_{T2} and S^o_{Tout} :

T_1	1459.67	^oR	R	1.987	Btu/lbmol-^oR
			MW	28.97	lb_m/lbmol

T (^oR)	S^o (Btu/lb_m-^oR)
1450	0.24808
1459.67	S^o_{T1}
1500	0.25705

Interpolation yields :

S^o_{T1}	0.24981	Btu/lb_m-^oR
S^o_{T2}	0.13500	Btu/lb_m-^oR
S^o_{Tout}	0.02323	Btu/lb_m-^oR

Now, we can use S^o_{T2} and S^o_{Tout} and the Ideal Gas Property Tables to determine both T_2 and T_{out} and then U_1, U_2 and U_{out} by interpolation :

T (^oR)	U^o (Btu/lb_m)
1450	167.28
1459.67	U^o_1
1500	177.07

Interpolation yields :

U^o_1	169.17	Btu/lb_m

T (^oR)	U^o (Btu/lb_m)	S^o (Btu/lb_m-^oR)
930	69.166	0.13406
T_2	U^o_2	0.13500
940	70.985	0.13674

Interpolation yields :

T_2	933.51	^oR
U^o_2	69.804	Btu/lb_m

And at the turbine outlet :

T (^oR)	H^o (Btu/lb_m)	S^o (Btu/lb_m-^oR)
590	49.533	0.022643
T_{out}	H^o_{out}	0.02323
600	51.934	0.026678

T_{out}	591.44	^oR
H^o_{out}	49.880	Btu/lb_m

We can plug all of the given and determined values back into **Eqns 3**, **7**, **8** & **9** to evaluate $\mathbf{n_1}$, $\mathbf{n_2}$, $\mathbf{\Delta n}$, and finally, **V** :

R	10.7316	**psia-ft^3 / lbmol-^{o}R**	**V**	**2.979**	**ft^3**
$\mathbf{n_1}$ =	**0.07606**	**lbmol**	$\mathbf{m_1}$ =	**2.204**	**lb$_m$**
$\mathbf{n_2}$ =	**0.02230**	**lbmol**	$\mathbf{m_2}$ =	**0.646**	**lb$_m$**
$\mathbf{\Delta n}$ =	**-0.05376**	**lbmol**	$\mathbf{\Delta m}$ =	**-1.558**	**lb$_m$**

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an isentropic process :

$$\frac{P_r(T_2)}{P_r(T_1)} = \frac{P_2}{P_1} \qquad \textbf{Eqn 14}$$

Where $\mathbf{P_r}$ is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look up in the Ideal Gas Property Table for air.

We can solve **Eqn 14** For $\mathbf{P_r(T_2)}$, as follows :

$$P_r(T_2) = \frac{P_2}{P_1} P_r(T_1) \qquad \textbf{Eqn 15}$$

Look-up $\mathbf{P_r(T_1)}$ and use it in **Eqn 15** To determine $\mathbf{P_r(T_2)}$:

T (^{o}R)	$\mathbf{P_r}$
1450	**37.310**
1459.67	$\mathbf{P_r(T_1)}$
1500	**42.521**

$\mathbf{P_r(T_1)}$	**38.318**
$\mathbf{P_r(T_2)}$	**7.185**

Once we know $\mathbf{P_r(T_2)}$ we can determine $\mathbf{T_2}$ by interpolation on the the Ideal Gas Property Table.
We can then use $\mathbf{T_1}$ and $\mathbf{T_2}$ to determin $\mathbf{U_1}$ and $\mathbf{U_2}$ from the Ideal Gas Property Tables.

T (^{o}R)	**U^o (Btu/lb$_m$)**
1450	**167.28**
1459.67	$\mathbf{U^o_1}$
1500	**177.07**

T (^{o}R)	$\mathbf{P_r}$	**U^o (Btu/lb$_m$)**
930	**7.0696**	**69.166**
$\mathbf{T_2}$	**7.185**	$\mathbf{U^o_2}$
940	**7.3513**	**70.985**

Interpolation yields :

$\mathbf{U^o_1}$	**169.17**	**Btu/lb$_m$**

Interpolation yields :

$\mathbf{T_2}$	**934.08**	**^{o}R**
$\mathbf{U^o_2}$	**69.909**	**Btu/lb$_m$**

Because the turbine is also an isentropic process, we can determine the **relative pressure** of the turbine effluent:

$$\frac{P_r(T_{out})}{P_r(T_1)} = \frac{P_{out}}{P_1} \qquad \textbf{Eqn 16}$$

Rearranging: $$P_r(T_{out}) = \frac{P_{out}}{P_1} P_r(T_1) \qquad \textbf{Eqn 17}$$

$\mathbf{P_r(T_{out})}$	**1.4082**

Now, we can use $\mathbf{P_r(T_{out})}$ to determine $\mathbf{T_2}$ and then $\mathbf{H_{out}}$ using the Ideal Gas Property Tables :

T (^{o}R)	$\mathbf{P_r}$	**H^o (Btu/lb$_m$)**
590	**1.3914**	**49.533**
$\mathbf{T_{out}}$	**1.4082**	$\mathbf{H^o_{out}}$
600	**1.4758**	**51.934**

Interpolation yields :

$\mathbf{T_{out}}$	**591.99**	**^{o}R**
$\mathbf{H^o_{out}}$	**50.010**	**Btu/lb$_m$**

We can plug all of the given and determined values back into **Eqns 3**, **7**, **8** & **9** to evaluate **n_1**, **n_2**, **Δn**, and finally,**V** :

R	10.7316	**psia-ft^3 / lbmol-^oR**	**V**	**2.982**	**ft^3**
n_1 =	**0.07614**	**lbmol**	**m_1 =**	**2.206**	**lb_m**
n_2 =	**0.02231**	**lbmol**	**m_2 =**	**0.646**	**lb_m**
Δn =	**-0.05383**	**lbmol**	**Δm =**	**-1.560**	**lb_m**

Verify: The ideal gas assumption needs to be verified.

We need to determine the **specific volume** at each **state** and check **if**:

$$\tilde{V} > 80 \ \ ft^3 / lbmol$$

Air can be considered a diatomic gas.

Solving the Ideal Gas EOS for **molar volume** yields :

$$\tilde{V} = \frac{RT}{P}$$

Use : **R** 10.7316 **psia-ft^3 / lbmol-^oR**

V_1	**39.16**	**ft^3/lbmol**			
V_2	**133.57**	**ft^3/lbmol**	**V_3**	**431.78**	**ft^3/lbmol**

The **specific volume** at **state 2** and at the turbine effluent is greater than **80 ft^3/lbmol**. Air can be considered to be a diatomic gas, so the ideal gas assumption is valid here. The ideal gas assumption is not valid in **state 1** and this makes the solution somewhat questionable, but we were instructed to make the ideal gas assumption in the problem statement.

Answers :

		Method 1	Method 2	
The volume of the tank is:	**V**	**2.979**	**2.982**	**ft^3**

The difference between the **two** methods is caused by the following issues (ranked from most important to least important).

1 - Errors associated with linearly interpolating between values of a functions that are not really linear.

3 - Round-off error in the Ideal Gas Property Tables.

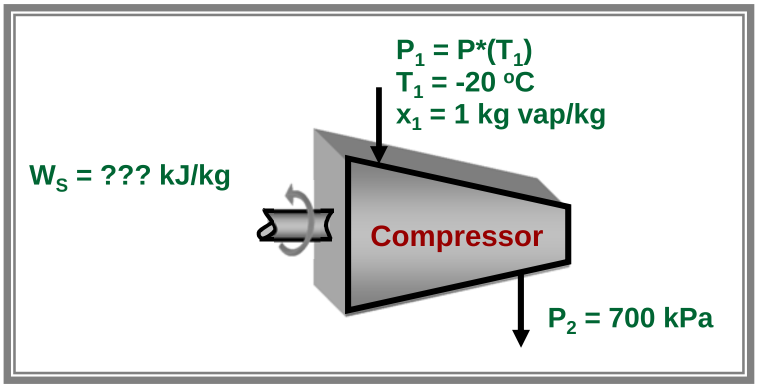
7E-1 Minimum Work for Compression of R-134a 4 pts

R-134a vapor enters an adiabatic compressor at **-20°C** and leaves at **700 kPa**. If the **R-134a** is saturated when it enters the compressor, determine the minimum **shaft work** required by the compressor in **kJ/kg**.

Read : We must apply the 1st Law to the compressor. We can get H_1 from the R-134a tables or the **NIST Webbook**, but we do not know H_2. The key to solving this problem is that a process that requires the minimum shaft work is an isentropic process. Knowing that $S_2 = S_1$ gives us the value of a 2nd intensive variable for **state 2**. This allows us to use the R-134a tables or **NIST Webbook** to detemine H_2. We can then plug H_2 into the 1st Law to determine the **work** requirement per **kg** of R-134a.

Given:				**Find:**			
	T_1	-20	°C		W_S	???	kJ/kg
	x_1	1	kg vap/kg				
	P_2	700	kPa				

Diagram:



Assumptions:

1 - The compressor is isentropic.
2 - The compressor operates at steady-state.
3 - Changes in kinetic and potential energies are negligible.
4 - **Shaft work** and **flow work** are the only types of **work** that cross the system boundary.

Equations / Data / Solve:

Apply the 1st Law to the compressor to determine the **shaft work** requirement.

For a steady-state, single-inlet, single outlet system with no **heat transfer** and negligible kinetic and potential energy changes, the 1st Law is:

$$\hat{W}_S = \frac{\dot{W}_S}{\dot{m}} = \left(\hat{H}_1 - \hat{H}_2\right) \qquad \text{Eqn 1}$$

We can get H_1 from the R-134a tables or the **NIST Webbook** because we know the **temperature** and we know it is a saturated vapor:

H_1 386.6 kJ/kg

The compressor is isentropic, so $S_2 = S_1$ and we can get S_1 from the R-134a tables or the **NIST Webbook**.

S_1 1.7413 kJ/kg-K

S_2 1.7413 kJ/kg-K

Now, we know the values of two intensive properties at **state 2**, so we can use the R-134a tables or the **NIST Webbook** to evaluate any other properties by interpolation. Here, we are interested in H_2.

At **P = 700 kPa**:

T (^{o}C)	H (kJ/kg)	S^o (kJ/kg-K)
30	416.60	1.7269
T_2	H_2	1.7413
40	426.72	1.7598

T_2 34.39 ^{o}C

H_2 421.0 kJ/kg

Now, we can plug values back into **Eqn 1** :

W_S -34.49 kJ/kg

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

W_S	-34.5	kJ/kg

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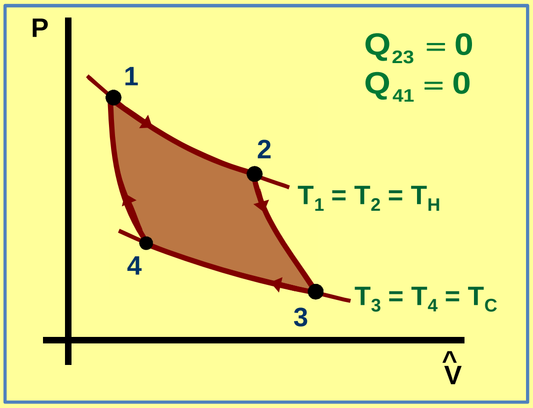
7E-2 PVT Relationships for Isentropic, IG Processes 8 pts

Consider the Carnot Power Cycle shown in the **PV Diagram**, below. The working fluid is **air** and the specific heat ratio, γ, is constant.



Show that…

a.) $V_2/V_1 = V_3/V_4$

b.) $(T_2\ /\ T_3)^{\gamma} = (P_2\ /\ P_3)^{(\gamma-1)}$

c.) $T_2\ /\ T_3 = (V_3\ /\ V_2)^{(\gamma-1)}$

Read : **(a)** Note that $\eta = W_{cycle}/Q_{in} = (W_{12} + W_{34})/Q_{12}$ because Q_{23} and Q_{41} are equal and have opposite signs. Determine W_{12} in terms of V_1, V_2, and T_H (**temperature** of high temp reservoir) and also W_{34} in terms of V_3, V_4, and T_C. Note the relationship between Q_{12} and W_{12} determined from an energy balance during **step 1-2**. Compare η obtained this way with the Carnot cycle efficicency and you will arrive at the desired conclusion.

(c) Easier to do **part (c)** before **(b)**. Apply the 1st Law to **step 2-3**. Note that $dU = m\ C_V\ dT$, $C_V = (R/MW)/(\gamma-1)$, $PV = nRT$ and $dW = P\ dV$. You will arrive at the form $(1/T)\ dT$ and $(1/V)\ dV$ on both sides. Integrate to obtain the desired result.

(b) Just use the result from **part (c)** along with the ideal gas EOS to convert **V** to **P**.

Given: Q_{23}	0	kJ	**Carnot Cycle**	$T_1 = T_2 = T_H$
Q_{41}	0	kJ		$T_3 = T_4 = T_C$

Step:	
1-2	Isothermal Expansion
2-3	Adiabatic Exapansion
3-4	Isothermal Compression
4-1	Adiabatic Compression

Find: **Show that:** **(a)** $V_4V_2 = V_1V_3$ **(b)** $T_2/T_3 = (P_2/P_3)^{((\gamma-1)/\gamma}$ **(c)** $T_2/T_3 = (V_3/V_2)^{\gamma-1}$

Diagram: Given in the problem statement.

Assumptions:

1 - The system consists of an ideal gas.
2 - The **specific heat ratio** is constant (required in **part (b)** only).
3 - The cycle is executed in a closed system (not required, but it makes the solution simpler).
4 - Changes in kinetic and potential energies are negligible.
5 - **Boundary work** is the only type of **work** that crosses the system boundary.
6 - The system undergoes a Carnot cycle (reversible).

Equations / Data / Solve:

Part a.) It may be hard to determine where to start with the proof but following the provided **hints** will help you.

Starting with the **thermal effiicency**:

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{12} + W_{34}}{Q_{12}}$$

Eqn 1

Now we need to determine W_{12} in terms of V_1, V_2, and T_H and also W_{34} in terms of V_3, V_4, and T_C.

PV work done by the system during the isothermal expansion and compression processes can be evaluated as follows:

$$W_{12} = \int_1^2 P\,dV$$

Eqn 2

Assuming the system consists of an ideal gas substitute **P = nRT/V**:

$$W_{12} = \int_1^2 \frac{nRT_H}{V}\,dV$$

Eqn 3

$$W_{34} = \int_3^4 \frac{nRT_C}{V}\,dV$$

Eqn 5

Integrating:

$$W_{12} = nRT_H\,Ln\left(V_2 / V_1\right)$$

Eqn 4

$$W_{34} = nRT_C\,Ln\left(V_4 / V_3\right)$$

Eqn 6

Apply the 1st Law for a closed system with negligible changes in kinetic and potential energies to get Q_{12}:

$$\Delta U_{12} = Q_{12} - W_{12}$$

Eqn 7

Since the **internal energy** of an ideal gas depends on **temperature** only and the **temperature** is constant along **Process 1-2**, $U_2 = U_1$ and the energy balance reduces to:

$$Q_{12} = W_{12}$$

Eqn 8

We already determined W_{12} in **Eqn 4**.

Substituting expressions for W_{12}, W_{23} and $Q_{12} = W_{12}$ into the **thermal efficiency** equation, **Eqn 1**, yields:

$$\eta = \frac{W_{12} + W_{34}}{Q_{12}} = \frac{W_{12} + W_{34}}{W_{12}}$$
$$= 1 + \frac{W_{34}}{W_{12}} = 1 + \frac{nRT_C\,Ln(V_4 / V_3)}{nRT_H\,Ln(V_2 / V_1)} = 1 + \left(\frac{Ln(V_4 / V_3)}{Ln(V_2 / V_1)}\right)\frac{T_C}{T_H}$$

Eqn 9

Recall that the **thermal efficiency** of a Carnot Cycle is:

$$\eta = 1 - \frac{T_C}{T_H}$$

Eqn 10

Substituting **Eqn 10** into **Eqn 9** yields:

$$1-\frac{T_C}{T_H}=1-\left(\frac{Ln(V_4\,/\,V_3)}{Ln(V_2\,/\,V_1)}\right)\frac{T_C}{T_H} \qquad \text{Eqn 11}$$

Eqn 11 simplifies to:

$$\frac{Ln(V_4\,/\,V_3)}{Ln(V_2\,/\,V_1)}=-1 \qquad \text{Eqn 12}$$

A little algebra finishes the job:

$$Ln(V_4\,/\,V_3)=Ln(V_1\,/\,V_2) \qquad \text{Eqn 13}$$

$$V_4\,/\,V_3=V_1\,/\,V_2 \qquad \text{Eqn 14}$$

$$\boxed{V_4\,V_2=V_3\,V_1} \qquad \text{Eqn 15}$$

Part c.) First we will apply the 1st Law to adiabatic **process 2-3** with no changes in kinetic or potential energy.

$$\Delta U_{23}=-W_{23} \qquad \text{Eqn 16}$$

Put **Eqn 16** into differential form:

$$dU_{23}=-\delta W_{23} \qquad \text{Eqn 17}$$

Substitute the definitions of **boundary work** and **heat capacity**:

$$dU_{23}=n\tilde{C}_V^o\,dT \qquad \text{Eqn 18}$$

$$\delta W_{23}=P\,dV \qquad \text{Eqn 19}$$

But, for an ideal gas:

$$\tilde{C}_V^o=\frac{R}{\gamma-1} \qquad \text{Eqn 20}$$

And the ideal gas EOS tells us that:

$$P=\frac{nRT}{V} \qquad \text{Eqn 21}$$

Now, plug **Eqns 18 - 20** back into **Eqn 17** to get :

$$n\frac{R}{\gamma-1}dT=-\frac{nRT}{V}dV \qquad \text{Eqn 22}$$

Rearrange **Eqn 22** to get :

$$\left(\frac{1}{\gamma-1}\right)\frac{1}{T}dT=-\frac{1}{V}dV \qquad \text{Eqn 23}$$

Integrate **Eqn 23** from **state 2** to **state 3** :

$$\left(\frac{1}{\gamma-1}\right)Ln\left(T_3\,/\,T_2\right)=-Ln\left(V_3\,/\,V_2\right) \qquad \text{Eqn 24}$$

Simplify algebraically :

$$Ln\left(T_3\,/\,T_2\right)=Ln\left(V_2\,/\,V_3\right)^{\gamma-1} \qquad \text{Eqn 25}$$

$$\boxed{\frac{T_3}{T_2}=\left(\frac{V_2}{V_3}\right)^{\gamma-1}} \qquad \text{Eqn 26}$$

Part b.) Substitute the ideal gas EOS in the form:

$$V = \frac{n\,R\,T}{P} \qquad \textbf{Eqn 27}$$

into the result from **part (c)**, **Eqn 26** to get:

$$\frac{T_3}{T_2} = \left(\frac{\cancel{n}\cancel{R}T_2 \,/\, P_2}{\cancel{n}\cancel{R}T_3 \,/\, P_3} \right)^{\gamma-1} \qquad \textbf{Eqn 28}$$

Cancelling terms in **Eqn 28** yields:

$$\frac{T_3}{T_2} = \left(\frac{T_2\,P_3}{T_3\,P_2} \right)^{\gamma-1} \qquad \textbf{Eqn 29}$$

Multiply through by $(T_3 \,/\, T_2)^{1-\gamma}$ to get :

$$\left(\frac{T_3}{T_2} \right)^{\gamma} = \left(\frac{P_3}{P_2} \right)^{\gamma-1} \qquad \textbf{Eqn 30}$$

A little more algebra yields :

$$\boxed{\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}}} \qquad \textbf{Eqn 31}$$

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers :

Part a.) $\boxed{V_4\,V_2 = V_3\,V_1}$

Part b.) $\boxed{\frac{T_3}{T_2} = \left(\frac{V_2}{V_3} \right)^{\gamma-1}}$

Part c.) $\boxed{\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}}}$

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7E-3 Work and ΔS for IGs Undergoing Isothermal, Polytropic and Adiabatic Processes 8 pts

Argon gas is compressed in a piston and cylinder device from **20 psia** and **55°F** to **120 psia**. The compression is internally reversible and the **helium** behaves as an ideal gas with a constant **heat capacity** of $C_P = (5/2)\,R$.

Determine the **work** in **Btu/lb$_m$** and **ΔS** in **Btu/lb$_m$-°R** and sketch the process path on both **PV** and **TS Diagrams** assuming the compression is…

a.) Polytropic with **$\delta = 1.5$**
b.) Adiabatic
c.) Isothermal

Read : For **part (a)** start with the equation for **PV work** for internally reversible, polytropic processes for ideal gases. When determining **ΔS** assume the **heat capacity** is constant. Since argon is monatomic use **$C_p = (5/2)\,R$**.

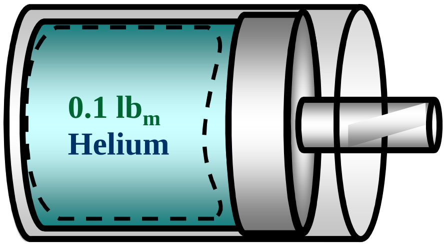
For **part (b)** determine **work** by applying an energy balance (where **Q = 0**). Assume constant **heat capacity** **$\Delta U = C_V\,\Delta T$** where **$C_V = C_P - R$** and **$C_p = (5/2)\,R$**. Determine **T_2** from an entropy balance. Recall that internally reversible, adiabatic processes are also isentropic.

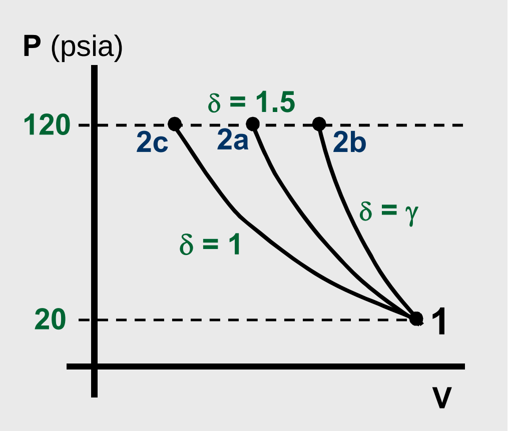
For **part (c)** start with the definition of **PV work**, substitute in the ideal gas EOS for **pressure** and integrate. Remember the process is isothermal (this simplifies the analysis of both the **work** and change in **entropy**).

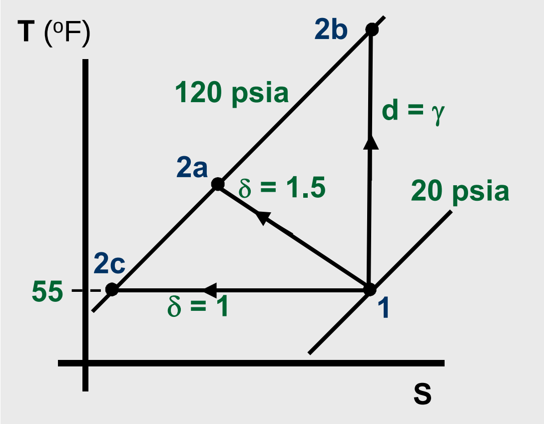
Given:	**P_1**	20	**lb$_f$ / in^2**		**P_2**	120	**lb$_f$ / in^2**
	T_1	55	**°F**		**m**	8.4	**lb$_m$**
		514.67	**°R**	**Part (a)**	δ	1.5	

Find:	**Part (a) - (c) :**	**W**	**?**	**Btu**	**ΔS**	**?**	**Btu/°R**

Diagram:







Assumptions:

1 - As shown in the diagram, the system is the gas.
2 - The gas is modeled as an ideal gas.
3 - The compression is internally reversible.
4 - **Boundary work** is the only form of **work** that crosses the system boundary.
5 - Changes in kinetic and potential energies are negligible.
6 - Argon has a constant **heat capacity** of **C_P = (5/2) R**.

R	**1.986**	**Btu/lbmol-°R**
MW	**39.948**	**lb_m/lbmol**

Equations / Data / Solve:

Part a.) In **part (a)** we must determine the **work** and the change in **entropy** for a polytropic process.

For polytopic, internally reversible processes with ideal gases:

$$W_{12} = \frac{m}{MW}\frac{R}{1-\delta}(T_2 - T_1) \qquad \text{Eqn 1}$$

The problem at this point is that we do not know **T_2**. But, we do know that the process is polytropic !

In Lesson 7E we learned that :

$$T_1 P_1^{(1-\delta)/\delta} = T_2 P_2^{(1-\delta)/\delta} \qquad \text{Eqn 2}$$

We can solve **Eqn 2** for **T_2** :

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\delta-1)/\delta} \qquad \text{Eqn 3}$$

Substitute **Eqn 3** for **T_2** into **Eqn 1** and rearrange the result to get :

$$W_{12} = \frac{m}{MW}\frac{RT_1}{1-\delta}\left(\left(\frac{P_2}{P_1}\right)^{(\delta-1)/\delta} - 1\right) \qquad \text{Eqn 4}$$

Now, we can plug values into **Eqns 12** & **10** or **Eqn 13** :

n	**0.2103**	**lb_m**
T_2	**935.22**	**°R**
T_2	**475.55**	**°F**
W_{12}	**-351.2**	**Btu**

ΔS can be determined by applying the 2nd Gibbs Equation for ideal gases:

$$\Delta S = \frac{m}{MW}\int_1^2 \tilde{C}_P \frac{dT}{T} - \frac{m}{MW} R \,Ln\left(\frac{P_2}{P_1}\right) \qquad \text{Eqn 5}$$

If we assume the **heat capacity** is constant:

$$\Delta S = \frac{m}{MW}\left[\tilde{C}_P \, Ln\frac{T_2}{T_1} - R\, Ln\frac{P_2}{P_1}\right] \qquad \text{Eqn 6}$$

Since argon is a montomic gas we can assume :

$$\tilde{C}_P^o = (5/2)R \qquad \text{Eqn 7}$$

C_P	4.965	Btu/lbmol-°R

Subtituting C_P and other values into **Eqn 6** yields :

ΔS	-0.12471	Btu/°R

Part b.) Here we must determine the **work** and the change in **entropy** for an adiabatic process.

The 1st Law for an adiabatic process with negligible changes in kinetic and potential energies is :

$$W_{12} = -m\,\Delta \hat{U}_{12} \qquad \text{Eqn 8}$$

We can evaluate the change in the **internal energy** using C_V and :

$$\Delta \hat{U}_{12} = \int_{T_1}^{T_2} \hat{C}_V^o \, dT \qquad \text{Eqn 9}$$

Assuming the **heat capacity** is constant, **Eqn 9** simplifies to :

$$\Delta \hat{U}_{12} = \hat{C}_V^o\,(T_2 - T_1) = \frac{\tilde{C}_V^o}{MW}\,(T_2 - T_1) \qquad \text{Eqn 10}$$

The following relationship applies to ideal gases :

$$\tilde{C}_P^o = \tilde{C}_V^o + R \qquad \text{Eqn 11}$$

We can combine this with **Eqn 7** to get :

$$\tilde{C}_P^o = (3/2)R \qquad \text{Eqn 12}$$

C_V	2.979	Btu/lbmol-°R

At this point, the only obstacle to using **Eqns 18** & **16** to evaluate W_{12} is that we do not know T_2.

We need to make use of the fact that the process is adiabatic and internally reversible to determine T_2.

An adiabatic process that is also internally reversible is isentropic :

ΔS	0	Btu/°R

We can use this fact with the 2nd Gibbs Equation for ideal gases with constant **heat capacities** to determine T_2 as follows :

$$\Delta S = \frac{m}{MW}\left[\tilde{C}_P \, Ln\frac{T_2}{T_1} - R\, Ln\frac{P_2}{P_1}\right] \qquad \text{Eqn 13}$$

Solve **Eqn 13** for T_2 using **ΔS = 0** :

$$0 = \frac{m}{MW}\left[\tilde{C}_P \, Ln\frac{T_2}{T_1} - R\, Ln\frac{P_2}{P_1}\right] \qquad \text{Eqn 14}$$

$$\tilde{C}_P \, Ln\frac{T_2}{T_1} = R\, Ln\frac{P_2}{P_1} \qquad \text{Eqn 15}$$

$$Ln\frac{T_2}{T_1} = Ln\left(\frac{P_2}{P_1}\right)^{R/\tilde{C}_P} = Ln\left(\frac{P_2}{P_1}\right)^{(\tilde{C}_P - \tilde{C}_V)/\tilde{C}_P} \qquad \text{Eqn 16}$$

Hey, we already KNEW this !

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \qquad \text{Eqn 17}$$

Where :
$$\gamma = \frac{\tilde{C}_P^o}{\tilde{C}_V^o} = \frac{(5/2)R}{(5/2)R} = \frac{5}{3}$$ **Eqn 18**

γ	**1.667**	
T_2	**1053.88**	**°R**
ΔU_{12}	**40.21**	**Btu/lb_m**
W_{12}	**-337.76**	**Btu**

Now, we can plug T_2 into **Eqn 10** and ΔU_{12} into **Eqn 8** :

Part c.) Determine the **work** from the definition of **boundary work** :
$$W_{12} = \int_1^2 P\,dV$$ **Eqn 19**

For an ideal gas substitute **P = nRT/V** into **Eqn 19** :
$$W_{12} = \int_1^2 \frac{m}{MW}\frac{RT}{V}dV$$ **Eqn 20**

Integrate **Eqn 2** (the process is isothermal, $T_1 = T_2 = T$) :
$$W_{12} = \frac{m}{MW} R\,T\,Ln\frac{V_2}{V_1}$$ **Eqn 21**

We don't know V_1 or V_2 but we can determine the values from the Ideal Gas EOS ($T_1 = T_2 = T$):

$$V_1 = \frac{m}{MW}\frac{RT}{P_1}$$ **Eqn 22**

$$V_2 = \frac{m}{MW}\frac{RT}{P_2}$$ **Eqn 23**

Dividing **Eqn 22** by **Eqn 23** we obtain :
$$V_2 / V_1 = P_1 / P_2$$ **Eqn 24**

Now, substitute **Eqn 24** back into **Eqn 21** to get :
$$W_{12} = \frac{m}{MW} RT\,Ln\frac{P_1}{P_2}$$ **Eqn 25**

Plug values into **Eqn 25** :

W_{12}	**-385.10**	**Btu**

Δ**S** can be determined by applying the 2nd Gibbs Equation for ideal gases:
$$\Delta S = \frac{m}{MW}\int_1^2 \tilde{C}_P \frac{dT}{T} - \frac{m}{MW} R\,Ln\left(\frac{P_2}{P_1}\right)$$ **Eqn 26**

For an isothermal process **Eqn 26** reduces to:
$$\Delta S = -\frac{m}{MW} R\,Ln\left(\frac{P_2}{P_1}\right)$$ **Eqn 27**

Now, we can plug values into **Eqn 27** :

ΔS	**-0.74824**	**Btu/°R**

Verify: The ideal gas assumption needs to be verified.

We need to determine the specific volume at each state and check if
$$\tilde{V} > 80 \ ft^3 / lbmol$$
(Argon is a noble gas).

Solving the Ideal Gas EOS for **molar volume** yields :
$$\tilde{V} = \frac{RT}{P}$$

Use : **R** 10.7316 **psia-ft^3 / lbmol-°R**

$V_1 = V_{2C}$	**276.16**	**ft^3/lbmol**
V_{2A}	**83.64**	**ft^3/lbmol**
V_{2B}	**94.25**	**ft^3/lbmol**

The **specific volume** at each state is greater than **80 ft^3/lbmol**, therefore the ideal gas assumption is reasonable.

Answers : **a.)**

W_{12}	-351	Btu
ΔS	-0.125	Btu/^{o}R

b.)

W_{12}	-338	Btu
ΔS	0	Btu/^{o}R

The isentropic process requires the least work !

c.)

W_{12}	-385	Btu
ΔS	-0.748	Btu/^{o}R

The isothermal process requires the most work !

How can **ΔS** be negative in **parts (a)** and **(c)** ?

Heat transfer from the system to the surroundings occurs. So, although **$\Delta S_{system} < 0$**, **$\Delta S_{surr} > 0$** by an even larger amount so that **$\Delta S_{universe} > 0$** and the 2nd Law is not violated.

7E-4 Performance of an Ideal Gas Cycle 10 pts

An ideal gas contained in a piston-and-cylinder device undergoes a thermodynamic cycle made up of **three** quasi-equilibrium processes.

Step 1-2: Adiabatic compression from **20°C** and **110 kPa** to **400 kPa**

Step 2-3: Isobaric cooling

Step 3-1: Isothermal expansion

a.) Carefully draw this process in a traditional piston-and-cylinder schematic

b.) Sketch the process path for this cycle on a **PV Diagram**.

Put a point on the diagram for each state and label it. Be sure to include and label **all** the important features for a complete **PV Δiagram** for this system

c.) Calculate **Q**, **W**, **ΔU** and **ΔH**, in **J/mole**,

for each step in the process and for the entire cycle. Assume that **C_P = (5/2) R**.

d.) Is this cycle a power cycle or a refrigeration cycle? Explain. Calculate the **thermal efficiency** or **COP** of the cycle, whichever is appropriate.

Read : Sketch carefully. Understanding what is going on in the problem is half the battle. Apply the 1st Law, the definitions of boundary **work**, **C_P** and **C_V** to a cycle on an ideal gas with constant **heat capacities**. Take advantage of the fact that **step 1-2** is <u>both</u> adiabatic and reversible, so it is isentropic. Power cycles produce a <u>net</u> amount of **work** and proceed in a <u>clock-wise</u> direction on a **PV Diagram**.

Given:

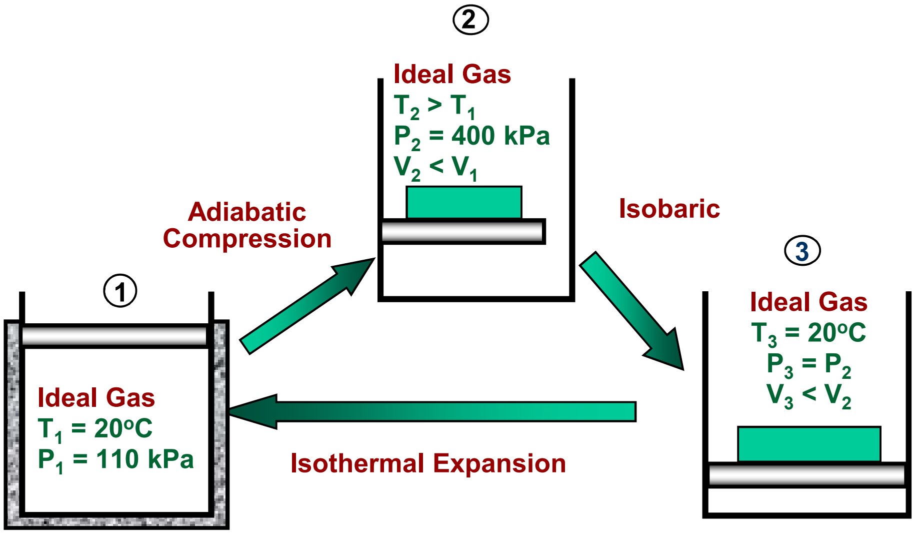
T_1	20	°C	P_1	110	kPa
	293.15	K	P_2	400	kPa
T_3	20	°C	P_3	400	kPa
	293.15	K	R	8.314	J/mole-K
Q_{12}	0	J/mole	C_P	20.785	J/mole-K

Find: For <u>each</u> of the three **steps** and for the <u>entire</u> cycle:

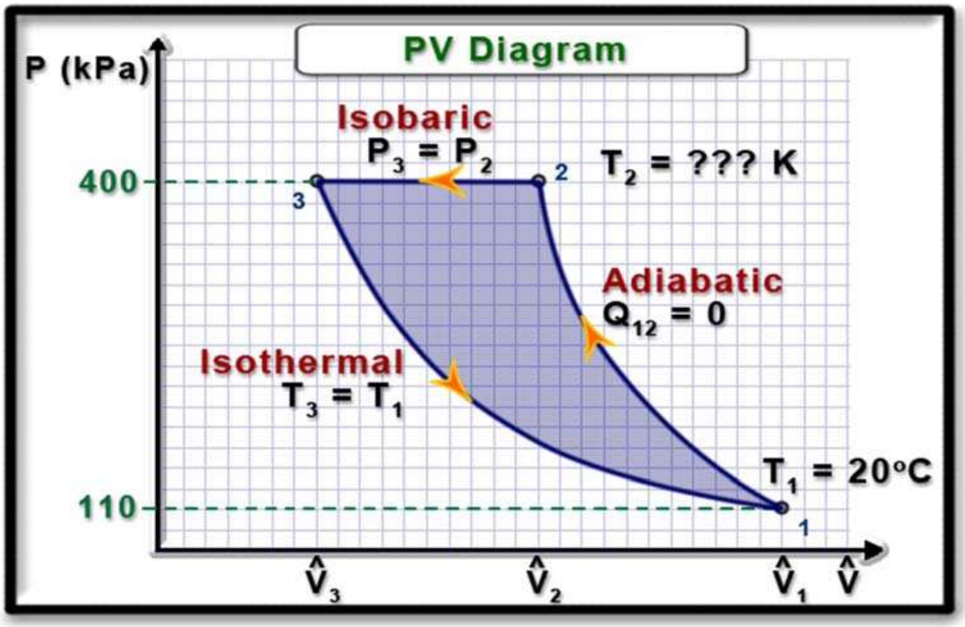
ΔU	???	J/mole
ΔH	???	J/mole
Q	???	J/mole
W	???	J/mole

Diagram:

Part a.)



Part b.)



Assumptions:

1 - **Step 1-2** is adaibatic, **Step 2-3** is isobaric, **Step 3-1** is isothermal.

2 - The entire cycle and all of the **steps** in the cycle are internally reversible.

3 - Changes in kinetic and potential energies are negligible.

4 - **Boundary work** is the only form of **work** interaction during the cycle.

5 - The **PVT** behavior of the system is accurately described by the ideal gas EOS.

Equations / Data / Solve:

Part c.) Let's begin by analyzing **step 1-2**, the adiabatic compression.

Begin by applying the 1st Law for closed systems to each **step** in the **Carnot Cycle**. Assume that changes in kinetic and potential energies are negligible.

$$\tilde{W}_{12} = -\Delta \tilde{U} = \tilde{U}_1 - \tilde{U}_2 \qquad \textbf{Eqn 1}$$

Because internal energy is not a function of **pressure** for an ideal gas, we can determine **ΔU** by integrating the equation which defines the **constant volume heat capacity**. The integration is simplified by the fact that the **heat capacity** for the gas in this problem has a constant value.

$$\tilde{C}_V^o = \left(\frac{d\tilde{U}}{dT}\right)_V \qquad \textbf{Eqn 2}$$

$$\tilde{U}_1 - \tilde{U}_2 = \int_{T_2}^{T_1} \tilde{C}_V^o \, dT = \tilde{C}_V^o \left(T_1 - T_2\right) \qquad \textbf{Eqn 3}$$

Combining **Eqns 1 & 3** yields:

$$\tilde{W}_{12} = \tilde{C}_V^o \left(T_1 - T_2\right) \qquad \textbf{Eqn 4}$$

The problem is that we do not know **T_2**. So, our next task is to determine **T_2**.

Since the entire cycle is reversible and this step is also adiabatic, this step is isentropic.

The fastest way to determine **T_2** is to use one of the **PVT** relationships for isentropic processes.

$$T_1 \, P_1^{\frac{1-\gamma}{\gamma}} = T_2 \, P_2^{\frac{1-\gamma}{\gamma}} \qquad \textbf{Eqn 5}$$

Solve **Eqn 5** for T_2 :

$$T_2 = T_1 \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} \qquad \textbf{Eqn 6}$$

Now, we need to evaluate γ :

$$\gamma = \frac{\tilde{C}_P^o}{\tilde{C}_V^o} \qquad \textbf{Eqn 7}$$

But for ideal gases :

$$\tilde{C}_P^o = \tilde{C}_V^o + R \qquad \textbf{Eqn 8}$$

Solving **Eqn 8** for **C_V** yields :

$$\tilde{C}_V^o = \tilde{C}_P^o - R \qquad \textbf{Eqn 9}$$

Plugging values into **Eqn 9** and then **Eqn 7** yields :

C_V	**12.471**	**J/mole-K**
γ	**1.667**	

Now, plug values into **Eqn 5** to get **T_2** and plug that into **Eqn 4** to get **W_{12}** :

Plugging values into **Eqn 1** yields :

T_2	**491.31**	**K**
W_{12}	**-2471.3**	**J/mole**
ΔU_{12}	**2471.3**	**J/mole**

Now, we can get Δ**H** from its definition :

$$\Delta \tilde{H} = \Delta \tilde{U} + \Delta\left(P \tilde{V}\right)$$

Eqn 10

But, the gas is an ideal gas:

$$P \tilde{V} = R T$$

Eqn 11

Combining **Eqns 10** & **11** gives us :

$$\Delta \tilde{H} = \Delta \tilde{U} + R\left(T_2 - T_1\right)$$

Eqn 12

Now, we can plug values into **Eqn 12** :

ΔH_{12}	**4118.8**	**J/mole**

Next, let's analyze **step 2-3**, isobaric cooling.

T_2	**491.31**	**K**	**P_2**	**400**	**kPa**
T_3	**293.15**	**K**	**P_3**	**400**	**kPa**

The appropriate form of the 1st Law is:

$$\tilde{Q}_{23} - \tilde{W}_{23} = \Delta \tilde{U}_{23}$$

Eqn 13

Because we assumed that **boundary work** is the <u>only</u> form of **work** that crosses the system boundary, we can determine **work** from its definition.

$$\tilde{W}_{23} = \int_2^3 P\, d\tilde{V}$$

Eqn 14

Isobaric process:

$$\tilde{W}_{23} = P\left(\tilde{V}_3 - \tilde{V}_2\right)$$

Eqn 15

Because the system contains and ideal gas:

$$\tilde{W}_{23} = R\left(T_3 - T_2\right)$$

Eqn 16

W_{23}	**-1647.5**	**J/mole**

Next we can calculate Δ**U** by applying **Eqn 3** to **step 2-3**:

$$\tilde{U}_3 - \tilde{U}_2 = \tilde{C}_V^{\circ}\left(T_3 - T_2\right)$$

Eqn 17

ΔU_{23}	**-2471.3**	**J/mole**

Now, solve **Eqn 13** to determine **Q_{23}** :

$$\tilde{Q}_{23} = \tilde{W}_{23} + \Delta \tilde{U}_{23}$$

Eqn 18

Q_{23}	**-4118.8**	**J/mole**

Now, we apply **Eqn 12** to **step 2-3** to determine Δ**H** :

$$\Delta \tilde{H} = \Delta \tilde{U} + R\left(T_3 - T_2\right) = \Delta \tilde{U} + \tilde{W}_{23} = \tilde{Q}_{23}$$

Eqn 19

ΔH_{23}	**-4118.8**	**J/mole**

Next, we analyze **step 3-1**, isothermal expansion.

For ideal gases, **U** and **H** are functions of **T** <u>only</u>. Therefore :

ΔU_{31}	**0.0**	**J/mole**
ΔH_{31}	**0.0**	**J/mole**

The appropriate form of the 1st Law is:

$$\tilde{Q}_{31} - \tilde{W}_{31} = \Delta\tilde{U}_{31}$$

Eqn 20

But since $\Delta U_{31} = 0$, **Eqn 20** becomes :

$$\tilde{Q}_{31} = \tilde{W}_{31}$$

Eqn 21

Again, because we assumed that boundary **work** is the <u>only</u> form of **work** that crosses the system boundary, we can determine **work** from its definition.

$$\tilde{W}_{31} = \int_3^1 P\,d\tilde{V}$$

Eqn 22

Ideal Gas EOS :

$$P\tilde{V} = R\,T$$

Eqn 23

Solve **Eqn 23** for **P** and substitute the result into **Eqn 22** to get :

$$P = \frac{R\,T}{\tilde{V}}$$

Eqn 24

$$\tilde{W}_{31} = \int_3^1 \frac{R\,T}{\tilde{V}}\,d\tilde{V}$$

Eqn 25

Integrating **Eqn 25** yields :

$$\tilde{W}_{31} = R\,T_1\,\mathrm{Ln}\left[\frac{\tilde{V}_1}{\tilde{V}_3}\right]$$

Eqn 26

We can use the Ideal Gas EOS to avoid calculating V_1 and V_3 as follows:

Apply **Eqn 23** to both **states 3** and **1** :

$$\frac{P_1\,\tilde{V}_1 = R\,T_1}{P_3\,\tilde{V}_3 = R\,T_3}$$

Eqn 27

Cancelling terms and rearranging leaves :

$$\frac{\tilde{V}_1}{\tilde{V}_3} = \frac{P_3}{P_1}$$

Eqn 28

Use **Eqn 27** to eliminate the **V**'s from **Eqn 25** :

$$\tilde{W}_{31} = R\,T_1\,\mathrm{Ln}\left[\frac{P_3}{P_1}\right]$$

Eqn 29

Now, plug values into **Eqn 28** and then **Eqn 20** :

W_{31}	3146.5	J/mole
Q_{31}	3146.5	J/mole

Finally, we can calculate Q_{cycle} and W_{cycle} from :

$$\tilde{W}_{cycle} = \tilde{W}_{12} + \tilde{W}_{23} + \tilde{W}_{31}$$

Eqn 30

$$\tilde{Q}_{cycle} = \tilde{Q}_{12} + \tilde{Q}_{23} + \tilde{Q}_{31}$$

Eqn 31

Plugging values into **Eqns 29 & 30** yields :

W_{cycle}	-972.3	J/mole
Q_{cycle}	-972.3	J/mole

This result confirms what an application of the 1st Law to the <u>entire</u> cycle tells us: $Q_{cycle} = W_{cycle}$

Part d.) The cycle is a refrigeration cycle because both $\mathbf{W_{cycle}}$ and $\mathbf{Q_{cycle}}$ are negative.

The coefficient of performance of a refrigeration cycle is defined as :

$$\text{COP}_R = \frac{\tilde{Q}_C}{\tilde{W}}$$ **Eqn 32**

$\mathbf{Q_C}$ is the heat absorbed by the system during the cycle. In this case, $\mathbf{Q_C = Q_{31}}$.
W is the net **work** input to the system during the cycle. In this case, $\mathbf{W = - W_{cycle}}$.

Therefore :

$\mathbf{Q_C}$ **3146.5 J/mole**
W 972.3 J/mole

Plug values into **Eqn 31** to get :

$\mathbf{COP_R}$ **3.236**

Verify: The ideal gas assumption needs to be verified.

$$\tilde{V} = \frac{RT}{P}$$ **Eqn 33**

We need to determine the **specific volume** at each **state** and check if :

$$\tilde{V} > 5 \text{ L/mol}$$ **Eqn 34**

V_1	22.16	L/mol
V_2	10.21	L/mol
V_3	6.09	L/mol

The **specific volume** at each **state** is greater than **5 L/mol** for all **states** and the working fluid is a diatomic gas, so the ideal gas assumption is valid.

Answers :

a.) **See diagram above.**

b.) **See diagram above.**

c.)

Step	ΔU	ΔH	Q	W
1 - 2	2471	4119	0	-2471
2 - 3	-2471	-4119	-4119	-1648
3 - 1	0	0	3146	3146
Cycle	0	0	-972	-972

All values in this table are in **J/mole**.

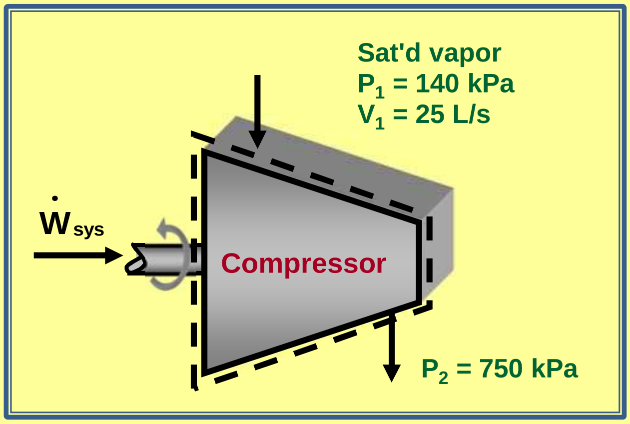
d.) **Refrigeration or Heat Pump Cycle.**

$\mathbf{COP_R}$ **3.2**

$\mathbf{COP_{HP}}$ **4.2**

7E-5 Power Input for an Internally Reversible, Polytropic Compressor 6 pts

Consider the internally reversible **ammonia** compressor shown below. The compression process is polytropic with δ **= 1.27**.



Determine W_s and **Q** in **kW**.

Read : The path equation given in the problem statement tells you that the compression process follows a polytropic path with δ **= 1.27**. All the properties of **state 1** can be determined using the Ammonia Tables or the **NIST Webbook**. The polytropic path equation allows us to determine the **specific volume** at **state 2**. This is the 2nd known intensive property at **state 2** , so we can evaluate the other properties using the Ammonia Tables or the **NIST Webbook**. Determine the **shaft work** based on the polytropic path and then apply the 1st Law to evaluate **Q**.

Given:

P_1	140	kPa
$V_{1,dot}$	25	L/s
x_1	1	kg vap/kg
P_2	750	kPa
δ	1.27	

Diagram: See the problem statement.

Find: a.) $(W_s)_{int\ rev}$??? kW b.) Q ??? kW

Assumptions: 1 -

1 - The compressor operates at steady-state.
2 - Changes in kinetic and potential energies are negligible.
3 - The compression is internally reversible.
4 - The compression process follows a polytropic process path with δ **= 1.08**.
5 - **Shaft work** and **flow work** are the only forms of **work** that cross the system boundary.

Equations / Data / Solve:

Part a.) **Work** for an internally reversible, polytropic process is given by :

$$(\dot{W}_S)_{int\ rev} = -\dot{m}\,\frac{\delta}{\delta - 1}\left[P_2\,\hat{V}_2 - P_1\,\hat{V}_1\right] \qquad \text{Eqn 1}$$

We can determine the **mass flow rate** from the **volumetric flow rate** using:

$$\dot{m} = \frac{\dot{V}_1}{\hat{V}_1} \qquad \text{Eqn 2}$$

We can use the Ammonia Tables or the **NIST Webbook** to evaluate $\mathbf{V_1}$ because it is a saturated vapor at a known pressure of **140 kPa**.

T_1	-26.682	°C

V_1	0.83074	m^3/kg
H_1	1409.0	kJ/kg

Next, plug values back into **Eqn 2** :

m_{dot}	0.0301	kg/s

Now, we need to determine $\mathbf{V_2}$. We can make use of the fact that the compression process follows a polytropic process path with δ **= 1.08**.

$$P_2 \hat{V}_2^{\delta} = P_1 \hat{V}_1^{\delta} \qquad \text{Eqn 3}$$

Solve **Eqn 3** for $\mathbf{V_2}$:

$$\hat{V}_2 = \left(\frac{P_1}{P_2}\right)^{1/\delta} \hat{V}_1 \qquad \text{Eqn 4}$$

Now, we can plug numbers into **Eqn 4** and then **Eqn 1** to complete this part of the problem.

V_2	0.22156	m^3/kg

$(W_s)_{int\ rev}$	-7.059	kW

Part b.) To determine the **heat transfer rate** for the compressor, we must apply the 1st Law for steady-state, **SISO** processes. For this compressor, changes in kinetic and potential energies are negligible and only **flow work** and **shaft work** cross the system boundaries. The appropriate form of the 1st Law for this compressor is :

$$\dot{Q} - \dot{W}_s = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) \qquad \text{Eqn 5}$$

We can solve **Eqn 5** for the **heat transfer rate**:

$$\dot{Q} = \dot{W}_s + \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) \qquad \text{Eqn 6}$$

In **part (a)** we evaluated all of the unknowns on the right-hand side of **Eqn 6** except $\mathbf{H_2}$. So, now we need to evaluate $\mathbf{H_2}$.

For **state 2**, we know the values of two intensive properties: $\mathbf{P_2}$ and $\mathbf{V_2}$. Therfore, we can use the Ammonia Tables or the **NIST Webbook** to evaluate any other properties of interest, in this case, $\mathbf{H_2}$.

We begin by determining the phases present. At **P = 750 kPa** :

$V_{sat\ liq}$	0.0016228	m^3/kg
$V_{sat\ vap}$	0.169798	m^3/kg

Since $\mathbf{V_2 > V_{sat\ vap}}$, **state 2** is a superheated vapor.

T (°C)	V (m^3/kg)	H (kJ/kg)
75	0.21661	1611.9
T_2	0.22156	H_2
100	0.23469	1672.2

T_2	81.85	°C
H_2	1628.4	kJ/kg

Finally, we can plug values back into **Eqn 5** to evaluate **Q** and complete this problem:

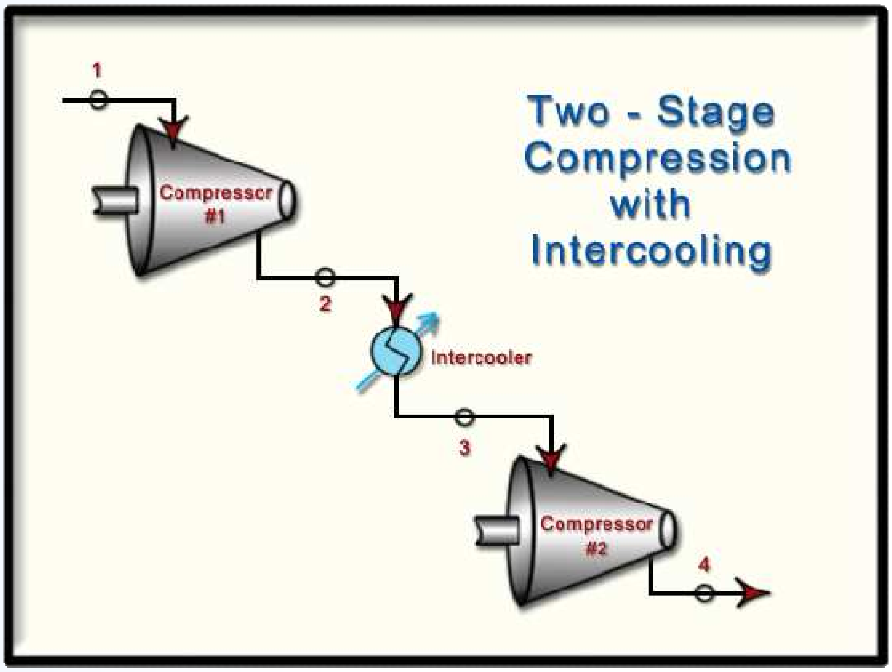
Q	-0.4586	kW

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

a.)	$(W_s)_{int\ rev}$	-7.06	kW
b.)	Q	-0.459	kW

Chapter 8



Thermodynamics of Flow Processes

The concept of entropy generation allows us to write entropy balance equations for both closed and open systems. Combination of the 1st and 2nd Laws and the 2nd Gibbs Equation leads to the Mechanical Energy Balance Equation, the Bernoulli Equation and a surprising relationship between shaft work, pressure and volume. Equations are derived for the shaft work of various polytropic processes.

This leads to the definition of isentropic efficiencies for turbines, nozzles and compressors. Isentropic efficiency is best visualized on an HS Diagram. Multi-stage compression with intercooling is discussed and represented on an HS Diagram.

The utility of the concept of entropy generation lies in its relationship to lost work. Equations relating entropy generation to lost work are derived for processes and cycles. Second Law Efficiency of a process is defined and discussed.

Entropy Balance: Closed System

- **1st Law:** $\mathbf{dU = \delta Q - \delta W}$
- **2nd Law, Internally Reversible Processes:** $\mathbf{\delta Q = T\,dS}$
- **Boundary Work, Internally Reversible Processes:** $\mathbf{W_b = P\,dV}$
- **Gibbs 1st Equation:** $\mathbf{dU = T\,dS - p\,dV}$
- **Entropy Balance Equation**
  - ◊ Integral Form: $\mathbf{S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}}$
  - ◊ Differential Form: $\mathbf{dS = \frac{\delta Q}{T} + \delta S_{gen}}$
  - ◊ Rate Form: $\mathbf{\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{S}_{gen}}$

- We start with closed systems, just to be thorough. The interesting application of the 2nd Law is for open systems.
- This is a quick run down of all the equations that apply to closed systems.
- The new part is the entropy balance equation.
  - ◊ The integral form is the one we will use most for closed systems.
    - It is essentially the definition of entropy generation.
    - It is not really new.
  - ◊ The rate form is a nice lead in to entropy balances on open systems.

Entropy Balance: Open System

- **General:** $$\mathbf{\frac{dS_{sys}}{dt} = \sum^{inlets} \dot{m}_{in}\,\hat{S}_{in} - \sum^{outlets} \dot{m}_{out}\,\hat{S}_{out} + \sum \frac{\dot{Q}}{T_{sys}} + \dot{S}_{gen}}$$
- **Steady-state, SISO:** $$\mathbf{\dot{m}\left(\hat{S}_{out} - \hat{S}_{in}\right) = \sum \frac{\dot{Q}}{T_{sys}} + \dot{S}_{gen}}$$
- **Entropy generation within the system:** $$\mathbf{\dot{S}_{gen} = \dot{m}\left(\hat{S}_{out} - \hat{S}_{in}\right) - \sum \frac{\dot{Q}}{T_{sys}}}$$

- The general entropy balance equation can be simplified a great deal for a SISO system operating at steady-state.
- Often, our objective is to determine the rate of entropy generation in a system.
  - ◊ You will see why this is a key quantity later in this chapter.
- It is very important to realize that this is ONLY the entropy generation <u>within the system</u>.
- What about the reservoirs and the surroundings ?
  - ◊ Irreversible heat exchange leads to additional <u>external</u> entropy generation.

- The general form of the entropy balance equation states that the entropy of the system changes for four different reasons.
  - ◊ Mass entering and leaving the system carries entropy with it.
    - The rate at which the entropy of the system changes as a result is equal to the rate at which mass flow carries entropy into the system minus the rate at which mass flow carries entropy out of the system.
  - ◊ Heat transfer to and from the system results in a change in the entropy of the system.
    - Each place in the system where heat transfer occurs changes the entropy of the system at a rate of $\mathbf{Q_{dot} / T}$, where **T** is the temperature of the system at the particular location where heat transfer occurs.
    - The net rate at which heat transfer changes the entropy of the system is the sum of all the individual $\mathbf{Q_{dot} / T}$ terms.For example: $\mathbf{Q_H / T_{sys,H} - Q_C / T_{sys,C}}$ for a heat engine with an evaporator operating at $\mathbf{T_{sys,H}}$ and a condenser operating at $\mathbf{T_{sys,C}}$.
- **Note:** These **T**'s are the temperatures within the system and <u>NOT</u> the temperatures of the

1st Law and Entropy

- **1st Law, Steady-state, Internally Reversible, SISO:**

$$\dot{Q} - \dot{W}_{\text{not b}} = \dot{m}\left[\Delta H + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z\right]$$

$$-\frac{\dot{W}_{\text{not b}}}{\dot{m}} = -\frac{\dot{Q}}{\dot{m}} + \left[\Delta H + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z\right]$$

- **From the definition of entropy:**

$$\hat{Q} = \frac{\dot{Q}}{\dot{m}} = \int_{in}^{out} T\,d\hat{S}$$

- **Gibbs 2nd Equation:**

$$T\,d\hat{S} = d\hat{H} - \hat{V}\,dP$$

- **Integrating for an open system:**

$$\int_{in}^{out} T\,d\hat{S} = \Delta\hat{H} - \int_{in}^{out} \hat{V}\,dP$$

- When we apply the 1st Law to a SISO, internally reversible, steady-state process, the work term includes all forms of work OTHER THAN boundary work.
- In this course, that usually this means shaft work.
- Our goal here is to derive an equation that will let us determine the shaft work for the process, so we begin by solving for $\mathbf{W}_{\text{not b}}$.
- We divide by m-dot to get the specific work on the left-hand side of the equation.
- The heat transfer is the key that connects the 1st Law to the 2nd Law.
- We can manipulate the definition of entropy and use it to eliminate Q from the 1st Law.
- Then, we can use Gibbs 2nd Equation to express **T dS** as **dH – V dP**.
- This helps us get S out of the equation. This is a good thing because entropy is notoriously difficult to measure.
- On the next slide, we put all of these equations together to eliminate **Q** and then **S**.

Mechanical Energy Balance Equation

- **Combine all the equations from the previous slide:**

$$-\frac{\dot{W}_{\text{not b}}}{\dot{m}} = -\left[\cancel{\Delta\hat{H}} - \int_{in}^{out} \hat{V}\,dP\right] + \left[\cancel{\Delta\hat{H}} + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z\right]$$

- **The MEBE:**

$$\boxed{-\frac{\dot{W}_{\text{not b}}}{\dot{m}} = \int_{in}^{out} \hat{V}\,dP + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z}$$

Reversible Steady-State SISO

- **If $W_{\text{not b}} = 0$:**

$$\boxed{0 = \int_{in}^{out} \hat{V}\,dP + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z}$$

Bernoulli Equation

$$\boxed{0 = \frac{\Delta P}{\rho} + \frac{\Delta v^2}{2\,g_C} + \frac{g}{g_C}\Delta z}$$

Bernoulli Equation (incompressible fluid)

- **If $\Delta E_{kin} = \Delta E_{pot} = 0$:**

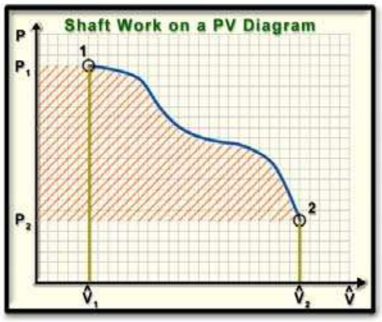
$$\boxed{-\frac{\dot{W}_{\text{not b}}}{\dot{m}} = -\hat{W}_{\text{not b}} = \int_{in}^{out} \hat{V}\,dP}$$

Usually, $W_{\text{not b}} = W_{sh}$

- The first cool thing that happens is that **ΔH** drops out.
  - ◊ This is good because **H** is also a bit difficult to measure.
- The result is the Mechanical Energy Balance Equation or MEBE.
  - ◊ This is a very useful equation in fluid mechanics.
  - ◊ But it does not take friction into account because we started this analysis by assuming that the process was internally reversible.
- Another famous equation is the Bernoulli Equation.
  - ◊ It is just the MEBE with no work at all.
  - ◊ Bernoulli is especially useful for analyzing flow in pipes.
  - ◊ If the fluid flowing through the pipes is an incompressible liquid, then Bernoulli can be simplified even further.
    - In this case it is customary to use $\boldsymbol{\rho = 1 / V_{hat}}$.
- The result that we will use most frequently in this course applies when changes in kinetic and potential energies are negligible.
- This is a surprising and all too often confusing result.
  - ◊ Look at it VERY carefully.
  - ◊ It says that the specific non-boundary (usually SHAFT) work is equal to the integral of $\mathbf{V}_{\text{hat}}$ **dP** from the inlet to the outlet pressure.
  - ◊ This is CONFUSING sometimes because we found a similar looking result earlier in this course:
    - Specific BOUNDARY work is the integral of **P** $\mathbf{dV}_{\text{hat}}$.
  - ◊ This is NOT a typo !
  - ◊ What does this result look like on a PV Diagram ?

Shaft Work and PV Diagrms

$$-\frac{\dot{W}_{sh}}{\dot{m}} = -\hat{W}_{sh} = \int_{in}^{out} \hat{V}\, dP$$



- **Polytropic Processes**

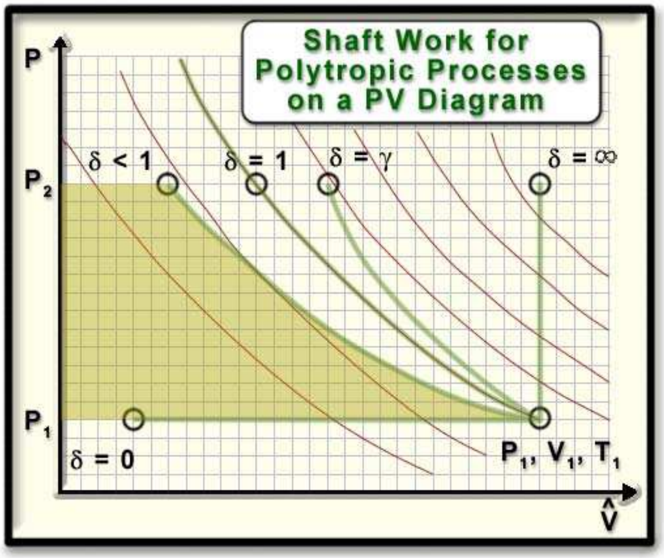
$$P\,\hat{V}^{\delta} = C \qquad \textbf{or:} \qquad \hat{V} = \left(\frac{C}{P}\right)^{1/\delta}$$

$$\hat{W}_{sh} = -\int_{in}^{out}\left(\frac{C}{P}\right)^{1/\delta} dP = -C^{1/\delta}\int_{in}^{out}\frac{dP}{P^{1/\delta}}$$

- $\delta \neq 1$: $\boxed{\hat{W}_{sh} = -\frac{\delta}{\delta-1}\left(P_2\,\hat{V}_2 - P_1\,\hat{V}_1\right)}$
- $\delta = 1$: $\boxed{\hat{W}_{sh} = -P_1\,\hat{V}_1\,\mathrm{Ln}\left[\frac{P_2}{P_1}\right]}$

- On a PV diagram, shaft work is equal to the area to the LEFT of the process path FOR…
  - ◊ Steady-state.
  - ◊ Internally reversible
  - ◊ SISO
  - ◊ Shaft work and flow work only
  - ◊ $\mathbf{\Delta E_{kin} = \Delta E_{pot} = 0}$
- For a known process path, such as a polytropic process path, we can evaluate the shaft work by direct integration.
  - ◊ Results are shown here for $\delta = \mathbf{1}$ and $\delta \neq \mathbf{1}$.
  - ◊ Keep in mind that $\delta = \mathbf{1}$ does not necessarily imply an isothermal process here !
  - ◊ The results shown here apply when the assumptions listed above are valid: SISO, SS, Int. Rev, $\mathbf{\Delta E_{kin} = \Delta E_{pot} = 0}$ & polytropic.
    - The equations apply for all fluids.
      - ◊ Liquids, real gases, and ideal gases.
      - ◊ **If** the fluid is an **ideal gas <u>and</u> d = 1**, **then** the process is **isothermal**.

PV Diagram for Polytropic Processes



- Assumptions
  - ◊ Steady-state.
  - ◊ Internally reversible
  - ◊ SISO
  - ◊ Shaft work and flow work only
  - ◊ $\mathbf{\Delta E_{kin} = \Delta E_{pot} = 0}$
- This diagram shows how the value of δ effects the shape of the polytropic process path on a **PV** Diagram.
- As δ increases, the process path becomes more steep and the specific volume changes by a smaller and smaller amount for the same change in pressure.
- Observations
  - ◊ I have shown $\delta = \mathbf{1}$ following the red isotherm.
  - ◊ This is ONLY the case for an ideal gas with constant heat capacities.
  - ◊ The path for $\delta = \gamma$ is only isentropic if the fluid is an ideal gas and its heat capacities are constant.
  - ◊ The isochoric process requires the most shaft work to raise the pressure from $\mathbf{P_1}$ to $\mathbf{P_2}$. With all of the assumptions we have made, no shaft work can be done in an isobaric process.
  - ◊ Keep in mind that boundary work and flow work can still be done.
- In the next slide, I present equations for specific $\mathbf{W_{Sh}}$ for various polytropic processes for real fluids and ideal gases.

Summary: W_{sh} Polytropic Processes

Process Type	Real Fluids	Ideal Gases
$\delta = 0$	$\hat{W}_{sh} = 0$	$\hat{W}_{sh} = 0$
$\delta = 1$	$\hat{W}_{sh} = -P_1 \hat{V}_1 \, Ln\left[\frac{P_2}{P_1}\right]$	Isothermal $\hat{W}_{sh} = -\frac{R\,T_1}{MW}\, Ln\left[\frac{P_2}{P_1}\right]$
$\delta \neq 1$: **Polytropic**	$\hat{W}_{sh} = -\frac{\delta}{\delta - 1}\left(P_2 \hat{V}_2 - P_1 \hat{V}_1\right)$	$\hat{W}_{sh} = -\frac{\delta}{\delta - 1}\frac{R}{MW}\left(T_2 - T_1\right)$
$\delta = \gamma =$ **const.**	$\hat{W}_{sh} = -\frac{\gamma}{\gamma - 1}\left(P_2 \hat{V}_2 - P_1 \hat{V}_1\right)$	Isentropic $\hat{W}_{sh} = -\frac{\gamma}{\gamma - 1}\frac{R}{MW}\left(T_2 - T_1\right)$
$\delta = \infty$: **Isochoric**	$\hat{W}_{sh} = -\hat{V}\left(P_2 - P_1\right)$	$\hat{W}_{sh} = -\frac{R}{MW}\left(T_2 - T_1\right)$

- **W_{sh} is actually all work other than flow work.**

- Here are all of the most useful equations for calculating shaft work for polytropic processes.
- Once again, the assumptions we made are:
  - ◊ Steady-state.
  - ◊ Internally reversible
  - ◊ SISO
  - ◊ Shaft work and flow work only
  - ◊ $\Delta E_{kin} = \Delta E_{pot} = 0$
- The ideal gas column adds the assumption that the fluid behaves as an ideal gas.
  - ◊ In order for $\delta = \gamma$ to mean that an IG is undergoing an isentropic process, the heat capacities (and therefore γ) must be constant.

Isentropic Efficiency: η_S

- **Compare work input or output of a real device to to that of an isentropic device.**
- **Adiabatic Turbines:**

$$\eta_{S,turb} = \frac{\dot{W}_{Sh,act}}{\dot{W}_{Sh,isen}} \qquad \left\{\begin{matrix} Q = 0 \\ \Delta E_{pot} = \Delta E_{kin} = 0 \end{matrix}\right\} \qquad \eta_{S,turb} = \frac{\hat{H}_{in} - \hat{H}_{out,act}}{\hat{H}_{in} - \hat{H}_{out,isen}}$$

- **Adiabatic Nozzles:**

$$\eta_{S,noz} = \frac{v^2_{out,act} / 2g_C}{v^2_{out,isen} / 2g_C} \qquad \left\{\begin{matrix} A_1 >> A_2 \\ v_1 << v_2 \end{matrix}\right\} \qquad \eta_{S,noz} \approx \frac{\hat{H}_{in} - \hat{H}_{out,act}}{\hat{H}_{in} - \hat{H}_{out,isen}}$$

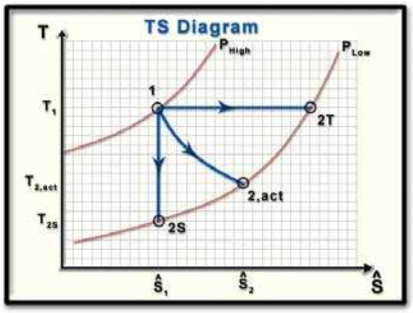
- **Adiabatic Compressors:**

$$\eta_{S,comp} = \frac{-\dot{W}_{Sh,isen}}{-\dot{W}_{Sh,act}} \qquad \left\{\begin{matrix} Q = 0 \\ \Delta E_{pot} = \Delta E_{kin} = 0 \end{matrix}\right\} \qquad \eta_{S,comp} = \frac{\hat{H}_{out,isen} - \hat{H}_{in}}{\hat{H}_{out,act} - \hat{H}_{in}}$$

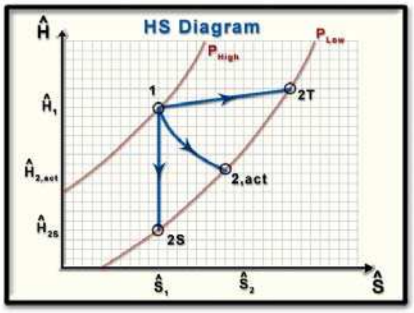
- Isentropic efficiency is a bit odd for nozzles because there is no shaft work involved
  - ◊ The goal of a nozzle is to increase the kinetic energy of the flowing fluid.
  - ◊ So, we define the isentropic efficiency of a nozzle as the ratio of the specific kinetic energy of the effluent for an actual or real nozzle to the specific kinetic energy of the effluent from an isentropic nozzle.
    - Real nozzles are not isentropic primarily because of friction within the fluid and between the fluid and the wall of the nozzle.
    - IF the outlet velocity is MUCH MUCH greater than the inlet velocity, then we get a simpler form of the equation for the isentropic efficiency of a nozzle in terms of the enthalpy of the fluid.

- We will apply the concept of isentropic efficiency to turbines, compressors, pumps and nozzles.
- Pumps are not listed here because the equation is the same as the one for the compressor.
- An isentropic turbine produces the most work possible beginning from the real inlet state and ending at the same outlet pressure as the real turbine.
- An isentropic compressor requires the least work possible beginning from the real inlet state and ending at the same outlet pressure as the real compressor.
- We define the isentropic efficiency of a turbine as the ratio of the actual work output to the work output of an isentropic turbine with the same inlet conditions and same outlet pressure.
- IF the REAL turbine is also adiabatic, then The 1st Law is: $\mathbf{W_{S,act} = m_{dot} (H_{in} - H_{out,act})}$.
- The isentropic efficiency is the ratio of the change in the specific enthalpy for the actual or real turbine to the change in the specific enthalpy of the isentropic turbine.
- We define the isentropic efficiency of a compressor as the ratio of the isentropic work input to the actual work input of a compressor with the same inlet conditions and same outlet pressure.
- IF the REAL compressor is also adiabatic, then:
- The 1st Law is: $\mathbf{W_{S,act} = m_{dot} (H_{in} - H_{out,act})}$.
- The isentropic efficiency is the ratio of the change in the specific enthalpy for the isentropic compressor to the change in the specific enthalpy of the actual or real compressor.

TS & HS Diagrams for Turbines



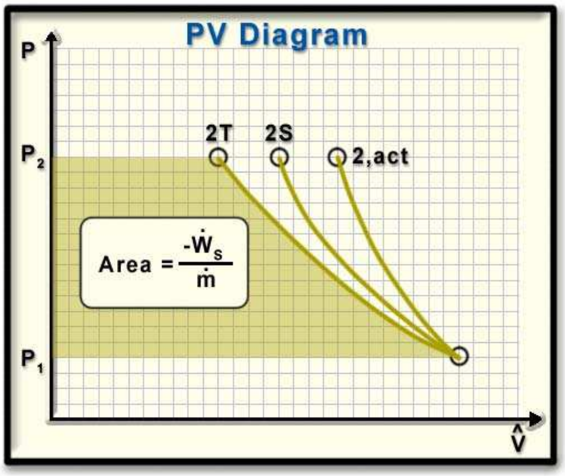
$$\dot{S}_{gen} = \dot{m}\left(\hat{S}_{out} - \hat{S}_{in}\right) - \sum \frac{\dot{Q}}{T_{sys}}$$



- **Only states to the right of path 1-2 are “accessible” states.**

- **TS** & **HS** Diagrams look deceptively similar for turbines, compressors pumps and nozzles.
- If the turbine is adiabatic, then **S^** cannot decrease because that would result in **S_{gen} < 0**.
- Therefore, the effluent of an adiabatic turbine can only occupy states to the right of **S_1^**.
- The isentropic path is the best case.
- The isothermal path is the WORST case.
  - ◊ The enthalpy actually goes UP ! How does that happen ?
    - Irreversibilities within the turbine are so bad that you have to put shaft work INTO the system !
    - It isn’t really a turbine anymore. But it isn’t really a compressor either.
    - It is not a device you would want to buy.
    - It converts shaft work and **PV** work into relatively low value internal energy.

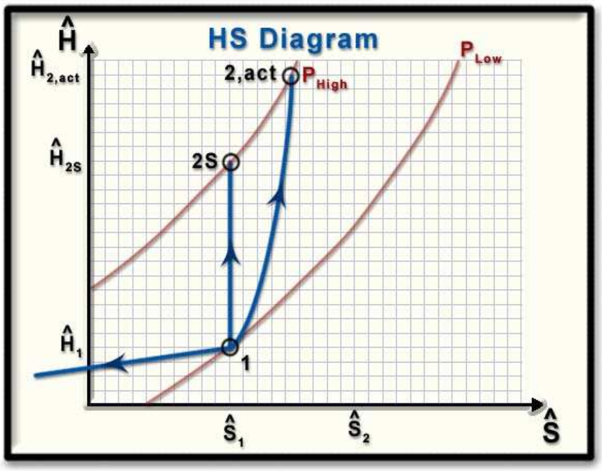
PV Diagram for a Compressor



- **Isothermal process requires less shaft work for an adiabatic compressor.**

◊ An isentropic compressor requires less shaft work input than an actual, adiabatic compressor.But, if we could build an isothermal compressor, that was also adiabatic, it would require even LESS work input than an isentropic compressor.I hope that seems suspicious to you.

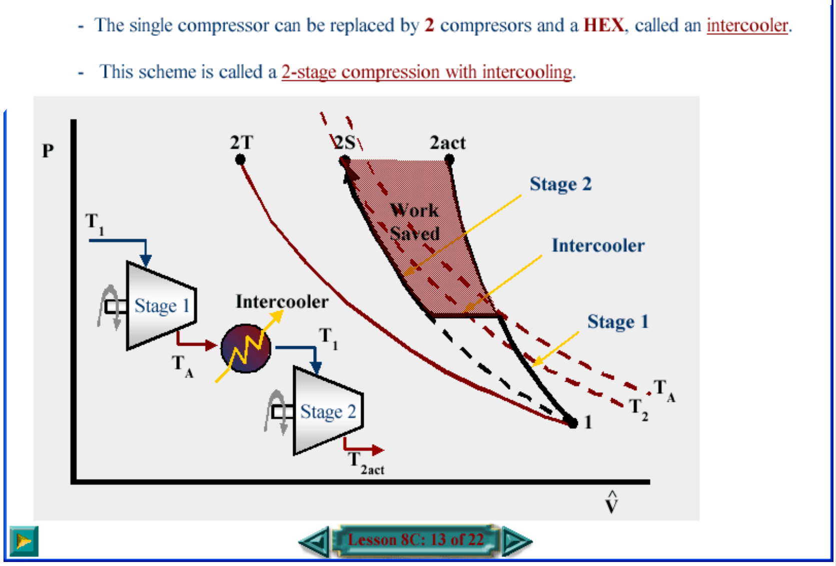
HS Diagram for a Compressor



- **Isothermal process for an adiabatic compressor violates the 2nd Law !**

- The problem is that a compressor that is BOTH isothermal AND ADIABATIC is not possible.
  - ◊ Its process path moves into states that are not “accessible”.
  - ◊ An isothermal, adiabatic compressor leads to negative values for $\mathbf{S_{gen}}$ and that is a violation of the 2nd Law.
- Unlike turbines, we DON’T WANT our compressor to be adiabatic.
  - ◊ It seems weird that we actually WANT our compressor to LOSE heat so that its performance and efficiency improve.
  - ◊ But we do !
  - ◊ IN the BEST case, we can RECOVER this heat and use it somewhere else in our car/airplane/factory/system.
- We can let our compressors lose heat to the air that surrounds them.
- We can give them a “cooling jacket” with cool water circulating through it.
- Or we can use multiple compressors and cool the working fluid between the compression steps.

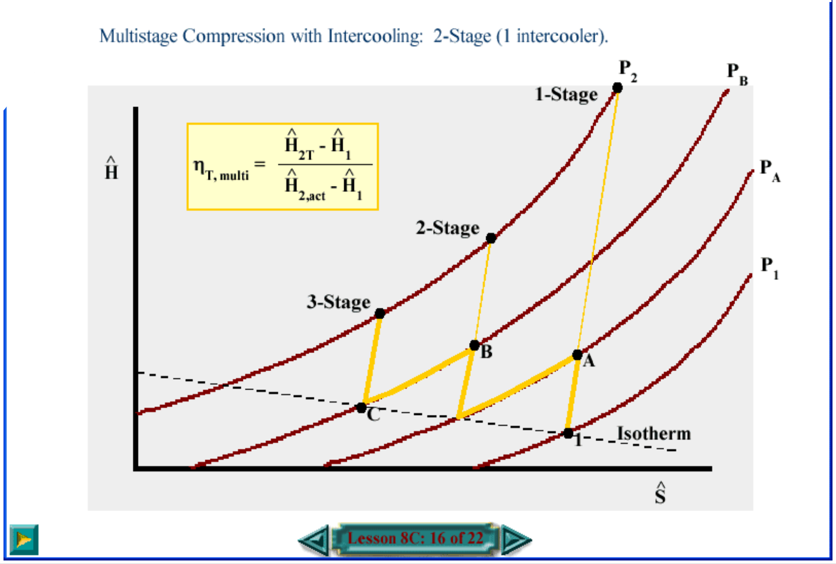
Two-Stage Compression with Intercooling



- The single compressor can be replaced by **2** compressors and a **HEX**, called an intercooler.
- This scheme is called a 2-stage compression with intercooling.

- It is easy to visualize the savings in shaft work when a 2-stage compressor train is used instead of 1 big compressor.
- The intercooler keeps the specific volume of the gas lower.
- 3-stage compression trains are common when you must get a gas up to a very high pressure.

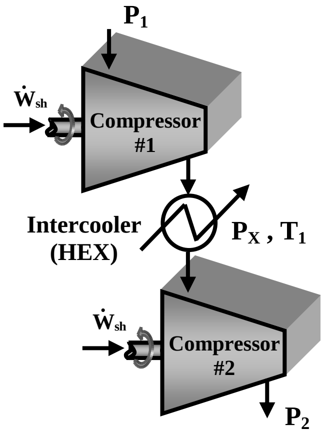
HS Diagram for Multi-Stage Compression



- Remember that the increase in enthalpy is how much shaft work you must put into the compression process.
- This HS Diagram makes it very clear why multi-stage compression can dramatically reduce the shaft work requirement for a compression process.
- Isothermal Efficiency
  - ◊ Only used for multi-stage compressors.
  - ◊ Compare a multi-stage compressor to one isothermal compressor.

2-Stage Compressor Power Requirement

- **Assume:**
  - ◊ Each compressor is isentropic (polytropic with $\delta = \gamma$)
  - ◊ The working fluid is an ideal gas in all states in the process.
    - The heat capacities of the working fluid are constant.
    - Intercooler returns the fluid to $\mathbf{T_1}$.



Compressor #1

$$\frac{-\dot{W}_{S,tot}}{\dot{m}} = \boxed{\frac{\gamma}{\gamma-1}\left(\frac{R}{MW}\right)T_1\left[\left(\frac{P_X}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]} + \ldots$$

$$+\boxed{\frac{\gamma}{\gamma-1}\left(\frac{R}{MW}\right)T_1\left[\left(\frac{P_2}{P_X}\right)^{\frac{\gamma-1}{\gamma}}-1\right]}$$

Compressor #2

- With a few assumptions we can determine the shaft work requirement and the total shaft work requirement for the system.
- The equation can be simplified if we use the intercooler to return the fluid temperature to the temperature of the feed, $\mathbf{T_1}$.
- The big question is how to decide how much shaft work to put into the 1st compressor and how much to put into the 2nd one ?
  - ◊ More to the point, what is $\mathbf{P_X}$?
  - ◊ Do we make $\mathbf{P_X}$ = average of $\mathbf{P_1}$ and $\mathbf{P_2}$?
  - ◊ Why ? How do we decide ?

Two-Stage Compressor Design

- Calculus to the rescue ! $\frac{d}{d P_X}\left(\frac{-\dot{W}_{Sh,tot}}{\dot{m}}\right) = 0$
- Solve for P_X !
  - ◊ (check the 2nd derivative to be sure you found a minimum and not a maximum !)
- Solution: $\frac{P_1}{P_X} = \frac{P_X}{P_2}$ or: $P_X = \sqrt{P_1 P_2}$

- We determine the value of P_X that minimizes the specific shaft work requirement for the entire process by…
  - ◊ Taking the derivative of the specific shaft work requirement for the entire process with respect to P_X…
  - ◊ Setting the derivative equal to zero…
  - ◊ And solving the resulting equation for P_X.
- We need to make sure that the extreme point that we found is a minimum and not a maximum by making sure that the sign of the 2nd derivative is positive.
- The results are nice and simple. That seems surprising.
- We do NOT set P_X at the algebraic average of P_1 and P_2.
- We set PX at the GEOMETRIC average of P_1 and P_2.
  - ◊ The square-root of P_1 times P_2 is called the geometric average.
- Real compressors are not isentropic and frequently the gases are not ideal, but the geometric average pressure is a good starting point for choosing the intermediate pressure in a 2-stage compressor system.
- Are there 3-stage compressor trains ? Yes, they are fairly common.
- Are there 4-stage compressor trains ? Very few.
  - ◊ The costs of all the extra equipment and its maintenance usually outweigh the savings due to improved efficiency.

Lost Work

- Definition: $\dot{W}_{S,lost} = \dot{W}_{S,rev} - \dot{W}_{S,act}$
  - ◊ Applies to any device, so watch the signs carefully.
  - ◊ $W_{S,rev}$
    - Rate at which work is done by or on a completely reversible process operating between the same initial state **AND** the same final state as the actual process.
    - Assume for now that heat exchange only occurs with the surroundings and **not** with a thermal reservoir.

- Lost work is the difference between the work of a reversible process between the real states 1 and 2 and the work of the real process between real states 1 and 2.

2^{nd} Law Efficiency

- 2^{nd} Law Efficiency: $\eta_{ii,comp} = \dfrac{\dot{W}_{Sh,rev}}{\dot{W}_{Sh,act}}$ $\eta_{ii,turb} = \dfrac{\dot{W}_{Sh,act}}{\dot{W}_{Sh,rev}}$
  - ◊ More fair than η_S because initial and final states are the same.
  - ◊ Best measure of performance for processes that produce or consume work.

◊ In terms of Lost Work:

$$\eta_{ii,comp} = \frac{\dot{W}_{Sh,act} + \dot{W}_{Sh,lost}}{\dot{W}_{Sh,act}} \qquad \eta_{ii,turb} = \frac{\dot{W}_{Sh,act}}{\dot{W}_{Sh,act} + \dot{W}_{Sh,lost}}$$

- The 2^{nd} Law efficiency compares the work of a real process to the work of a reversible process
  - ◊ Both processes begin at state 1 and end at the exact same state 2.
  - ◊ In this case we are comparing apples to apples, so to speak.
- For the isentropic efficiency, we compared apples and oranges.
  - ◊ We compared a real process from state 1 to state 2 to an isentropic process that began at state 1, but ended at a different state, state 2S.
- The isentropic efficiency is a much more fair and reasonable measure of the performance of a process than isentropic efficiency.
- Unfortunately, 2^{nd} Law Efficiency is only defined for processes that produce or consume work, like pumps, compressors, turbines and thermodynamic cycles !
- As we will see in a minute, Lost Work is relatively easy to calculate, so it is often convenient to express the 2^{nd} Law Efficiency in terms of lost work.
- Now, let's see how to evaluate lost work.

- We begin by analyzing a real process from state 1 to state 2 that only exchanges heat with the surroundings and NOT with any thermal reservoir.
  - ◊ This is just to make things simpler.
  - ◊ Later we will consider processes that exchange heat with reservoirs.

Apply 1^{st} law & Definition of Entropy

- **1^{st} Law:** $\dot{Q} - \dot{W}_{Sh} = \dot{m}\,\Delta\hat{H}$
- **Solve for W_S:** $\dot{W}_{Sh} = \dot{Q} - \dot{m}\,\Delta\hat{H}$
- **From the definition of entropy:** $\dfrac{\dot{Q}_{int\,rev}}{\dot{m}} = \displaystyle\int_1^2 T\,d\hat{S} = T\,\Delta\hat{S} = T_{surr}\,\Delta\hat{S}$

  ($T_{sys} = T_{surr}$, <u>completely</u> reversible)
- **For a reversible process:** $\dot{W}_{Sh,rev} = \dot{m}\left(T_{surr}\,\Delta\hat{S} - \Delta\hat{H}\right)$
- **For the actual process:** $\dot{W}_{Sh,act} = \dot{Q}_{act} - \dot{m}\,\Delta\hat{H}$
- **Lost Work:** $\dot{W}_{Sh,lost} = \left(\dot{m}\,T_{surr}\,\Delta\hat{S} - \dot{m}\,\Delta\hat{H}\right) - \left(\dot{Q}_{act} - \dot{m}\,\Delta\hat{H}\right)$
- **Canceling terms gives:** $\boxed{\dot{W}_{Sh,lost} = \dot{m}\,T_{surr}\,\Delta\hat{S} - \dot{Q}_{act}}$

- Here, we apply the 1^{st} Law to both the reversible and actual processes from state 1 to state 2.
- Entropy enters into the equations in order to eliminate $\mathbf{Q_{rev}}$.
  - ◊ Here, it is absolutely critical to understand that heat transfer must ALSO be reversible in the completely reversible process.
  - ◊ Therefore, $\mathbf{T_{sys} = T_{surr}}$ in the completely reversible process.
- Once we eliminate $\mathbf{Q_{rev}}$ from the 1^{st} Law for the reversible process, we are ready to determine $\mathbf{W_{Sh,lost}}$.
- We solve the two 1^{st} Law equations (reversible and actual) for $\mathbf{W_{Sh}}$.
- Lost work is just the reversible work minus the actual work.
- When we put all of the equations together, a cool thing happens.
- The $\mathbf{\Delta H}$ terms cancel.
- This leaves us with the equation in the box.
- We can compute $\mathbf{W_{Sh,lost}}$ from the actual heat exchange with the surroundings, $\mathbf{\Delta S}$ for the process and the temperature of the surroundings.
- This is a useful equation, but we know that $\mathbf{\Delta S}$ is related to entropy generation, so we need to work the 2^{nd} Law into this analysis

2^{nd} Law and External S_{gen}

- **2^{nd} Law:**

$$\dot{m}\,\Delta\hat{S} = \frac{\dot{Q}_{act}}{T_{act}} + \dot{S}_{gen,int}$$

- **Algebraic slight of hand:**

$$\frac{\dot{Q}_{act}}{T_{act}} = \dot{Q}_{act}\left[\frac{1}{T_{surr}} + \frac{1}{T_{act}} - \frac{1}{T_{surr}}\right]$$

- **More algebra:**

$$\frac{\dot{Q}_{act}}{T_{act}} = \frac{\dot{Q}_{act}}{T_{surr}} + \dot{Q}_{act}\left[\frac{1}{T_{act}} - \frac{1}{T_{surr}}\right]$$

- **Substitute back into the 2^{nd} Law:**

$$\dot{m}\,\Delta\hat{S} = \frac{\dot{Q}_{act}}{T_{surr}} + \dot{Q}_{act}\left[\frac{1}{T_{act}} - \frac{1}{T_{surr}}\right] + \dot{S}_{gen,int}$$

- **External Entropy Generation:**

$$\dot{S}_{gen,ext} = \dot{Q}_{act}\left[\frac{1}{T_{act}} - \frac{1}{T_{surr}}\right]$$

- **A more robust statement of the 2^{nd} Law:**

$$\boxed{\dot{m}\,\Delta\hat{S} = \frac{\dot{Q}_{act}}{T_{surr}} + \dot{S}_{gen,ext} + \dot{S}_{gen,int}}$$

- Here is our old friend the 2^{nd} Law.
  - ◊ Notice that I have replaced the integral of **dQ / T** with **Q_{act} / T_{act}**.
  - ◊ **This is only allowed when the temperature of the system at which heat exchange occurs is CONSTANT, at Tact.**
- Now, things get a bit weird.
- I have done some algebra to split **Q_{act} / T_{act}** into two parts.
- I claim that **Q_{act}** times **[$1/T_{act}$ – $1/T_{surr}$]** is the <u>external</u> entropy generation due to irreversible heat transfer.
- Why is this true ?
  - ◊ Look at the boxed equation.
  - ◊ If heat exchange between the system and the surroundings is reversible, then **T_{act} = T_{surr}** .
  - ◊ Consequently, **S_{gen} = 0**.
  - ◊ The equation in the box becomes the 1st equation on this page…the 2^{nd} Law !
  - ◊ I will show why this is true using a different approach in an example at the end of this lesson.

Lost Work

- **2^{nd} Law:**

$$\dot{m}\,\Delta\hat{S} = \frac{\dot{Q}_{act}}{T_{surr}} + \dot{S}_{gen,ext} + \dot{S}_{gen,int} = \frac{\dot{Q}_{act}}{T_{surr}} + \dot{S}_{gen,tot}$$

- **Total Entropy Generation:**

$$\dot{S}_{gen,tot} = \dot{m}\,\Delta\hat{S} - \frac{\dot{Q}_{act}}{T_{surr}}$$

- **Lost Work revisited:**

$$\dot{W}_{S,lost} = \dot{m}\,T_{surr}\,\Delta\hat{S} - \dot{Q}_{act}$$

- **Algebra:**

$$T_{surr}\,\dot{S}_{gen,tot} = \dot{m}\,T_{surr}\,\Delta\hat{S} - \dot{Q}_{act}$$

- **BIG result:**

$$\boxed{\dot{W}_{S,lost} = T_{surr}\,\dot{S}_{gen,tot}}$$

- What does **T_{surr}** have to do with lost work.
  - ◊ **T_{surr}** is the temperature of a reservoir in the dead state.
  - ◊ This reservoir has infinite capacity to give and receive heat without changing in temperature.
  - ◊ Heat exchange with this reservoir has no cost.
  - ◊ Anything that has no cost and infinite supply has NO VALUE.
  - ◊ Hence the name “dead state”.

- Now, let’s make the connection between entropy generation and lost work.
- Define the total entropy generation as the sum of the internal and external entropy generation.
- Solve the form of the 2^{nd} Law that we derived on the previous page for the total entropy generation.
- Next, recall the equation we derived for lost work (2 slides back).
- When we compare the equation for the total entropy generation with the equation for lost work we find an extraordinarily simple relationship.
- Lost work is the product of the temperature of the surroundings and the lost work !
  - ◊ This equation applies even for complicated processes or cycles that interact with reservoirs !
  - ◊ The boxed equation even applies for processes that do NOT exchange heat with the surroundings !
  - ◊ The example at the end of this lesson should shed some light on why this is true.
  - ◊ This is so cool, so simple and so elegant !
  - ◊ This is the reason we care about calculating **S_{gen}** !
  - ◊ We can also use **$W_{Sh,lost}$** to compute the 2^{nd} Law Efficiency.

Lost Work for a Cycle

- **Total entropy generation for a cycle:**

$$\dot{S}_{gen,tot} = \dot{m}\,\Delta\hat{S} - \sum_{i}^{Processes} \frac{\dot{Q}_{act,i}}{T_{res,i}}$$

- **Lost Work for a cycle:**

$$\dot{W}_{Sh,lost} = T_{surr} \sum_{i}^{Processes} \left[\dot{S}_{gen,tot} \right]_i$$

- **BIG Result:**

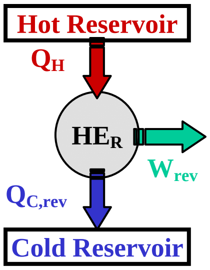
$$\boxed{\dot{W}_{Sh,lost} = T_{surr} \sum_{i}^{Processes} \frac{-\dot{Q}_{act,i}}{T_{res,i}}}$$

- **Typical HE: (adiabatic pump, turbine compressor etc. HEX only with thermal reservoirs.)**

$$\boxed{\dot{W}_{Sh,lost} = T_{surr} \left(\frac{\dot{Q}_{C,act}}{T_{C,res}} - \frac{\dot{Q}_{H,act}}{T_{H,res}} \right)}$$

- The lost work for a cycle is particularly useful to us and it isn't very difficult to calculate.
- We just apply our new and improved form of the 2nd Law in terms of the total entropy generation to the entire cycle.
- **ΔS_{cycle} = 0**. It's a cycle !
- The result is the equation in the box.
- All you need to do to compute the lost work is evaluate all the heat transfer terms for the cycle.
  - ◊ And know the **T**'s for the reservoirs and the surroundings, of course.
  - ◊ And you know **$W_{S,lost}$**.
  - ◊ This is cool and relatively straightforward.
  - ◊ You have just GOT to watch the signs !
    - The **Q**'s here are NOT all positive.
    - They follow the sign convention !
    - Next, we'll try a simple example that will clear this up.

Example #1



- **Consider the HE shown here:**
  - ◊ **Q_H = 800 J Q_C = 300 J $W_{Sh,act}$ = 500 J**
  - ◊ **T_H = 1000 K T_C = 300 K T_{surr} = 300 K**
- **Determine both the reversible and the lost work.**

◊ **Method #1:**

$$\eta_{rev} = \frac{W_{Sh,rev}}{Q_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7$$

$$W_{Sh,rev} = 0.7\ Q_H = 0.7\left(800\ J\right) = 560\ J$$

$$\boxed{W_{Sh,lost}} = W_{S,rev} - W_{S,act} = 560\ J - 500\ J = \boxed{60\ J}$$

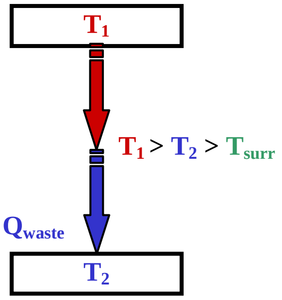
◊ **Method #2:**

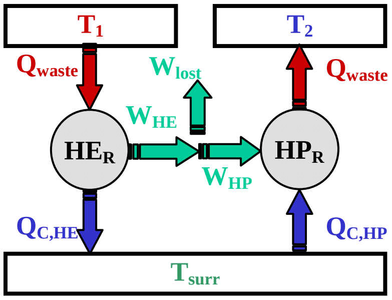
$$\dot{W}_{Sh,lost} = T_{surr}\left(\frac{\dot{Q}_{C,act}}{T_{C,res}} - \frac{\dot{Q}_{H,act}}{T_{H,res}} \right)$$

$$\boxed{\dot{W}_{Sh,lost}} = 300\,K\left(\frac{300\ J}{300\,K} - \frac{800\ J}{100\,K} \right) = 300\ \left(1 - 0.8\right) J = \boxed{60\ J}$$

Example #2

- **Why is there lost work for irreversible heat transfer ?**
- **How can we calculate the lost work for irreversible heat transfer ?**





- **By allowing the heat to flow spontaneously from T_1 to T_2, we lost the opportunity to recover work.**
- **We call this "Lost Work".** $W_{HE} > W_{HP}$ $W_{lost} > 0$

- We can determine the lost work by building the hypothetical system, shown at right.
- This system includes 2 reversible cycles… one HE and one HP.
- The HE is more efficient than the reversed version of the HP because $T_1 > T_2$.
- As a result, the HE produces more work than the HP requires.
- Remember that both of these cycles are reversible !
- The difference between W_{HE} and W_{HP} is work that we COULD have gotten out of Q_{waste} if we had used it instead of letting it spontaneously dribble away into the cold reservoir.
- We call this lost work.
- $W_{lost} = W_{HE} - W_{HP}$.
- Now, let's determine exactly how much work we lost.

Lost Work

- **1st Law, Reversible HE:**

$$W_{HE} = Q_{waste} - Q_{C,HE} = Q_{waste}\left(1 - \frac{Q_{C,HE}}{Q_{waste}}\right) = Q_{waste}\left(1 - \frac{T_{surr}}{T_1}\right)$$

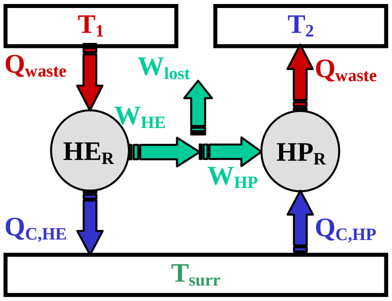
- **1st Law, Reversible HP:**

$$W_{HP} = Q_{waste} - Q_{C,HP} = Q_{waste}\left(1 - \frac{Q_{C,HP}}{Q_{waste}}\right) = Q_{waste}\left(1 - \frac{T_{surr}}{T_2}\right)$$

- **Lost Work:**

$$W_{lost} = W_{HE} - W_{HP}$$

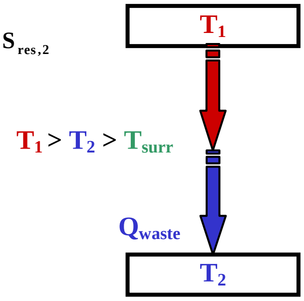
$$\boxed{W_{lost} = Q_{waste}\, T_{surr}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]}$$



$T_1 > T_2 > T_{surr}$

- We can determine W_{HE} and W_{HP} in terms of **Q**'s by applying the 1st Law to the HE and the HP, as shown here.
- The catch is that we now apply Kelvin's Principle, remembering to use absolute T when it comes time to plug in numbers.
- Now, we substitute these equations for W_{HE} and W_{HP} into our definition of lost work.
- The result is simple, sweet and familiar !
- No tricks. This is how much WORK we could have gotten out of Q_{waste} if we didn't let it spontaneously dribble away into the cold reservoir.
- Now, let's look at this problem from another angle.

S_{gen} and ΔS_{univ}



- **Definition:** $S_{gen} = \Delta S_{univ} = \Delta S_{res,1} + \Delta S_{res,2}$
- **Reservoirs are isothermal and internally reversible:**

$$\Delta S_{res,1} = \frac{-Q_{waste}}{T_1} \qquad \Delta S_{res,2} = \frac{Q_{waste}}{T_2}$$

- **Algebra:** $S_{gen} = \Delta S_{univ} = \frac{-Q_{waste}}{T_1} + \frac{Q_{waste}}{T_2}$
- **Big result:** $\boxed{S_{gen} = \Delta S_{univ} = Q_{waste}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]}$

- In this system, the universe is made up of just two reservoirs.
  - No heat is exchanged with the surroundings.
- We can easily calculate **ΔS** for each reservoir because they are isothermal.
- The result is the same one presented on slide #5.
  - This is the entropy generation associated with heat exchange.
  - It is often considered to be $\mathbf{S_{gen,ext}}$ in more complicated processes.

Conclusion

- **Lost Work:** $\boxed{W_{lost} = Q_{waste}\, T_{surr}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]}$
- **Entropy Generation:** $\boxed{S_{gen} = \Delta S_{univ} = Q_{waste}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]}$
- **Conclusion:** $\boxed{W_{lost} = T_{surr}\, S_{gen}}$

- Now, we put everything in this approach together.
- We found the 1st equation for lost work two slides back.
- On the previous slide we found the second boxed equation for $\mathbf{S_{gen}}$.
- The big conclusion is the last boxed equation.
- Lost work really IS the product of $\mathbf{T_{surr}}$ and the total entropy generation !
- Even if the process does not exchange heat with the surroundings !!
- This is weird and very cool !

8A-1 Entropy Generation and Thermal Efficiency in Power Cycles 5 pts

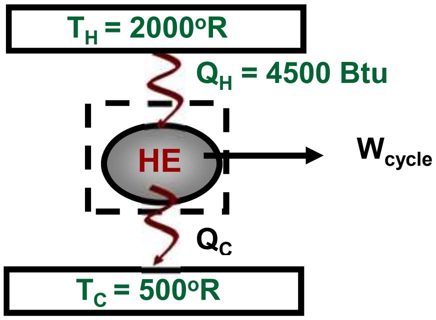
A power cycle exchanges heat with only **two** thermal reservoirs at **500°R** and **2000°R**. **Q_H = 4500 Btu/h**.
For each of the following, calculate the **rate of entropy generation** in **Btu/°R** and state whether the power cycle is internally reversible, internally irreversible or impossible. **a.)** **η = 83%**, **b.)** **η = 75%**, **c.)** **η = 44%**

Read : The key equations for this problem are the defintion of entropy generation and thermal efficiency. Because the process operates in a cycle, **ΔS = 0**. The other key point is that we are only asked to decide whether the process is **internally** reversible. That means we don't have to worry about whether heat exchange with the reservoirs is reversible. The main point is that the **temperatures** of the regions that exchange heat with the reservoirs must remain constant. This allows us to directly evaluate the integral of **δQ/T** and evaluate the entropy generation.

Given:

T_H	2000	°R	a.)	η_A	0.83
Q_H	4500	Btu	b.)	η_B	0.75
T_C	500	°R	c.)	η_C	0.44

Diagram:



Find: Parts a-c.) S_{gen} ??? Btu/°R

Internally Reversible ? Internally Irreversible ? Impossible ?

Assumptions:

1 - The system undergoes a power cycle while receiving **Q_H** at **T_H** and discharging **Q_C** at **T_C**.

2 - The region of the system that receives **heat** from the hot reservoir remains at a constant **temperature** of **T_H = 2000°R**.

3 - The region of the system that rejects **heat** to the cold reservoir remains at a constant **temperature** of **T_C = 500°R**.

Equations / Data / Solve:

Entropy generation is defined by:

$$\Delta S = \int \left(\frac{\delta Q}{T} \right) + S_{gen} \qquad \textbf{Eqn 1}$$

We can solve **Eqn 1** for S_{gen} :

$$S_{gen} = \Delta S - \int \left(\frac{\delta Q}{T} \right) \qquad \textbf{Eqn 2}$$

Since we are dealing with a cycle,
$\Delta S = 0$ and **Eqn 2** becomes:

$$S_{gen} = -\int \left(\frac{\delta Q}{T} \right) \qquad \textbf{Eqn 3}$$

In our process, the system receives **heat**, Q_H, at a constant **temperature**, T_H , and rejects **heat**, Q_C, at a constant **temperature**, T_C. Because the **temperatures** are constant, they can be pulled out of the integral in **Eqn 3** leaving :

$$S_{gen} = -\left(\frac{\int \delta Q_H}{T_H} - \frac{\int \delta Q_C}{T_C} \right) = -\left(\frac{Q_H}{T_H} - \frac{Q_C}{T_C} \right) \qquad \textbf{Eqn 4}$$

In **Eqn 4**, notice that δQ becomes $+\delta Q_H$ and $-\delta Q_C$ because of the sign convention that **heat transfer** into the system is positive.

The only variable in **Eqn 4** that we don't already know is Q_C. But we are given the value of the thermal efficiency of the power cycle in each part of this problem.

The definition of thermal efficiency is:

$$\eta = 1 - \frac{Q_C}{Q_H} \qquad \textbf{Eqn 5}$$

Rearranging **Eqn 5** to solve for Q_C yields :

$$Q_C = (1 - \eta) Q_H \qquad \textbf{Eqn 6}$$

Now, we can plug numbers into **Eqn 6** to determine Q_C and then plug Q_C and the given values of Q_H, T_H and T_C into **Eqn 4** to complete the solution.

Part (a)	Q_C	765	Btu	S_{gen}	-0.72	Btu/°R
Part (b)	Q_C	1125	Btu	S_{gen}	0.00	Btu/°R
Part (c)	Q_C	2520	Btu	S_{gen}	2.79	Btu/°R

We can now determine whether the cycle in each part of the problem is internally reversible, reversible or impossible using the following rules based on the definition of entropy generation.

If $S_{gen} = 0$	the cycle is internally reversible.
If $S_{gen} > 0$	the cycle is internally irreversible.
If $S_{gen} < 0$	the cycle is impossible.

Verify: The assumptions made in this solution cannot be verified with the given information.

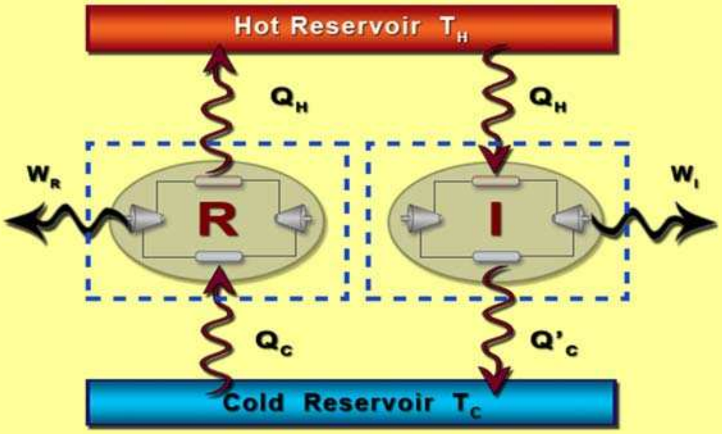
Answers :

Part (a)	S_{gen}	-0.72	Btu/°R
Part (b)	S_{gen}	0.00	Btu/°R
Part (c)	S_{gen}	2.79	Btu/°R

Part (a) is impossible.
Part (b) is internally reversible.
Part (c) is internally irreversible.

8A-2 Heat, Work and Entropy Generation 5 pts

Two power cycles operate between the same two thermal reservoirs, as shown below. **Cycle R** is reversible and **cycle I** is irreversible.



They each absorb the same amount of **heat** from the hot reservoir, Q_H, but produce different amounts of **work**, W_R and W_I, and reject different amounts of **heat** to the cold reservoir, Q_C and Q'_C.

a.) Derive an equation for S_{gen} for the irreversible cycle in terms of W_I, W_R, and T_C only.

b.) Show that W_I W_R **and** Q'_C Q_C.

Read : Start with the equation for the **entropy generated** and do an energy balance on both the reversible and irreversible cycles. Put the equations together and simplify to get an equation in the desired terms.

Given: A reversible power cycle, **R**, and an irreversible power cycle, **I**, operate between the same two reservoirs.

Find:

Part (a) Evaluate S_{gen} for cycle **I** in terms of W_I, W_R, and T_C.

Part (b) Show that: $W_I < W_R$ and $Q'_C > Q_C$.

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - The systems shown undergo power cycles. **R** is reversible and **I** is irreversible.

2 - Each system receives Q_H at a constant **temperature** region at T_H from the hot reservoir and rejects **heat,** Q_C, at a constant **temperature** region at T_C to the cold reservoir.

Equations / Data / Solve:

Part a.) Let's begin with the defintion of entropy generation:

$$\Delta S = \int \left(\frac{\delta Q}{T} \right) + S_{gen}$$ **Eqn 1**

We can solve **Eqn 1** for S_{gen} :

$$S_{gen} = \Delta S - \int \left(\frac{\delta Q}{T} \right)$$ **Eqn 2**

Since we are dealing with a cycle,
ΔS = 0 and **Eqn 2** becomes:

$$S_{gen} = -\int \left(\frac{\delta Q}{T} \right)$$ **Eqn 3**

In the irreversible process, the system receives **heat**, Q_H, at a constant **temperature**, T_H , and rejects **heat**, Q'_C, at a constant **temperature**, T_C. Because the **temperatures** are constant, they can be pulled out of the integral in **Eqn 3** leaving :

$$\Delta S = -\left(\frac{Q_H}{T_H} - \frac{Q'_C}{T_C} \right)$$ **Eqn 4**

The 1st Law for cycles is:

$$Q_{cycle} = W_{cycle}$$ **Eqn 5**

We can apply **Eqn 5** to both the reversible and the irreversible cycles, as follows :

$$Q_H = W_R + Q_C$$ **Eqn 6**

$$Q_H = W_I + Q'_C$$ **Eqn 7**

We can combine **Eqns 6** & **7** to obtain :

$$W_R + Q_C = W_I + Q'_C$$ **Eqn 8**

Now, solve **Eqn 8** for Q'_C :

$$Q'_C = Q_C + W_R - W_I$$ **Eqn 9**

Next, we can use **Eqn 9** to
eliminate Q'_C from **Eqn 4** to get :

$$S_{gen} = -\left(\frac{Q_H}{T_H} - \frac{Q_C + W_R - W_I}{T_C} \right)$$ **Eqn 10**

We can rearrange **Eqn 10** slightly to
make it more clear how to proceed :

$$S_{gen} = -\left[\left(\frac{Q_H}{T_H} - \frac{Q_C}{T_C} \right) - \frac{W_R - W_I}{T_C} \right]$$ **Eqn 11**

Because **R** is a reversible cycle and
we use the Kelvin Temperature Scale :

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$ **Eqn 12**

Eqn 12 can be rearranged to help simplify **Eqn 11** :

$$\frac{Q_C}{T_C} = \frac{Q_H}{T_H}$$ **Eqn 13**

This yields :

$$S_{gen} = \frac{W_R - W_I}{T_C}$$ **Eqn 14**

Part b.) Because irreversibilities are present in cycle **I** :

$$S_{gen} = \frac{W_R - W_I}{T_C} > 0 \qquad \textbf{Eqn 15}$$

Rearranging **Eqn 15** gives us :

$$W_R > W_I \qquad \textbf{Eqn 16}$$

Finally, we can rearrange **Eqn 9** to help us determine whether Q'_C or Q_C is larger :

$$Q'_C - Q_C = W_R - W_I \qquad \textbf{Eqn 17}$$

Since **Eqn 16** tells us that $W_R > W_I$, **Eqn 17** tells that :

$$Q'_C > Q_C \qquad \textbf{Eqn 18}$$

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers : **Part a.)**

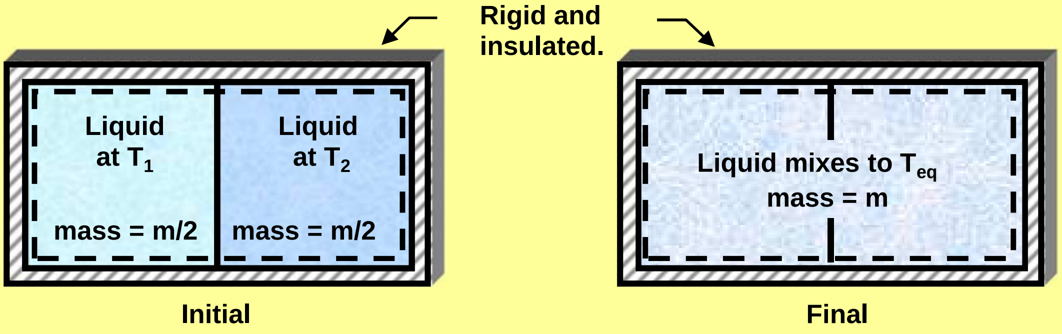
$$\boxed{S_{gen} = \frac{W_R - W_I}{T_C}}$$

Part b.)

$$\boxed{W_R > W_I} \qquad \boxed{Q'_C > Q_C}$$

8A-3 Entropy Production of Mixing Two Liquids at Different Temperatures 8 pts

The initial and final states of a sealed, insulated, rigid tank are shown below.Each side of the tank contains a different incompressible liquid at a different **temperature**, T_1 and T_2.



The **mass** of liquid initally on each side of the tank is the same: $m_1 = m_2 = m/2$. The barrier between the two sides of the tank is removed and the two liquids mix and eventually reach the final equilibrium state.

Assume each liquid has a constant **heat capacity** and there are no thermal effects due to the mixing of the fluids.

a.) Show that S_{gen} is given by the following equation:

$$S_{gen} = m\,C\,Ln\left[\frac{T_1 + T_2}{2\,(T_1T_2)^{1/2}}\right]$$

b.) Show that S_{gen} must be positive.

Read : For **part (a)** Perform an entropy balance to determine an equation for S_{gen}. Then perform an energy balance to determine an expression for the final **temperature** and substitute the expression into S_{gen} and simplify.

Given:

Initial State:		Final State :	
Chamber 1 :	T_1	**Chamber 1 :**	T_{eq}
Chamber 2 :	T_2	**Chamber 2 :**	T_{eq}

Incompressible fluids with $C_P = C_V = C$.

Find: **Part (a)** Show that the amount of **entropy generated** is:

$$S_{gen} = m\,C\,Ln\left[\frac{T_1 + T_2}{2\,(T_1T_2)^{1/2}}\right] \qquad \text{Eqn 1}$$

Part (b) Demonstrate that S_{gen} must be positive.

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - The system consists of the total **mass** of liquid in the entire tank.
2 - The system is isolated (adiabatic and closed).
3 - The liquid is incompressible with constant **specific heat**, **C**.
4 - No **work** crosses the sytem boundary.

Equations / Data / Solve:

Part a.)

Let's begin with the defintion of **entropy generation**:

$$\Delta S = \int \left(\frac{\delta Q}{T} \right) + S_{gen}$$

Eqn 2

We can solve **Eqn 2** for S_{gen} :

$$S_{gen} = \Delta S - \int \left(\frac{\delta Q}{T} \right)$$

Eqn 3

Since the system is isolated,
there is no **heat transferred**:

$$\int \left(\frac{\delta Q}{T} \right) = 0$$

Eqn 4

We can use **Eqn 4** to simplify **Eqn 3**, yielding :

$$S_{gen} = \Delta S$$

Eqn 5

The change in the **entropy** of the system is :

$$\Delta S = m \hat{S}_{final} - m \hat{S}_{init}$$

Eqn 6

$$\Delta S = m \hat{S}_{final} - \left[\frac{m}{2} \hat{S}_1 + \frac{m}{2} \hat{S}_2 \right]$$

Eqn 7

We can rearrange **Eqn 7** to show that the total change in **entropy** for the system is the sum of the changes in **entropy** of each of the two fluids.

$$\Delta S = \frac{m}{2} \left[\left(\hat{S}_{final} - \hat{S}_1 \right) + \left(\hat{S}_{final} - \hat{S}_2 \right) \right]$$

Eqn 8

The **entropy change** for an incompressible fluid
depends only on **temperature**.

$$\Delta \hat{S} = \int_{T_{init}}^{T_{final}} \frac{\hat{C}}{T} dT$$

Eqn 9

Because the **heat capacity** in this problem is a constant,
it is relatively easy to integrate **Eqn 9** to get:

$$\Delta \hat{S} = \hat{C}_{avg} \, Ln \left[\frac{T_{final}}{T_{init}} \right]$$

Eqn 10

Next, apply **Eqn 10** to determine the **entropy change** of each fluid in this process and substitute the result into **Eqn 8** :

$$\Delta S = \frac{m}{2} \hat{C} \left[Ln \frac{T_{final}}{T_1} + Ln \frac{T_{final}}{T_2} \right]$$

Eqn 11

Properties of logarithms let us rearrange **Eqn 11** to :

$$\Delta S = \frac{m}{2} \hat{C} Ln \left[\frac{T_{final}^2}{T_1 T_2} \right]$$

Eqn 12

Combining **Eqn 12** with **Eqn 5** gives us :

$$S_{gen} = \frac{m}{2} \hat{C} Ln \left[\frac{T_{final}^2}{T_1 T_2} \right]$$

Eqn 13

To complete this derivation, we must eliminate T_{final} from **Eqn 13**. We can determine T_{final} in terms of T_1 and T_2 by applying the 1st Law to this process.

$$\Delta U = Q - W$$

Eqn 14

No **work** or **heat** crosses the system boundary, so **Eqn 14** becomes :

$$\Delta U = 0$$

Eqn 15

Now, use the constant **specific heat** of the incompressible fluid to determine Δ**U** :

$$\frac{m}{2}\left[\left(\hat{U}_{final}-\hat{U}_1\right)+\left(\hat{U}_{final}-\hat{U}_2\right)\right]=0$$ **Eqn 16**

$$\frac{m}{2}\,\hat{C}\left[\left(T_{final}-T_1\right)+\left(T_{final}-T_2\right)\right]=0$$ **Eqn 17**

Now, solve **Eqn 17** for **T_{final}** :

$$T_{final}=\frac{T_1+T_2}{2}$$ **Eqn 18**

Now, we can use **Eqn 18** to eliminate **T_{final}** from **Eqn 13** :

$$S_{gen}=\frac{m}{2}\,\hat{C}\,Ln\left[\frac{1}{T_1T_2}\left(\frac{T_1+T_2}{2}\right)^2\right]$$ **Eqn 19**

Simplify **Eqn 19** algebraically :

$$S_{gen}=\frac{m}{2}\,\hat{C}\left[2\cdot Ln\left[\frac{1}{\left(T_1T_2\right)^{1/2}}\left(\frac{T_1+T_2}{2}\right)\right]\right]$$ **Eqn 20**

Finally :

$$S_{gen}=m\,\hat{C}\,Ln\left[\frac{T_1+T_2}{2\left(T_1T_2\right)^{1/2}}\right]$$ **Eqn 21**

Part b.) **Entropy generation** is non-negative when :

$$m\,\hat{C}\,Ln\left[\frac{T_1+T_2}{2\left(T_1T_2\right)^{1/2}}\right]\geq 0$$ **Eqn 22**

The values of **m** and **C** must be positive so, **S_{gen}** is non-negative when :

$$\frac{T_1+T_2}{2\left(T_1T_2\right)^{1/2}}\geq 1$$ **Eqn 23**

Simplify **Eqn 23** by algebraic manipulation, as follows :

$$T_1+T_2\geq 2\left(T_1T_2\right)^{1/2}$$ **Eqn 24**

Squaring both sides of **Eqn 24** yields :

$$\left(T_1+T_2\right)^2\geq 4\left(T_1T_2\right)$$ **Eqn 25**

(This is **OK** because **T_1 > 0 K** and **T_2 > 0 K**)

Expand the left-hand side of **Eqn 25** :

$$T_1^2+2T_1T_2+T_2^2\geq 4T_1T_2$$ **Eqn 26**

$$T_1^2-2T_1T_2+T_2^2\geq 0$$ **Eqn 27**

Finally, we get :

$$\left(T_1-T_2\right)^2\geq 0$$ **Eqn 28**

The inequality in **Eqn 28** is satisfied for either **T_1 > T_2** or **T_2 > T_1**.
The equality in **Eqn 28** is satisfied only when **T_1 = T_2**.

Verify: The assumptions made in this solution cannot be verified with the given information.

Answers : **Part a.)**

$$\boxed{S_{gen}=m\,\hat{C}\,Ln\left[\frac{T_1+T_2}{2\left(T_1T_2\right)^{1/2}}\right]}$$

Part b.)

$$\boxed{S_{gen}\geq 0 \qquad \textbf{when :}\quad \left(T_1-T_2\right)^2\geq 0 \qquad \textbf{which is ALWAYS true !}}$$

8A-4 Entropy Change For R-134a Compression in Piston-and-Cylinder Device 6 pts

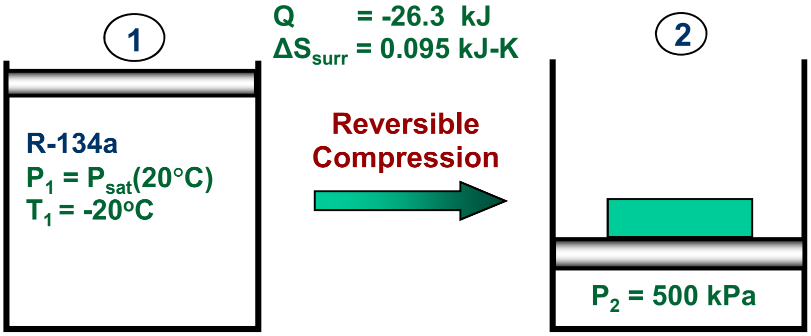
Four kilograms Saturated **R-134a** vapor at **-20°C** is compressed in a piston-and-cylinder device until the **pressure** reaches **500 kPa**.

During the process, **26.3 kJ** of **heat** is lost to the surroundings, resulting in an increase in the **specific entropy** of the surroundings of **0.095 kJ/K**.

Assuming the process is completely reversible, calculate the **work** for this compression process in **kJ**.

Read : Use the 2nd Law and the fact that the process is completely reversible to determine S_2. This gives you the second intensive property you need to evaluate U_2. Then, use the 1st Law to determine W_b for the compression process.

Diagram:



Given:	m	4	kg	P_2	500	kPa
	x_1	1	kg vap/kg	Q	-26.3	kJ
	T_1	-20	°C	ΔS_{surr}	0.095	kJ/K
	P_1	132.73	kPa			

Find: W_b ??? kJ

Assumptions:

1 - As shown in the diagram, the system is the **R-134a** inside the cylinder.
2 - **Boundary work** is the only form of **work** that crosses the system boundary.
3 - Changes in kinetic and potential energies are negligible.
4 - The compression process is completely reversible, so there is no **entropy generated** and no **entropy change** of the universe.

Equations / Data / Solve:

To determine the **work** required we need to apply the 1st Law for closed systems:

$$Q - W = \Delta U + \Delta E_K + \Delta E_P \qquad \textbf{Eqn 1}$$

When we assume that changes in kinetic and potential energies are negligible and we assume **boundary work** is the only form of **work**, **Eqn 1** becomes:

$$Q - W_b = \Delta U = m \cdot \left(\hat{U}_2 - \hat{U}_1\right) \qquad \textbf{Eqn 2}$$

We can now solve **Eqn 2** for $\mathbf{W_b}$:

$$W_b = Q - m\cdot\left(\hat{U}_2 - \hat{U}_1\right)$$ **Eqn 3**

All we need to do is determine $\mathbf{U_2}$ and $\mathbf{U_1}$ and then we can use **Eqn 3** to calculate $\mathbf{W_b}$ and complete this problem.

Start with $\mathbf{U_1}$ because we were given $\mathbf{T_1}$ and and it is a saturated vapor, so we can immediately look-up $\mathbf{U_1}$ in the **Saturated R-134a Table**.

$\mathbf{U_1}$ **366.99** kJ/kg

We know $\mathbf{P_2}$, but we need to know the value of <u>two</u> intensive properties before we can use the **R-134a Tables** to look-up $\mathbf{U_2}$.

Because the process is <u>completely</u> reversible:

$$S_{gen} = \Delta S_{univ} = 0$$ **Eqn 4**

In this case, the universe consists of <u>two</u> parts: the system and the surroundings. As a result, **Eqn 4** becomes:

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr} = 0$$ **Eqn 5**

We were given $\mathbf{\Delta S_{surr}}$, so we need to consider $\mathbf{\Delta S_{sys}}$ further in order to use **Eqn 5**.

We can express the **entropy change** of the system in terms of the <u>initial</u> and <u>final</u> **states** as follows.

$$\Delta S_{sys} = m\cdot\left(\hat{S}_2 - \hat{S}_1\right)$$ **Eqn 6**

Next, we can <u>combine</u> **Eqn 5** and **Eqn 6** and solve for $\mathbf{S_2}$.

$$m\cdot\left(\hat{S}_2 - \hat{S}_1\right) + \Delta S_{surr} = 0$$ **Eqn 7**

$$\hat{S}_2 = \hat{S}_1 - \frac{\Delta S_{surr}}{m}$$ **Eqn 8**

We can evaluate $\mathbf{S_1}$ because we were given $\mathbf{T_1}$ and and it is a saturated vapor.

$\mathbf{S_1}$ **1.7413** **kJ/kg-K**

Now, we can plug values into **Eqn 8** to evaluate $\mathbf{S_2}$:

$\mathbf{S_2}$ **1.7176** **kJ/kg-K**

This gives us the value of a <u>second</u> intensive property for **state 2** which allows us to calculate $\mathbf{U_2}$.

At **P = 500 kPa** :

$\mathbf{S_{sat\,liq}}$ **1.0759** **kJ/kg-K**

$\mathbf{S_{sat\,vap}}$ **1.7197** **kJ/kg-K**

Since $\mathbf{S_{sat\,liq} < S_2 < S_{sat\,vap}}$, **state 2** is a saturated mixture.

Determine $\mathbf{x_2}$ from the **specific entropy**, using:

$$x_2 = \frac{\hat{S}_2 - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}}$$ **Eqn 8**

$\mathbf{x_2}$ **0.9967** **kg vap/kg**

Then, we can use the **quality** to determine $\mathbf{H_{5S}}$, using:

$$\hat{U}_2 = x_2\,\hat{U}_{sat\,vap} + \left(1 - x_2\right)\hat{U}_{sat\,liq}$$ **Eqn 9**

At **P = 20 psia** :

$\mathbf{U_{sat\,liq}}$ **221.10** $\mathbf{Btu/lb_m}$

$\mathbf{U_{sat\,vap}}$ **386.91** $\mathbf{Btu/lb_m}$

$\mathbf{U_2}$ **386.36** $\mathbf{Btu/lb_m}$

Now, we can use $\mathbf{U_2}$ in **Eqn 3** to evaluate $\mathbf{W_b}$ and finish this problem.

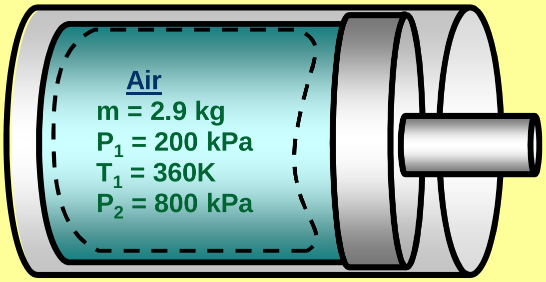
$\mathbf{W_b}$ **-103.80** **kJ**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : $\mathbf{W_b}$ **-104** **kJ**

8A-5 Entropy Production for the Adiabatic Compression of Air 6 pts

Air is compressed in an adiabatic piston-and-cylinder device, as shown below, from **200 kPa** and **360 K** to **800 kPa**.



a.) Calculate the final **temperature**, T_2, and the boundary **work** if the process is internally reversible.

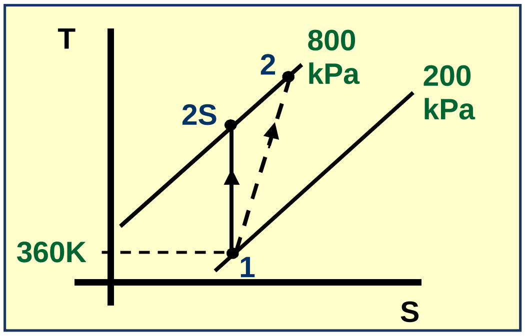
b.) Calculate T_2 and the **entropy generation** if a real piston-and-cylinder device requires **15%** more **work** than the internally reversible device.

Read : Assume ideal gas behavoir for air. Apply an energy balance and an entropy balance.
Notice that in **part (a)** the problem asks for the "work **required**", therefore our answer will be positive.
To get **S** and **U** data, use the 2nd Gibbs Equation in terms of the Ideal Gas Entropy Function.

Given:	T_1	360	K		m	2.9	kg
	P_1	200	kPa		Q	0	KJ
	P_2	800	kPa	**Part (b)**	$W_{part\ (b)}$ =	15%	> $W_{part\ (a)}$

Find:	**Part (a)**	T_{2S}	???	K	**Part (b)**	T_2	???	K
		$-W_b$	???	kJ		S_{gen}	???	kJ/K
						W_{lost}	???	kJ

Diagram:



Assumptions:

1 - As shown in the diagram, the system is the air inside the cylinder.
2 - Air is modeled as an ideal gas.
3 - No **heat transfer** occurs.
4 - **Boundary work** is the only form of **work** that crosses the system boundary.
5 - Changes in kinetic and potential energies are negligible.
6 - For **Part (a)**, there is no **entropy generated**.

Equations / Data / Solve:

Part a.) To determine the **work** required we need to apply the 1st Law for closed systems:

$$Q - W = \Delta U + \Delta E_K + \Delta E_P \qquad \textbf{Eqn 1}$$

Because the process is adiabatic and we assume that changes in kinetic and potential energies are negligible and we assume **boundary work** is the only form of **work**, **Eqn 1** becomes:

$$\Delta U = -W_b \qquad \textbf{Eqn 2}$$

or:

$$-W_b = m\left(\hat{U}_2 - \hat{U}_1\right) \qquad \textbf{Eqn 3}$$

So, in order to answer **part (a)**, we need to determine **U** for both the initial and final states.
Use the **Ideal Gas Property Table** for air to evaluate U_1 and U_2, but first we must know T_1 and T_2.

Because this process is both adiabatic and internally reversible, the process is **isentropic**.
In this problem, we have air and we assume it behaves as an ideal gas.
We can solve this problem using the ideal gas entropy function.

The 2nd Gibbs Equation in terms of the S^o is:

$$\hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW} \ln \frac{P_2}{P_1} \qquad \textbf{Eqn 4}$$

Since **part (a)** is an isentropic process, **Eqn 4** becomes:

$$0 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW} \ln \frac{P_2}{P_1} \qquad \textbf{Eqn 5}$$

The final **temperature**, T_2, can be determined from determining $S^o(T_2)$ and then interpolating on the **Ideal Gas Properties Table** for air.

Solving **Eqn 5** for $S^o(T_2)$ yields:

$$\hat{S}^o_{T2} = \hat{S}^o_{T1} + \frac{R}{MW} \ln \frac{P_2}{P_1} \qquad \textbf{Eqn 6}$$

Properties for **state 1** are determined from **Ideal Gas Properties Table** for air.

$S^o(T_1)$	0.189360	kJ/kg-K
$U(T_1)$	44.3940	kJ/kg

Now, we can plug values into Eqn 3:

R	8.314	kJ/kmol K
MW	28.97	kg/kmol
$S^o(T_2)$	0.58721	kJ/kg-K

Now, we can go back to the **Ideal Gas Properties Table** for air and determine T_2 and U_2 by interpolation.

T (K)	U^o kJ/kg	S^o kJ/kg-K
520	163.42	0.56803
T_2	U_2	0.58721
530	171.04	0.58802

T_2	529.59	K
U_2	170.73	kJ/kg

Put values into **Eqn 3** to finish this part of the problem:

$-W_b$	**366.38**	kJ

Part b.)

The actual **work** is **15%** greater than the **work** determined in **Part (a)**:

$$-W_{part\,(b)} = 1.2\left(-W_{part\,(a)}\right) \qquad \text{Eqn 7}$$

$-W_b$ **421.33** **kJ**

In this part of the problem, we know the actual **work**, but we don't know **T_2** or **U_2**.

We can solve **Eqn 3** for **U_2** interms of the known variables **m**, **W_b** and **U_1** :

$$\hat{U}_2 = \frac{-W_{part\,(b)}}{m} + \hat{U}_1 \qquad \text{Eqn 8}$$

Plugging values into **Eqn 7** yields :

U_2 **189.68** **kJ/kg**

Next, we can determine **T_2** and **$S^o(T_2)$** by interpolating on the **Ideal Gas Properties Table** for air.

U^o kJ/kg	T (K)	S^o kJ/kg-K
186.36	550	0.62702
189.68	T_2	$S^o(T_2)$
194.05	560	0.64605

T_2	554.3	K
$S^o(T_2)$	0.63524	kJ/kg-K

The 2nd Law in terms of the **entropy generated** is:

$$S_{gen} = \Delta S - \int\left(\frac{\delta Q}{T}\right) \qquad \text{Eqn 9}$$

Since there is no **heat transfer** in this problem:

$$S_{gen} = \Delta S = m \cdot \Delta\hat{S} \qquad \text{Eqn 10}$$

The **entropy change** can be determined from **Eqn 4**:

$$\Delta\hat{S} = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\,\mathrm{Ln}\,\frac{P_2}{P_1} \qquad \text{Eqn 11}$$

Now, we can plug values into **Eqns 9** & **10** to determine the **entropy generated**:

S_{gen} **0.04803** **kJ/K**

Verify:

The ideal gas assumption needs to be verified.

$$\tilde{V} = \frac{RT}{P} \qquad \text{Eqn 12}$$

We need to determine the **specific volume** at each **state** and check if :

$$\tilde{V} > 5 \;\; L/mol \qquad \text{Eqn 13}$$

V_1	14.97	L/mol
V_{2A}	5.50	L/mol
V_{2B}	5.76	L/mol

The **specific volume** at each **state** is greater than **5 L/mol** for all **states** and the working fluid can be treated as a diatomic gas, so the ideal gas assumption is valid.

Answers :

Part a.)

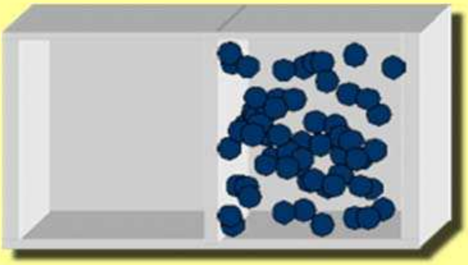
T_{2S}	530	K
$-W_b$	366	kJ

Part b.)

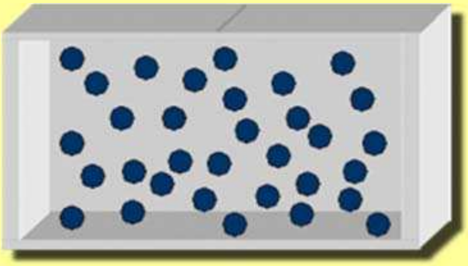
T_2	554	K
S_{gen}	0.0480	kJ/K

8A-6 Entropy Change as Compressed Liquid Water Expands 5 pts

Consider the rigid tank shown below. It is divided into two equal **volumes** by a barrier. The left-hand side (**LHS**) is a perfect vacuum and the right-hand side (**RHS**) contains **5 kg** of **ammonia** at **300 kPa** and **-10°C**.



When the barrier is removed, the **ammonia** expands and fills the entire tank.



When the **ammonia** reaches equilibrium, the **pressure** in the tank is **200 kPa**. Calculate **ΔS** for the **ammonia** and **Q** for this process.

Read : The key to this problem is that the **mass** of ammonia in the system does not change and the **volume** doubles. We can use the Ammonia Tables to determine the **specific volume** and **specific entropy** at **state 1** because we know **T_1** and **P_1**. We can use the **specific volume** at **state 1** and the known **mass** and **volume** relationships to determine the **specific volume** at **state 2**. This gives us the values of two intensive properties at **state 2**, **P_2** and **specific volume**, and allows us to use the Ammonia Tables to determine the **specific entropy** and the total **entropy** at **state 2**. **$\Delta S = S_2 - S_1$** and we are done.

The 2nd Law and the fact that **entropy generation** must be positive will allow us to determine the direction of **heat transfer** or if the process could be adiabatic.

Given:

m	5	kg	P_2	200	kPa
T_1	-10	°C	$V_2 = 2\,V_1$		Eqn 1
P_1	300	kPa			

Find: ΔS ??? kJ/K **Determine whether:** **Q = 0**, **Q > 0** or **Q < 0**

Diagram: See the problem statement.

Assumptions:

1 - The system is the contents of the entire tank.
2 - No **work** or **mass** crosses the system boundary.
3 - Changes in kinetic and potential energies are negligible.

Equations / Data / Solve:

The <u>change</u> in **entropy** can be calculated using:

$$\Delta S = m\left(\hat{S}_2 - \hat{S}_1\right) \qquad \textbf{Eqn 2}$$

We know <u>both</u> $\mathbf{T_1}$ and $\mathbf{P_1}$, so we can look up $\mathbf{S_1}$ in the Subcooled Liquid Table of the Ammonia Tables or in the **NIST Webbook**.

$\mathbf{S_1}$ **0.54252** **kJ/kg-K**

At **state 2**, we <u>only</u> know the value of <u>one</u> intensive variable, $\mathbf{P_2}$. So, we need to determine the value of <u>another</u> intensive variable before we can use the Ammonia Tables to determine $\mathbf{S_2}$.

We can determine the **specific volume** at **state 2** as follows:

$$V_1 = m\,\hat{V}_1 \qquad \textbf{Eqn 3}$$

We can obtain **specific volume** at **state 1** from the Ammonia Tables or the **NIST Webbook** :

$\mathbf{V_1}$ **0.0015336** $\mathbf{m^3/kg}$ $\mathbf{V_1}$ **0.0076680** $\mathbf{m^3}$

Then, we can use the given relationship in **Eqn 1** to determine $\mathbf{V_2}$: $\mathbf{V_2}$ **0.015336** $\mathbf{m^3}$

Then, determine the **specific volume** at **state 2** using:

$$\hat{V}_2 = \frac{V_2}{m} \qquad \textbf{Eqn 4}$$

$\mathbf{V_2}$ **0.0030672** $\mathbf{m^3/kg}$

Now, we know the values of <u>two</u> intensive variables, so we can go back to the Ammonia Tables or **NIST Webbook** and determine $\mathbf{S_2}$ by <u>interpolation</u>.

At P_2 = 200 kPa :

$\mathbf{V_{sat\ liq}}$ **0.0015068** $\mathbf{m^3/kg}$

$\mathbf{V_{sat\ vap}}$ **0.5946** $\mathbf{m^3/kg}$

Since $\mathbf{V_{sat\ liq} < V_2 < V_{sat\ vap}}$, **state 2** is a saturated mixture.

We can determine $\mathbf{x_2}$ from the **specific volume**, using:

$$x_2 = \frac{\hat{V}_2 - \hat{V}_{sat\,liq}}{\hat{V}_{sat\,vap} - \hat{V}_{sat\,liq}} \qquad \textbf{Eqn 5}$$

$\mathbf{x_2}$ **0.002631** **kg vap/kg**

Then, we can use the **quality** to determine $\mathbf{S_2}$, using:

$$\hat{S}_2 = x_2\,\hat{S}_{sat\,vap} + (1 - x_2)\,\hat{S}_{sat\,liq} \qquad \textbf{Eqn 6}$$

At P_2 = 200 kPa :

$\mathbf{S_{sat\ liq}}$ **0.387505** **kJ/kg-K**

$\mathbf{S_{sat\ vap}}$ **5.5998** **kJ/kg-K** $\mathbf{S_2}$ **0.40122** **kJ/kg-K**

Finally, we can plug values into **Eqn 2** : $\mathbf{\Delta S}$ **-0.70651** **kJ/K**

The 2nd Law tells us that:

$$\Delta S = \int\left(\frac{\delta Q}{T}\right) + S_{gen} \qquad \textbf{Eqn 7}$$

where $\mathbf{S_{gen} > 0}$ and $\mathbf{T > 0}$ because it is a thermodynamic **temperature** scale, such as the Kelvin scale.

Therforefore, the <u>only</u> way for Δ**S** to be negative is <u>if</u> δ**Q < 0**.

We conclude that **heat** <u>must</u> have been <u>transferred</u> <u>**out**</u> of the system during this process !

We <u>could</u> apply the 1st Law to evaluate **Q**, but it is not required. I got **Q = -184.6 kJ**.

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

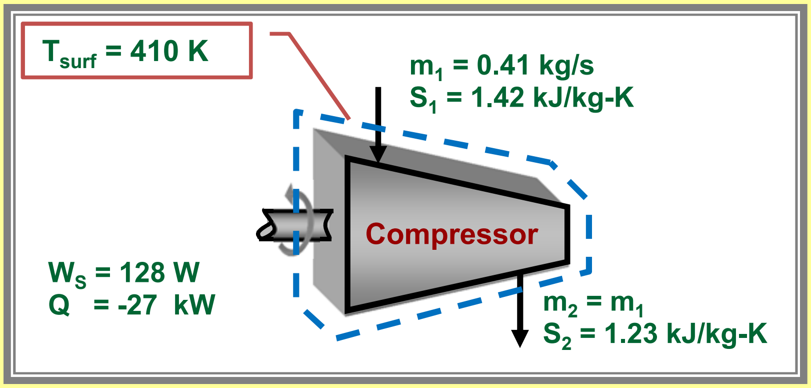
Answers :

Part a.) **ΔS** **-0.707** **kJ/K**

Part b.) **Q < 0** **Heat was transferred <u>out</u> of the system during this process !**

8B-1 Entropy Generation in a Compressor 2 pts

Consider the compressor shown below.



Determine the **entropy generation rate** within the compressor in **kW/K**. Is this compressor internally reversible, internally irreversible or impossible?

Read : This is a straightforward application of the 2nd Law in the form of an entropy balance on an open system. Use $\mathbf{T_{surf}}$ in the 2nd Law equation to determine $\mathbf{S_{gen}}$ for only the turbine. Finish this problem by using the sign of $\mathbf{S_{gen}}$ to determine whether the compressor is impossible, internally reversible or internally rreversible.

Diagram: See the problem statement.

Given:

m	0.41	kg/s	Q	-27	kW
S_1	1.42	kJ/kg-K	T_{surf}	410	K
S_2	1.23	kJ/kg-K	W_S	128	kW

Find: S_{gen} ??? kW/K

Is this compressor **internally reversible**, **internally irreversible** or **impossible**?

Assumptions:

1 - The compressor operates at steady-state.

2 - Kinetic and potential energy changes are negligible.

3 - **Shaft work** and **flow work** are the only forms of **work** that cross the system

4 - **Heat** loss from the compressor occurs at a constant and uniform **temperature** of 410 K.

Equations / Data / Solve:

We can determine the **entropy generation** from an entropy balance on the compressor.

The entropy balance equation for a **SISO** process operating at steady-state that exchanges **heat** only with the surroundings is:

$$\left(\hat{S}_{gen}\right)_{comp} = \hat{S}_2 - \hat{S}_1 - \frac{\hat{Q}}{T_{HT}} \qquad \textbf{Eqn 1}$$

Because we are interested only in the entropy generation **inside** the turbine, the **temperature** at which heat transfer occurs is the surface **temperature** of the turbine, **410 K**. If we used $T_{HT} = T_{surr}$, we would obtain the total **entropy generation** for the process. This would include both the **entropy generated** inside the turbine and the entropy generated due to the irreversible nature of **heat transfer** through a finite **temperature** difference, that is between T_{surf} and T_{surr}.

We were given the values of all the variables on the right-hand side of **Eqn 1**, so we can immediately evaluate $(S_{gen})_{comp}$.

$(S_{gen})_{comp}$	**-0.01205**	**kJ/kg-K**

If the $(S_{gen})_{comp}$ is …

... negative, the compressor is impossible

... **zero**, the compressor is reversible

... positive, the compressor is irreversible

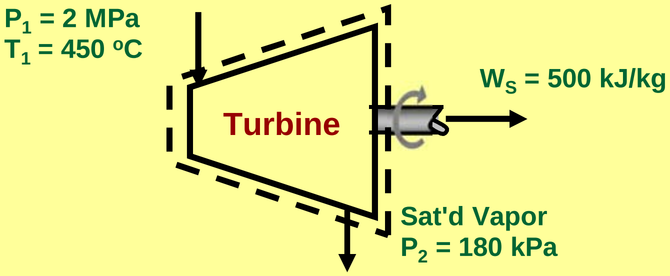
This compressor is impossible because $\Delta(S_{gen})_{comp} < 0$.

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **This compressor is impossible because $\Delta(S_{gen})_{comp} < 0$.**

8B-2 Entropy Generation in a Steam Turbine 5 pts

The outer surface of a steam turbine is at an average **temperature** of 160°C and the surroundings are at 20°C. Calculate the internal, external and total **entropy generation** for the turbine in **kJ/kg-K**.
The operating parameters for the turbine are given in the figure below.



Read : Apply the 1st Law to determine **Q** and the 2nd Law to get $\mathbf{S_{gen}}$. Properties come from the Steam Tables or the **NIST Webbook**. The key is that the **heat** losses occur at the constant, average surface **temperature** and this must be taken into account when evaluating $\mathbf{S_{gen}}$ for the turbine.

Given:	P_1	2000	kPa	x_2	1	
	T_1	450	°C	T_{HT}	160	°C
	P_2	180	kPa	T_{surr}	20	°C
				W_S	500	kJ/kg
Find:	$(S_{gen})_{turb}$	???	kJ/kg-K			

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - The turbine operates at steady-state.
2 - Kinetic and potential energy changes are negligible.
3 - **Shaft work** and **flow work** are the only forms of **work** that cross the system boundary.
4 - **Heat** loss from the turbine occurs at a constant and uniform **temperature** of 160°C.

Equations / Data / Solve:

We can determine the **entropy generation** from an entropy balance on the turbine.

The entropy balance equation for a **SISO** process operating at steady-state that exchanges **heat** only with the surroundings is:

$$\left(\hat{S}_{gen}\right)_{turb} = \hat{S}_2 - \hat{S}_1 - \frac{\hat{Q}}{T_{HT}} \qquad \textbf{Eqn 1}$$

Because we are interested only in the entropy generation **inside** the turbine, the **temperature** at which heat transfer occurs is the surface **temperature** of the turbine, **160°C**. If we used **T_{HT} = T_{surr}**, we would obtain the total **entropy generation** for the process. This would include both the **entropy generated** inside the turbine and the entropy generated due to the irreversible nature of **heat transfer** through a finite **temperature** difference, that is between **T_{HT}** and **T_{surr}**.

We can lookup **S_1** and **S_2** in the Steam Tables or the **NIST Webbook** because **states 1** and **2** are completely determined by the information given in the problem statement.

S_1	**7.2866**	**kJ/kg-K**	**S_2**	**7.1621**	**kJ/kg-K**

Now, use the 1st Law for a steady-state process with negligible changes in kinetic and potential energies to determine **Q**.

$$\hat{Q} - \hat{W}_S = \Delta\hat{H} \qquad \textbf{Eqn 2}$$

Solve **Eqn 2** for **Q** :

$$\hat{Q} = \hat{W}_S + \hat{H}_2 - \hat{H}_1 \qquad \textbf{Eqn 3}$$

We can now lookup **H_1** and **H_2** in the Steam Tables or the **NIST Webbook** because, again, **states 1** and **2** are completely determined by the information given in the problem statement.

H_1	**3358.2**	**kJ/kg**	**H_2**	**2701.4**	**kJ/kg**

Now we can plug values into **Eqn 2** to evaluate **Q** : **Q** **-156.81** **kJ/kg**

We can now evaluate the **entropy generation** within the turbine using **Eqn 1**.

$(S_{gen})_{turb}$ **0.2375** **kJ/kg-K**

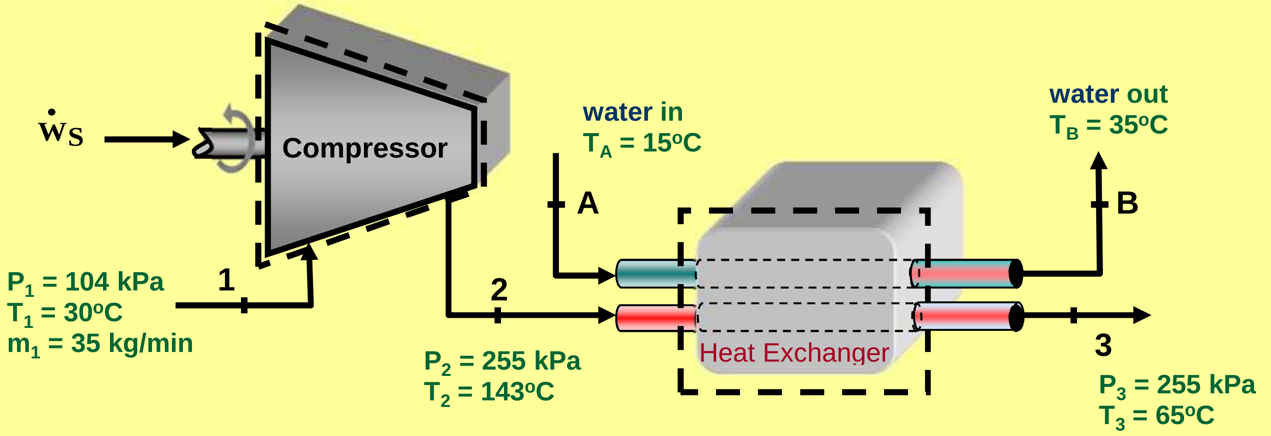
Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

Q	**-157**	**kJ/kg**	**S_{gen}**	**0.237**	**kJ/kg-K**

8B-3 Ideal Gas Compressor and Heat Exchanger Combination 8 pts

The effluent from an **air** compressor is cooled by contacting with **water** in a heat exchanger. The operating parameters for this system are given in the diagram, below.



Assume **air** behaves as an ideal gas and heat losses to the surroundings are negligible.

a.) Calculate the **power** requirement for the compressor and the required cooling water **mass flow rate** for the heat

b.) Calculate the **entropy production rate** for the compressor and for the heat exchanger separately.

Read : Use the ideal gas EOS and the **volumetric flow rate** to determine the **mass flow rate**.
Use an energy balance to determine the **work** for the compressor in **kW**.
To determine the water **flow rate**, draw the control volume enclosing the heat exchanger. This control volume has four **mass flows** entering or leaving but no **Q** or **W**. An energy balance on this control volume yields the water **flow rate**.

Very important point: the air and the water DO NOT MIX in the heat exchanger!

For the **enthalpy** of the cooling water, use **H(T)** for saturated liquid water from the Steam Tables.

Part (b) Don't forget about the cooling water when you calculate the **entropy generated**. Use **S(T)** for saturated liquid water from the Steam Tables.

Given:

T_A	15	°C	P_2	255	kPa
T_B	35	°C	T_2	143	°C
P_1	104	kPa	P_3	255	kPa
T_1	30	°C	T_3	65	°C
m_{air}	35	kg/min			

Find:

Part (a)	W_S	???	kW	Part (b)	$S_{gen, comp}$	???	kW/K
	m_{cw}	???	kg/s		$S_{gen, HEX}$	???	kW/K

Diagram: Diagram already provided in the problem statement.

Assumptions:

1 - Both the compressor and heat exchanger operate at steady-state.
2 - Heat exchange between the equipment in this process and the surroundings is negligible.
3 - There is no **shaft work** in the heat exchanger.
4 - Kinetic and potential energy changes are negligible.
5 - The air behaves as an ideal gas.
6 - The properties of the cooling water are the same as the properties of saturated liquid water at the same **temperature**.

Equations / Data / Solve:

Part a.)

R **8.314** **kJ/kmol K**
MW **28.97** **kg/kmol**
m_{air} **0.5833** **kg/s**

We can determine the properties of air at all three **states** by interpolating on the **Ideal Gas Property Tables** for air because all three **temperatures** are given in the problem statement.

T_1 **303.15** **K**
T_2 **416.15** **K**
T_3 **338.15** **K**

T (K)	H° (kJ/kg)	S° (kJ/kg-K)			
300	87.410	0.0061681			
310	97.396	0.038914	H_1	90.556	kJ/kg
410	198.63	0.32178			
420	208.88	0.34649	H_2	204.93	kJ/kg
330	117.45	0.10159			
340	127.51	0.13163	H_3	125.65	kJ/kg

Now, we can plug values back into **Eqn 1** to evaluate **W_S**:

W_S **-66.72** **kW**

The **mass flow rate** of the cooling water can be determined by an energy balance on the heat exchanger. For a steady-state process with negligible heat transfer, kinetic and potential energy changes and no **shaft work**:

$$\cancel{Q} - \cancel{W}_s = \sum_{j=1}^{\text{outlets}} m_{out,j}\,\hat{H}_{out,j} - \sum_{i=1}^{\text{inlets}} m_{in,i}\,\hat{H}_{in,i} \qquad \textbf{Eqn 3}$$

Eqn 3 can be simplified because no **shaft work** crosses the system boundary and when we use the entire **HEX** as the system, there is no **heat transfer** across the system boundary either.

$$0 = \dot{m}_{air}\left(\hat{H}_2 - \hat{H}_3\right) + \dot{m}_{cw}\left(\hat{H}_A - \hat{H}_B\right) \qquad \textbf{Eqn 4}$$

Now, we can solve **Eqn 4** for **m_{cw}** :

$$\dot{m}_{cw} = \left(\frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_B - \hat{H}_A}\right)\dot{m}_{air} \qquad \textbf{Eqn 5}$$

We do not know the **pressure** of the cooling water, so we cannot look up its properties. Therefore, we assume that the **enthalpy** of the cooling water is the same as the **enthalpy** of saturated liquid water at the same **temperature**. This assumption is accurate as long as water is nearly an incompressible liquid. The properties of saturated liquid water were determined from **NIST WebBook**:

Saturated liquid water at **T_A**: **H_A** **62.981** **kJ/kg**
Saturated liquid water at **T_B**: **H_B** **146.63** **kJ/kg**

Now, we can plug values into **Eqn 5** and evaluate **m_{cw}**:

m_{cw} **0.5529** **kg/s**

Part b.) **Entropy generation** is defined by:

$$S_{gen} = \Delta S - \int\left(\frac{\delta Q}{T}\right) \qquad \text{Eqn 6}$$

No **heat transfer** crosses the system boundary for either the compressor or the **HEX**, so **Eqn 6** simplifes to:

$$S_{gen} = \Delta S \qquad \text{Eqn 7}$$

We can determine the **entropy** change for air (ideal gas) in the compressor from the 2nd Gibbs Equation:

$$\hat{S}_{gen} = \Delta\hat{S} = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1} \qquad \text{Eqn 8}$$

Therefore, the **rate** at which **entropy** is **generated** in the compressor is:

$$(\dot{S}_{gen})_{comp} = \dot{m}_{air}\,\hat{S}_{gen} = \dot{m}_{air}\left[\hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1}\right] \qquad \text{Eqn 9}$$

We can determine the properties of air at all three **states** by interpolating on the **Ideal Gas Property Tables** for air because all three **temperatures** are given in the problem statement.

$S^o(T_1)$	0.016483	kJ/kg-K			
$S^o(T_2)$	0.33698	kJ/kg-K	$S^o(T_3)$	0.12607	kJ/kg-K

Now, we can plug values back into **Eqn 9** : $S_{gen,\ comp}$ 0.03681 kW/K

The **entropy generated** in the **HEX** must take into account the **entropy change** of BOTH the air and the cooling water.

$$(\dot{S}_{gen})_{HE} = \dot{m}_{air}\left[\hat{S}^o_{T3} - \hat{S}^o_{T2} - \frac{R}{MW}\,\mathrm{Ln}\frac{P_3}{P_2}\right] + \dot{m}_{cw}\left[\hat{S}_{cw}(T_B) - \hat{S}_{cw}(T_A)\right] \qquad \text{Eqn 10}$$

Again, we assume that the **entropy** of the cooling water is the same as the **entropy** of saturated liquid water at the same **temperature**. The properties of saturated liquid water were determined from **NIST WebBook**:

Saturated liquid water at T_A:	$S(T_A)$	0.22446	kJ/kg-K
Saturated liquid water at T_B:	$S(T_B)$	0.50513	kJ/kg-K

Now, we can plug value into Eqn 10 : $S_{gen,\ HEX}$ 0.03215 kW/K

Verify: Check the Ideal Gas Assumption:

$$\tilde{V} = \frac{R\,T}{P}$$

V_1 =	24.23	L/mole
V_2 =	13.57	L/mole
V_3 =	11.03	L/mole

Since air can be considered to be a diatomic gas and all three **molar volumes** are greater than **5 L/mole**, it is accurate to treat the air as an ideal gas.

Answers : **Part a.)**

W_S	-66.7	kW
m_{cw}	0.553	kg/s

Part b.)

$S_{gen,\ comp}$	0.0368	kW/K
$S_{gen,\ HEX}$	0.0321	kW/K

8B-4 Polytropic Compression of Air 6 pts

Calculate **Q** & $\mathbf{W_S}$, in **kJ/kg**, when ambient **air** at **104 kPa** and **320 K** is compressed polytropically to **950 kPa**.
Assume δ **= 1.38** for this process path and that **air** behaves as an ideal gas.

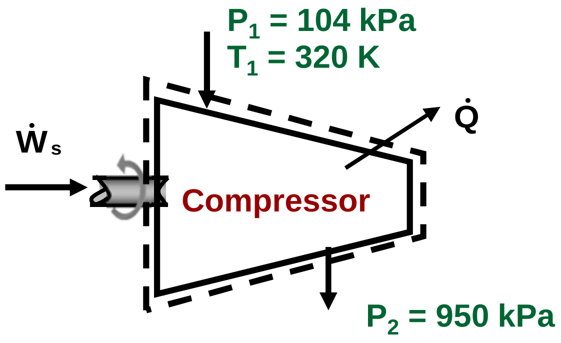
Read : The key to this problem is the fact that the process is polytropic and that the air can be assumed to be an ideal gas. Because the process is polytropic, we can determine $\mathbf{T_2}$ and $\mathbf{W_S}$. Because the gas is ideal, we can use the **Ideal Gas Property Tables** to evaluate $\mathbf{H_1}$ and $\mathbf{H_2}$. Finish by using $\mathbf{W_S}$, $\mathbf{H_1}$ and $\mathbf{H_2}$ to evaluate **Q**.

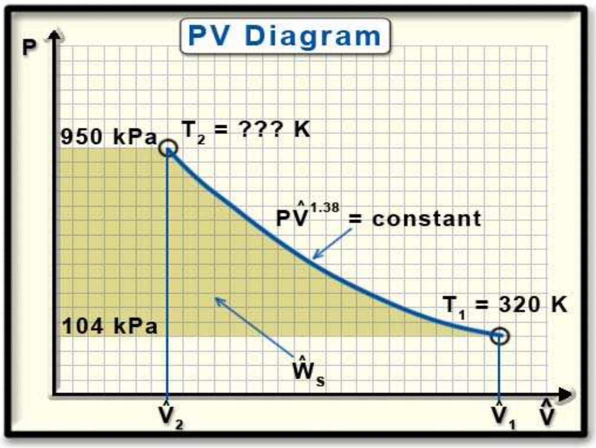
Given:				**Find:**			
	$\mathbf{P_1}$	**104**	**kPa**		$\mathbf{W_S}$	**???**	**kJ/kg**
	$\mathbf{T_1}$	**320**	**K**		**Q**	**???**	**kJ/kg**
	δ	**1.38**					
	$\mathbf{P_2}$	**950**	**kPa**				

Assumptions:

1 - The compressor operates at steady-state.
2 - Kinetic and potential energy changes are negligible.
3 - **Shaft work** and **flow work** are the only forms of **work** that cross the system boundary.
4 - Air is modeled as an ideal gas.

Diagram:





Equations / Data / Solve:

We can determine the **shaft work** for a polytropic process on an ideal gas using:

$$\hat{W}_s = -\frac{\delta}{\delta - 1}\frac{R}{MW}\left(T_2 - T_1\right)$$

Eqn 1

We can determine T_2 using the following **PVT** relationship for polytropic processes:

$$T_1\,P_1^{\frac{1-\delta}{\delta}} = T_2\,P_2^{\frac{1-\delta}{\delta}} = \text{constant}$$

Eqn 2

Solve Eqn 2 for T_2 :

$$T_2 = T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-\delta}{\delta}}$$

Eqn 3

We can now either evaluate T_2 or use Eqn 3 to eliminate T_2 from Eqn 1.

$$\hat{W}_s = -\frac{\delta}{\delta - 1}\frac{R\,T_1}{MW}\left(\left(\frac{P_1}{P_2}\right)^{\frac{1-\delta}{\delta}} - 1\right)$$

Eqn 4

Now, we can plug values into Eqns 3 & 4 to complete the first part of this problem.

MW	29.0	**g/mol**	T_2	588.42	**K**
R	8.314	**J/mol-K**	W_S	-279.75	**kJ/kg**

In order to determine the **specific heat transfer** for the compressor, we must apply the 1st Law for steady-state, SISO processes. For this compressor, changes in kinetic and potential energies are negligible and only **flow work** and **shaft work** cross the system boundaries. The appropriate form of the 1st Law for this compressor is :

$$\hat{Q} - \hat{W}_s = \hat{H}_2 - \hat{H}_1$$

Eqn 5

Solve Eqn 5 for **Q** :

$$\hat{Q} = \hat{W}_s + \hat{H}_2 - \hat{H}_1$$

Eqn 6

Because we know both T_1 and T_2 and we assumed that air behaves as an ideal gas in this process, we can use the **Ideal Gas Property Table** for air to evaluate H_1 and H_2.

T (K)	H^o (kJ/kg)
590	386.57
588.42	H_2
600	397.21

H_1	107.41	**kJ/kg**
H_2	384.89	**kJ/kg**

Now, we can plug values back into

Q -2.272 **kJ/kg**

Verify:

Check the Ideal Gas Assumption:

$$\tilde{V} = \frac{RT}{P}$$

V_1	25.58	**L/mole**	V_2 =	5.15	**L/mole**

Since air can be considered to be a diatomic gas and both **molar volumes** are greater than **5 L/mole**, it is acceptable to consider the air an ideal gas.

Answers :

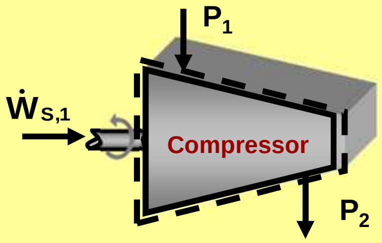
Part a.)

W_S	-280	kJ/kg
Q	-2.27	kJ/kg

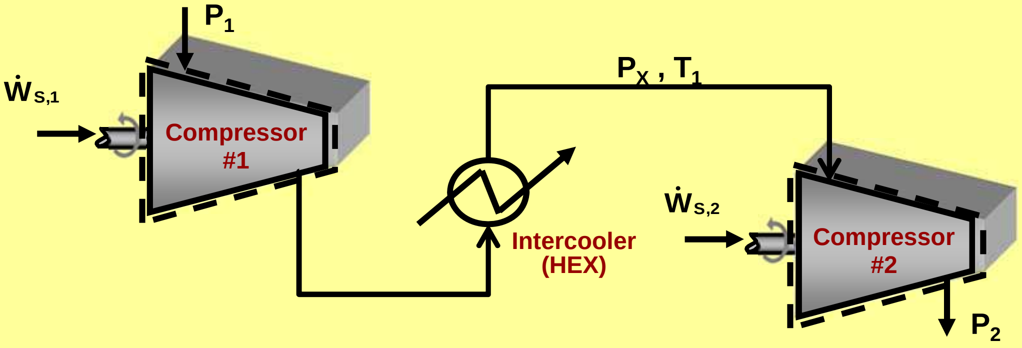
Q > 0, the compressor loses **heat** to the surroundings.

8C-1 Shaft Work Requirement for Different Compression Systems 7 pts

Determine the **specific shaft work** required for the **air** compressor shown below if the compression is…



a.) Isothermal
b.) Isentropic
c.) Polytropic with **δ = 1.24**
d.) A 2-stage compressor with intercooling with **δ = 1.24** is used instead. Calculate the **isothermal efficiency** of this system.



Assume **air** behaves as an ideal gas with constant **heat capacities** and all the compressors are internally reversible.

Read : **Parts (a)** through **(c)** are direct applications of equations derived for **shaft work** in polytropic processes. **Part (a)** requires us to assume the fluid is an ideal gas with constant **heat capacities** so that we can assume $\delta = \gamma$. **Part (d)** is the application of equations for the internally reversible, polytropic compression of of an ideal gas. The key is to to determine the optimal intermediate **pressure**, **P_X**, and use it to to determine the **shaft work** for each compressor.

Given:

P_1	90	kPa	(isothermal)	δ_A	1
T_1	310	K	(isentropic)	γ_B	1.4
P_2	1250	kPa		δ_C	1.24
			(ideal, 2-stage w/ intercooling)	δ_D	1.24

Find: **For each part of the problem…** **$-W_S / m_{dot}$** **???** kJ/kg **Compare results.**

Diagram: See the problem statement.

Assumptions:

1 - All compressors operate at steady-state.
2 - Air behaves as an ideal gas.
3 - The **heat capacities** of the **air** are constant.
4 - The intercooler in **part (d)** returns the **air** to the inlet **temperature**, T_1.
5 - All compressors are internally reversible.

Equations / Data / Solve:

The key equation for **parts (a)**, **(c)** and **(d)** is the equation for the **specific shaft work** in steady-state, polytropic processes.

$$\frac{-\dot{W}_S}{\dot{m}} = \frac{\delta}{\delta - 1}\left(\frac{R}{MW}\right)T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\delta-1}{\delta}} - 1\right] \qquad \text{Eqn 1}$$

Part a.) When $\delta = 1$ for a polytropic process on an ideal gas, the process is isothermal.
In this case, **Eqn 1** does not apply. Instead, we must use :

$$\frac{-\dot{W}_S}{\dot{m}} = -\frac{R\,T_1}{MW}\,Ln\left[\frac{P_2}{P_1}\right] \qquad \text{Eqn 2}$$

Plugging values into **Eqn 2** yields :

$-W_S / m_{dot}$	234.1	kJ/kg

Part b.) Plug values into **Eqn 1** :

R	8.314	J/mole-K
MW	28.97	g/mole
$-W_S / m_{dot}$	349.0	kJ/kg

Part c.) Plug values into **Eqn 1** :

$-W_S / m_{dot}$	305.2	kJ/kg

Part d.) The total **work** per unit **mass** of flowing fluid (**air** in this case) in a 2-stage compression process is the sum of the **specific work** for each compressor. The resulting equation is just the application of **Eqn 1** to each compressor.

$$\frac{-\dot{W}_{S,tot}}{\dot{m}} = \frac{\delta}{\delta - 1}\left(\frac{R}{MW}\right)T_1\left[\left(\frac{P_X}{P_1}\right)^{\frac{\delta-1}{\delta}} - 1\right] + \frac{\delta}{\delta - 1}\left(\frac{R}{MW}\right)T_1\left[\left(\frac{P_2}{P_X}\right)^{\frac{\delta-1}{\delta}} - 1\right] \qquad \text{Eqn 3}$$

Where P_X is the intermediate **pressure** between the two compressors.

The optimal value of the intermediate **pressure** can be determined using:

$$P_X = \sqrt{P_1 P_2} \qquad \text{Eqn 4}$$

Now, we can plug values into **Eqns 4** & **3** :

	P_X	335.4	kPa
Compressor #1:	$-W_S / m_{dot}$	133.29	kJ/kg
Compressor #2:	$-W_S / m_{dot}$	133.29	kJ/kg
Total :	$-W_S / m_{dot}$	266.6	kJ/kg

Notice that when the optimal value of $\mathbf{P_X}$ is used the compression ratio across each compressor is the same.

$$\frac{P_1}{P_X} = \frac{P_X}{P_2} \qquad \textbf{Eqn 5}$$

As a result, the **specific shaft work** for each compressor is the same as well.

The **isothermal efficiency** of the 2-stage compressor can be determined from :

$$\eta_{T,multi} = \frac{-\dot{W}_{S,iso-T}}{-\dot{W}_{S,act}} \qquad \textbf{Eqn 6}$$

Plugging values into **Eqn 6** yields :

η_T	**87.81%**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **Part a.)**

$-W_S / m_{dot}$	**234**	**kJ/kg**

This is the standard against which the other compressors are compared. The problem is that an isothermal compressor cannot be built.

Part b.)

$-W_S / m_{dot}$	**349**	**kJ/kg**

This value seems high, but this compressor does not require any heat exchange because it is adiabatic.

Part c.)

$-W_S / m_{dot}$	**305**	**kJ/kg**

This polytropic compressor must reject some **heat** to accomplish the compression with less **power** input than the isentropic compressor.

Part d.)

$-W_S / m_{dot}$	**267**	**kJ/kg**

η_T	**87.8%**

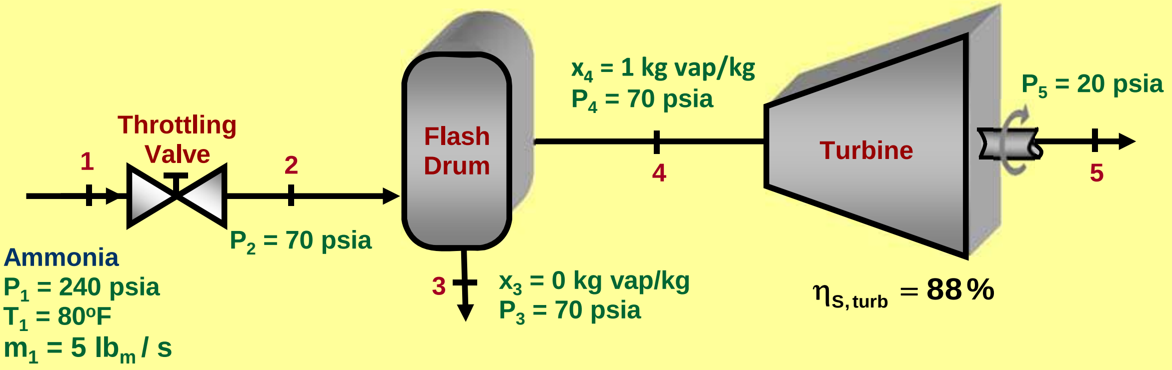
The 2-stage compressor with intercooling reduces the **power** requirement by about **13%** compared to the compressor in **part (c)**.

8C-2 Power & Entropy Generation in Turbine With a Flash Drum 8 pts

A flash drum is a vessel in which gravity is allowed to separate a liquid and a gas. A throttling device can be used to reduce the **pressure** on a liquid stream to produce a two-phase mixture.

The two-phase mixture enters the flash drum where the liquid settles to the bottom and the vapor rises to the top. Because the vapor and liquid phases are allowed to reach equilibrium, the vapor and liquid leaving a flash drum

are both saturated.
Consider the system in the diagram shown below.



Calculate the **power** output of the turbine and the **entropy generation rate** for the valve, for the flash drum and for the turbine. Which unit or units generate a large amount of entropy?

Read : This is a complicated problem just because it is an ensemble of **three** processes. Drawing a good **flow diagram** that includes all the given information is essential. Fortunately, the problem statement includes a good **flow diagram**.

Start from **State 1** because you know a lot of information about this stream. The valve behaves as a throttling valve. Apply **mass** and **energy** balances to the flash drum to determine $\mathbf{m_3}$ and $\mathbf{m_4}$. Use the **isentropic efficiency** to calculate $\mathbf{H_5}$ and then use $\mathbf{H_5}$ to get $\mathbf{x_5}$. Application of the 1st Law to the turbine yields $\mathbf{W_{turb}}$.

To get the correct **entropy generated**, you must be very accuarate in your calculations. Do not round off until the very end. Save the intermediate results in your calculator memory or use **Excel**.

Given:

P_1	240	lb_f/in^2	P_2	70	lb_f / in^2
T_1	80	°F	P_3	70	lb_f / in^2
m_1	5	lb_m/s	P_4	70	lb_f / in^2
$\eta_{S,\,turb}$	0.88		P_5	20	lb_f / in^2

Find:

a.)	W_{turb}	?	Btu/s	b.)	$(S_{gen})_{valve}$	?	Btu / s-°R
					$(S_{gen})_{flash}$	?	Btu / s-°R
					$(S_{gen})_{turb}$	?	Btu / s-°R

Diagram: The necessary flow diagram was provided in the problem statement.

Assumptions:

1 - Each component operates at steady-state with negligible **heat transfer** between the flowing fluid and the surroundings.

2 - Kinetic and potential energy changes are negligible.

3 - The expansion across the valve is an isenthalpic throttling process.

Equations / Data / Solve:

Part a.) Let's begin by applying the steady-state mass balance equation to the valve, the flash drum and the turbine, one unit at a time.

$$\dot{m}_1 = \dot{m}_2 \qquad \textbf{Eqn 1}$$

$$\dot{m}_2 = \dot{m}_3 + \dot{m}_4 \qquad \textbf{Eqn 2}$$

$$\dot{m}_4 = \dot{m}_5 \qquad \textbf{Eqn 3}$$

Next, apply the 1st Law to the turbine. The turbine is a steady-state, **SISO** process with negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{Q}_{turb} - \dot{W}_{turb} = \dot{m}_5 \hat{H}_5 - \dot{m}_4 \hat{H}_4 = \dot{m}_4 \left(\hat{H}_5 - \hat{H}_4\right) \qquad \textbf{Eqn 4}$$

In **Eqn 4**, **m_5** was eliminated using **Eqn 3**. Because the turbine is also assumed to be adiabatic, **$Q_{turb} = 0$** and **Eqn 4** becomes:

$$\dot{W}_{turb} = -\dot{m}_4 \left(\hat{H}_5 - \hat{H}_4\right) \qquad \textbf{Eqn 5}$$

We can lookup **H_4** in the Ammonia Tables or **NIST Webbook** because we know it is a saturated vapor at **70 psia**.

H_4	622.25	Btu/lb_m

We must use the **isentropic efficiency** of the turbine to determine **H_5** because we only know the value of one intensive variable at **state 5** (**P_5**).

Isentropic efficiency applied to our turbine is defined by:

$$\eta_{S,turb} = \frac{\hat{H}_4 - \hat{H}_5}{\hat{H}_4 - \hat{H}_{5S}} \qquad \textbf{Eqn 6}$$

We can solve **Eqn 6** for **H_5**, as follows:

$$\hat{H}_5 = \hat{H}_4 - \eta_{S,turb} \left(\hat{H}_4 - \hat{H}_{5S}\right) \qquad \textbf{Eqn 7}$$

H_{5S} is the **enthalpy** of the effluent (at **P_2**) from an adiabatic, isentropic turbine that has the same feed as the actual turbine. Because this hypothetical turbine is isentropic: **$S_{5S} = S_4$**

We can look up **S_4** in the Ammonia Tables or the **NIST Webbook** :

S_4	1.2651	Btu lb_m-^oR
S_{5S}	1.2651	Btu lb_m-^oR

Now, we know the values of two intensive properties at **state 5S**, so we can determine the values of other properties at this **state**, such as **T_{5S}** and **H_{5S}**, by interpolating on the Ammonia Tables or the **NIST Webbook**. We begin by determining the phases present.

At **P = 20 psia** :

$S_{sat\ liq}$	0.057608	Btu/lb_m-^oR
$S_{sat\ vap}$	1.3691	Btu/lb_m-^oR

Since **$S_{sat\ liq} < S_5 < S_{sat\ vap}$**, **state 5S** is a saturated mixture.

Determine **x_{5S}** from the **specific entropy**, using:

$$x_{5S} = \frac{\hat{S}_{5S} - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}} \qquad \textbf{Eqn 8}$$

x_{5S}	0.9207	lb_m vap/lb_m

Then, we can use the **quality** to determine **H_{5S}**, using:

$$\hat{H}_{5S} = x_{5S}\,\hat{H}_{sat\,vap} + (1 - x_{5S})\,\hat{H}_{sat\,liq}$$ **Eqn 9**

At **P = 20 psia** :

$H_{sat\,liq}$	24.887	**Btu/lb$_m$**
$H_{sat\,vap}$	605.98	**Btu/lb$_m$**

H_{5S}	**559.90**	**Btu/lb$_m$**

Now, we can use **Eqn 7** to evaluate **H_5** :

H_5	**567.38**	**Btu/lb$_m$**

Next, we need to evaluate **m_4**. Combine **Eqns 1** & **2** to get :

$$\dot{m}_4 = \dot{m}_1 - \dot{m}_3$$ **Eqn 10**

We know **m_1**, so we need to find **m_3** to calculate **m_4**.

We can determine **m_3** by applying the 1st Law to the flash drum. The flash drum is adiabatic, operates at steady-state, no **shaft work** crosses it boundaries and changes in kinetic and potential energies are negligible. Therefore, the appropriate form of the 1st Law is:

$$0 = \dot{m}_2\,\hat{H}_2 - \dot{m}_3\,\hat{H}_3 - \dot{m}_4\,\hat{H}_4$$ **Eqn 11**

Use **Eqn 10** to eliminate **m_4** from **Eqn 11** and use **Eqn 1** to eliminate **m_2** from **Eqn 11** and you are left with:

$$0 = \dot{m}_1\,\hat{H}_2 - \dot{m}_3\,\hat{H}_3 - (\dot{m}_1 - \dot{m}_3)\,\hat{H}_4$$ **Eqn 12**

We can solve **Eqn 12** for **m_3** , as follows:

$$\dot{m}_3 = \dot{m}_1\left[\frac{\hat{H}_1 - \hat{H}_4}{\hat{H}_3 - \hat{H}_4}\right]$$ **Eqn 13**

We can lookup the **specific enthalpies** in **states 1** and **3** in the Ammonia Tables or the **NIST Webbook** and we already know **H_4**. Then, we can plug these values into **Eqn 13** to evaluate **m_3** :

H_1	131.96	**Btu/lb$_m$**
H_3	84.109	**Btu/lb$_m$**

m_3	**4.56**	**lb$_m$/s**

Next we can evaluate **m_4** from **Eqn 10**:

m_4	**0.44**	**lb$_m$/s**

At last, we can plug values back into **Eqn 5** to complete **part (a)** :

W_{turb}	**24.39**	**Btu/s**

Part b.) The entropy generation rate is defined in the 2nd Law as:

$$\dot{S}_{gen} = \sum^{outlets} \dot{m}\,\hat{S} - \sum^{inlets} \dot{m}\,\hat{S} - \int \frac{\delta \dot{Q}}{T}$$ **Eqn 14**

Since we assumed that each of our three processes was adiabatic, **Eqn 14** simplifies to:

$$\dot{S}_{gen} = \dot{m}\,\Delta\hat{S}$$ **Eqn 15**

Apply **Eqn 15** to each of the three processes:

$$(\dot{S}_{gen})_{valve} = \dot{m}_1(\hat{S}_2 - \hat{S}_1)$$ **Eqn 16**

$$(\dot{S}_{gen})_{flash} = \dot{m}_4\,\hat{S}_4 + \dot{m}_3\,\hat{S}_3 - \dot{m}_2\,\hat{S}_2$$ **Eqn 17**

$$(\dot{S}_{gen})_{turb} = \dot{m}_4(\hat{S}_5 - \hat{S}_4)$$ **Eqn 18**

At this point, we have determined the **state** of all five streams in this process, so we can use the Ammonia Tables or the **NIST Webbook** to evaluate the **entropy** of each.

The **specific entropies** of **streams 1**, **3** and **4** come straight out of the Ammonia Tables or **NIST Webbook**.

S_1 **0.27391** **Btu / lb_m -^oR**

S_3 **0.18317** **Btu / lb_m -^oR**

S_4 **1.2651** **Btu / lb_m -^oR**

For S_5, we must first determine the **quality** using:

$$x_5 = \frac{\hat{H}_5 - \hat{H}_{sat\,liq}}{\hat{H}_{sat\,vap} - \hat{H}_{sat\,liq}} \qquad \textbf{Eqn 19}$$

x_5 **0.9336** **lb_m vap/lb_m**

Then we can evaluate S_5 using :

$$\hat{S}_5 = x_5\,\hat{S}_{sat\,vap} + (1 - x_5)\,\hat{S}_{sat\,liq} \qquad \textbf{Eqn 20}$$

S_5 **1.2820** **Btu / lb_m -^oR**

In order to determine S_2, we must apply the 1st Law to the valve. We assume the valve operates at steady-state, is adiabatic, exhibits negligible changes in kinetic or potential energies and involves no **shaft work**. Under these conditions, the 1st Law tells us that the valve is an isenthalpic throttling device.

$$\hat{H}_2 = \hat{H}_1 \qquad \textbf{Eqn 21}$$

H_2 **131.96** **Btu/lb_m**

Now, we know the values of two intensive properties at **state 2**, so we can determine the values of other properties at this state, such as S_2 , by interpolating on the Ammonia Tables or the **NIST Webbook**. We begin by determining the phases present.

At **P = 70 psia** :

$H_{sat\,liq}$ **84.109** **Btu/lb_m**

$H_{sat\,vap}$ **622.25** **Btu/lb_m**

Since $H_{sat\,liq} < H_2 < H_{sat\,vap}$, **state 2** is a saturated mixture.

Determine x_2 from the **specific entropy**, using:

$$x_2 = \frac{\hat{H}_2 - \hat{H}_{sat\,liq}}{\hat{H}_{sat\,vap} - \hat{H}_{sat\,liq}} \qquad \textbf{Eqn 22}$$

x_2 **0.08892** **lb_m vap/lb_m**

Then, we can use the **quality** to determine S_2, using:

$$\hat{S}_2 = x_2\,\hat{S}_{sat\,vap} + (1 - x_2)\,\hat{S}_{sat\,liq} \qquad \textbf{Eqn 23}$$

At **P = 70 psia** :

$S_{sat\,liq}$ **0.18317** **Btu/lb_m-^oR**

$S_{sat\,vap}$ **1.2651** **Btu/lb_m-^oR**

S_2 **0.27938** **Btu/lb_m-^oR**

Finally, we have all the values necessary to plug into Eqns 16-18 to evaluate the entropy generation in each device and then Eqn 24 to evaluate the lost work in each device.

$(S_{gen})_{valve}$ **0.02734** **Btu / s-^oR**

$(S_{gen})_{flash}$ **0.00000** **Btu / s-^oR**

$(S_{gen})_{turb}$ **0.00751** **Btu / s-^oR**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers:

a.)

W_{turb}	**24.4**	**Btu/s**

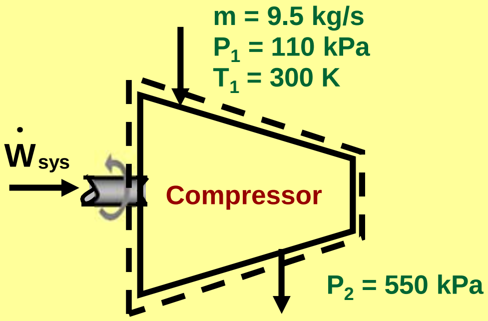
b.)

$(S_{gen})_{valve}$	**0.0273**	**Btu / s-^oR**
$(S_{gen})_{flash}$	**0.00000**	**Btu / s-^oR**
$(S_{gen})_{turb}$	**0.00751**	**Btu / s-^oR**

The expansion valve generates the most entropy.

8C-3 Isentropic Efficiency of an Ideal Gas Compressor 7 pts

Consider the adiabatic **air** compressor shown below.



a.) Calculate the minimum **power** input required and $\mathbf{T_2}$

b.) The outlet **temperature** from a real, adiabatic compressor that accomplishes the same compression is **520K**. Calculate the actual **power** input and the isentropic efficiency of the real compressor.

Read : Determine $\mathbf{S^o(T_2)}$ for an isentropic process and then interpolate to obtain both $\mathbf{T_{2S}}$ and $\mathbf{H_{2S}}$. Then, an energy balance will give you $\mathbf{(W_S)_{min}}$. Use the **isentropic efficiency** and $\mathbf{(W_S)_{min}}$ to determine $\mathbf{(W_S)_{act}}$.

Given:	**m**	**9.5**	**kg/s**	**Find:**	**a.)**	$\mathbf{(W_S)_{min}}$	**???**	**kW**
	$\mathbf{P_1}$	**110**	**kPa**			$\mathbf{T_{2S}}$	**???**	**K**
	$\mathbf{T_1}$	**310**	**K**		**b.)**	$\mathbf{(W_S)_{act}}$	**???**	**kW**
	$\mathbf{P_2}$	**550**	**kPa**			$\eta_{S,\ comp}$	**???**	
	$\mathbf{T_{2,\ part\ (b)}}$	**520**	**K**					

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - The compressor operates at steady-state and there is no significant **heat transfer**.

2 - Kinetic and potential energy changes are negligible.

3 - Air is modeled as an ideal gas.

Equations / Data / Solve:

Part a.) An isentropic compressor requires the minimum **power** input.

We can determine the isentropic **work** by applying the 1st Law to an isentropic compressor that takes in the same feed and yields an effluent at the same pressure.

For a steady-state, single-inlet, single outlet system with negligible **heat transfer**, kinetic and potential energy changes, the 1st Law is:

$$(\dot{W}_S)_{min} = (\dot{W}_S)_{isen} = \dot{m}\left(\hat{H}_1 - \hat{H}_{2S}\right) \qquad \text{Eqn 1}$$

The **entropy change** for this process can determined using the 1st Gibbs Equation in terms of the Ideal Gas Entropy Function.

$$\hat{S}_2-\hat{S}_1=\hat{S}^o_{T2}-\hat{S}^o_{T1}-\frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1}\qquad \textbf{Eqn 2}$$

We can also apply **Eqn 2** to our hypothetical, isentropic compressor:

$$0=\hat{S}^o_{T2S}-\hat{S}^o_{T1}-\frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1}\qquad \textbf{Eqn 3}$$

We can solve **Eqn 3** for the unknown $\mathbf{S^o_{T2S}}$:

$$\hat{S}^o_{T2S}=\hat{S}^o_{T1}+\frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1}\qquad \textbf{Eqn 4}$$

We can evaluate $\mathbf{S^o_{T1}}$ using the **Ideal Gas Property Tables**: $S^o(T_1)$ **0.0061681** kJ/kg-K

We can get $\mathbf{H^o_{T1}}$ while we are looking in the **Ideal Gas Property Tables** because we will need it later when we evaluate **Eqn 1**.

H_1	87.410	kJ/kg

Now, we can plug values into Eqn 4 :

R	8.314	kJ/kmol-K
MW	28.97	kg/kmol
$S^o(T_{2S})$	0.46806	kJ/kg-K

Now, we can use $\mathbf{S^o_{T2S}}$ and the **Ideal Gas Property Tables** to determine $\mathbf{T_{2S}}$ and $\mathbf{H_{2S}}$ by interpolation.

T (K)	H (kJ/kg)	S^o (kJ/kg-K)
470	260.49	0.46258
T_{2S}	H_{2S}	0.46806
480	270.88	0.48445

T_{2S}	472.50	K
H_{2S}	263.09	kJ/kg

Now, we can plug values back into **Eqn 1** :

$(W_S)_{min}$	-1669.0	kW

Part b.) We can determine the actual **power** input for the compressor by applying the 1st Law to the real compressor, just as we did in **Eqn 1** for the isentropic compressor.

$$\dot{W}_{S,act}=\dot{m}\left(\hat{H}_1-\hat{H}_2\right)\qquad \textbf{Eqn 5}$$

We can evaluate $\mathbf{H^o_{T2}}$ using the **Ideal Gas Property Tables**:

$T_{2,\text{ part (b)}}$	520	K
H_2	312.65	kJ/kg

Now, we can evaluate $\mathbf{W_{S,act}}$ using **Eqn 5**:

$(W_S)_{act}$	-2139.78	kW

The isentropic efficiency of a compressor is defined by:

$$\eta_{S,comp}=\frac{\dot{W}_{S,isen}}{\dot{W}_{S,act}}\qquad \textbf{Eqn 6}$$

Since we determined the isentropic **work** in **part (a)** and the actual **work** in **part (b)**, we are ready to plug numbers into **Eqn 6** and wrap up this problem.

$\eta_{S,\text{ comp}}$	78.00%

Verify: Check the Ideal Gas assumption:

$$\tilde{V}=\frac{RT}{P}$$

V_1 =	23.43	L/mole
V_2 =	7.86	L/mole

Since **air** can be considered to be a diatomic gas and both **molar volumes** are greater than **5 L/mole**, it is accurate to treat the **air** as an ideal gas.

Answers : **Part a.)**

$(W_S)_{min}$	-1670	kW
T_{2S}	473	K

Part b.)

$(W_S)_{act}$	-2140	kW
$\eta_{S,\text{ comp}}$	78.0%	

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8D-1 Lost Work Associated with Heat Transfer 5 pts

A thermal reservoir at **1550 K** transfers **10,000 kJ** of **heat** to a thermal reservoir at **350 K**. The **temperature** of the surroundings is **298 K**. Determine the **lost work** for this process based on…

a.) The performance characteristics of Carnot Cycles.
b.) The total **entropy generation** of this process.

Read : **Heat transfer** through a finite **temperature** difference is irreversible. It results in **entropy generation** and represents a lost opportunity to do **work**. **Part (a)** is more challenging because we need to build a hypothetical process out of reversible HE's, Ref's and HP's that accomplishes the same net **heat transfer** as the real process and then determine how much **work** we COULD have obtained using these reversible devices. **Part (b)** is a straightforward application of the definition of **entropy generation**, the **entropy change** for an isothermal process such as a thermal reservoir and the relationship between **lost work** and **entropy generation**.

Given:						
	Q	10000	kJ	T_H	1550	K
	T_{surr}	298	K	T_C	350	K

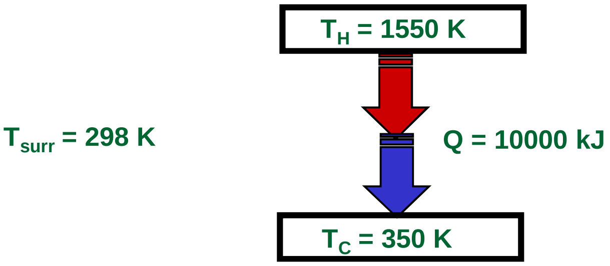
Find:			
	$W_{s,lost}$	???	kJ

Assumptions:

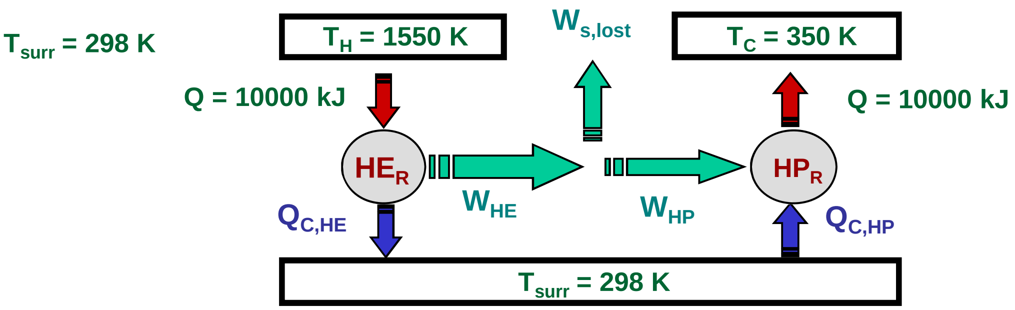
1 - No **shaft work** is obtained in the actual process.

2 - Both reservoirs are true thermal reservoirs whose **temperature** does not change as **heat** is added or removed.

Diagram: **Actual Process**



Hypothetical Process



Equations / Data / Solve:

Part a.) The definition of **lost work** is :

$$W_{S,lost} = W_{S,rev} - W_{S,act} \qquad \text{Eqn 1}$$

In our actual process, no **shaft work** is produced, so : $W_{s,act}$ **0** **kJ**

So, we need to evaluate the reversible **work** in order to determine the **lost work**.

In order to evaluate the reversible **work**, we must setup a reversible process that accomplishes the same thing as the actual process.

The hypothetical process must result in a transfer of **10000 kJ** of **heat** from the hot reservoir to the cold reservoir.

We can build the hypothetical process from **Carnot Cycles**. The diagram of the hypothetical process I have chosen includes a heat engine and a heat pump, both of which are reversible.

The **HE** must absorb the **1000 kJ** from the hot reservoir and the **HP** must reject **10000 kJ** to the cold reservoir. In this way, the hypothetical process does indeed accomplish the same thing as the real process. The **work** that this hypothetical (reversible) process produces **is** the reversible **work** and **Eqn 1** tells us that is also the **lost work** because the actual **work** is **zero**.

So, now we need to use the **Carnot Efficiency** and **COP** to evaluate the **lost work**.

The **Carnot Efficiency** of our **HE** is :

$$\eta_R = 1 - \frac{T_{surr}}{T_H} = \frac{W_{HE}}{Q_H} \qquad \text{Eqn 2}$$

η_R **0.8077**

Solving **Eqn 2** for the **work** produced by the **HE** yields :

$$W_{HE} = \eta_R \, Q_H \qquad \text{Eqn 3}$$

W_{HE} **8077** **kJ**

The **COP** of a Carnot HP is :

$$COP_R = \frac{1}{1 - \frac{T_{surr}}{T_C}} = \frac{Q_H}{W_{HP}} \qquad \text{Eqn 4}$$

COP_R **6.731**

Solving **Eqn 4** for the **work** required by the **HP** yields :

$$W_{HP} = \frac{Q_H}{COP_R} \qquad \text{Eqn 5}$$

W_{HP} **1486** **kJ**

The reversible **work**, and therefore the **lost work**, is equal to the difference between the **work** produced by the reversible **HE** and the **work** required by the reversible **HP**.

$W_{s,rev}$ **6592** **kJ**

$\boxed{W_{s,lost} \quad 6592 \quad kJ}$

Part b.) The **entropy generation** by the real process is equal to the **entropy change** of the universe resulting from the process. The **entropy change** of the universe is made up of the **entropy** increase of the cold reservoir and the **entropy** decrease of the hot reservoir because of the transfer of **1000 kJ**.

$$S_{gen} = \Delta S_{univ} = \Delta S_{hot} + \Delta S_{cold} = -\frac{Q}{T_H} + \frac{Q}{T_C}$$

Eqn 6

S_{gen} **22.12** **kJ/K**

Lost work is related to the total **entropy generation** by :

$$\dot{W}_{S,lost} = T_{surr}\,\dot{S}_{gen}$$

Eqn 7

$W_{s,lost}$ **6592** **kJ**

We can combine **Eqns 6** & **7** to obtain a convenient equation for calculating the **lost work** associated with **heat transfer** through a finite **temperature** difference.

$$W_{S,lost} = Q\ T_{surr}\left\{\frac{T_H - T_C}{T_H\,T_C}\right\}$$

Eqn 8

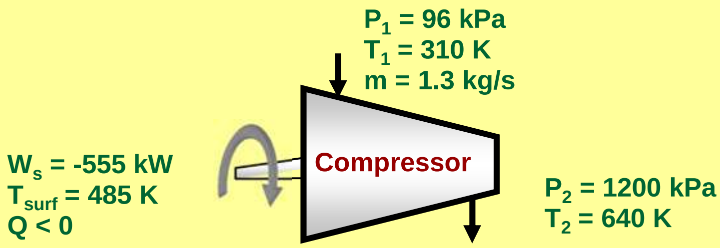
Note: In this equation **Q** is the absolute value of the amount of **heat transferred** (a positive quantity).

Verify: None of the assumptions made in this problem solution can be verified.

Answers : **Parts a & b :** $W_{s,lost}$ **6592** **kJ**

8D-2 Sgen and Wlost for a Compressor with Heat Losses 7 pts

The outer surface of a air compressor is at an average **temperature** of 485 K and the surroundings are at 300 K. The operating parameters for the compressor are given in the figure below.



a.) Calculate the internal, external and total **entropy generation rates** for the compressor in **kW/K**.
b.) Calculate the internal, external and total **lost work** for the compressor in **kW**.

c.) Show that the total **lost work** determined in **part (b)** is the difference between the power requirement of a completely reversible compressor that accomplishes the same change in state of the **air** stream and the actual **work** for the real compressor.

Read : Equations for evaluating total and external **entropy generation** can be obtained from **Lesson 8D**. You could just take the difference between these to determine the internal **entropy generation**. But it is useful to understand how the position of the system boundary can be manipulated to directly yield an equation for the internal **entropy generation**.

The keys are to use the 1st Law to determine **Q** and to use the 2nd Gibbs Equation and the Ideal Gas Entropy Function to evaluate Δ**S**.

Once the **entropy generation rates** are known, it is easy to determine the **lost work rates**.

In order to verify the definition of **lost work**, we must determine the amount of **work** a reversible compressor would require to accomplish the same compression process. Q_{rev} must be different from Q_{act}. The key to this part of the problem is that total **entropy generation rate** for the reversible compressor must be **zero**.

Given:						
	P_1	96	kPa	T_1	310	K
	P_2	1200	kPa	T_2	640	K
	m	1.30	kg/s	T_{surf}	485	K
	$W_{s,act}$	-555	kW	T_{surr}	300	K

Find:								
	a.)	$S_{gen,int}$	???	kW/K	b.)	$W_{lost,int}$	???	kW
		$S_{gen,ext}$	???	kW/K		$W_{lost,ext}$	???	kW
		$S_{gen,tot}$	???	kW/K		$W_{lost,tot}$	???	kW

c.) **Show that :**

$$\dot{W}_{S,lost} = \dot{W}_{S,rev} - \dot{W}_{S,act} \qquad \text{Eqn 1}$$

Assumptions:

1 - The surface **temperature** of the compressor remains constant. Therefore, it can be treated as a thermal reservoir.

2 - The compressor operates at steady-state with negligible changes in kinetic and potential energies.

3 - **Air** behaves as an ideal gas.

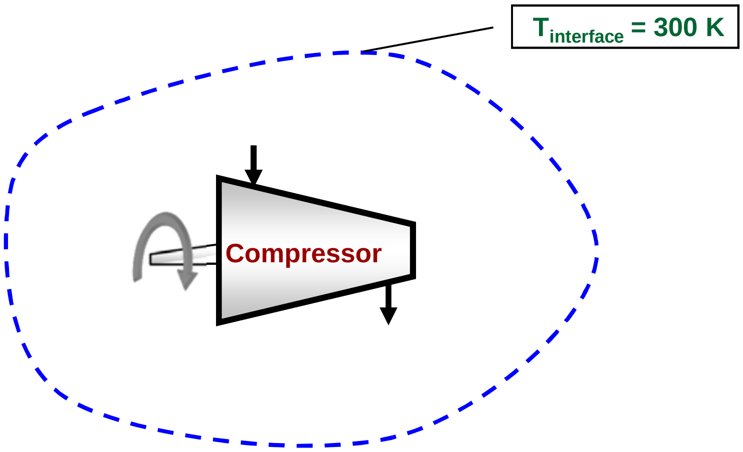
Diagram: The diagram in the problem statement is adequate.

Equations / Data / Solve:

Part a.) We can use the following equations to evaluate the internal and total entropy generation rates.

$$\dot{S}_{gen,tot} = \dot{m}\,\Delta\hat{S} - \frac{\dot{Q}_{act}}{T_{surr}} \qquad \text{Eqn 2}$$

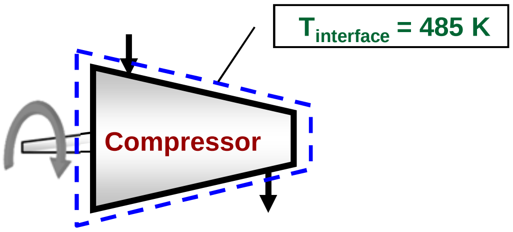
If we place the system boundary far from the surface of the compressor, then ALL of the irreversibilities are inside the system because the **temperature** at the system boundary is the same as the **temperature** of the surroundings. So, **heat exchange** between this big system and the surroundings is reversible.



This explains why using T_{surr} in **Eqn 2** tells us that we are computing the TOTAL **entropy generation rate**. Because with this big system, there are no irreversibilities OUTSIDE the system boundary. Notice that **heat transfer** from this system does occur, but it occurs with **zero temperature** driving force, so it is reversible !

$$\dot{S}_{gen,int} = \dot{m}\Delta\hat{S} - \frac{\dot{Q}_{act}}{T_{surf}} \qquad \text{Eqn 3}$$

Using our usual system boundary, right at the surface of the compressor, **heat exchange** with the surroundings is irreversible. So, by using **Eqn 3**, with T_{surf} instead of T_{surr}, we have excluded the external irreversibility due to **heat transfer** through a finite **temperature** difference. So, this **entropy generation** equation gives us the INTERNAL **entropy generation** only !



The external **entropy generation rate** is equal to the rate at which the **entropy** of the universe (compressor and surroundings) changes due only to the **heat transfer** from the compressor to the surroundings.

$$\dot{S}_{gen,ext} = \dot{S}_{univ} = \dot{S}_{comp} + \dot{S}_{surr} = \frac{\dot{Q}}{T_{surf}} - \frac{\dot{Q}}{T_{surr}} \qquad \textbf{Eqn 4}$$

The minus sign appears in **Eqn 4** because the sign of **Q** from the perspective of the system is negative, but from the perspective of the surroundings, **Q > 0** because **heat** is entering the surroundings !

Eqn 4 can be rearranged to give us :

$$\dot{S}_{gen,ext} = \dot{Q}\left\{\frac{T_{surf} - T_{surr}}{T_{surf}\,T_{surr}}\right\} \qquad \textbf{Eqn 5}$$

Let's begin by determing ΔS for the working fluid using the 2nd Gibbs Equation and the Ideal Gas Property Table for **air**.

$$\Delta\hat{S} = \hat{S}^o_{T_2} - \hat{S}^o_{T_1} - \frac{R}{MW}\,Ln\left[\frac{P_2}{P_1}\right] \qquad \textbf{Eqn 6}$$

S^o_1	0.038914	kJ/kg-K	S^o_2	0.78826	kJ/kg-K
R	8.314	J/mole-K			
MW	28.97	g/mole	ΔS	0.02450	kJ/kg-K

Next, we need to evaluate **Q**, the rate of **heat** loss from the compressor. We can do this using the 1st Law for open systems operating at steady-state with negligible changes in kinetic and potential energies.

$$\dot{Q} - \dot{W}_s = \dot{m}\,\Delta\hat{H} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) \qquad \textbf{Eqn 7}$$

We can solve **Eqn 7** for **Q** :

$$\dot{Q} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) + \dot{W}_s \qquad \textbf{Eqn 8}$$

Now, we can use the **Ideal Gas Property Table** for **air** to evaluate **H_1** and **H_2**.

H^o_1	97.396	kJ/kg	H^o_2	439.98	kJ/kg
Plug values back into **Eqn 7** to evaluate **Q** :			Q	-109.64	kW

Now, we are able to plug values back into **Eqns 2, 3 & 5** to evaluate the entropy generation rates.

$S_{gen,int}$	0.25791	kW/K			
$S_{gen,ext}$	0.13941	kW/K	$S_{gen,tot}$	0.39731	kW/K

Double check your calculations using :

$$\dot{S}_{gen,tot} = \dot{S}_{gen,int} + \dot{S}_{gen,ext} \qquad \textbf{Eqn 9}$$

Part b.) Once we have completed **part (a)**, **part (b)** is a straightforward application of the relationship between **lost work** and **entropy generation**.

$$\dot{W}_{S,lost} = T_{surr}\,\dot{S}_{gen} \qquad \textbf{Eqn 10}$$

Eqn 10 applies for internal, external and total **entropy generation** and **lost work**.

$W_{lost,int}$	77.37	kW			
$W_{lost,ext}$	41.82	kW	$W_{lost,tot}$	119.19	kW

Part c.) Here we must verify that our answer from **part (b)** is consistent with the defintion of **lost work**.

$$W_{S,lost} = W_{S,rev} - W_{S,act} \qquad \textbf{Eqn 11}$$

In order to do this, we must evaluate $W_{S,rev}$.

The key to doing this is to understand that the reversible process still operates between **state 1** and **state 2**.

The values of **Q** and W_S are different from those for the real compressor.

The fact that ties this part of the problem together and allows us to determine **Q** and W_S for the reversible compressor is that :

$$\dot{S}_{gen,tot,rev} = 0 \qquad \textbf{Eqn 12}$$

We can use this fact and solve **Eqn 2** for Q_{rev} :

$$\dot{Q}_{rev} = \dot{m}\, T_{surr}\, \Delta\hat{S} \qquad \textbf{Eqn 13}$$

Q_{rev}	9.55	kW

Next, we can apply the 1st Law to the reversible compressor to evaluate $W_{S,rev}$.

We can solve **Eqn 7** for $W_{S,rev}$. The result is :

$$\dot{W}_{S,rev} = \dot{Q}_{rev} - \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) \qquad \textbf{Eqn 14}$$

Plugging values into **Eqn 14** yields :

$W_{S,rev}$	-435.81	kW

Finally, put values into the right-hand side of **Eqn 11** :

$W_{S,rev} - W_{S,act} =$	119.19	kW

This matches our result for **lost work** from **part (b)**. So, we have confirmed the relationship between reversible **work**, actual **work** and **lost work**.

Verify: None of the assumptions made in this problem solution can be verified.

Answers :

a.)

$S_{gen,int}$	0.258	kW/K
$S_{gen,ext}$	0.139	kW/K
$S_{gen,tot}$	0.397	kW/K

b.)

$W_{lost,int}$	77.4	kW
$W_{lost,ext}$	41.8	kW
$W_{lost,tot}$	119.2	kW

c.)

$W_{S,rev} - W_{S,act} =$	119.2	kW

8D-3 Isentropic and 2nd Law Efficiencies of a Steam Turbine 6 pts

A **steam** turbine lets **2000 psia steam** down to **60 psia**. The inlet **steam temperature** is **1500°F** and the **isentropic efficiency** is 88%.

a.) Calculate $\mathbf{W_{S,act}}$ in **Btu/lb$_m$**

b.) Calculate the **2nd Law Efficiency** of the turbine. Assume $\mathbf{T_{surr} = 75^oF}$.

Read : The key to this problem is to assume that the turbine is adiabatic.

We can calculate the isentropic **work** of the turbine because $\mathbf{S_2 = S_1}$ gives us the additional intensive variable value that we need to fix the **state** of the outlet stream. Then we can calculate the actual **work** from the isentropic **work** and the **isentropic efficiency**.

The **2nd Law Efficiency** is the ratio of the actual **work** that we found in **part (a)** to the reversible **work**. We need to know the actual **entropy** of the outlet stream in order to determine the reversible **work** for the turbine. We can use the actual **work** and the 1st Law to determine the actual **enthalpy** of the effluent. This gives us the second intensive property we need in order to use the **Steam Tables** to evaluate $\mathbf{S_2}$.

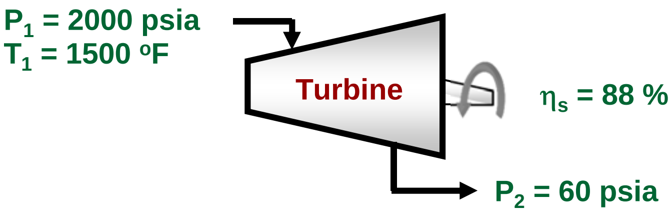
Given:

η_S	88%	
P_1	2000	psia
T_1	1500	°F
P_2	60	psia
T_{surr}	75	°F

Find:

$W_{S,act}$	???	Btu/lb$_m$
η_{ii}	???	%

Diagram:



Assumptions:

1 - The turbine is assumed to be adiabatic.
2 - Changes in kinetic and potential energies are negligible.

Equations / Data / Solve:

Part a.) The **isentropic efficiency** of an adiabatic turbine is defined by:

$$\eta_S = \frac{\hat{W}_{S,act}}{\hat{W}_{S,isen}} = \frac{\hat{H}_1 - \hat{H}_{2,act}}{\hat{H}_1 - \hat{H}_{2S}} \qquad \text{Eqn 1}$$

We can solve **Eqn 1** for $\mathbf{W_{S,act}}$:

$$\hat{W}_{S,act} = \eta_S \cdot \hat{W}_{S,isen} = \eta_S \left(\hat{H}_1 - \hat{H}_{2S}\right) \qquad \text{Eqn 2}$$

Because we know the values of two intensive properties at **state 1**, we can use the **Steam Tables** or the **NIST Webbook** to look-up H_1. Because $T_1 > T_{critical}$ **(1165.3°R)**, we need to look in the superheated vapor table for properties at **state 1**.

H_1 **1779.2** Btu/lb_m

The key to determining H_{2S} is the fact that $S_{2S} = S_1$ and we can determine S_1 from the **Steam Tables** or the **NIST Webbook**.

S_1 **1.7406** $Btu/lb_m\text{-}^oR$

S_{2S} **1.7406** $Btu/lb_m\text{-}^oR$

Now, we know the values of two intensive properties at **state 2S**, so we can determine the values of other properties at this state, such as T_{2S} and H_{2S}, by interpolating on the **Steam Tables** or the **NIST Webbook**. We begin by determining the phases present.

At **P = 60 psia** :

$S_{sat\ liq}$ **0.4276** $Btu/lb_m\text{-}^oR$

$S_{sat\ vap}$ **1.6454** $Btu/lb_m\text{-}^oR$

Since $S_2 > S_{sat\ vap}$, **state 2S** is a superheated vapor.

Interpolation within the **60 psia** superheated steam table is required.

At **P = 60 psia** :

T (°F)	H (Btu/lb_m)	S (Btu/lb_m-°R)
400	1234.5	1.7149
T_{2S}	H_{2S}	1.7406
600	1333.1	1.8181

T_{2S} **449.9** °F

H_{2S} **1259.1** Btu/lb_m

Now, we can plug values back into **Eqn 2** to evaluate $W_{S,act}$:

$W_{S,act}$	457.7	Btu/lb_m

Part b.) The **2nd Law Efficiency** of a turbine is defined as:

$$\eta_{ii,turb} = \frac{\hat{W}_{S,act}}{\hat{W}_{S,rev}}$$ **Eqn 3**

The reversible **work** can be determined from :

$$\hat{W}_{S,rev} = T_{surr}\,\Delta\hat{S} - \Delta\hat{H}$$ **Eqn 4**

We could determine S_2 if we knew H_2. We can determine H_2 from and $W_{S,act}$ by applying the 1st Law to the actual, adiabatic process where changes in kinetic and potential energies are negligible.

$$\hat{Q} - \hat{W}_S = \Delta\hat{H}$$ **Eqn 5**

or:

$$\hat{H}_2 = \hat{H}_1 - \hat{W}_{S,act}$$ **Eqn 6**

Plugging values into **Eqn 6** yields:

H_2 **1321.5** Btu/lb_m

Now that we know the values of two intensive variables at **state 2**, P_2 and H_2, we can interpolate on the **Steam Tables** or **NIST Webbook** data to determine S_2. Because $H_2 > H_{2S}$, and **state 2S** is a superheated vapor, we know that the actual **state 2** is a superheated vapor.

At **P = 60 psia** :

T (°F)	H (Btu/lb_m)	S (Btu/lb_m-°R)
400	1234.5	1.7149
T_2	1321.51	S_2
600	1333.1	1.8181

T_2 **576.5** °F

S_2 **1.8059** $Btu/lb_m\text{-}^oR$

Now, using $T_{surr} = 75^oF$, we can use **Eqn 4** to calculate $W_{S,rev}$ and then use **Eqn 3** to evaluate the **2nd Law Efficiency** of the turbine.

T_{surr} **534.67** °R

$W_{S,rev}$ **492.6** Btu/lb_m

η_{ii}	92.9%

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **Part a.)**

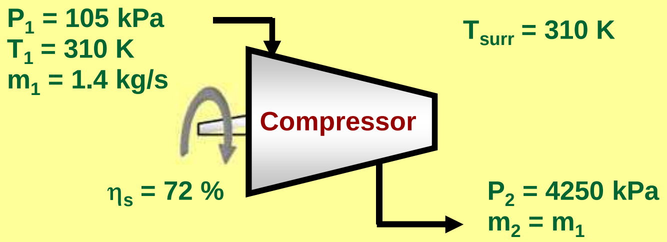
$W_{S,act}$	458	Btu/lb_m

Part b.)

η_{ii}	92.9%

8D-4 2nd Law Efficiency and Lost Work in an Air Compressor 6 pts

For the compressor shown below, calculate…



a.) The actual, reversible and lost **work** in **kW**

b.) The **2nd Law efficiency**

Assume **air** behaves as an ideal gas and the **temperature** of the surroundings is **310 K**.

Read : We can calculate the isentropic **work** requirement of the compressor because $S_2 = S_1$ gives us the additional intensive variable value that we need to fix the state of the effluent stream. Then we can calculate the actual **work** from the isentropic **work** using the **isentropic efficiency**.

We need to know the actual **entropy** of the effluent in order to determine the reversible **work** for the turbine. We can use the actual **work** and the 1st Law to determine the actual **enthalpy** of the effluent. This gives us the second intensive property we need in order to interpolate on the Ideal Gas Property Table for **air** to evaluate S^o_2. Once we know S^o_2, we can use the 2nd Gibbs Equation to determine S_2. Use S_2 to evaluate $W_{S,lost}$. The **2nd Law Efficiency** is the ratio of the actual **work** to the reversible **work** that we found in **part (a)**.

Given:

η_s	72%	
P_1	105	kPa
T_1	310	K
P_2	4250	kPa
m_{dot}	1.4	kg/s
T_{surr}	310	K

Find:

$W_{s,act}$	???	kW
$W_{s,rev}$	???	kW
$W_{s,lost}$	???	kW
η_{ii}	???	%

Diagram: The diagram in the problem statement is adequate.

Assumptions:

1 - The compressor operates at steady-state.
2 - The compressor is adiabatic.
3 - Changes in kinetic and potential energies are negligible.

Equations / Data / Solve:

Part a.) The **isentropic efficiency** of an adiabatic compressor is defined by:

$$\eta_S = \frac{-\hat{W}_{S,isen}}{-\hat{W}_{S,act}} = \frac{\hat{H}_{2S}-\hat{H}_1}{\hat{H}_{2,act}-\hat{H}_1} \qquad \text{Eqn 1}$$

We can solve **Eqn 1** for $\mathbf{W_{S,act}}$:

$$\hat{W}_{S,act} = \frac{\hat{W}_{S,isen}}{\eta_S} = \frac{\left(\hat{H}_1-\hat{H}_{2S}\right)}{\eta_S} \qquad \text{Eqn 2}$$

Because we assumed **air** is an ideal gas and we know $\mathbf{T_1}$, we can look-up $\mathbf{H_1}$ in the **Ideal Gas Property Table** for **air**.

H_1 **97.396** **kJ/kg**

Next, we need to determine $\mathbf{T_{2S}}$ so we can look-up $\mathbf{H_{2S}}$ in the **Ideal Gas Properties Table** for **air**.
We can either use the Ideal Gas Entropy Function or the Relative Pressure Method.

Method 1: Use the Ideal Gas Entropy Function.

The 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function is :

$$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2S} - \hat{S}^o_{T1} - \frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1} = 0 \qquad \text{Eqn 3}$$

We can solve **Eqn 3** for the unknown $\mathbf{S^o_{T2}}$:

$$\hat{S}^o_{T2S} = \hat{S}^o_{T1} + \frac{R}{MW}\,\mathrm{Ln}\frac{P_2}{P_1} \qquad \text{Eqn 4}$$

We can look up $\mathbf{S^o_{T1}}$ in the **Ideal Gas Property Table** for **air** and use it with the known **pressures** in **Eqn 4** to determine $\mathbf{S^o_{T2}}$.

R	**8.314**	**J/mol-K**	S^o_{T1}	**0.038914**	**kJ/kg-K**
MW	**28.97**	**g/mol**	S^o_{T2S}	**1.1010**	**kJ/kg-K**

Now, we can use $\mathbf{S^o_{T2}}$ and the **Ideal Gas Property Table** for **air** to determine $\mathbf{T_2}$ and then $\mathbf{H_2}$ by interpolation :

T (K)	S° (kJ/kg-K)	H° (kJ/kg)
840	1.0850	658.42
T_{2S}	1.1010	H_{2S}
860	1.1112	680.67

T_{2S}	**852.19**	**K**
H_{2S}	**671.98**	**kJ/kg**

Now, we can plug values back into **Eqn 2** to get :

$W_{s,act}$ **-798.04** **kJ/kg**

The rate at which **work** is actually done can be determined using :

$$\dot{W} = \dot{m}\,\hat{W}_S \qquad \text{Eqn 5}$$

$$\boxed{W_{s,act} \quad -1117.2 \quad \text{kW}}$$

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an isentropic process :

$$\frac{P_r(T_{2S})}{P_r(T_1)} = \frac{P_2}{P_1}$$ **Eqn 6**

Where $\mathbf{P_r}$ is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqn 6** For $\mathbf{P_r(T_{2S})}$, as follows :

$$P_r(T_{2S}) = \frac{P_2}{P_1} P_r(T_1)$$ **Eqn 7**

Look-up $\mathbf{P_r(T_1)}$ and use it in **Eqn 7** To determine $\mathbf{P_r(T_{2S})}$:

$P_r(T_1)$	1.145
$P_r(T_{2S})$	46.353

We can now determine $\mathbf{T_{2S}}$ by interpolation on the the **Ideal Gas Property Table** for **air**.
Then, we use $\mathbf{T_{2S}}$ to determine $\mathbf{H_{2S}}$ from the **Ideal Gas Property Table** for **air**.

T (K)	P_r	H^o (kJ/kg)
840	43.852	658.42
T_{2S}	46.353	H_{2S}
860	48.039	680.67

T_{2S}	851.95	K
H_{2S}	671.71	kJ/kg

Now, we can plug values back into **Eqn 2** to get :

$W_{s,act}$	-797.66	kJ/kg

Then, use **Eqn 5** to determine the rate at which **shaft work** is actually done :

$W_{s,act}$	-1116.73	kW

The two key equations for determining lost and reversible **work** are :

$$(\dot{W}_S)_{lost} = (\dot{W}_S)_{rev} - (\dot{W}_S)_{act}$$ **Eqn 8**

$$(\dot{W}_S)_{lost} = \dot{m}\, T_{surr}\, \hat{S}_{gen} = \dot{m}\, T_{surr}\, \Delta \hat{S} - \dot{Q}_{act}\left(\frac{T_{surr}}{T_{HT}}\right)$$ **Eqn 9**

Because the process is adiabatic, **Eqn 9** simplifies to :

$$(\dot{W}_S)_{lost} = \dot{m}\, T_{surr}\left(\hat{S}_2 - \hat{S}_1\right)$$ **Eqn 10**

We can determine $\mathbf{S_1}$ from the **Ideal Gas Property Table** for **air** but we still need to know $\mathbf{S_2}$.

Since we know $\mathbf{(W_s)_{act}}$, we can determine $\mathbf{H_{2,act}}$ by applying the 1st Law. The appropriate form of the 1st Law for this adiabatic, open system, operating at steady-state with negligible changes in kinetic and potential energies is :

$$-\left(\hat{W}_S\right)_{act} = \hat{H}_2 - \hat{H}_1$$ **Eqn 11**

Solve **Eqn 11** for $\mathbf{H_2}$:

$$\hat{H}_2 = \hat{H}_1 - \left(\hat{W}_S\right)_{act}$$ **Eqn 12**

Plug numbers into **Eqn 12** :

Method 1, S^o :	H_2	895.43	kJ/kg
Method 2, P_r :	H_2	895.06	kJ/kg

We already obtained S^o_{T1} from the **Ideal Gas Property Table** for **air** : S^o_{T1} **0.038914 kJ/kg**

Now, we can interpolate on the **Ideal Gas Property Table** for **air** to determine T_2 and S^o_{T2} :

T (K)	S° (kJ/kg-K)	H° (kJ/kg)
1040	1.3257	883.90
T_2	S^o_{T2}	H_2
1060	1.3475	906.80

	Method 1	Method 2	
T_2	1050.1	1049.7	K
S^o_{T2}	1.3367	1.3363	kJ/kg-K

Now, we apply **Eqn 3** to the actual turbine to evaluate ΔS :

$$\Delta \hat{S} = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW} \ln \frac{P_2}{P_1} \qquad \text{Eqn 13}$$

	Method 1	Method 2	
ΔS	0.23571	0.23535	kJ/kg-K

Now, plug values back into **Eqns 10** & **8** to get :

	Method 1	Method 2	
$W_{s,lost}$	102.30	102.14	kW
$W_{s,rev}$	-1015.0	-1015.1	kW

Part b.) The **2nd Law Efficiency** of a turbine is defined as:

$$\eta_{ii,comp} = \frac{\dot{W}_{S,rev}}{\dot{W}_{S,act}} \qquad \text{Eqn 14}$$

Plugging values into **Eqn 14** gives us:

	Method 1	Method 2
η_{ii}	90.84%	90.90%

Note: We could have determined the reversible **work** from :

$$\hat{W}_{S,rev} = T_{surr} \Delta \hat{S} - \Delta \hat{H} \qquad \text{Eqn 14}$$

Verify: Check the Ideal Gas Assumption:

$$\tilde{V} = \frac{R\,T}{P}$$

V_1 = 24.55 L/mole **V_2 = 2.05 L/mole**

Air can be considered to be a diatomic gas, but the **molar volume** at **state 2** is not greater than **5 L/mole**. So, it is not accurate to treat the **air** as an ideal gas.

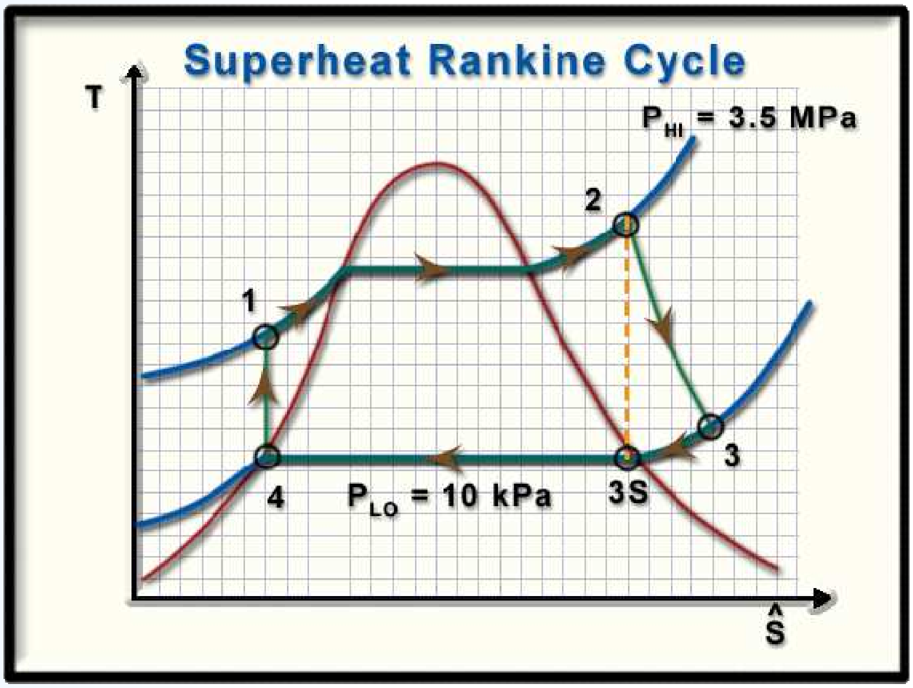
Answers : **Part a.)**

$W_{s,act}$	-1117	kW
$W_{s,lost}$	102.3	kW
$W_{s,rev}$	-1015	kW

Part b.)

η_{ii}	90.8%

Chapter 9



Power Systems

This chapter focuses on practical aspects of power generation using heat engines. In particular, we look at how the efficiency of power cycles can be increased.

The first part of the chapter covers the Rankine vapor power cycle. Five variations on the Rankine Cycle that increase the thermal efficiency of the cycle are discussed in detail.

In the second half of this chapter, gas power cycles are discussed. Many gas power cycles exist, but we limit our discussion to the Air-Standard Brayton Power Cycle. Four variations on the Air-Standard Brayton Power Cycle that increase the thermal efficiency of the cycle are discussed.

The impact of process irreversibilties on the thermal efficiencies of these cycles are also discussed.

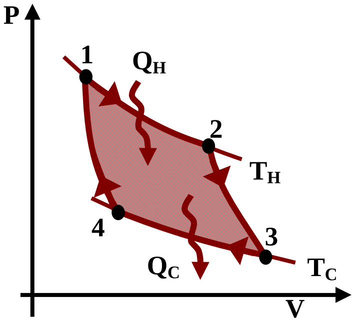
Vapor and Gas Power Systems

- **Vapor Cycles**
  - Use evaporator and condenser for isothermal heat transfer steps
- **Gas Cycles**
  - No phase changes
  - Use isobaric heat exchangers for heat transfer steps
- **Closed Cycles**
  - Not piston-and-cylinder devices
  - Working fluid remains inside the system
  - Use two HEX's to absorb and reject heat
  - Power generation turbines
- **Open Cycles**
  - Fresh working fluid drawn into the cycle
  - Spent working fluid discarded after 1 pass through the cycle
  - Usually air is the working fluid
  - Only 1 HEX because the fluid doesn't actually complete the cycle
    - Automobile and jet engines

- Power systems use a high-temperature heat source to produce work, usually shaft work.
- So far, we have only discussed closed cycles.
  - The working fluid is recycled.
  - Open cycles are often used in transportation systems, such as cars and planes.
    - The working fluid is almost always air.
    - This type of cycle eliminates the heat exchanger in which the working fluid is cooled or condensed to complete the cycle and return to the original state.
    - Instead, fresh working fluid is drawn into the system put through 3 steps of the cycle and then flows out of the system.
    - Think of automobile exhaust or jet engine contrails and you will have the right idea.
    - We will model open cycles as if they were really closed cycles.
      - This makes it easier to see the similarities between cycles.
      - It also simplifies the analysis somewhat.
      - More about open cycles in Lessons 9E & 9F.

PV Diagram: Power Cycles

- **This is a PV Diagram of a Gas Power Cycle**
- **A Vapor Power Cycle looks the same, but the 2-phase envelope is also shown on the plot.**



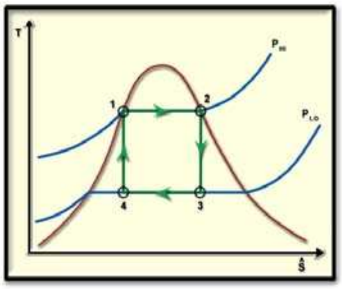
$$\frac{\dot{W}_{Sh,cycle}}{\dot{m}} = -\int_1^2 \hat{V}\,dP - \int_2^3 \hat{V}\,dP - \int_3^4 \hat{V}\,dP - \int_4^1 \hat{V}\,dP = \text{Enclosed Area}$$

- **In Vapor or Gas Power Cycles, the area enclosed by the cycle path is equal to the specific shaft work produced by the cycle.**

- We learned in Ch 8 that for internally reversible, SISO processes operating at steady-state with negligible changes in kinetic and potential energies…
  - The specific shaft work for a process is minus the integral of V^ dP.
- We apply this to each of the four processes that make up a Carnot Cycle and find that…
  - The area enclosed by the cycle path is equal to the specific shaft work produced by the cycle.
  - Remember that power cycles proceed clockwise around the cycle path on PV Diagrams.

TS Diagram: Power Cycles

- **This is a TS Diagram of a Vapor Power Cycle**
- **A Gas Power Cycle looks the same, but the 2-phase envelope is**



$$\dot{Q}_{cycle} = \dot{W}_{Sh,cycle}$$

$$\dot{m}\,d\hat{S} = \int \left(\frac{\delta \dot{Q}}{T} \right)_{int\,rev}$$

$$\frac{\dot{W}_{Sh,cycle}}{\dot{m}} = \int_1^2 T\,d\hat{S} + \int_2^3 T\,d\hat{S} + \int_3^4 T\,d\hat{S} + \int_4^1 T\,d\hat{S} = \text{Enclosed Area}$$

- **In Vapor or Gas Power Cycles, the area enclosed by the cycle path is, once again, equal to the specific shaft work produced by the cycle.**

- The 1st Law for a cycle tells us that $Q_{cycle} = W_{cycle}$.
- The definition of entropy allows us to evaluate Q for each step in our reversible Carnot Cycle as the integral of T dS.
- When we put these equations together and integrate all the way around the cycle…
  - ◊ We again find that the area enclosed by the cycle path is equal to the specific shaft work produced by the cycle.
  - ◊ Remember that power cycles proceed clockwise around the cycle path on TS Diagrams as well.

Carnot Is Not Practical

- **Why is the Carnot Vapor Power Cycle impractical ?**
  - Most pumps do not handle vapor-liquid mixtures well
    - Pumps that do are more expensive and less efficient.
  - Turbines do not work well with low quality
    - Quality must be greater than 90% at the outlet
    - This is difficult to achieve without superheating in the reboiler
  - Using strictly isothermal heating prevents…
    - Subcooled liquid feed to the boiler which would make the pump more efficient
    - Superheating in the boiler effluent which would increase turbine effluent quality

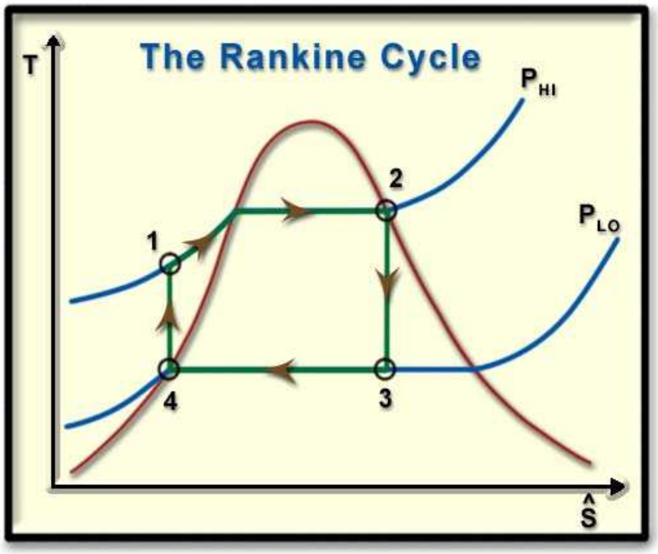
- The types of pumps and turbines that are most efficient are easily damaged by 2-phase mixtures.
- Also, pump efficiency drops significantly when vapor is present.
- We need to be able to use subcooled liquid feed to the boiler to avoid cavitation (vapor bubbles) in the pump.
- We need to produce superheated vapor in the boiler to improve the quality of the turbine effluent.
- The problem is that both of these practical improvements prevent the boiler from operating isothermally !
- This makes it almost impossible to make heat transfer in the boiler reversible because the inlet and outlet temperatures are different.
  - ◊ No matter what temperature you use for your hot reservoir, there is going to be a finite temperature difference and therefore irreversibility and lost work.
- This is a trade-off that we must accept to make our pump and turbine work well.

The Rankine Cycle

- **A practical vapor power cycle**
  - Minimizes cavitation problems in the pump
  - Allows for the possibility of superheating in the boiler
- **The Rankine Cycle**
  - Step 1-2: Boiler Heat added at constant pressure
  - Step 2-3 Turbine Isentropic expansion
  - Step 3-4: Condenser Heat rejected at constant pressure
  - Step 4-1: Pump Isentropic compression
- **Internally Reversible**
  - External irreversibility due to heat transfer through a finite temperature difference in the boiler

- In the Rankine Cycle, the boiler and condenser operate at constant pressure, but not necessarily at constant temperature.
- This allows the use of subcooled liquid boiler feed.
- In a modified version of the Rankine Cycle, we will also use superheating in the boiler.
- More about this in the next lesson.
- The irreversible heat transfer in the boiler means that the Rankine Cycle is NOT as efficient as the Carnot Cycle.
- We **cannot** calculate its efficiency based only on the reservoir temperatures, because it is not completely reversible.
- In order to determine the efficiency of a Rankine Cycle, we must analyze each step in the process, calculate $\mathbf{W_{Sh,cycle}}$ and $\mathbf{Q_H}$and then use them to determine the thermal efficiency of the cycle from its definition: h = $\mathbf{W_{Sh,cycle}}$ **/** $\mathbf{Q_H}$.

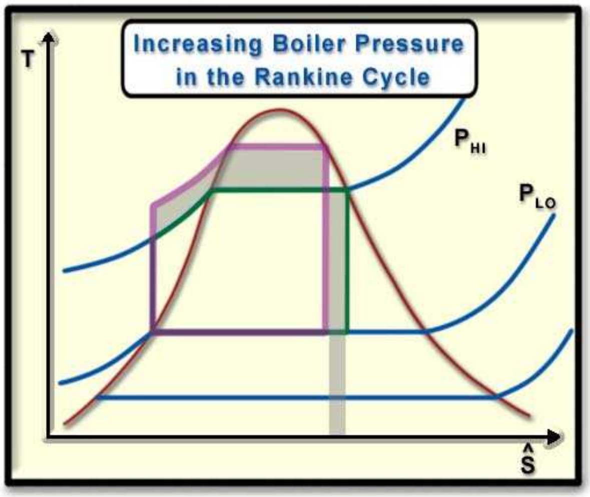
Rankine Cycle: TS Diagram



- The Rankine Cycle is internally reversible.
  - ◊ The pump and turbine are isentropic.
  - ◊ Heat transfer at the boiler is irreversible.
- The Rankine Cycle…
  - ◊ Addresses the problem with pumping a two-phase mixture.
  - ◊ Does not address the problem of low quality in the turbine effluent.
    - But allows for the possibility of fixing this problem by producing superheated vapor in the boiler.
    - This will be a modification / improvement to the Rankine Cycle that we will study in the next lesson.

Efficiency & Boiler Pressure

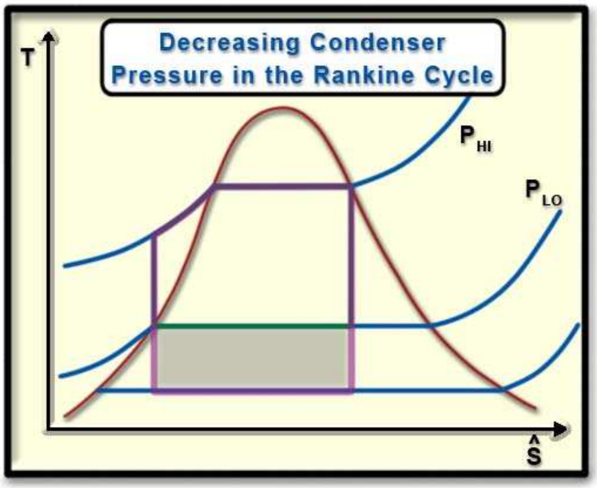
- W_{Sh} ↑↑
- Q_H ↑
- $\eta_{th} = \frac{W_{Sh}}{Q_H}$
- η_{th} ↑



- When we operate the boiler at a higher pressure, BOTH $\mathbf{W_{Sh}}$ and $\mathbf{Q_H}$ increase.
- The good news is that $\mathbf{W_{Sh}}$ increases by a larger amount.
- The net result is an INCREASE in the thermal efficiency of the cycle. Whoohoo !
- The bad news is that the quality of the turbine effluent drops even lower.
  - ◊ We will see in the next lesson that this problem can be addressed by superheating the turbine feed in the boiler.

Efficiency & Condenser Pressure

- W_{Sh} ↑↑
- Q_H: same
- $\eta_{th} = \frac{W_{Sh}}{Q_H}$
- η_{th} ↑



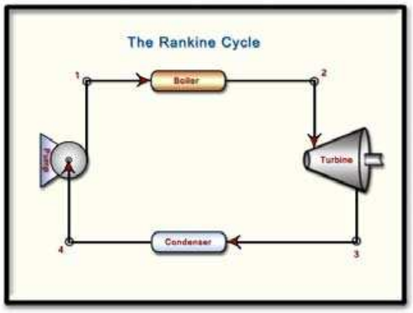
- When we operate the condenser at a lower pressure, $\mathbf{W_{Sh}}$ increases while $\mathbf{Q_H}$ remains the same.
- The result is an INCREASE in the thermal efficiency of the cycle. Whoohoo !
- Unfortunately, the quality of the turbine effluent drops.
- Again, we will see in the next lesson that this problem can be addressed by superheating the turbine feed in the boiler.

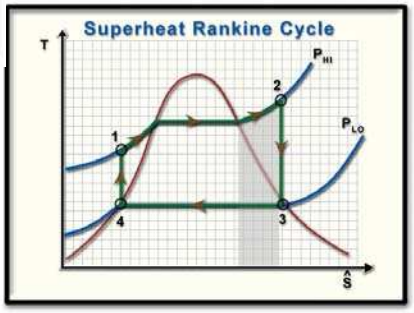
Improvements on the Rankine Cycle

- Superheat Rankine Cycle
  - ◊ Almost always used, improves η and turbine effluent quality
- Supercritical Rankine Cycle
  - ◊ Increases η. Not common b/c, for steam, T and P are very high & materials become very expensive.
- Reheat Rankine Cycle
  - ◊ Very common way to improve turbine effluent quality
  - ◊ η drops slightly unless regeneration is used as well.
- Regeneration Rankine Cycle
  - ◊ Preheating boiler feed reduces irreversibility of heat transfer
  - ◊ Increases η.
- Binary Rankine Cycle
  - ◊ Not very common.
  - ◊ Main advantage is T_H >> T_CBig increase in η, but also increases cost to build.
- Rankine Cycle with Cogeneration
  - ◊ Use some of the HP turbine effluent in another process.

- All of the improvements are designed to either..
  - ◊ Increase the quality of the turbine effluent.
  - ◊ Increase the thermal efficiency of the cycle.
  - ◊ OR BOTH !
- Cogeneration is not really an improvement to the Rankine Cycle.
  - ◊ It is more of an optional mode of operation.

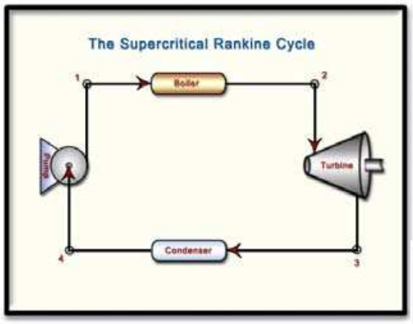
Superheat Rankine Cycle

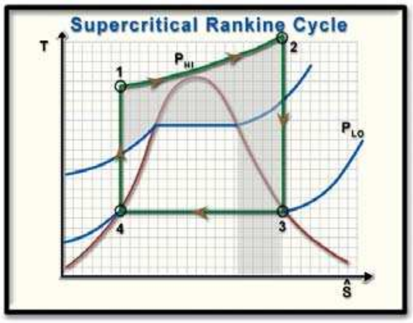




- Superheat is essentially a given.
- The improvement in both efficiency and turbine effluent quality more than makes up for the increase in irreversiblities in the heat transfer in the boiler.
  - ◊ Notice that T_H must be greater than T_2.
  - ◊ As a result, there is a relatively large **ΔT** between the how reservoir and both the subcooled and saturated liquid/vapor in the boiler.
  - ◊ This results in lost work due to irreversibilities.
  - ◊ This is one drawback of using superheat, but it is minor compared to the benefits.
  - ◊ The only other drawback is the increased operating temperature of the boiler and turbine.
    - The boiler and turbine must be made of materials that can withstand these higher temperatures without mechanical failure.

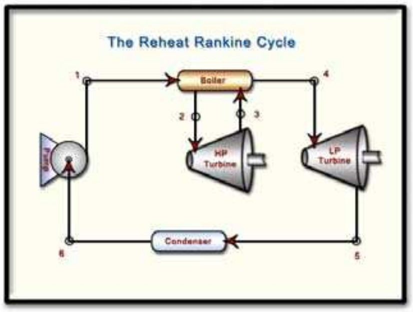
Supercritical Rankine Cycle

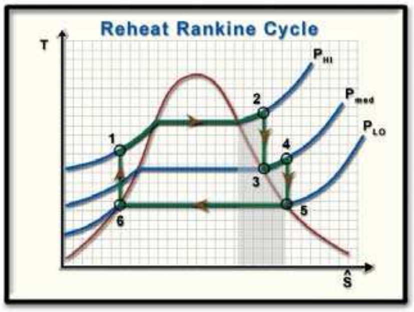




- Big boost to efficiency.
- Not so common when steam is the working fluid because the critical point of water is so high…**373.95°C**.
  - ◊ This forces the boiler to operate at very high temperatures.
  - ◊ This requires the boiler and turbine to made of very expensive materials.
- More useful when steam is not the working fluid.

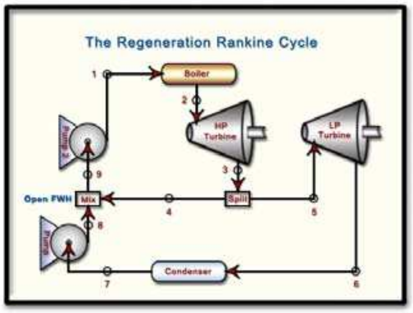
Reheat Rankine Cycle





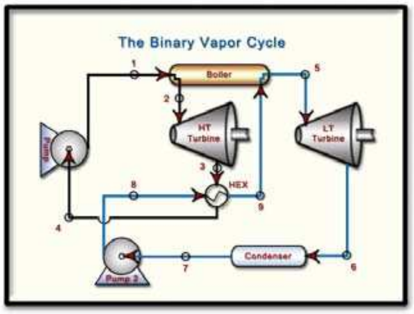
- Reheat is almost as common as superheat.
- Bad news:
  - ◊ It costs more to build a 2-stage turbine.
  - ◊ Reheat actually lowers the efficiency.
    - Unless you combine reheat with regeneration !
    - So, it is often used in combination with regeneration.
- Good news:
  - ◊ You can use high boiler pressures and low condenser pressures without worrying much about turbine effluent quality.

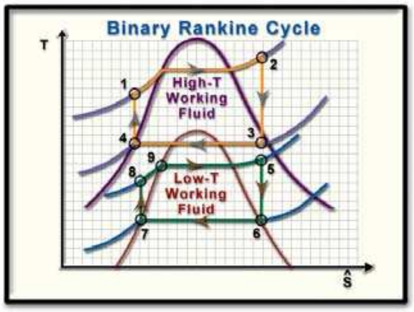
Regeneration Rankine Cycle



- Regeneration is more complicated than the other schemes.
- The purpose of regeneration is to reduce the lost work due to irreversible heat transfer in the boiler by PRE-HEATING the subcooled liquid boiler feed.
  - ◊ The boiler feed can be pre-heated in an open feedwater heater: streams 4 and 8 mix to form stream 9.
  - ◊ Or in a closed feedwater heater: this is just a heat exchanger.
- The cost of construction is not much more than reheat alone.
  - ◊ The extra pump, tank and plumbing are pretty cheap.
- The result is that reheat can be used AND efficiency can still be better than in the superheated Rankine Cycle.

Binary Rankine Cycle





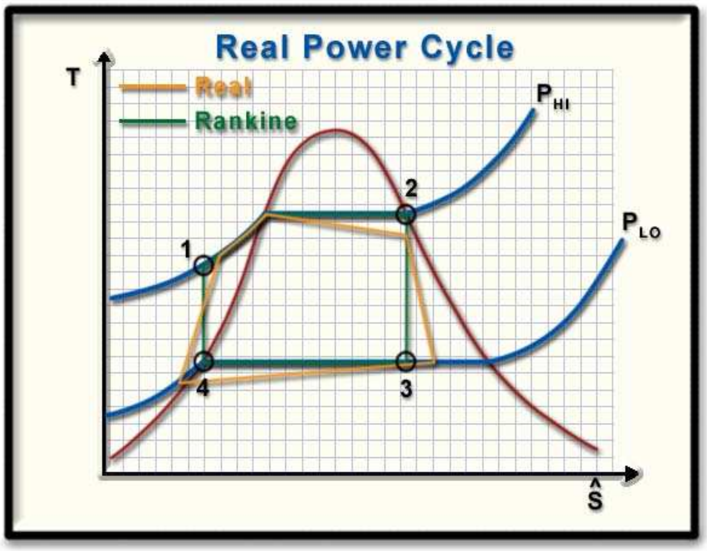
- •Use two working fluid in two separate superheat Rankine Cycles.
  - ◊A common choice has been mercury and water.
  - ◊ Mercury is a good high-temperature fluid, but it is relatively toxic.
- The key that connects the two cycles is a heat exchanger.
  - ◊ As the high-temperature working fluid condenses, it gives up its heat to boil the low-temperature fluid.
  - ◊ Cool, eh ?
- A binary cycle yields a very high efficiency, but costs almost twice as much as our standard superheat Rankine Cycle.

Irreversibilities

- **4 main sources of irreversibilty in a real power cycle:**
  - ◊ Heat losses to the surroundings
    - Effects every process as well as the pipes that connect them.
  - ◊ Fluid friction
    - Effects every process as well as the pipes that connect them.
    - Result is pressure drop. This causes the temperature to drop as well in the boiler and condenser.
  - ◊ Mechanical losses
    - Friction & rapid expansion & compression)
    - Effects the turbine and the pump the most.
    - Causes entropy to increase.
  - ◊ Subcooling in the condenser
    - Necessary to avoid cavitation in the pump.

- Consider a real power cycle that is setup to follow the Rankine Cycle as close as possible.
- Irreversibilities make it impossible for the real cycle to be a Rankine Cycle.
- This is a short list of the main sources of irreversibility.
- On the next page we will see what the real cycle path looks like on a **TS** Diagram.

Irreversibilities on a TS Diagram



- Every step in the process path is skewed.
- Every form of irreversiblity adds to the entropy generation and the lost work.

Irreversibilities & Lost Work

- **From Lesson 8D:**

$$\dot{S}_{gen,tot} = \dot{m}\,\Delta\hat{S} - \sum_{i}^{Processes} \frac{\dot{Q}_{act,i}}{T_{res,i}}$$

$$\dot{W}_{Sh,lost} = T_{surr} \sum_{i}^{Processes} \left[\dot{S}_{gen,tot}\right]_i$$

$$\dot{W}_{Sh,lost} = T_{surr} \sum_{i}^{Processes} \frac{-\dot{Q}_{act,i}}{T_{res,i}}$$

$$\dot{W}_{Sh,lost} = T_{surr} \left(\frac{\dot{Q}_{C,act}}{T_{C,res}} - \frac{\dot{Q}_{H,act}}{T_{H,res}} \right)$$

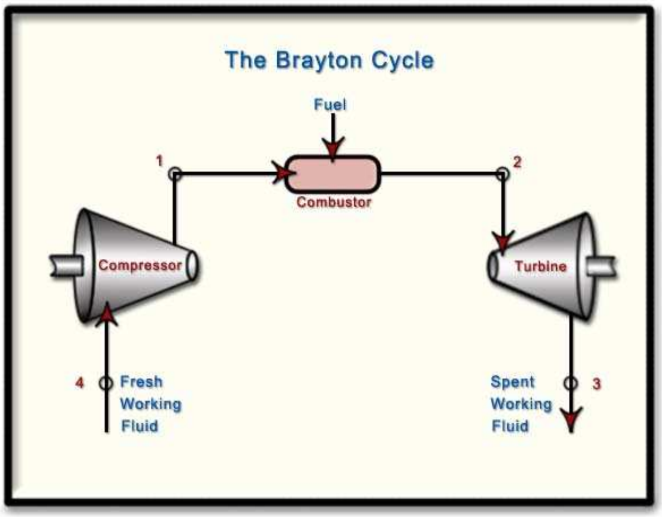
- **Now, include the effect of direct heat lost to the surroundings:**

$$\boxed{\dot{W}_{Sh,lost} = T_{surr} \left(\frac{\dot{Q}_{C,act}}{T_{C,res}} - \frac{\dot{Q}_{H,act}}{T_{H,res}} \right) + \dot{Q}_{lost}}$$

- The algebra is shown in Thermo-CD.
- The bottom line on heat lost is simple.
- Any heat lost results in a DIRECT increase in the lost work.
  - ◊ This not surprising.
  - ◊ Any energy that is transferred to the surroundings (the DEAD state) is LOST !

The Brayton Cycle

- **The ideal gas cycle for gas-turbine engines**



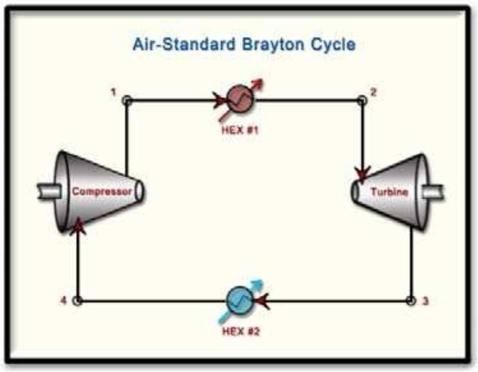
- Gas-turbine engines are used for transportation.
  - ◊ Warships
  - ◊ Abrams tanks
- Gas turbines are also commonly used for electrical power generation.
- Other types of gas power cycles are better suited to other applications.
  - ◊ The Diesel Cycle is a reasonable approximation of the cycles used in many trucks, trains and ships.
  - ◊ The Otto Cycle is a reasonable approximation of the cycles used in gasoline powered automobile engines.
  - ◊ The Brayton Cycle is the only gas power cycle we will consider in this course.
- The Brayton Cycle is an open cycle because fresh working fluid is draw into the cycle and spent working fluid is rejected from the cycle.
  - ◊ This almost invariably means the working fluid is air.

- The Brayton Cycle is an internal combustion engine.
  - ◊ This means heat is not added to the cycle in a HEX.
  - ◊ Instead, a chemical reaction (combustion) is carried out within the cycle.
  - ◊ The energy released by the chemical reaction increases the temperature of the working fluid (air-fuel-combustion products mixture).
  - ◊ This complicates the system a great deal because we no longer have a pure working fluid.

The Air-Standard Brayton Cycle

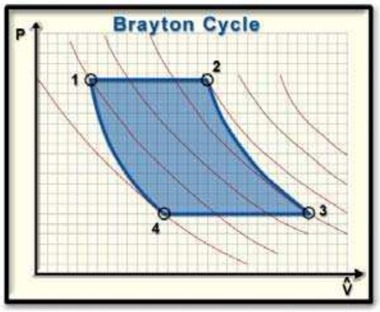
- **Air-Standard Assumptions**
  - ◊ Air is the working fluid and it behaves as an ideal gas.
  - ◊ The Brayton Cycle is modeled as as a closed cycle.
  - ◊ The combustor is replaced by HEX #1. (External Combustion)
  - ◊ All processes are internally reversible.

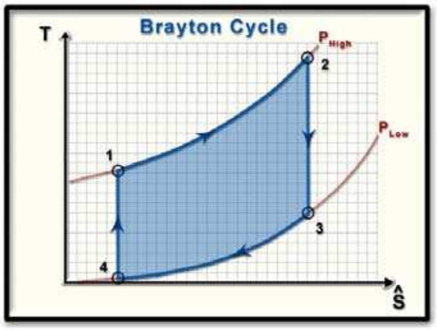
- Step 1-2: Isobaric heating
- Step 2-3: Isentropic expansion
- Step 3-4: Isobaric cooling
- Step 4-1: Isentropic compression



- It is much easier to analyze the performance of the Air-Standard Brayton Cycle.
- We lose some accuracy by assuming the air is an ideal gas.
  - ◊ But we can still learn a great deal about how Brayton Cycles work and how different operating parameters effect their efficiency.
- In an internally reversible Brayton Cycle, the Compressor and Turbine are adiabatic and therefore, isentropic.
  - ◊ But, in Ch 7 and Ch 8 we learned how to use isentropic efficiency to take irreversibilities into account.
  - ◊ We will use isentropic efficiencies for the compressor and turbine in Brayton Cycles to develop slightly more realistic models of their performance.
- We will always consider the heat exchangers to be isobaric.

Brayton Cycle: PV & TS Diagrams





- **PV** and **TS** Diagrams for the internally reversible Brayton Cycle are pretty simple.

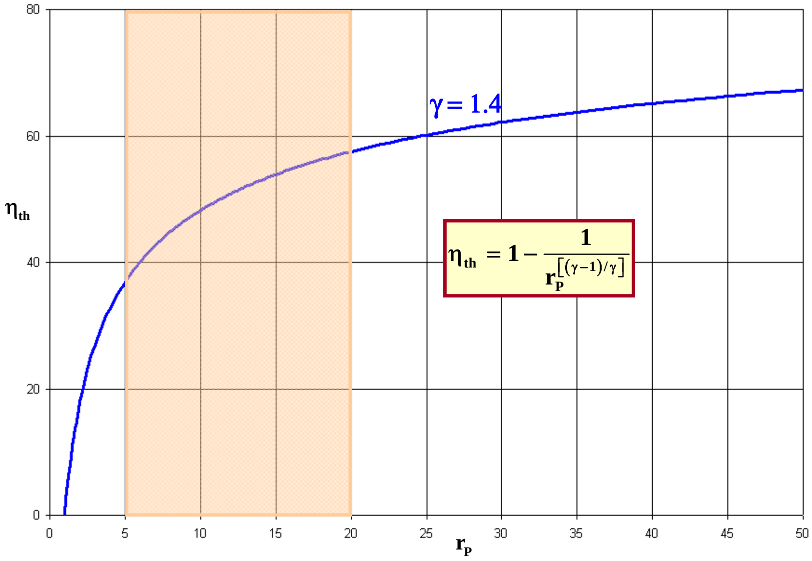
The Cold Air-Standard Assumption

- **The heat capacities of air are <u>constant</u> and always have the values determined at <u>25^oC</u>.**
- **Compression Ratio:** $r_P = \frac{P_1}{P_4}$
- **Thermal efficiency of an internally reversible, cold air-standard Brayton Cycle:**

$$\boxed{\eta_{th} = 1 - \frac{1}{r_P^{[(\gamma-1)/\gamma]}}}$$

- The cold air-standard assumption makes analysis of the Brayton Cycle pretty straightforward.
  - ◊ **C_P** and **C_V** are both assumed to be constant. Nothing complicated like the Shomate Equation is used in this model.
  - ◊ **C_P** and **C_V** are both evaluated at **25^oC**.
    - Because gas turbines generally do not operate anywhere near **25^oC**, this assumption introduces some very significant error.
    - Still, the cold air-standard model helps us understand the trends observed in a real gas power cycle without all the tedious calculations.
    - We are also able to understand how the Brayton Cycle can be improved.
- The derivation of the thermal efficiency of the air-standard Brayton Cycle is in Thermo-CD.
  - ◊ It isn't long or hard.
  - ◊ Take the time to understand it and you will have a good grasp of how to analyze Brayton Cycles.
  - ◊ The boxed equation is the result.

Cold Air-Standard Brayton Cycle Efficiency



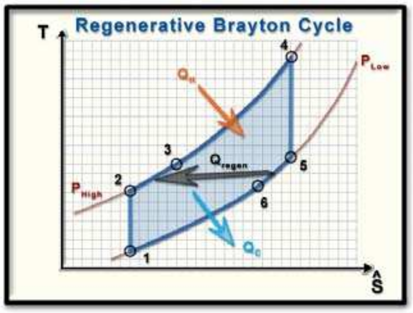
- ◊ The shaded band represents the range of compression ratios that are commonly used.
- ◊ The graph shows that the corresponding range of thermal efficiency is from about **37%** to about **58%**.
- ◊ That is not bad when you consider the efficiency of most automobile engines is less than **35%**.

Improvements to the Brayton Cycle

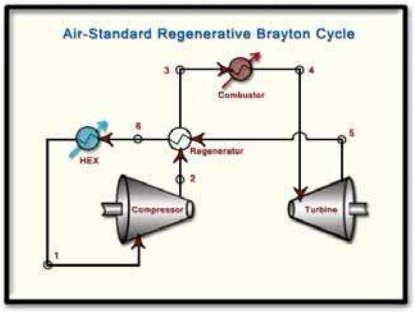
- **Regeneration**
  - ◊ Use the hot turbine effluent to preheat the feed to the combustor.
- **Reheat**
  - ◊ Use a 2-stage turbine and reheat the effluent from the HP turbine before putting into the LP turbine.
- **Intercooling**
  - ◊ Use a 2-stage compressor with an intercooler.
- **Regeneration with Reheat and Intercooling**
  - ◊ Use all of the techniques listed above to achieve high efficiency.

- Regeneration and reheat work in the same way that they do in a vapor power cycle.
- Regeneration can improve the efficiency if the proper compression ratio is used.
- Reheat reduces the efficiency of the cycle unless it is used with regeneration.
  - ◊ Reheat can help keep operating temperatures down.
  - ◊ This can reduce equipment costs.
- A 2-stage compressor costs more and actually lowers the thermal efficiency of the cycle…
  - ◊ Unless it is used in combination with regeneration !
- When you use regeneration with both a 2-stage turbine with reheat and a 2-stage compressor with intercooling, the thermal efficiency increases substantially !
  - ◊ In fact, if you could use an infinite number of compressor stages with intercoolers…
  - ◊ And an infinite number of turbine stages with reheat…
  - ◊ You would have a CARNOT cycle and you would get maximum efficiency !
  - ◊ You would also have infinite cost !

Regenerative Brayton Cycle



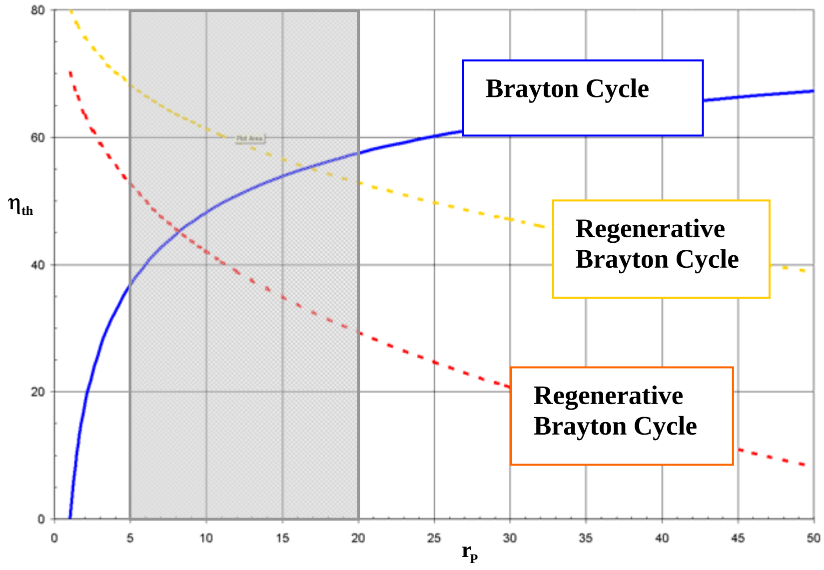
$$r_P = \frac{P_2}{P_1}$$



$$\boxed{\eta_{th} = 1 - \frac{T_1}{T_4}\, r_P^{[(\gamma-1)/\gamma]}}$$

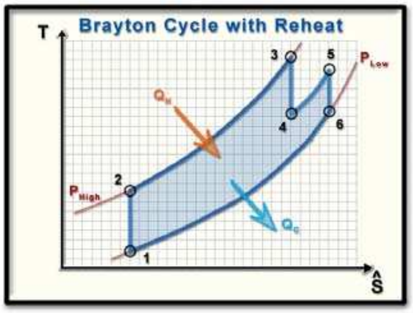
- The boxed equation applies for an internally reversible air-standard Brayton Cycle with Regeneration.
- A closed cycle is shown here because this is the way we will analyze Brayton Cycles, even though they are usually open.
- The hot turbine effluent, stream 5, is used to preheat the compressed air that enters the combustor, stream 3.
- Regeneration does not effect the power generated by the cycle, but reduces **Q_H** and **Q_C** and thereby increases the efficiency of the cycle.

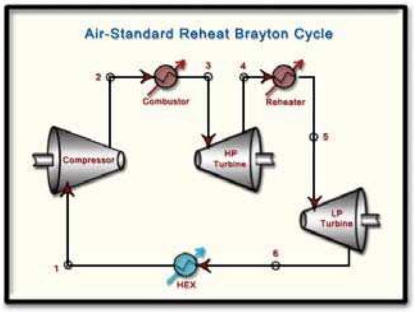
Regenerative Brayton Cycle Efficiency



- For certain compression ratios, the regeneration cycle is significantly more efficient.
- The bonus is that the regenerative cycle is more efficient at LOW compression ratios !
    - The result is that the compressor is less expensive to purchase and to operate.

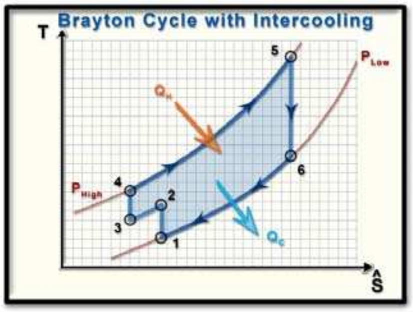
Reheat Brayton Cycle

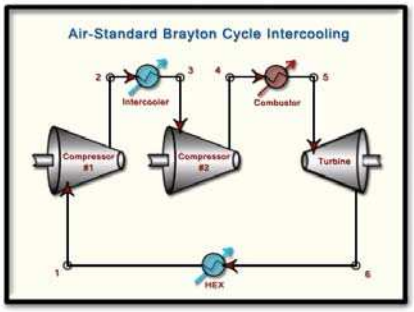




- Reheat increases $\mathbf{Q_H}$ and $\mathbf{Q_C}$, but DECREASES the thermal efficiency.
- It does help avoid very high operating temperatures which drive up the costs of the equipment.
- Reheat is never used without regeneration in a Brayton Cycle.

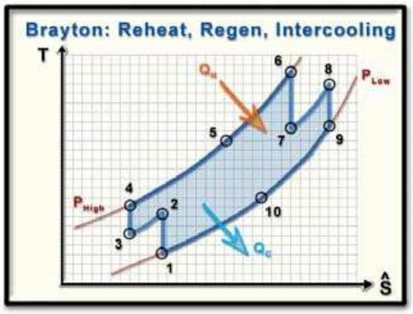
Brayton Cycle with Intercooling

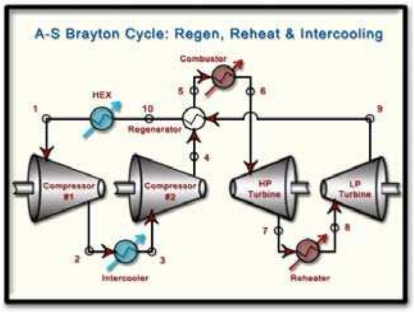




- Intercooling reduces the power requirement for compression.
- It also increases $\mathbf{Q_H}$ and $\mathbf{Q_C}$. But it increases $\mathbf{Q_C}$by more !
- The net result is a slight DECREASE in the thermal efficiency.
- Intercooling is never used without regeneration.

Regeneration, Reheat & Intercooling

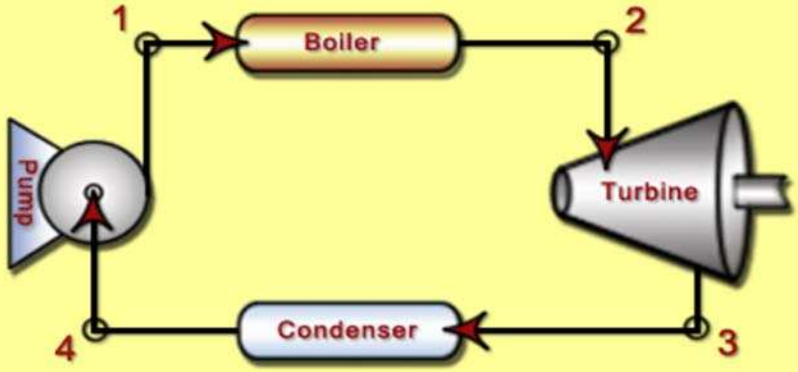




- This is the way real gas turbine power cycles are operated.
- The increase in thermal efficiency is substantial when all the improvements are combined.
- The intercooler reduces the compressor work.
- The reheater keeps the maximum operating temperature down while increasing $\mathbf{Q_H}$.
- The regenerator facilitates the intercooler and reheater while maintaining a high thermal efficiency.
- This is the top of the line.
- It is also the most complex system we will analyze in this course.

9B-1 **Ideal Rankine Cycle Efficiency as a Function of Condenser Pressure** **8 pts**

Consider the ideal Rankine Cycle with superheat using **water** as the working fluid.



Construct plots of the **net power output** of the cycle and the **thermodynamic efficiency** as functions of the operating **pressure** of the condenser. Consider condenser **pressures** from **10 kPa** to **200 kPa**.

Data: P_1 **= 10 MPa**, T_2 **= 550°C**, **m = 75 kg/s**

Read : The key is that the cycle is an ideal **Rankine Cycle**. This means that the pump and turbine operate isentropically and that the condenser effluent is a saturated liquid.

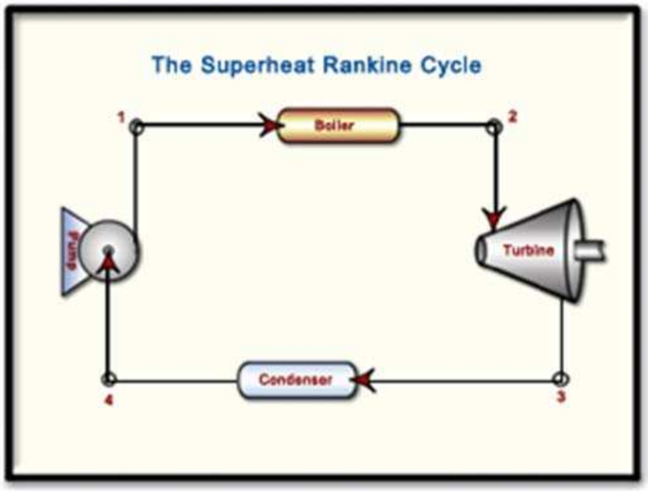
Constructing the plots requires looking-up a lot of **data**. This can be done most efficiently using the **NIST Webbook** to generate **data** tables that can be copied and pasted into **Excel**.

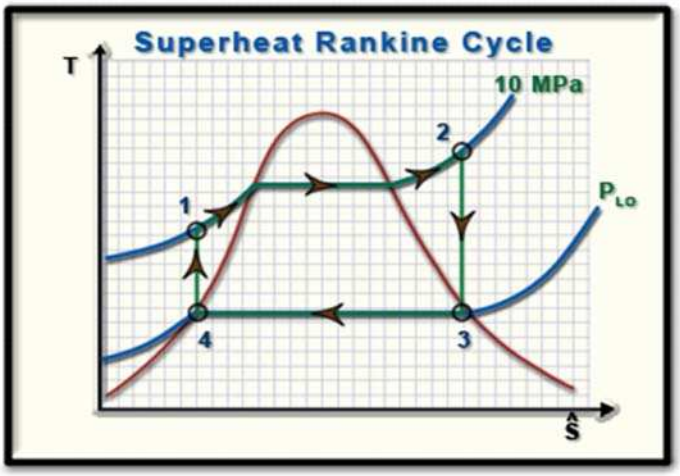
Given:

$P_1 = P_2$	10	MPa	m	75	kg/s
T_2	550	°C			

Find: Plot W_{cycle} and η_{th} as functions of P_4.

Diagram:





Assumptions:	**1 -**	The pump and the turbine are adiabatic and reversible and, therefore, isentropic.
	2 -	Only **flow work** crosses the boundary of the boiler and condenser.

Equations / Data / Solve:

Let's begin with a very detailed analysis of the problem for a single condenser **pressure** of **10 kPa**.

Then, we can present a table of results for all of the other condenser **pressures** so we can construct the plot that is required.

Let's organize the **data** that we need to collect into a table. This will make it easier to keep track of the values we have looked up and the values we have calculated.

Stream	State	T (^{o}C)	P (kPa)	X	H (kJ/kg)	S (kJ/kg-K)
1	Sub Liq	**46.14**	10000	N/A	**201.88**	**0.64920**
2	Super Vap	550	10000	N/A	3502.0	6.7585
3	**VLE**	45.81	10	**0.8146**	**2140.4**	**6.7585**
4	Sat Liquid	45.81	10	0	191.81	0.64920

Additional **data** that may be useful.

State	T (^{o}C)	P (kPa)	X	H (kJ/kg)	S (kJ/kg-K)
Sat Vap	45.81	10	1	2583.9	8.1488
Sat Liquid	45.81	10	0	191.81	0.64920
Sat Vap	311.06	10000	1	2725.5	5.6160
Sat Liquid	311.06	10000	0	1408.1	3.3606

The values in the table that are shown in **bold** with a **yellow** background are the values we will determine in the following solution.

One approach to solving cycle problems of this nature is to work your way around the cycle until you have evaluated all the properties to complete the **table** shown above. Then, you can go back and apply the 1st Law to each process in the cycle to evaluate **Q** and $\mathbf{W_s}$ as need. That is the approach I will take.

In this problem, it makes the most sense to begin at either **state 2** or **state 4** because these **states** are completely fixed (this means we know the values of two intensive properties and we can use them to determine the values of any other intensive properties using the **NIST Webbook**. I will begin at **state 2**.

H_2 **3502.0** **kJ/kg** **S_2** **6.7585** **kJ/kg-K**

Because the turbine is isentropic, we know that $\mathbf{S_3 = S_2}$: **S_3** **6.7585** **kJ/kg-K**

Now, we know the values of two intensive properties at **state 3**, so this **state** is completely fixed and we can look-up all of its properties.

We can see from the additional **data** table that $\mathbf{S_{sat\ liq} < S_3 < S_{sat\ vap}}$. Therefore, **state 3** is a two-phase mixture, $\mathbf{T_3 = T_{sat}}$ at **10 kPa** and we must determine the **quality** so that we can determine the **enthalpy**.

$$x_3 = \frac{\hat{S}_3 - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}}$$ **Eqn 1**

T_3 **45.81** **^{o}C** **x_3** **0.8146** **kg vap/kg**

$$\hat{H}_3 = x_3 \hat{H}_{sat\,vap} + (1 - x_3)\hat{H}_{sat\,liq}$$ **Eqn 2**

H_3 **2140.4** **kJ/kg**

We can look-up all the properties at **state 4** because the **state** is completely fixed by the fact that the **fluid** is a saturated liquid leaving the condenser in an Ideal Rankine Cycle and because we know the pressure is **10 kPa**.

Next, we proceed to **state 1** using the fact that the pump is also isentropic: $\mathbf{S_1 = S_4}$.

S_1	**0.64920**	**kJ/kg**

Now, we know the values of two intensive properties at **state 4**, so this **state** is completely fixed and we can look-up all of its properties.

We can see from the additional **data** table, above, that $\mathbf{S_4 < S_{sat\ liq}}$. Therefore, **state 4** is a subcooled liquid. We can look-up its properties in the **NIST Webbook**. The **NIST Webbook** require interpolation or repeated **temperature** range selection to zero-in on the precise property values for **state 4**.

By zooming-in on a very narrow **temperature** range, I found :

T_1	**46.14**	**°C**
H_1	**201.88**	**kJ/kg**

Interpolation on a wider **temperature** range is shown below. The results are very similar.

At **10 MPa** :

T (°C)	**H (kJ/kg)**	**S(kJ/kg-K)**
45	**197.15**	**0.63436**
T_1	H_1	**0.64920**
50	**217.94**	**0.69920**

T_1	**46.14**	**°C**
H_1	**201.91**	**kJ/kg**

Now, we have all the information we need to apply the 1st Law to each device in the cycle to determine **Q** and $\mathbf{W_S}$ for each device. This process is made easier by our assumptions that no **shaft work** crosses the boundary of the boiler or condenser and that both the pump and the turbine are adiabatic.

The four relevant forms of the 1st Law are :

$$\hat{Q} - \hat{W}_S = \Delta\hat{H} \qquad \textbf{Eqn 4}$$

Boiler :

$$\hat{Q}_{boil} = \hat{H}_2 - \hat{H}_1 \qquad \textbf{Eqn 5}$$

Turbine :

$$\hat{W}_{S,turb} = \hat{H}_2 - \hat{H}_3 \qquad \textbf{Eqn 6}$$

Condenser :

$$\hat{Q}_{cond} = \hat{H}_4 - \hat{H}_3 \qquad \textbf{Eqn 7}$$

Pump :

$$\hat{W}_{S,pump} = \hat{H}_4 - \hat{H}_1 \qquad \textbf{Eqn 8}$$

Plugging values into **Eqns 5 - 8** yields :

Q_{boil}	**3300.1**	**kJ/kg**	W_{turb}	**1361.6**	**kJ/kg**
Q_{cond}	**-1948.6**	**kJ/kg**	W_{pump}	**-10.071**	**kJ/kg**
Q_{boil}	**247.5**	**MW**	W_{turb}	**102.12**	**MW**
Q_{cond}	**-146.1**	**kW**	W_{pump}	**-755.3**	**kW**

W_{cycle}	**101.36**	**MW**

Finally, we can calculate the **thermal efficiency** of this cycle.

$$\eta_{th} = \frac{W_{cycle}}{Q_{boil}} \qquad \textbf{Eqn 9}$$

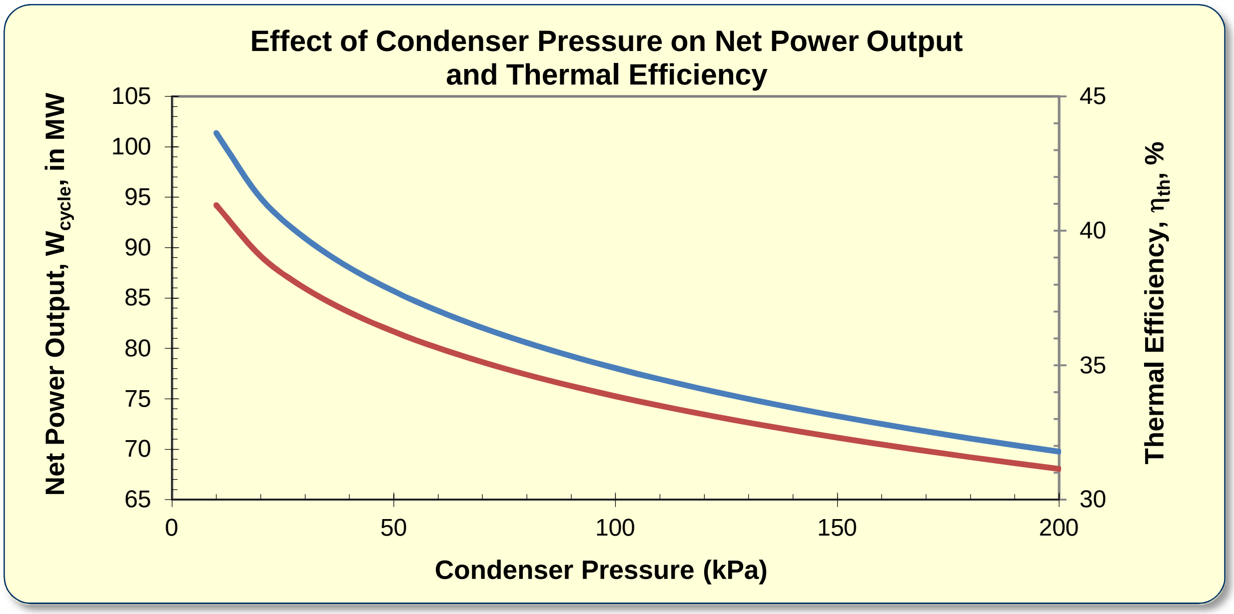
η_{th}	**40.95%**

Repeating this analysis for a variety of condenser pressures yield the following table of results.

P_{cond} (kPa)	H_3 (kJ/kg)	H_4 (kJ/kg)	S_4 (kJ/kg-K)	H_1 (kJ/kg)	W_{cycle} (MW)	Q_H (MW)	η_{th}
10	2140.4	191.81	0.64920	201.88	**101.4**	247.5	**41.0**
20	2226.2	251.42	0.83202	261.55	**94.9**	243.0	**39.1**
30	2279.2	289.27	0.94407	299.44	**90.9**	240.2	**37.9**
40	2318.3	317.62	1.0261	327.83	**88.0**	238.1	**37.0**
50	2349.4	340.54	1.0912	350.77	**85.7**	236.3	**36.3**
60	2375.4	359.91	1.1454	370.15	**83.7**	234.9	**35.6**
70	2397.8	376.75	1.1921	387.02	**82.0**	233.6	**35.1**
80	2417.5	391.71	1.2330	401.99	**80.6**	232.5	**34.7**
90	2435.2	405.20	1.2696	415.49	**79.2**	231.5	**34.2**
100	2451.1	417.50	1.3028	427.81	**78.0**	230.6	**33.8**
110	2465.8	428.84	1.3330	439.15	**76.9**	229.7	**33.5**
120	2479.3	439.36	1.3609	449.68	**75.9**	228.9	**33.2**
130	2491.8	449.19	1.3868	459.52	**75.0**	228.2	**32.9**
140	2503.5	458.42	1.4110	468.76	**74.1**	227.5	**32.6**
150	2514.5	467.13	1.4337	477.47	**73.3**	226.8	**32.3**
160	2524.9	475.38	1.4551	485.73	**72.5**	226.2	**32.1**
170	2534.7	483.22	1.4753	493.58	**71.8**	225.6	**31.8**
180	2544.0	490.70	1.4945	501.06	**71.1**	225.1	**31.6**
190	2552.8	497.85	1.5127	508.22	**70.4**	224.5	**31.4**
200	2561.3	504.70	1.5302	515.07	**69.8**	224.0	**31.1**

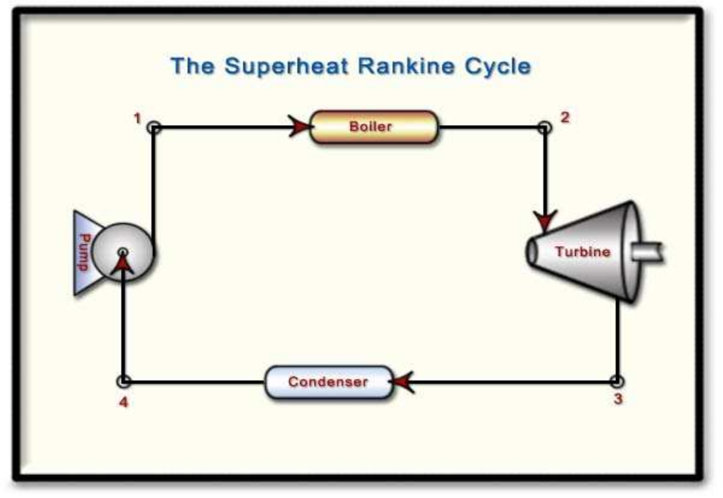
Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :



9B-2 Steam Power Plant Operating on the Rankine Cycle 9 pts

Consider the Rankine Power Cycle shown below. **Steam** is the working fluid. The hot and cold thermal reservoirs are at **500°C** and **10°C**, respectively. The boiler operates at **12 MPa** and the condenser operates at **100 kPa**. The



pump is isentropic and the turbine has an **isentropic efficiency** of **84%**. The pump and turbine are adiabatic. The **temperature** of the surroundings is **T_{surr} = 300 K**.

a.) Construct a neat, fully labelled **TS Diagram** of the cycle.

Calculate…

b.) **Q** & **W** for **each** unit in the cycle, in **kJ/kg**.

c.) The **thermal efficiency** of the power cycle.

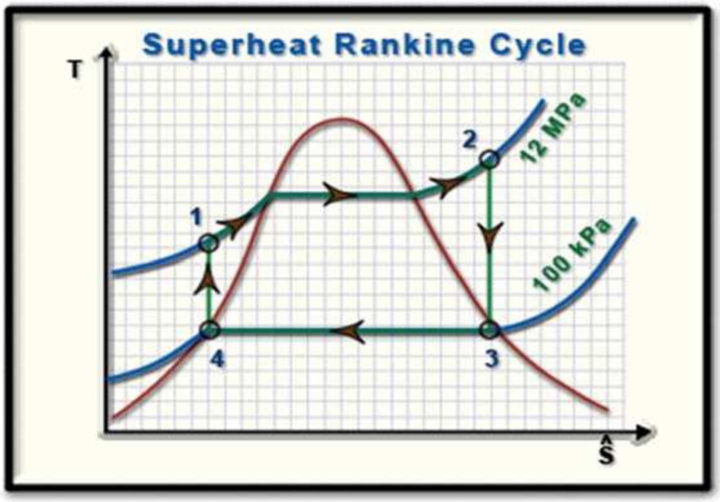
d.) The total **entropy generation**in **kJ/kg-K** and the total **lost work** in **kJ/kg** for the cycle.

e.) The net **work** that would be produced if this cycle were completely reversible and the state of all four streams remained the same as in the actual cycle.

Read : The **TS Diagram** is pretty standard. The pump is isentropic, but the turbine is not. In order to determine the **Q**'s and **W_S**'s we will need to determine all the **H**'s. **H_2** and **H_4** come straight from the **Steam Tables**. Then use **entropy** to determine **H_1** and **H_3**. Plug the **H**'s into the 1st Law for each unit to determine all the **Q**'s and **W**'s. Once we have all the **Q**'s and **W**'s in **part (b)**, we can calculate the **thermal efficiency** from its definition. The key to **part (d)** is that, for a cycle, **$\Delta S = 0$**. So, **$S_{gen} = \Delta S_{univ}$** is just **$\Delta S_{furn} + \Delta S_{cw}$**. Because the furnace and cooling water behave as thermal reservoirs, we can evaluate the change in their **entropy** from the definition of **entropy**. **Lost work** is just **$T_{surr}\ S_{gen}$**. The easiest way to evaluate **$W_{S,rev}$** is to use the definition of **$W_{S,lost}$** and the values of **$W_{S,act}$** and **$W_{S,lost}$** that were determined in **parts (b)** and **(d)**, repectively. **$W_{S,rev} = W_{S,act} + W_{S,lost}$**.

Given:	P_2	12000	kPa	$Q_{41} = Q_{23} =$	0	kJ/kg
	T_2	450	°C	T_{furn}	500	°C
	$\eta_{s,turb}$	84%			773.15	K
	P_3	100	kPa	T_{cw}	10	°C
	x_4	1	kg vap / kg		283.15	K
				T_{surr}	300	K

Diagram:



Let's organize the **data** that we need to collect into a table. This will make it easier to keep track of the values we have looked up and the values we have calculated.

Stream	State	T (^oC)	P (kPa)	X	H (kJ/kg)	S (kJ/kg-K)
1	Sub Liq	100.42	12000	N/A	429.9	1.3028
2	Super Vap	450	12000	N/A	3209.8	6.3028
3S	Sat Mix	99.61	100	0.8256	2281.3	6.3028
3	Sat Mix	99.61	100	0.8914	2429.9	6.7013
4	Sat Liq	99.61	100	1	417.50	1.30276

Additional **data** that may be useful.

State	T (^oC)	P (kPa)	X	H (kJ/kg)	S (kJ/kg-K)
Sat Vap	324.68	12000	1	2685.4	5.4939
Sat Liquid	324.68	12000	0	1491.46	3.49671
Sat Vap	99.63	100	1	2674.9	7.3588
Sat Liquid	99.63	100	0	417.5	1.3028

Part b.) In order to evaluate **Q** and $\mathbf{W_S}$ for each process in the cycle, we will apply the 1st Law to each process. All the devices operate at steady-state and any changes in kinetic and potential energies are negligible. The boiler and condenser have no **shaft work** interaction and the pump and turbine are both adiabatic. Therefore, the relevant forms of the 1st Law for the four devices are:

Boiler :

$$\hat{Q}_{12} = \hat{H}_2 - \hat{H}_1 \qquad \text{Eqn 1}$$

Turbine :

$$\hat{W}_{S,turb} = \hat{H}_2 - \hat{H}_3 \qquad \text{Eqn 2}$$

Condenser :

$$\hat{Q}_{34} = \hat{H}_4 - \hat{H}_3 \qquad \text{Eqn 3}$$

Pump :

$$\hat{W}_{S,pump} = \hat{H}_4 - \hat{H}_1 \qquad \text{Eqn 4}$$

In order to evaluate all the **W**'s and **Q**'s in **Eqns 1 - 4**, we must first determine **H** for all four streams. We can immediately evaluate $\mathbf{H_2}$ and $\mathbf{H_4}$ because we know both $\mathbf{P_2}$ and $\mathbf{T_2}$ and we know $\mathbf{P_4}$ and we know that it is a saturated liquid, $\mathbf{x_4 = 0}$. So, we can lookup $\mathbf{H_2}$ and $\mathbf{H_4}$ in the **NIST Webbook**.

H_2	3209.8	kJ/kg	H_4	417.50	kJ/kg

In order to determine H_1, we must make use of the fact that the pump is adiabatic and internally reversible and that changes in kinetic and potential energies are negligible. Under these conditions, the **shaft work** done at the pump can be determined from the Mechanical Energy Balance Equation:

$$-\hat{W}_{S,pump} = \int_4^1 \hat{V}\,dP + \cancel{\frac{\Delta v^2}{2g_C}} + \cancel{\frac{g}{g_C}}\Delta z \qquad \textbf{Eqn 5}$$

Cancelling terms yields :

$$-\hat{W}_{S,pump} = \int_4^1 \hat{V}\,dP \qquad \textbf{Eqn 6}$$

Because the liquid **water** flowing through the pump is incompressible, the **specific volume** is constant, and **Eqn 6** simplifies to:

$$-\hat{W}_{S,pump} = \hat{V}_4\left(P_1 - P_4\right) = \hat{V}_4\left(P_2 - P_3\right) \qquad \textbf{Eqn 7}$$

Now, we can look-up $\mathbf{V_4}$ and use it in **Eqn 7** to evaluate $\mathbf{W_{S,pump}}$:

V_4 0.0010432 m^3/kg

$W_{S,pump}$ -12.414 kJ/kg

Now, we can use $\mathbf{W_{S,pump}}$ and **Eqn 4** to determine $\mathbf{H_1}$.

$$\hat{H}_1 = \hat{H}_4 - \hat{W}_{S,pump} \qquad \textbf{Eqn 8}$$

H_1 429.9 kJ/kg

Next, we need to use the **isentropic efficiency** of the turbine to determine $\mathbf{H_3}$.

$$\eta_{S,turb} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_2 - \hat{H}_{3S}} \qquad \textbf{Eqn 9}$$

Solve **Eqn 9** for $\mathbf{H_3}$:

$$\hat{H}_3 = \hat{H}_2 - \eta_{S,turb}\left(\hat{H}_2 - \hat{H}_{3S}\right) \qquad \textbf{Eqn 10}$$

Now, we must evaluate $\mathbf{H_{3S}}$ before we can use **Eqn 10** to determine $\mathbf{H_3}$. $\mathbf{H_{3S}}$ is the **enthalpy** of the turbine effluent **IF** the turbine were isentropic. Therefore, $\mathbf{S_{3S} = S_2}$.

S_2 6.3028 kJ/kg-K

S_{3S} 6.3028 kJ/kg-K

Now, we know the values of two intensive properties at **state 3S**: $\mathbf{S_{3S}}$ and $\mathbf{P_{3S}}$, so we can fix this **state** and determine $\mathbf{H_{3S}}$ by interpolation in the **NIST Webbook**. First we must determine the phase of **state 3S**.

At **100 kPa** : T_{sat} **99.63** °C

	H (kJ/kg)	S (kJ/kg-K)
Sat Liq:	417.50	1.30276
Sat Vap :	2674.9	7.3588

Since $\mathbf{S_{sat\ liq} < S_{3S} < S_{sat\ vap}}$, **state 3S** is a saturated mixture.

T_{3S} 99.63 °C

Determine $\mathbf{x_{3S}}$ from the **specific entropy**, using:

$$x_{3S} = \frac{\hat{S}_3 - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}} \qquad \textbf{Eqn 11}$$

x_{3S} 0.8256 kg vap/kg

Then, we can use the **quality** to determine $\mathbf{H_{3S}}$, using:

$$\hat{H}_{3S} = x_3\,\hat{H}_{sat\,vap} + \left(1 - x_3\right)\hat{H}_{sat\,liq} \qquad \textbf{Eqn 12}$$

H_{3S} 2281.3 kJ/kg

Now, plug $\mathbf{H_{3S}}$ back into **Eqn 10** to determine $\mathbf{H_3}$:

H_3	2429.86	kJ/kg

Now that we know the **specific enthalpy** of all four streams, we can use **Eqns 1 - 3** to evaluate the $\mathbf{W_{S,turb}}$ and the two remaining **Q**'s.

$W_{S,turb}$	779.97	kJ/kg
Q_{12}	2779.9	kJ/kg
Q_{34}	-2012.4	kJ/kg

Part c.) The **thermal efficiency** of a power cycle is defined by:

$$\eta = \frac{\hat{W}_{cycle}}{\hat{Q}_{in}} = \frac{\hat{W}_{cycle}}{\hat{Q}_{12}} \qquad \text{Eqn 13}$$

Where :

$$\hat{W}_{cycle} = \hat{W}_{S,turb} + \hat{W}_{S,pump} \qquad \text{Eqn 14}$$

Now, we can plug values into **Eqns 13** & **14** to complete this part of the problem.

W_{cycle}	767.55	kJ/kg
η	27.61	%

Part d.) The total **entropy generation** for the cycle is equal to the **entropy change** of the universe caused by the cycle.

$$\hat{S}_{gen} = \Delta\hat{S}_{univ} = \cancel{\Delta\hat{S}}_{cycle} + \Delta\hat{S}_{furn} + \Delta\hat{S}_{cw} \qquad \text{Eqn 15}$$

Because the furnace and the cooling water are isothermal (they behave as thermal reservoirs) we can evaluate the change in their **entropy** directly from the definition of **entropy**.

$$\Delta\hat{S}_{furn} = \int \frac{\delta\hat{Q}_{furn}}{T_{furn}} = \frac{-\hat{Q}_{12}}{T_{furn}} \qquad \text{Eqn 16}$$

$$\Delta\hat{S}_{cw} = \int \frac{\delta\hat{Q}_{cw}}{T_{cw}} = \frac{-\hat{Q}_{34}}{T_{cw}} \qquad \text{Eqn 17}$$

Now, we can put values into **Eqns 16**, **17** & **15** :

ΔS_{furn}	-3.5956	kJ/kg-K
ΔS_{cw}	7.1070	kJ/kg-K
S_{gen}	3.5115	kJ/kg-K

We can calculate **lost work** using :

$$\hat{W}_{lost} = T_{surr}\ \hat{S}_{gen} \qquad \text{Eqn 18}$$

$W_{S,lost}$	1053.44	kJ/kg

Alternatively, we can compute the **lost work** using :

$$\hat{W}_{S,lost} = T_{surr}\left(\frac{\hat{Q}_{cw}}{T_{cw}} - \frac{\hat{Q}_{furn}}{T_{furn}}\right) = T_{surr}\left(\frac{-\hat{Q}_{34}}{T_{cw}} - \frac{\hat{Q}_{12}}{T_{furn}}\right) \qquad \text{Eqn 19}$$

$W_{S,lost}$	1053.44	kJ/kg

Part e.) The reversible **work** can be determined from the definition of **lost work** :

$$\hat{W}_{S,lost} = \hat{W}_{S,rev} - \hat{W}_{S,act} = \hat{W}_{S,rev} - \hat{W}_{cycle} \qquad \text{Eqn 19}$$

Solving for $W_{S,rev}$ yields :

$$\hat{W}_{S,rev} = \hat{W}_{S,lost} + \hat{W}_{cycle} \qquad \text{Eqn 20}$$

$W_{S,rev}$ 1821.0 kJ/kg

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **a.)** See the diagram at the beginning of this solution.

b.)

Device	Step	Q (kJ/kg)	W_S (kJ/kg)
Boiler	1 - 2	2780	0
Turbine	2 - 3	0	780
Condenser	3 - 4	-2010	0
Pump	4 - 1	0	-12.4

c.)

η	27.6	%

d.)

S_{gen}	3.51	kJ/kg-K
$W_{S,lost}$	1053	kJ/kg

e.)

$W_{S,rev}$	1820	kJ/kg

HEY ! Why is it that :

$$\frac{\hat{W}_{S,rev}}{\hat{Q}_H} \neq 1 - \frac{T_C}{T_H} \qquad ???$$

$W_{S,rev}$ / Q_H	65.5%
1-T_C/T_H	63.4%

Because the reversible cycle also exchanges **heat** at the turbine !

In fact, the reversible turbine absorbs **heat** reversibly from a third thermal reservoir. This reservoir is a very special thermal reservoir because it is always at the same **temperature** as the **working fluid** in the turbine. This is pretty tricky because the **temperature** of the **working fluid** changes as it passes through the turbine !

Consider the definition of **entropy generation**:

$$\Delta \hat{S}_{23} = \int \frac{\delta \hat{Q}_{23}}{T_{sys}} + \cancel{\hat{S}_{gen}} = \int \frac{\delta \hat{Q}_{23}}{T_{sys}} \qquad \text{Eqn 21}$$

But, T_{sys} is **NOT** constant !

$$\hat{Q}_{23} = \int_{\hat{S}2}^{\hat{S}3} T_{sys}\, d\hat{S} \qquad \text{Eqn 22}$$

The value of Q_{23} is equal to the area under the path for **Step 1-2** on the **TS Diagram**, but we cannot evaluate it because we do not know the equation of the **path**, **T = fxn(S)**.

We could evaluate Q_{23} by applying the 1st Law to the entire reversible cycle, because Q_{12}, Q_{34} and Q_{41} are the same for the reversible cycle as they were for the actual cycle. So, we can use the values calculated in **part (c)**.

1st Law, Reversible Cycle :

$$\hat{Q}_{12} + \hat{Q}_{23,rev} + \hat{Q}_{34} + \cancel{\hat{Q}_{41}} - \hat{W}_{S,rev} = 0 \qquad \text{Eqn 23}$$

$$\hat{Q}_{23,rev} = \hat{W}_{S,rev} - \hat{Q}_{34} - \hat{Q}_{12} \qquad \text{Eqn 24}$$

$Q_{23,rev}$ 1053.44 kJ/kg

Let's double check that this cycle is indeed reversible when it **GAINS heat** from the surroundings at the turbine.

$$\Delta \hat{S}_{univ} = \cancel{\Delta \hat{S}_{cycle}} + \Delta \hat{S}_{furn} + \Delta \hat{S}_{cw} + \Delta \hat{S}_{surr} \qquad \textbf{Eqn 25}$$

We already calculated $\Delta \mathbf{S}_{\mathbf{furn}}$ and $\Delta \mathbf{S}_{\mathbf{cw}}$, above. So, now we need to calculate $\Delta \mathbf{S}_{\mathbf{surr}}$.

$$\Delta \hat{S}_{surr} = \int \frac{\delta \hat{Q}_{surr}}{T_{surr}} = \frac{-\hat{Q}_{32,rev}}{T_{surr}} \qquad \textbf{Eqn 26}$$

Plug in values:

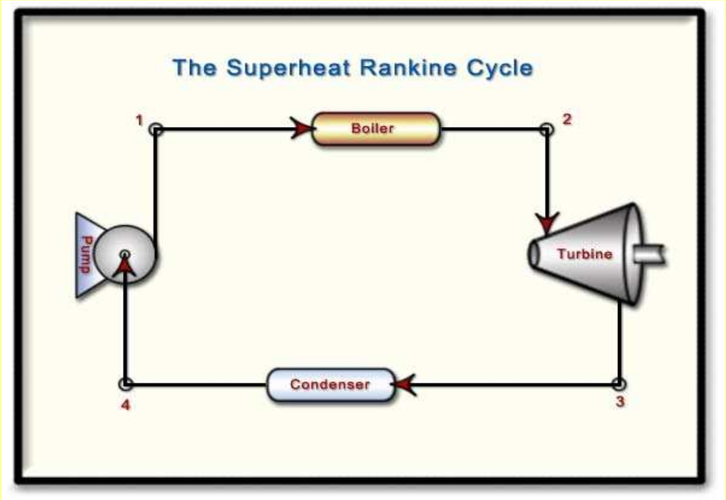
Now, plug values into **Eqn 26** :

Plug in values:	ΔS_{surr}	-3.5115	kJ/kg-K
Now, plug values into Eqn 26 :	ΔS_{univ}	0.0000	kJ/kg-K

There is one impossible aspect about our imaginary reversible cycle. The surroundings (at **300 K**) must reversibly supply **heat** to the **steam** inside the turbine (which is always at a higher **temperature** than **300 K**). That would require a heat pump and that would be a very complicated turbine, indeed ! This is ok because this is just a hypothetical reversible turbine anyway. It just seems strange.

9B-3 Vapor Power Cycle Based on Temperature Gradients in the Ocean 9 pts

A Rankine Power Cycle uses **water** at the surface of a tropical ocean as the heat source, **T_H = 82°F**, and cool **water** deep beneath the surface as the heat sink, **T_C = 48°F**. **Ammonia** is the working fluid.



The boiler produces saturated **ammonia** vapor at **80°F** and the condenser effluent is saturated liquid **ammonia** at **50°F**. The **isentropic efficiencies** of the pump and turbine are **75%** and **85%**, repectively.

Calculate the...

a.) **Thermal efficiency** of this Rankine Cycle

b.) **Thermal efficiency** of a Carnot Cycle operating between the same two thermal reservoirs

Read : Assume the process operates at steady-state and that changes in potential and kinetic energies are negligible.

We want the boiler **pressure** to be as high as possible and the condenser **pressure** to be as low as possible in order to maximize the **thermal efficiency** of the cycle. But the saturation temperature of the **ammonia** at the boiler **pressure** must be less than **82°F** in order to absorb **heat** from the warmer **seawater**. This explains why the **ammonia** boils at **80°F**. A similar argument regarding the vapor-liquid equilibrium in the condenser leads us to the choice of **50°F** for the saturation temperature of the **ammonia** in the condenser.

Given:						
	T_2	80	°F	T_C	48	°F
	x_2	1	lb_m vap/lb_m	T_H	82	°F
	T_3	50	°F	$(T_{sw, in})_{boil}$	80	°F
	x_4	0	lb_m vap/lb_m	$(T_{sw, in})_{cond}$	48	°F
	$\eta_{S,pump}$	75%		$\eta_{S,turb}$	85%	

Find: **a.)** η_{th} ??? % **b.)** η_{max} ??? %

Diagram: A good flow diagram was provided in the problem statement.

TS Diagram :

Assumptions:

1 - Each process in the cycle operates at steady-state.

2 - The power cycle operates on the **Rankine Cycle**.

3 - Changes in kinetic and potential energies are negligible.

4 - The pump and the turbine are both adiabatic.

Equations / Data / Solve:

Let's organize the **data** that we need to collect into a table. This will make it easier to keep track of the values we have looked up and the values we have calculated.

Stream	State	T (^oF)	P (psia)	X (lb_m vap/lb_m)	H (Btu/lb_m)	S (Btu/lb_m-^oR)
1S	Sub Liq	50.18	153.13	N/A	98.132	0.21024
1	Sub Liq	50.27	153.13	N/A	98.233	0.21044
2	Sat Vap	80	153.13	1	630.36	1.1982
3S	Sat Mix	50	89.205	0.9551	601.38	1.1982
3	Sat Mix	50	89.205	0.9633	605.73	1.2068
4	Sat Liq	50	89.205	0	97.828	0.21024

Additional **data** that may be useful.

State	T (^oF)	P (psia)	X (lb_m vap/lb_m)	H (Btu/lb_m)	S (Btu/lb_m-^oR)
Sat Vap	80	153.13	1	630.36	1.1982
Sat Liquid	80	153.13	0	131.86	0.27452
Sat Vap	50	89.205	1	625.07	1.2447
Sat Liquid	50	89.205	0	97.828	0.21024

Part a.) The **thermal efficiency** of a power cycle can be determined using :

$$\eta = 1 - \frac{\hat{Q}_C}{\hat{Q}_H} = 1 - \frac{-\hat{Q}_{34}}{\hat{Q}_{12}} \qquad \textbf{Eqn 1}$$

We need to evaluate $\mathbf{Q_{12}}$ and $\mathbf{Q_{34}}$ so we can use **Eqn 1** to evaluate the **thermal efficiency** of the cycle.

Apply the 1st Law to the boiler, assuming it operates at steady-state, changes in kinetic and potential energies are negligible and no **shaft work** crosses the boundary of the boiler.

$$\hat{Q}_H = \hat{Q}_{12} = \hat{H}_1 - \hat{H}_2 \qquad \textbf{Eqn 2}$$

We can get a value for $\mathbf{H_1}$ from the **Steam Tables** or **NIST Webbook** because we know that the boiler effluent in a **Rankine Cycle** is a saturated vapor at **80°F**.

P_2	**153.13**	**psia**	**H_2**	**630.36**	**Btu/lb_m**

In order to fix **state 1** and evaluate $\mathbf{H_1}$, we must use the **isentropic efficiency** of the pump. The feed to the pump is a saturated liquid at **80°F**. So, we can look-up $\mathbf{S_4}$ in the **Steam Tables** or **NIST Webbook**.

P_4	**89.205**	**psia**	**S_4**	**0.21024**	**Btu/lb_m-°R**
			H_4	**97.828**	**Btu/lb_m**

The definition of **isentropic efficiency** for a pump is :

$$\eta_{S,pump} = \frac{-\dot{W}_{S,isen}}{-\dot{W}_{S,act}} = \frac{\dot{m}\left(\hat{H}_{1S} - \hat{H}_4\right)}{\dot{m}\left(\hat{H}_1 - \hat{H}_4\right)} \qquad \textbf{Eqn 3}$$

We can solve **Eqn 2** for $\mathbf{H_1}$:

$$\hat{H}_1 = \hat{H}_4 + \frac{\hat{H}_{1S} - \hat{H}_4}{\eta_{S,pump}} \qquad \textbf{Eqn 4}$$

Next, we need to determine $\mathbf{H_{1S}}$. For an isentropic pump : $\mathbf{S_{1S}}$ **0.21024** $\mathbf{Btu/lb_m}$**-°R**

Now, we know the value of two intensive properties at **state 1S**: $\mathbf{S_{1S}}$ and $\mathbf{P_1}$ (because the boiler is isobaric in a **Rankine Cycle**, $\mathbf{P_1 = P_2}$.

At **P = 153.13 psia** :

T (°F)	H Btu/lb_m	S Btu/lb_m-°R
50	97.935	0.20985
T_{1S}	H_{1S}	0.21024
55	103.528	0.22077

T_{1S}	**50.18**	**°F**
H_{1S}	**98.13**	**Btu/lb_m**

Now ,we can plug values into **Eqn 4** to determine $\mathbf{H_1}$: $\mathbf{H_1}$ **98.23** $\mathbf{Btu/lb_m}$

Next, we can plug values into **Eqn 2** to evaluate $\mathbf{Q_{12}}$: $\mathbf{Q_{12}}$ **532.12** $\mathbf{Btu/lb_m}$

We can determine $\mathbf{Q_{34}}$ by applying the 1st Law, with all the same assumptions made about the boiler.

$$\hat{Q}_C = -\hat{Q}_{34} = \hat{H}_3 - \hat{H}_4 \qquad \textbf{Eqn 5}$$

We already know $\mathbf{H_4}$, so we need to evaluate $\mathbf{H_3}$. To do this, we use the **isentropic efficiency** of the turbine.

In order to fix **state 3** and evaluate $\mathbf{H_3}$, we must use the **isentropic efficiency** of the turbine. The feed to the turbine is a saturated vapor at **80°F**. So, we can look-up $\mathbf{S_2}$ in the **Steam Tables** or **NIST Webbook**.

P_4	**89.205**	**psia**	**S_2**	**1.1982**	**Btu/lb_m-°R**
			H_2	**630.36**	**Btu/lb_m**

The definition of **isentropic efficiency** for a turnine is :

$$\eta_{S,turb} = \frac{\dot{W}_{S,act}}{\dot{W}_{S,isen}} = \frac{\dot{m}\left(\hat{H}_2 - \hat{H}_3\right)}{\dot{m}\left(\hat{H}_2 - \hat{H}_{3S}\right)}$$

Eqn 6

We can solve **Eqn 6** for $\mathbf{H_3}$:

$$\hat{H}_3 = \hat{H}_2 - \eta_{S,pump}\left(\hat{H}_2 - \hat{H}_{3S}\right)$$

Eqn 7

Next, we need to determine $\mathbf{H_{3S}}$. For an isentropic turbine :

S_{3S}	1.1982	Btu/lb$_m$-°R

Now, we know the value of two intensive properties at **state 3S**: $\mathbf{S_{3S}}$ and $\mathbf{P_3}$ (because the condenser is isobaric in a **Rankine Cycle**, $\mathbf{P_3 = P_4}$.

At **P = 89.205 psia** :

$S_{sat\ liq}$	0.21024	Btu/lb$_m$-°R
$S_{sat\ vap}$	1.2447	Btu/lb$_m$-°R

Since $\mathbf{S_{sat\ liq} < S_{3S} < S_{sat\ vap}}$, **state 3S** is a saturated mixture.

Determine $\mathbf{x_{3S}}$ from the **specific entropy**, using:

$$x_{3S} = \frac{\hat{S}_{3S} - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}}$$

Eqn 8

x_{3S}	0.9551	lb$_m$ vap/lb$_m$

Then, we can use the **quality** to determine $\mathbf{H_{3S}}$, using:

$$\hat{H}_{3S} = x_3\,\hat{H}_{sat\,vap} + \left(1 - x_3\right)\hat{H}_{sat\,liq}$$

Eqn 9

At **P = 89.205 psia** :

$H_{sat\ liq}$	97.828	Btu/lb$_m$
$H_{sat\ vap}$	625.07	Btu/lb$_m$
H_{3S}	601.38	Btu/lb$_m$

Now ,we can plug values into **Eqn 7** to determine $\mathbf{H_1}$: H_3 605.73 Btu/lb$_m$

Next, we can plug values into **Eqn 5** to evaluate $\mathbf{Q_{34}}$: Q_{34} -507.90 Btu/lb$_m$

We can now plug values into **Eqn 1** to evaluate the **thermal efficiency** of the **Rankine Cycle**.

$$\boxed{\eta_{th} \qquad 4.55\%}$$

The maximum **thermal efficiency** for a power cycle operating between two thermal reservoirs at $\mathbf{T_H}$ and $\mathbf{T_C}$ is the Carnot Efficiency :

$$\eta_{max} = 1 - \frac{T_C}{T_H}$$

Eqn 10

T_C	507.67	°R
T_H	541.67	°R

$$\boxed{\eta_{max} \qquad 6.28\%}$$

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : a.) $\boxed{\eta_{th} \qquad 4.55\%}$ $\boxed{\eta_{max} \qquad 6.28\%}$

9C-1 Ideal Rankine Cycle with Reheat 9 pts

Water is the working fluid in an ideal Rankine cycle with **reheat**. The **steam** at the high-pressure turbine inlet is at **1500 psia** and **800^{o}F** and the effluent is saturated vapor.

The **steam** is reheated to **750^{o}F** before it enters the low pressure turbine where the **steam** is let down to **20 psia**. If the **mass flow rate** of **steam** is **126 lb$_m$/s**, determine...

a.) The **net power output** in million **Btu** per **hour** (**mmBtu/h**)
b.) The **heat transfer rate** in the reheat process in **mmBtu/h**
c.) The **thermal efficiency** of the cycle

Read : Determine the **specific enthalpy** of each **stream** in the process and then use the 1st Law to calculate **$(W_S)_{cycle}$**, **Q_{in}**, and η.

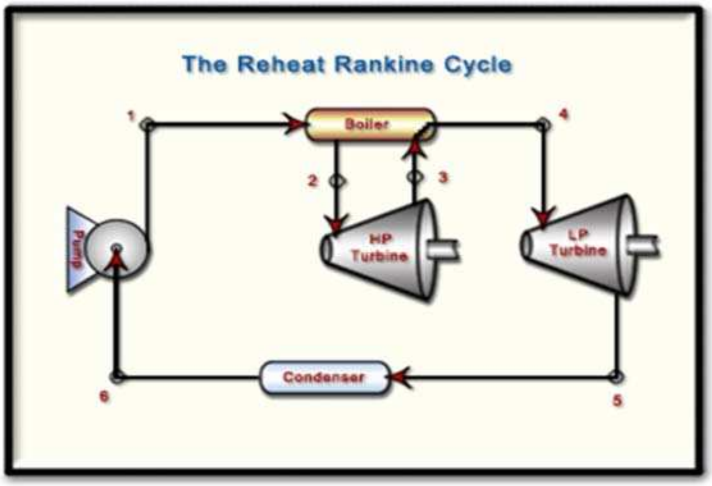
States 2 & **6** are completely determined from the given information. Use the fact that the pump and HP turbine are **isentropic** to fix **states 1** & **3** respectively. Once **state 3** is fixed, you know **P_3** and **$P_4 = P_3$**. **T_4** is given, so **state 4** is now fixed. Next, use the fact that the LP turbine is also **isentropic** to fix **state 5**.

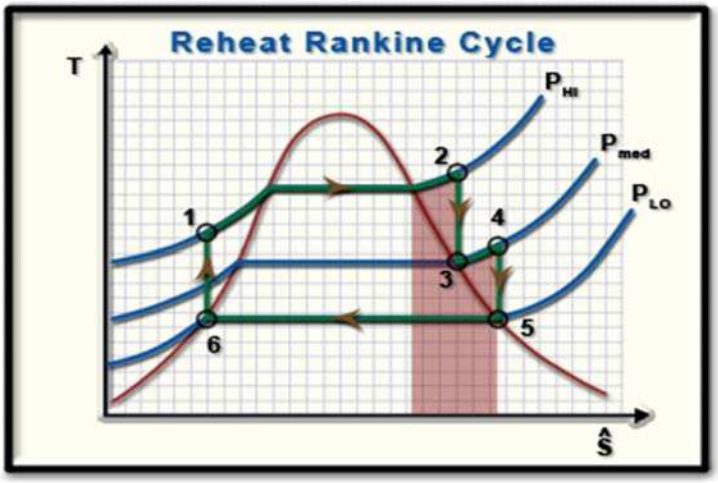
Once we know all the **H** values, we apply the 1st Law to the pump and turbines to determine **W_{cycle}**. Then, we apply the 1st Law to the boiler and the reheater to determine **Q_{in}**. Finally, we evaluate η from its definition.

Given:	**m_{dot}**	**126**	**lb$_m$/s**	**T_2**	**800**	**^{o}F**
		4.54E+05	**lb$_m$/h**	**T_4**	**750**	**^{o}F**
	P_1	**1500**	**psia**	**P_5**	**20**	**psia**
	P_2	**1500**	**psia**	**P_6**	**20**	**psia**

Find:	**a.)**	**W_{cycle}**	**???**	**Btu/h**				
	b.)	**Q_{34}**	**???**	**Btu/h**	**c.)**	η	**???**	**%**

Diagram:





Assumptions:

1 - Each component in the cycle is analyzed as an open system operating at steady-state.
2 - All of the processes are internally reversible.
3 - The turbine and pump operate adiabatically and are internally reversible, so they are also **isentropic**.
4 - Condensate exits the condenser as saturated liquid.
5 - The effluent from the HP turbine is a saturated vapor.
6 - No **shaft work** crosses the system boundary of the boiler or condenser.
7 - Changes in kinetic and potential energies are negligible.

Equations / Data / Solve:

Let's organize the **data** that we need to collect into a table. This will make it easier to keep track of the values we have looked up and the values we have calculated.

Stream	T (^oF)	P (psia)	X (lb_m vap/lb_m)	H (Btu/lb_m)	S (Btu/lb_m-^oR)	Phase
1	229.30	1500	N/A	201.002	0.33605	Sub. Liq.
2	800	1500	N/A	1364.0	1.5075	Super. Vap.
3	422.18	316.11	1	1204.6	1.5075	Sat'd Vap.
4	750	316.11	N/A	1395.1	1.6928	Super. Vap.
5	227.92	20	0.9712	1129.26	1.6928	VLE
6	227.92	20	0	196.400	0.33605	Sat'd Liq.

Additional **data** that may be useful.

State	T (^oF)	P (psia)	X (lb_m vap/lb_m)	H (Btu/lb_m)	S (Btu/lb_m-^oR)
Sat Vap	0	1500	1	1169.80	1.3372
Sat Liquid	0	1500	0	612.08	0.80900
Sat Vap	0	20	1	1156.96	1.7331
Sat Liquid	0	20	0	196.40	0.33605

Part a.) The net **shaft work** for the reheat cycle is:

$$\dot{W}_{cycle} = \dot{W}_{S,turb} + \dot{W}_{S,pump} \qquad \text{Eqn 1}$$

Now, apply the 1st Law to the LP and HP turbines, as well as the pump.
Assume each device is adiabatic, operating at steady-state and has negligible changes in kinetic and potential energies.

$$W_{S,turb} = \dot{m}\left[\left(\hat{H}_2 - \hat{H}_3\right) + \left(\hat{H}_4 - \hat{H}_5\right)\right] \qquad \text{Eqn 2}$$

$$W_{S,pump} = \dot{m}\left(\hat{H}_6 - \hat{H}_1\right) \qquad \text{Eqn 3}$$

So, we need to determine the **enthalpy** in every **stream** in the cycle in order to determine $\mathbf{W_{cycle}}$.

States 2 & **6** are the only streams that are completely determined by the given information, so let's look up the properties of those streams in the **Steam Tables** or **NIST Webbook** first.

T_2	800	^oF	T_6	227.92	^oF
H_2	1364.0	Btu/lb_m	H_6	196.400	Btu/lb_m
S_2	1.5075	Btu/lb_m-^oR	S_6	0.33605	Btu/lb_m-^oR

Now, because the pump and HP turbine are **isentropic**, $\mathbf{S_1 = S_6}$ and $\mathbf{S_3 = S_2}$.

Now, we know the values of two intensive properties at **state 1** and we know both S_3 and x_3, so we can fix these states and determine H_1 and H_3. Because **stream 1** is a subcooled liquid, it is easiest and most accurate to use the **NIST Webbook** instead of the **Steam Tables**.

At **1500 psia**:

T (°F)	H Btu/lb$_m$	S Btu/lb$_m$-°R
220	**191.667**	**0.32241**
T_1	H_1	**0.33605**
230	**201.708**	**0.33708**

Interpolation yields:

T_1	**229.30**	°F
H_1	**201.01**	Btu/lb$_m$

Sat. Vap. :

T (°F)	P (psia)	H Btu/lb$_m$	S Btu/lb$_m$-°R
417.35	**300**	**1204.1**	**1.5121**
T_3	P_3	H_3	**1.5075**
423.31	**320**	**1204.7**	**1.5064**

Interpolation yields:

P_3	**316.18**	psia
T_3	**422.18**	°F
H_3	**1204.6**	Btu/lb$_m$

Now that we know P_4, we can use it with T_4 to fix **state 4** and use the **Steam Tables** or **NIST Webbook** to evaluate H_4 and S_4.

H_4	**1395.1**	Btu/lb$_m$	S_4	**1.6928**	Btu/lb$_m$-°R

We now know S_4 and we know that the LP turbine is also **isentropic**, so $S_5 = S_4$.

S_5	**1.6928**	Btu/lb$_m$-°R

We now know the values of two intensive properties at **state 5**, so the state is fixed and we can evaluate its properties. We begin by determining which phases are present in **state 5**.

At **20 psia** :

T_{sat}	**227.92**	°F
$S_{sat\ liq}$	**0.33605**	Btu/lb$_m$-°R
$S_{sat\ vap}$	**1.7331**	Btu/lb$_m$-°R

Since $S_{sat\ liq} < S_5 < S_{sat\ vap}$, **state 5** is a saturated mixture.

T_5	**227.92**	°F

Determine x_{3S} from the **specific entropy**, using:

$$x_5 = \frac{\hat{S}_5 - \hat{S}_{sat\,liq}}{\hat{S}_{sat\,vap} - \hat{S}_{sat\,liq}} \qquad \textbf{Eqn 4}$$

x_5	**0.9712**	lb$_m$ vap/lb$_m$

Then, we can use the **quality** to determine H_{3S}, using:

$$\hat{H}_{3S} = x_3\,\hat{H}_{sat\,vap} + (1 - x_3)\,\hat{H}_{sat\,liq} \qquad \textbf{Eqn 5}$$

At **20 psia** :

$H_{sat\ liq}$	**196.400**	Btu/lb$_m$
$H_{sat\ vap}$	**1106.2**	Btu/lb$_m$

H_5	**1079.96**	Btu/lb$_m$

At last we know all the properties at all of the **states** in the reheat cycle and we can use **Eqns 1 - 3** to evalaute the **shaft work** for each device as well as the entire cycle.

$W_{S,pump}$	**-2.090E+06**	Btu/h
$W_{S,LP\text{-}turb}$	**1.430E+08**	Btu/h
$W_{S,HP\text{-}turb}$	**7.233E+07**	Btu/h

W_{cycle}	**2.132E+08**	Btu/h

Part b.) The amount of **heat** absorbed in the **reheat** step is $\mathbf{Q_{34}}$. We can determine it by applying the 1st Law to the reheater, **Step 3-4**. The reheater operates at steady-state, has no **shaft work** interaction and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{Q}_{34} = \dot{m}\left(\hat{H}_4 - \hat{H}_3\right) \qquad \textbf{Eqn 6}$$

Since we already know $\mathbf{m_{dot}}$ and all the **H** values, we can immediately plug values into **Eqn 6** :

Q_{34}	8.645E+07	Btu/h

Part c.) The **thermal efficiency** of a power cycle is defined by:

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{Q}_{12} + \dot{Q}_{34}} \qquad \textbf{Eqn 7}$$

In **parts (a)** and **(b)** we determined $\mathbf{W_{cycle}}$ and $\mathbf{Q_{34}}$, so here we need to evaluate $\mathbf{Q_{12}}$ so we can use **Eqn 7** to evaluate η.

We can determine $\mathbf{Q_{12}}$ by applying the 1st Law to the boiler, **Step 1-2**. The boiler operates at steady-state, has no **shaft work** interaction and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{Q}_{12} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) \qquad \textbf{Eqn 8}$$

Since we already know $\mathbf{m_{dot}}$ and all the **H** values, we can immediately plug values into **Eqn 8** :

Now, we can plug values into **Eqn 7** :

Q_{12}	5.275E+08	Btu/h
Q_{in}	6.140E+08	Btu/h
η	34.72%	

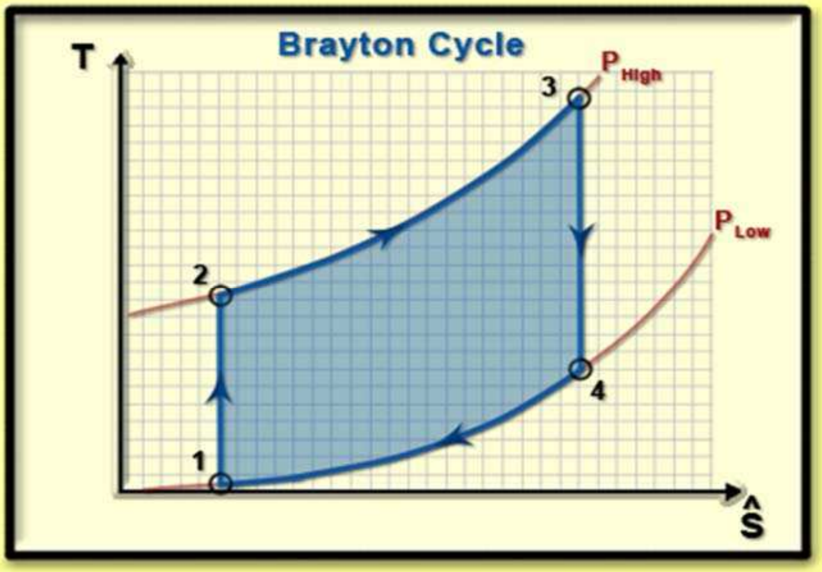
Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

a.)	W_{cycle}	2.13E+08	Btu/h
b.)	Q_{34}	8.65E+07	Btu/h
c.)	η	34.7%	

9E-1 Optimal Compressor Outlet Pressure for the Ideal Brayton Power Cycle 8 pts

Consider the **TS Diagram**, below, for an ideal Brayton Cycle.



The working fluid is **air** and the cold air-standard assumptions apply.

Using a given pair of thermal reservoirs, T_1 and T_3 are fixed. Show that the maximum **net work** from the cycle is obtained when the compressor effluent **temperature** is $T_2 = (T_1\ T_3)^{1/2}$.

Read : Use the 1st Law to write an equation for the net **work** produced by the cycle.

Use the constant **heat capacity** to eliminate **enthalpy** from your equation for W_{cycle}.

Differentiate your equation for W_{cycle} with respect to r_P and set the result equal to **zero**.

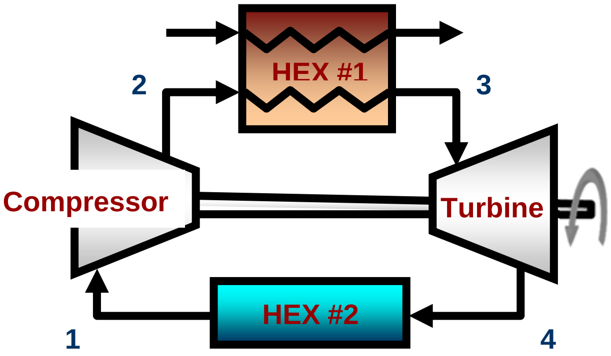
Simplify the result using the fact that the compressor and turbine are isentropic and the **heat capacity ratio**, γ, is constant.

Finally, calculate the 2nd derivative of W_{cycle} with respect to r_P and verify that it is positive at the point determined above so that the extremum you found is a maximum of W_{cycle} and not a minimum.

Given: An ideal **Brayton Cycle** is analyzed on a cold air-standard basis.

Find: Show that the compressor effluent **temperature** that maximizes net **work** per unit **mass** of **air** flow is given by $T_2 = (T_1 T_3)^{1/2}$.

Diagram:



Assumptions:

1 - Each component is an open system operating at steady-state.
2 - The turbine and compressor are isentropic.
3 - There are no **pressure** drops for flow through the heat exchangers.
4 - Kinetic and potential energy changes are negligible.
5 - The working fluid is **air** modeled as an ideal gas.
6 - The **specific heat**, C_P , and the **specific heat ratio**, γ , are constant.

Equations / Data / Solve:

The net **work** for the power cycle is:

$$\hat{W}_{cycle} = \hat{W}_C + \hat{W}_T \qquad \textbf{Eqn 1}$$

The 1st Law can be applied to the compressor and to the turbine, assuming they operate adiabatically at steady-state with negligible changes in kinetic and potential energies.

$$\hat{W}_S = -\Delta\hat{H} \qquad \textbf{Eqn 2}$$

Combining **Eqn 1** and **Eqn 2** yields:

$$\hat{W}_{cycle} = -\left(H_2 - H_1\right) - \left(H_4 - H_3\right) \qquad \textbf{Eqn 3}$$

The change in **enthalpy** of the **air** in the turbine and in the compressor can be determined as follows because we have assumed that the **heat capacity** is constant.

$$\Delta\hat{H}_{a-b} = \int_{T_a}^{T_b} \hat{C}_P \, dT = \hat{C}_P \cdot \left(T_b - T_a\right) \qquad \textbf{Eqn 4}$$

Now we can apply **Eqn 4** to the turbine and to the compressor and use the results to eliminate **enthalpy** from **Eqn 3**.

$$\hat{W}_{cycle} = -C_P \cdot \left(T_2 - T_1\right) C_P \cdot \left(T_4 - T_3\right) \qquad \textbf{Eqn 5}$$

The goal is to determine the maximum W_{cycle}, so we need to set $dW_{cycle}/dr_P = 0$ and solve for T_2. This will help us find an extremum, either a maximum or a minimum. Later, we will need to make sure the 2nd derivative, d^2W_{cycle}/dr^2_P , is positive so we can be sure this extremum is a maximum value of W_{cycle}.

$$\frac{d\hat{W}_{cycle}}{dr_P} = \frac{d}{dr_P}\left(-C_P \cdot \left(T_2 - T_1\right) - C_P \cdot \left(T_4 - T_3\right)\right) = 0 \qquad \textbf{Eqn 6}$$

We can simplify **Eqn 6** because T_1, T_3 and C_P are all constant.

$$\frac{d}{dr_P}\left(-T_2 - T_4\right) = 0 \qquad \textbf{Eqn 7}$$

A little algebra makes **Eqn 7** easier to work with.

$$\frac{dT_2}{dr_P} = -\frac{dT_4}{dr_P}$$ **Eqn 8**

Because T_2 and T_4 change as the compression ratio, r_P changes. We need to figure out the relationship between T_2 and T_4 and r_P in order to solve **Eqn 8**.

Next, we can take advantage of the fact that the compressor and the turbine are **isentropic** and use the following relationships from **Lesson 7E**, **page 6**.

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}}$$ **Eqn 9**

$$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}}$$ **Eqn 10**

Solve **Eqns 9** and **10** for T_2 and T_4, respectively.

$$T_2 = T_1 \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} = T_1 \cdot r_P^{\frac{\gamma-1}{\gamma}}$$ **Eqn 11**

$$T_4 = T_3 \left(\frac{P_3}{P_4}\right)^{\frac{1-\gamma}{\gamma}} = T_1 \cdot r_P^{\frac{1-\gamma}{\gamma}}$$ **Eqn 12**

Now, we can differentiate **Eqns 11** and **12** with respect to r_P so we can use the results in **Eqn 8**.

$$\frac{dT_2}{dr_P} = T_1 \cdot \left(\frac{\gamma-1}{\gamma}\right) r_P^{\frac{-1}{\gamma}}$$ **Eqn 13**

$$\frac{dT_4}{dr_P} = T_3 \cdot \left(\frac{1-\gamma}{\gamma}\right) r_P^{\frac{1-2\gamma}{\gamma}}$$ **Eqn 14**

Now, we can substitute **Eqns 13** and **14** into **Eqn 8**.

$$T_1 \cdot \left(\frac{\gamma-1}{\gamma}\right) r_P^{\frac{-1}{\gamma}} = -\left[T_3 \cdot \left(\frac{1-\gamma}{\gamma}\right) r_P^{\frac{1-2\gamma}{\gamma}}\right]$$ **Eqn 15**

When the minus sign is combined with the constant term, it is clear that the two constant terms, **(γ-1)/γ**, cancel.

$$T_1 \cdot \cancel{\left(\frac{\gamma-1}{\gamma}\right)} r_P^{\frac{-1}{\gamma}} = T_3 \cdot \cancel{\left(\frac{\gamma-1}{\gamma}\right)} r_P^{\frac{1-2\gamma}{\gamma}}$$ **Eqn 16**

This equation is more manageable, but we can simplify it a bit further.

$$T_1 \cdot r_P^{\frac{-1}{\gamma}} = T_3 \cdot r_P^{\frac{1-2\gamma}{\gamma}}$$ **Eqn 17**

The result is simple, but there is noT_2 in **Eqn 18**!

$$T_1 = T_3 \cdot r_P^{2 \cdot \frac{1-\gamma}{\gamma}}$$ **Eqn 18**

Here, we need to use **Eqn 11** again, but in a slightly different form.

$$T_1 = T_2 \cdot r_P^{\frac{1-\gamma}{\gamma}}$$ **Eqn 19**

Squaring **Eqn 19** yields :

$$T_1^2 = T_2^2 \cdot r_P^{2 \cdot \frac{1-\gamma}{\gamma}}$$ **Eqn 20**

A slight rearrangement of **Eqn 20** yields :

$$r_P^{2 \cdot \frac{1-\gamma}{\gamma}} = \left(\frac{T_1}{T_2}\right)^2$$ **Eqn 21**

Now, we can use **Eqn 21** to eliminate r_P from **Eqn 18**.

$$T_1 = T_3 \cdot \left(\frac{T_1}{T_2}\right)^2$$ **Eqn 22**

Now, solve for T_2 :

$$T_1 \cdot T_2^2 = T_3 \cdot T_1^2$$ **Eqn 23**

$$\boxed{T_2 = (T_1 T_3)^{1/2}}$$ **Eqn 24**

Now we need to make sure this is the maximum $\mathbf{W_{cycle}}$ and not the minimum. The criterion for a maximum is:

$$\frac{d^2 W_{cycle}}{d r_P^2} > 0 \qquad \text{Eqn 25}$$

Combining **Eqns 5** and **25** yields :

$$\frac{d^2}{d r_P^2}\left[-C_P \cdot (T_2 - T_1) C_P \cdot (T_4 - T_3)\right] \overset{?}{>} 0 \qquad \text{Eqn 26}$$

We can simplify **Eqn 26** because $\mathbf{T_1}$, $\mathbf{T_3}$ and $\mathbf{C_P}$ are all constant.

$$\frac{d^2}{d r_P^2}(-T_2 - T_4) = -\left[\frac{d^2 T_2}{d r_P^2} + \frac{d^2 T_4}{d r_P^2}\right] \overset{?}{>} 0 \qquad \text{Eqn 27}$$

Now, we need to differentiate **Eqns 13** and **14**, as follows.

$$\frac{d^2 T_2}{d r_P^2} = T_1 \cdot \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{-1}{\gamma}\right) r_P^{\frac{-1-\gamma}{\gamma}} \qquad \text{Eqn 28}$$

$$\frac{d^2 T_4}{d r_P^2} = T_3 \cdot \left(\frac{1 - \gamma}{\gamma}\right)\left(\frac{1 - 2\gamma}{\gamma}\right) r_P^{\frac{1-3\gamma}{\gamma}} \qquad \text{Eqn 29}$$

Combine **Eqns 27 - 29** :

$$-\left[T_1 \cdot \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{-1}{\gamma}\right) r_P^{\frac{-1-\gamma}{\gamma}} + T_3 \cdot \left(\frac{1 - \gamma}{\gamma}\right)\left(\frac{1 - 2\gamma}{\gamma}\right) r_P^{\frac{1-3\gamma}{\gamma}}\right] \overset{?}{>} 0 \qquad \text{Eqn 30}$$

Let the algebra fly :

$$T_1 \cdot \cancel{\left(\frac{1 - \gamma}{\gamma}\right)}\left(\frac{-1}{\gamma}\right) r_P^{\frac{-1-\gamma}{\gamma}} \overset{?}{>} T_3 \cdot \cancel{\left(\frac{1 - \gamma}{\gamma}\right)}\left(\frac{1 - 2\gamma}{\gamma}\right) r_P^{\frac{1-3\gamma}{\gamma}} \qquad \text{Eqn 31}$$

Multiply by γ . This does not change the **>** sign because γ **> 0**.

$$-T_1 \cdot r_P^{\frac{-1-\gamma}{\gamma}} \overset{?}{>} T_3 \cdot (1 - 2\gamma) r_P^{\frac{1-3\gamma}{\gamma}} \qquad \text{Eqn 32}$$

Combine the $\mathbf{r_P}$ terms :

$$-T_1 \overset{?}{>} T_3 \cdot (1 - 2\gamma) r_P^{\frac{1-3\gamma}{\gamma}} \cdot r_P^{\frac{1+\gamma}{\gamma}} = T_3 \cdot (1 - 2\gamma) \cdot r_P^{2 \cdot \frac{1-\gamma}{\gamma}} \qquad \text{Eqn 33}$$

Combine **Eqn 18** with **Eqn 33** to get : $-T_1 \overset{?}{>} T_1 \cdot (1 - 2\gamma)$ **Eqn 34**

We can divide **Eqn 34** by $\mathbf{T_1}$ without changing the **>** sign because $\mathbf{T_1 > 0}$ and then do some more algebra.

$$-1 \overset{?}{>} 1 - 2\gamma \qquad \text{Eqn 35}$$

$$2\gamma \overset{?}{>} 2 \qquad \text{Eqn 35}$$

Finally :

$$\gamma \overset{?}{>} 1 \qquad \text{Eqn 36}$$

The **heat capacity ratio**, γ, is always greater than **1**. So the 2nd derivative is positive and we have indeed found the maximum $\mathbf{W_{cycle}}$ and NOT the minimum !

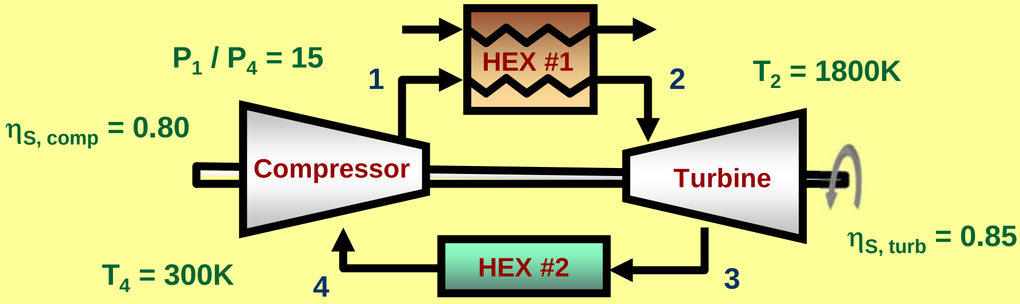
Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : $\boxed{T_2 = (T_1 T_3)^{1/2}}$

The optimal temperature of the compressor effluent in a Brayton Cycle is the geometric average of the temperatures of the compressor and turbine feed streams.

9E-2 Performance of a "Real" Brayton Cycle 10 pts

A real gas power cycle is similar to the Brayton Cycle, but the compressor and turbine are not isentropic. The compressor and turbine have **isentropic efficiencies** of 80% and 85%, respectively.



Use air-standard analysis to determine…

a.) Q_C and W_{net}, in **kJ/kg**

b.) The **thermal efficiency**

c.) Repeat **parts (a)** and **(b)** for an ideal Brayton Cycle (isentropic turbine and compressor).

Read : In order to evaluate all of the W_S and **Q** values that we need to answer all the **parts** of this question, we will need to know the **H** values at every **state**. In addition, we will need to know H_{1S} and H_{3S} because **part (c)** requires that we analyze the ideal cycle as well the actual cycle.

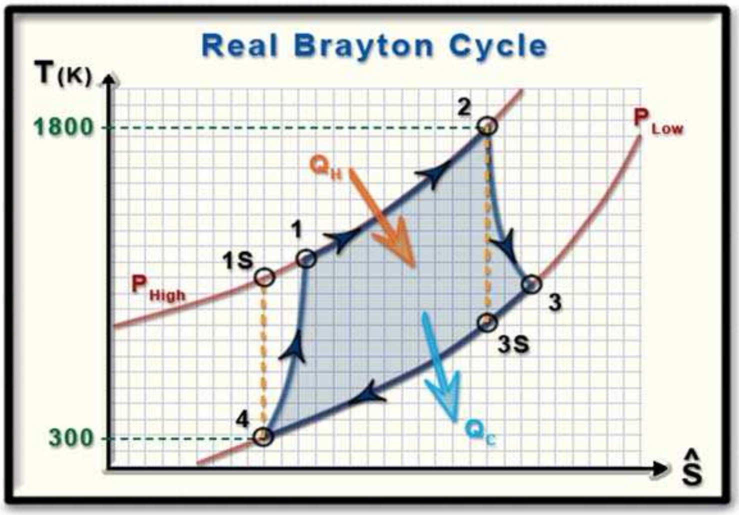
We can lookup H_2 and H_4 immediately, but we need to use the **isnetropic efficiency** and either the Ideal Gas Entropy Function and the 2nd Gibbs Equation or the Ideal Relative Pressure to determine H_1 and H_3. Whichever method we choose, we will compute H_{1S} and H_{2S} in the process, so when we are done with **part (a)**, we will have all the values we need to complete **part (c)**.

Once we know all the **H** values, it is a straight-forward process to apply the 1st Law to each process in the cycle in order to answer the questions in **parts (a)** & **(b)**. We repeat these calculations in **part (c)** using H_{1S} instead of H_1 and H_{3S} instead of H_3. Finally, calculate the **% Change** in each answer for our comparison.

Given:	$\eta_{S, comp}$	0.80	T_2	1800	K
	$\eta_{S, turb}$	0.85	T_4	300	K
	P_1/P_4	15			

Find:	**a.)**	W_{cycle}	???	kJ/kg	**c.)**	$(W_{cycle})_{ideal}$	???	kJ/kg
		Q_{out}	???	kJ/kg		$(Q_{out})_{ideal}$	???	kJ/kg
	b.)	η	???	%		η_{ideal}	???	%

Diagram: The flow diagram in the problem statement is adequate. A **TS Diagram** will also be useful.



Assumptions:

1 - Each component is an open system operating at steady-state.
2 - The turbine and compressor are adiabatic.
3 - There are no **pressure** drops for flow through the heat exchangers.
4 - Kinetic and potential energy changes are negligible.
5 - The working fluid is **air** modeled as an ideal gas.

Equations / Data / Solve:

Stream	T (K)	H^o (kJ/kg)	S^o (kJ/kg-K)
1	717.4	523.58	
1S	636.6	436.34	0.78254
2	1800	1791.5	1.9784
3	1068.4	916.49	
3S	932.7	762.08	1.20203
4	300	87.410	0.0061681

Part a.) Only the compressor and the turbine have **shaft work** interactions, so the net **work** for the cycle is given by:

$$\hat{W}_{cycle} = \hat{W}_{S,turb} + \hat{W}_{S,comp}$$ **Eqn 1**

Apply the 1st Law to the turbine and the compressor. They are adiabatic, operate at steady-state and changes in kinetic and potential energies are negligible.

$$\hat{W}_{S,turb} = \hat{H}_2 - \hat{H}_3$$ **Eqn 2**

$$\hat{W}_{S,comp} = \hat{H}_4 - \hat{H}_1$$ **Eqn 3**

We know T_2 and T_4, so we can look-up H_2 and H_4 in the **Ideal Gas Properties Table** for **air**.

H_2 1791.5 kJ/kg

H_4 87.410 kJ/kg

We can determine T_1 and T_3 using either the Ideal Gas Entropy Function and the 2nd Gibbs Equation or we can use Relative Properties. Both **methods** are presented here.

Method 1: Use the Ideal Gas Entropy Function and the 2nd Gibbs Equation.

The 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function is :

$$\Delta\hat{S} = \hat{S}_{3S} - \hat{S}_2 = \hat{S}^o_{T3S} - \hat{S}^o_{T2} - \frac{R}{MW}\,Ln\frac{P_3}{P_2} = 0 \qquad \textbf{Eqn 4}$$

$$\Delta\hat{S} = \hat{S}_{1S} - \hat{S}_4 = \hat{S}^o_{T1S} - \hat{S}^o_{T4} - \frac{R}{MW}\,Ln\frac{P_1}{P_4} = 0 \qquad \textbf{Eqn 5}$$

We can solve **Eqns 4** & **5** for the unknowns $\mathbf{S^o_{T1}}$ & $\mathbf{S^o_{T3}}$:

$$\hat{S}^o_{T3S} = \hat{S}^o_{T2} + \frac{R}{MW}\,Ln\frac{P_3}{P_2} \qquad \textbf{Eqn 6}$$

$$\hat{S}^o_{T1S} = \hat{S}^o_{T4} + \frac{R}{MW}\,Ln\frac{P_1}{P_4} \qquad \textbf{Eqn 7}$$

We can look up $\mathbf{S^o_{T2}}$ and $\mathbf{S^o_{T4}}$ in the **Ideal Gas Property Table** for **air** and use it with the known **compression ratio** in **Eqns 6** & **7** to determine $\mathbf{S^o_{T3}}$ and $\mathbf{S^o_{T1}}$. We can do this because the HEX's are isobaric. $\mathbf{P_1 = P_2}$ and $\mathbf{P_3 = P_4}$.

R	8.314	**J/mol-K**	**MW**	29.00	**g/mol**
$\mathbf{S^o_{T2}}$	**1.9784**	**kJ/kg-K**	$\mathbf{S^o_{T4}}$	**0.0061681**	**kJ/kg-K**
$\mathbf{S^o_{T3S}}$	**1.2020**	**kJ/kg-K**	$\mathbf{S^o_{T1S}}$	**0.78254**	**kJ/kg-K**

Now, we can use $\mathbf{S^o_{T1S}}$ and $\mathbf{S^o_{T3S}}$ and the **Ideal Gas Property Table** for **air** to determine $\mathbf{T_{1S}}$ and $\mathbf{T_{3S}}$ and then $\mathbf{H_{1S}}$ and $\mathbf{H_{3S}}$ by interpolation :

T (K)	**H° (kJ/kg)**	**S° (kJ/kg-K)**				
630	**429.25**	**0.77137**				
$\mathbf{T_{1S}}$	$\mathbf{H_{1S}}$	**0.78254**	Interpolation yields :	$\mathbf{T_{1S}}$	**636.61**	**K**
640	**439.98**	**0.78826**		$\mathbf{H_{1S}}$	**436.34**	**kJ/kg**

T (K)	**H° (kJ/kg)**	**S° (kJ/kg-K)**				
920	**747.82**	**1.1867**				
$\mathbf{T_{3S}}$	$\mathbf{H_{3S}}$	**1.2020**	Interpolation yields :	$\mathbf{T_{3S}}$	**932.67**	**K**
940	**770.33**	**1.2109**		$\mathbf{H_{3S}}$	**762.08**	**kJ/kg**

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_{3S})}{P_r(T_2)} = \frac{P_3}{P_2} \qquad \textbf{Eqn 8} \qquad\qquad \frac{P_r(T_{1S})}{P_r(T_4)} = \frac{P_1}{P_4} \qquad \textbf{Eqn 9}$$

Where $\mathbf{P_r}$ is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqns 8** & **9** For $\mathbf{P_r(T_3)}$ and $\mathbf{P_r(T_1)}$, as follows :

$$P_r(T_{3S}) = \frac{P_3}{P_2}\,P_r(T_2) \qquad \textbf{Eqn 10} \qquad\qquad P_r(T_{1S}) = \frac{P_1}{P_4}\,P_r(T_4) \qquad \textbf{Eqn 11}$$

Look-up $\mathbf{P_r(T_2)}$ and $\mathbf{P_r(T_2)}$ and use them in **Eqns 10** & **11**, respectively, To determine $\mathbf{P_r(T_3)}$ and $\mathbf{P_r(T_1)}$:

$\mathbf{P_r(T_2)}$	**986.20**	$\mathbf{P_r(T_4)}$	**1.0217**
$\mathbf{P_r(T_{3S})}$	**65.747**	$\mathbf{P_r(T_{1S})}$	**15.326**

We can now determine T_{3S} and T_{1S} by interpolation on the the **Ideal Gas Property Table** for **air**.

Then, we use T_{3S} and T_{1S} to determine H_{3S} and H_{1S} from the **Ideal Gas Property Table** for **air**.

T (K)	P_r	H^o (kJ/kg)
630	14.7	429.25
T_{1S}	15.326	H_{1S}
640	15.591	439.98

Interpolation yields :			
	T_{1S}	637.02	K
	H_{1S}	436.78	kJ/kg

T (K)	P_r	H^o (kJ/kg)
920	62.489	747.82
T_{3S}	65.747	H_{3S}
940	67.990	770.33

Interpolation yields :			
	T_{3S}	931.84	K
	H_{3S}	761.15	kJ/kg

Since the two methods differ by only about **0.1%**, I will use the results from **Method 1** in the remaining calculations of this problem.

Next, we use the **isentropic efficiencies** of the compressor and the turbine to determine the actual **T** and **H** of **states 1** and **3**.

$$\eta_{S,turb} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_2 - \hat{H}_{3S}} \qquad \textbf{Eqn 12}$$

$$\eta_{S,comp} = \frac{\hat{H}_4 - \hat{H}_{1S}}{\hat{H}_4 - \hat{H}_1} \qquad \textbf{Eqn 13}$$

Solve **Eqns 12** & **13** for H_3 and H_1, respectively :

$$\hat{H}_3 = \hat{H}_2 - \eta_{S,turb}\left(\hat{H}_2 - \hat{H}_{3S}\right) \qquad \textbf{Eqn 14}$$

$$\hat{H}_1 = \hat{H}_4 - \frac{\left(\hat{H}_4 - \hat{H}_{1S}\right)}{\eta_{S,turb}} \qquad \textbf{Eqn 15}$$

Plugging values into **Eqns 14** & **15** gives:

H_1	523.58	kJ/kg
H_3	916.49	kJ/kg

And by interpolation on the **Ideal Gas Property Tables**:

T (K)	H^o (kJ/kg)
710	515.58
T_1	523.58
720	526.46

T (K)	H^o (kJ/kg)
1060	906.80
T_3	916.49
1080	929.77

T_1 717.35 K

T_3 1068.44 K

Now that we have fixed all the **states** and determined the values of all the **H**'s, we can plug values back into **Eqns 1 - 3** and complete **part (a)**.

$W_{S,turb}$	875.01	kJ/kg
$W_{S,comp}$	-436.17	kJ/kg

W_{cycle} 438.84 kJ/kg

Heat tranfer out of the system occurs in **step 3-4**. We can determine Q_{34} by appplying the 1st Law to HEX #2. The HEX operates at steady-state, has no **shaft work** interaction and changes in kinetic and potential energies are negligible. So, the appropriate form of the 1st Law is:

$$\hat{Q}_{34} = \hat{H}_4 - \hat{H}_3 \qquad \textbf{Eqn 16}$$

Plugging values into **Eqn 16** gives us:

Q_{34} -829.08 kJ/kg

Part b.) We can calculate the **thermal efficiency** of the cycle from:

$$\eta = \frac{\hat{W}_{cycle}}{\hat{Q}_{12}} \qquad \textbf{Eqn 17}$$

Heat tranfer into of the system occurs in **step 1-2**. We can determine Q_{12} by appplying the 1st Law to HEX #1. The HEX operates at steady-state, has no **shaft work** interaction and changes in kinetic and potential energies are negligible. So, the appropriate form of the 1st Law is:

$$\hat{Q}_{12} = \hat{H}_2 - \hat{H}_1 \qquad \textbf{Eqn 18}$$

Plugging values into Eqn 18 gives us:	Q_{12}	1267.92	kJ/kg
Plugging values into Eqn 17 gives us:	η	34.61%	

Part c.) In the ideal cycle, the compressor and turbine are **isentropic**. So, all we need to do to complete this part of the problem is use H_{1S} and H_{3S} instead of H_1 and H_3 when we calculate $W_{S,comp}$, $W_{S,turb}$, W_{cycle}, Q_{34}, Q_{12} and η.

The equations from **parts (a) - (c)** become:

$$\hat{W}_{cycle} = \hat{W}_{S,turb} + \hat{W}_{S,comp} \qquad \textbf{Eqn 19}$$

$$\hat{W}_{S,turb} = \hat{H}_2 - \hat{H}_3 \qquad \textbf{Eqn 20}$$

$$\hat{W}_{S,comp} = \hat{H}_4 - \hat{H}_1 \qquad \textbf{Eqn 21}$$

$$\hat{Q}_{34} = \hat{H}_4 - \hat{H}_3 \qquad \textbf{Eqn 22}$$

$$\hat{Q}_{12} = \hat{H}_2 - \hat{H}_1 \qquad \textbf{Eqn 23}$$

$$\eta = \frac{\hat{W}_{cycle}}{\hat{Q}_{12}} \qquad \textbf{Eqn 24}$$

Plugging values into **Eqns 19 - 24** yields the values in the following table. The "**% Change**" is defined as :

$$\%\text{Change} = \frac{\text{Real} - \text{Ideal}}{\text{Ideal}} \times 100\% \qquad \textbf{Eqn 25}$$

		Real Cycle	Ideal Cycle	% Change
$W_{S,turb}$	(kJ/kg)	875.0	1029.4	-15.0%
$W_{S,comp}$	(kJ/kg)	-436.2	-348.9	25.0%
W_{cycle}	(kJ/kg)	438.8	680.5	-35.5%
Q_{34}	(kJ/kg)	-829.1	-674.7	22.9%
Q_{12}	(kJ/kg)	1267.9	1355.2	-6.4%
η		34.6%	50.2%	-31.1%

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

			Real Cycle
	$W_{S,turb}$	(kJ/kg)	875.0
	$W_{S,comp}$	(kJ/kg)	-436.2
a.)	W_{cycle}	(kJ/kg)	438.8
	Q_{34}	(kJ/kg)	-829.1
	Q_{12}	(kJ/kg)	1267.9
b.)	η		34.6%

c.)

Ideal Cycle	%Change
1029.4	-15.0%
-348.9	25.0%
680.5	-35.5%
-674.7	22.9%
1355.2	-6.4%
50.2%	-31.1%

Although the isentropic efficiencies of the compressor and turbine are very high, 80% and 85%, they reduce the work output by 35% and reduce the efficiency by 31%. This shows the enormous significance of the these isentropic efficiencies in the overall performance of the power cycle.

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9F-1 Air-Standard Brayton Cycle With and Without Regeneration 10 pts

Consider three air-standard power cycles operating between the same two thermal reservoirs. All three cycles have the same **pressure ratio**, 12, and the same maximum and minimum **temperatures**, 2500°R and 560°R, respectively. In each cycle, the **mass flow rate** of air is 25,000 lb_m/h and the **pressure** at the compressor inlet is 14.7 psia. **Cycle A** is an ideal Brayton Cycle. In **Cycle B**, the compressor and turbine have isentropic efficiencies of 85% and 90%, respectively. **Cycle C** uses the same compressor and turbine as **Cycle B**, but also incorporates a **regenerator** with an **effectiveness** of 75%. Calculate the **net power output**, in **hP**, and **thermal efficiency** of each cycle.

Read : We will need to know all of the **H**'s in order to determine both W_{cycle} and η. We can get H_1 and H_3 immediately from the **Ideal Gas Property Table** for **air**.

Then, for each part of the problem, use the given **isentropic** compressor and turbine **efficiencies** to evaluate H_2 and H_4. Then, calculate W_{cycle} and Q_{in} for each part and finally the **thermal efficiency**.

In **Cycle C**, re-number the streams carefully so you can easily use most of the **H**'s from **Cycle B**. The key to **Cycle C** is to use the regenerator **effectiveness** to determine the **H** of the combustor feed. Once you have this, you can compute Q_{in}. W_{cycle} is the same as in **Cycle B**. So, calculate η from its definition.

Given:

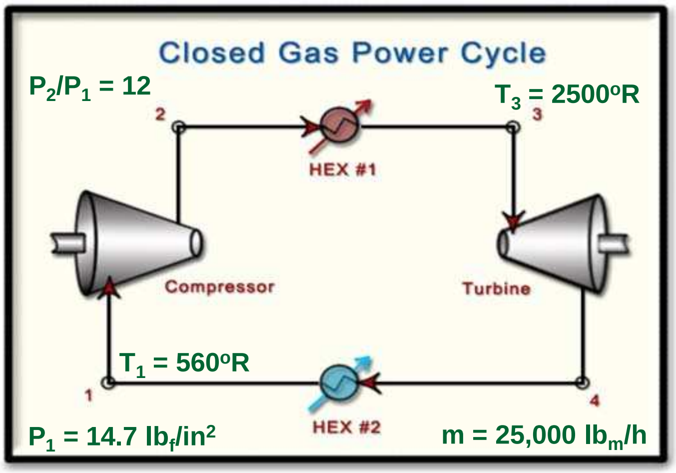
P_2/P_1	12		T_3	2500	°R
P_1	14.7	psia	m	25,000	lb_m/h
T_1	560	°R			

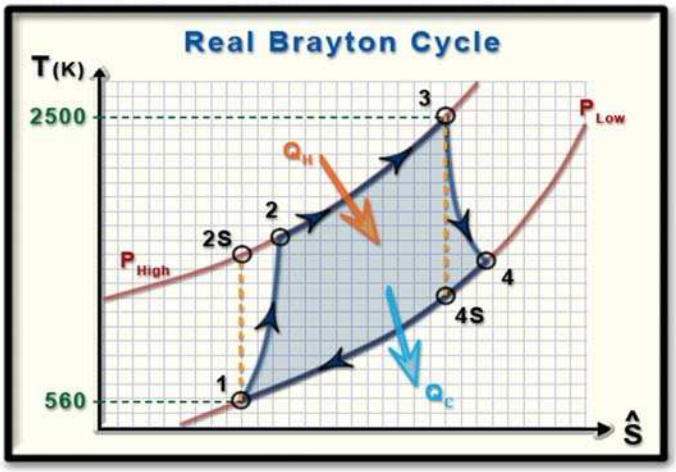
Cycle A	$\eta_{S,\ turb}$	1.00	**Cycle B**	$\eta_{S,\ turb}$	0.90
	$\eta_{S,\ comp}$	1.00		$\eta_{S,\ comp}$	0.85
Cycle C	$\eta_{S,\ turb}$	0.90			
	$\eta_{S,\ comp}$	0.85			
	η_{regen}	0.75			

Find: For each cycle :

η	???	%	W_{cycle}	???	hp

Diagram: **Cycle A** and **Cycle B**





Assumptions:

1 - Each component is an open system operating at steady-state.
2 - The turbine and compressor are adiabatic.
3 - There are no **pressure** drops for flow through the heat exchangers.
4 - Kinetic and potential energy changes are negligible.
5 - The working fluid is **air** modeled as an ideal gas.

Equations / Data / Solve:

Stream	T (°R)	P (psia)	H° (Btu/lb$_m$)	S° (Btu/lb$_m$-°R)	P$_r$
1	560	14.7	42.351	0.010149	1.1596
2		176.4	204.58		
2S	1117.2	176.4	180.24	0.010149	13.915
3	2500	176.4	555.34	0.39732	329.12
4		14.7	269.21		
4S	1338.5	14.7	237.42	0.39732	27.427

We can calculate the **thermal efficiency** of the cycle when the compressor and turbine are isentropic using :

$$\eta = 1 - \frac{\hat{Q}_{out}}{\hat{Q}_{in}} = 1 - \frac{-\hat{Q}_{41}}{\hat{Q}_{23}} \qquad \text{Eqn 1}$$

We can determine $\mathbf{Q_{12}}$ and $\mathbf{Q_{34}}$ by applying the 1st Law to HEX #1 and HEX #2, respectively.

Each HEX operates at steady-state, involves no **shaft work** and has negligible changes in kinetic and potential energies. The appropriate forms of the 1st Law are:

$$\hat{Q}_{23} = \hat{H}_3 - \hat{H}_2 \qquad \text{Eqn 2}$$

$$\hat{Q}_{41} = \hat{H}_1 - \hat{H}_4 \qquad \text{Eqn 3}$$

In order to use **Eqns 1 - 3**, we must first evaluate **H** at each **state** in the cycle.

Let's begin with **states 4** and **2** because they are completely fixed by the given data.

We know $\mathbf{T_1}$ and $\mathbf{T_3}$, so we can look-up $\mathbf{H_1}$ and $\mathbf{H_3}$ in the **Ideal Gas Property Table** for **air**.

$\mathbf{H_1}$	42.35	**Btu / lb$_m$**	$\mathbf{H_3}$	555.3	**Btu / lb$_m$**

The two remaining **H** values depend on the **isentropic efficiency** of the compressor and the turbine, so they will be different depending on which **part** of the problem is being considered.

We can determine W_{cycle} by applying the 1st Law to the entire cycle.

$$W_{cycle} = Q_{cycle}$$ **Eqn 4**

Because the compressor and turbine are assumed to be adiabatic, **Eqn 4** simplifies to:

$$W_{cycle} = Q_{cycle} = Q_{23} + Q_{41}$$ **Eqn 5**

So, once we determine Q_{23} and Q_{41} for each **part** of the problem, we can use **Eqn 5** to evaluate W_{cycle}.

Part a.) Both the compressor and the turbine are isentropic. This changes **Eqns 2** & **3** to:

$$\hat{Q}_{23} = \hat{H}_3 - \hat{H}_{2S}$$ **Eqn 6**

$$\hat{Q}_{41} = \hat{H}_1 - \hat{H}_{4S}$$ **Eqn 7**

We can determine T_{1S} and T_{3S} using either the **Ideal Gas Entropy Function** and the **2nd Gibbs Equation** or we can use **Relative Properties**. Both methods are presented here.

Method 1: Use the **Ideal Gas Entropy Function** and the **2nd Gibbs Equation**.

The 2nd Gibbs Equation for Ideal Gases in terms of the **Ideal Gas Entropy Function** for the compressor and the turbine are :

$$\Delta\hat{S} = \hat{S}_{2S} - \hat{S}_1 = \hat{S}^o_{T2S} - \hat{S}^o_{T1} - \frac{R}{MW}\,Ln\frac{P_2}{P_1} = 0$$ **Eqn 8**

$$\Delta\hat{S} = \hat{S}_{4S} - \hat{S}_3 = \hat{S}^o_{T4S} - \hat{S}^o_{T3} - \frac{R}{MW}\,Ln\frac{P_4}{P_3} = 0$$ **Eqn 9**

We can solve **Eqns 8** & **9** for the unknowns S^o_{T2S} & S^o_{T4S} :

$$\hat{S}^o_{T2S} = \hat{S}^o_{T1} + \frac{R}{MW}\,Ln\frac{P_2}{P_1}$$ **Eqn 10**

$$\hat{S}^o_{T4S} = \hat{S}^o_{T3} + \frac{R}{MW}\,Ln\frac{P_4}{P_3}$$ **Eqn 11**

We can look up S^o_{T1} and S^o_{T3} in the **Ideal Gas Property Table** for **air** and use it with the known **compression ratio** in **Eqns 10** & **11** to determine S^o_{T2S} and S^o_{T4S}. We can do this because the HEX's are isobaric. $P_2 = P_3$ and $P_4 = P_1$.

R	1.987	Btu/lbmole-°R	MW	29.00	lb_m / lbmole
S^o_{T1}	0.010149	Btu / lb_m °R	S^o_{T3}	0.39732	Btu / lb_m °R
S^o_{T2S}	0.18041	Btu / lb_m °R	S^o_{T4S}	0.22706	Btu / lb_m °R

Now, we can use S^o_{T2S} and S^o_{T4S} and the **Ideal Gas Property Table** for **air** to determine T_{2S} and T_{4S} and then H_{2S} and H_{4S} by interpolation :

T (°R)	H^o (Btu/lb_m)	S^o (Btu/lb_m°R)
1100	175.86	0.17647
T_{2S}	H_{2S}	0.18041
1120	180.97	0.18106

Interpolation yields :

T_{2S}	1117.16	°R
H_{2S}	180.24	Btu / lb_m

T (°R)	H^o (Btu/lb_m)	S^o (Btu/lb_m°R)
1300	227.38	0.21948
T_{4S}	H_{4S}	0.22706
1350	240.41	0.22932

Interpolation yields :

T_{4S}	1338.52	°R
H_{4S}	237.42	Btu / lb_m

Method 2: Use the **Ideal Gas Relative Pressure**.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_{2S})}{P_r(T_1)} = \frac{P_2}{P_1}$$ **Eqn 12**

$$\frac{P_r(T_{4S})}{P_r(T_3)} = \frac{P_4}{P_3}$$ **Eqn 13**

Where P_r is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqns 12** & **13** For $P_r(T_3)$ and $P_r(T_1)$, as follows :

$$P_r(T_{2S}) = \frac{P_2}{P_1} P_r(T_1)$$ **Eqn 14**

$$P_r(T_{4S}) = \frac{P_4}{P_3} P_r(T_3)$$ **Eqn 15**

Look-up $P_r(T_1)$ and $P_r(T_3)$ and use them in **Eqns 14** & **15**, respectively, To determine $P_r(T_{2S})$ and $P_r(T_{4S})$:

$P_r(T_1)$	1.1596	$P_r(T_3)$	329.12
$P_r(T_{2S})$	13.915	$P_r(T_{4S})$	27.427

We can now determine T_{2S} and T_{4S} by interpolation on the the **Ideal Gas Property Table** for **air**.

Then, we use T_{2S} and T_{4S} to determine H_{2S} and H_{4S} from the **Ideal Gas Property Table** for **air**.

T (°R)	P_r	H^o (Btu/lb$_m$)
1100	13.124	175.86
T_{2S}	13.915	H_{2S}
1120	14.034	180.97

Interpolation yields :

T_{2S}	1117.39	°R
H_{2S}	180.30	Btu / lb$_m$

T (°R)	P_r	H^o (Btu/lb$_m$)
1300	24.581	227.38
T_{4S}	27.427	H_{4S}
1350	28.376	240.41

Interpolation yields :

T_{4S}	1337.49	°R
H_{4S}	237.15	Btu / lb$_m$

Since the two methods differ by less than **0.1%**, I will use the results from **Method 1** in the remaining calculations of this problem.

Now, that we have values for all of the **H**'s, we can plug values back into **Eqns 2**, **3, 5** & **1** to complete our analysis of **Cycle A**.

Q_{23}	375.10	Btu / lb$_m$
Q_{41}	-195.07	Btu / lb$_m$
1 hp	2544.5	Btu/h

W_{cycle}	4.501E+06	Btu / h
W_{cycle}	1769	hP
η	48.00%	

Cycle B $\eta_{S,\,turb}$ **0.90** $\eta_{S,\,comp}$ **0.85**

We use the **isentropic efficiencies** of the compressor and the turbine to determine the actual **T** and **H** of **states 1** and **3**.

$$\eta_{S,comp} = \frac{\hat{H}_1 - \hat{H}_{2S}}{\hat{H}_1 - \hat{H}_2}$$ **Eqn 16**

$$\eta_{S,turb} = \frac{\hat{H}_3 - \hat{H}_4}{\hat{H}_3 - \hat{H}_{4S}}$$ **Eqn 17**

Solve **Eqns 12** & **13** for H_2 and H_4, respectively :

$$\hat{H}_2 = \hat{H}_1 - \frac{\left(\hat{H}_1 - \hat{H}_{2S}\right)}{\eta_{S,turb}}$$ **Eqn 18**

$$\hat{H}_4 = \hat{H}_3 - \eta_{S,turb}\left(\hat{H}_3 - \hat{H}_{4S}\right)$$ **Eqn 19**

Plugging values into **Eqns 18** & **19** gives:

H_2	204.58	Btu / lb_m
H_4	269.21	Btu / lb_m

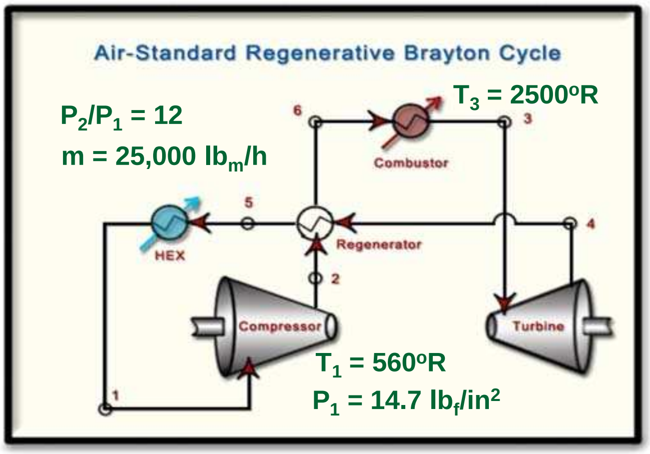
We have all of the **H**'s, so we can plug values back into **Eqns 2**, **3**, **5** & **1** to complete our analysis of **Cycle B**.

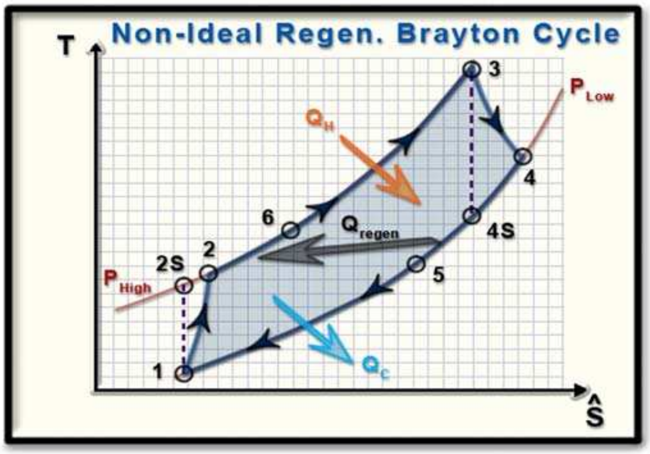
Q_{23}	350.76	Btu / lb_m
Q_{41}	-226.86	Btu / lb_m
W_{cycle}	123.90	Btu / lb_m

W_{cycle}	3.098E+06	Btu / h
W_{cycle}	1217	hP
η	35.32%	

Cycle C $\eta_{S, turb}$ 0.90 $\eta_{S, comp}$ 0.85 $\varepsilon_{S, regen}$ 0.75

Diagram:





Stream	T (°R)	P (psia)	H° (Btu/lbm)	S° (Btu/lbm-°R)	P_r
1	560	14.7	42.351	0.010149	1.1596
2		176.4	204.58		
2S	1117.2	176.4	180.24	0.010149	13.915
3	2500	176.4	555.34	0.39732	329.12
4		14.7	269.21		
4S	1338.5	14.7	237.42	0.39732	27.427
5					
6			253.05		

We can determine the **thermal efficiency** of the regenerative cycle using:

$$\eta_{regen} = \frac{W_{cycle}}{Q_{in}} = \frac{W_{cycle}}{Q_{63}} \qquad \text{Eqn 20}$$

The addition of a regenerator does not effect the value of W_{cycle}. It is the same as in **Cycle B**.

W_{cycle} 123.902 Btu / lb_m

The regenerator does reduce the magnitude of both Q_{63} and Q_{51}.
We can determine Q_{12} by applying the 1st Law to the combustor.

The combustor operates at steady-state, involves no **shaft work** and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\hat{Q}_{63} = \hat{H}_6 - \hat{H}_3 \qquad \text{Eqn 21}$$

From **Cycle B** the following values of **H** do not change:

H_2	**204.58**	**Btu / lb$_m$**			
H_3	**555.34**	**Btu / lb$_m$**	H_4	**269.21**	**Btu / lb$_m$**

So, we need to use the **effectiveness** of the regenerator to determine H_6.

The **effectiveness** of the regenerator is given by:

$$\varepsilon_{regen} = \frac{\hat{H}_6 - \hat{H}_2}{\hat{H}_4 - \hat{H}_2} \quad \text{Eqn 22}$$

We can solve **Eqn 22** for H_3 :

$$\hat{H}_6 = \hat{H}_2 + \varepsilon_{regen}\left(\hat{H}_4 - \hat{H}_2\right) \quad \text{Eqn 23}$$

Now, we can plug values back into **Eqns 23**, **21** & **20** :

H_6	**253.05**	**Btu / lb$_m$**
Q_{62}	**302.29**	**Btu / lb$_m$**
W_{cycle}	**3.098E+06**	**Btu / h**

W_{cycle}	**1217**	**hP**
η	**40.99%**	

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

Cycle A

W_{cycle}	**1770**	**hP**
η	**48.0%**	

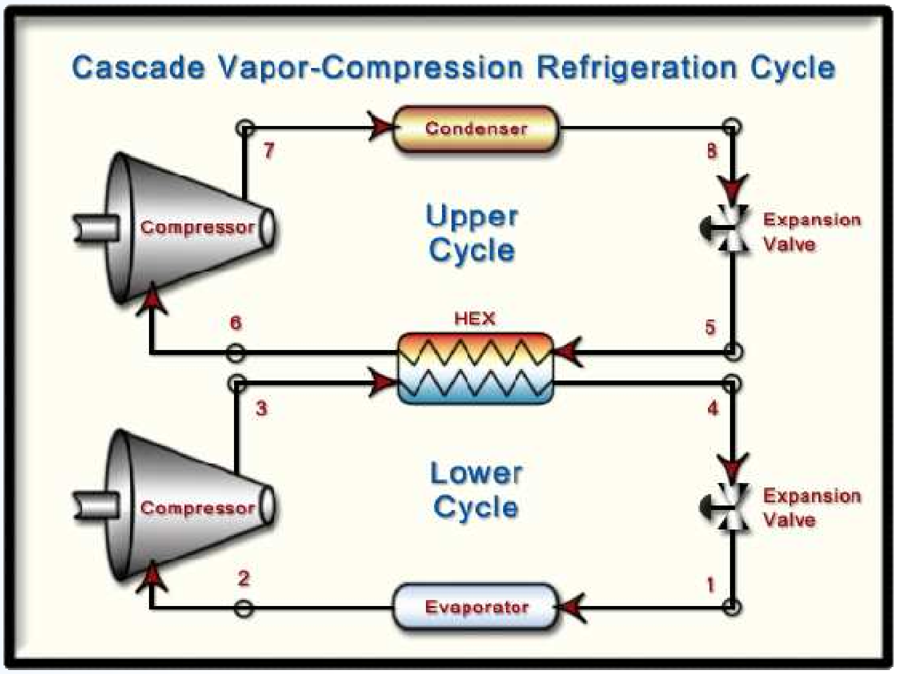
Cycle B

W_{cycle}	**1220**	**hP**
η	**35.3%**	

Cycle C

W_{cycle}	**1220**	**hP**
η	**41.0%**	

Chapter 10



Refrigeration and Heat Pump Systems

In this chapter, we discuss refrigeration and heat pump systems in a manner analogous to our discussion of power systems in Chapter 9.

We begin with a discussion of ideal vapor-compression refrigeration (VCR) and follow with a discussion of the impact of irreversibilities on the coefficient of performance. Refrigerant selection criteria are discussed, with special attention to the vapor pressure—temperature relationship. Cascade and multi-stage VCR cycles are covered. Heat pumps and the key reversing valve are discussed.

The chapter concludes with a discussion of Air-Standard Gas Refrigeration Cycles. Regeneration can be used to increase the coefficient of performance. Irreversibilities decrease the coefficient of performance.

Applications of Refrigeration Systems

- **V-C Refrigeration Cycle**
  - Food preservation
  - Air conditioning
  - Industrial cooling processes
- **Gas Refrigeration Cycle**
  - Air conditioning on passenger aircraft
  - Cryogenic refrigeration (very low temperatures, < 150 K)
- **Thermoelectric Refrigeration Systems**
  - Peltier Effect
  - Very small and lightweight
    - Very low efficiency

- Refrigeration revolutionized food distribution.
  - Limits the growth of bacteria in the food supply.
- Air-conditioning dramatically improved life in the tropical and subtropical regions.
  - Facilitated the economic development of the southern US.
- Gas refrigeration is not as efficient as V-C refrigeration.
  - But, it is lightweight. This makes it well-suited to use in passenger aircraft.
  - The other great attribute of gas refrigeration cycles is that they can reach very, very low temperatures, even into the cryogenic range.
- Thermoelectric refrigerators make use of the Peltier Effect.
  - When a current is passed through the junction between two wires made of different metals, heat is either released or absorbed, depending on the direction of the electrical current.
  - This is also the principle upon which thermocouples operate.
  - Thermoelectric refrigerators are lightweight and inexpensive, but not very efficient. (Less than 10% of Carnot Efficiency, so generally less than 5%.)

Ammonia Absorption Refrigeration

- **Ammonia Absorption Refrigeration**
  - No moving parts
  - No work input
  - Well-suited to remote and portable applications
    - Locations where electric power is not available
    - RV's and campers
    - Invented by a student in Sweden
- **See external flow diagram or visit: http://www.nh3tech.org/absorption.html**

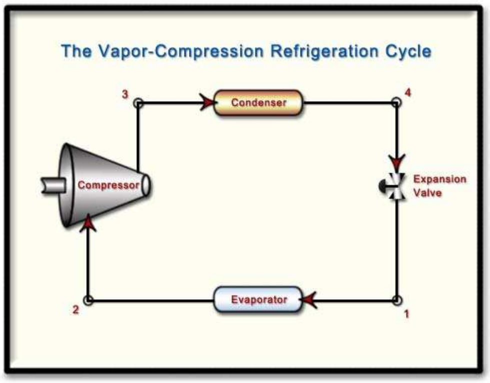
- Harrison Ford builds a giant ammonia absorption refrigeration system in the film Mosquito Coast !
- The refrigeration system in the VW Eurovan and most campers is an ammonia refrigeration system!
  - It is VERY important to park the Eurovan or camper on level ground in order to make the refrigerator work.
  - See why by visiting the Ammonia Refrigeration Technicians Association website.

Ideal V-C Refrigeration Cycle

- **The Ideal V-C Refrigeration Cycle**

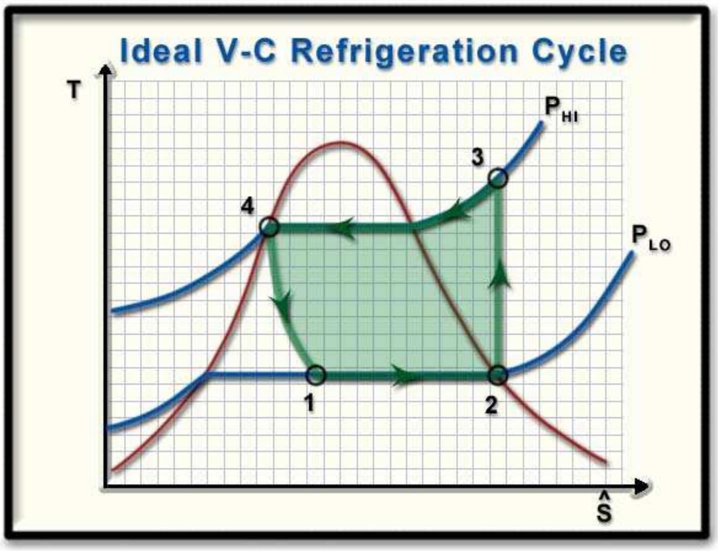
Step 1-2:	Evaporator	Heat absorbed at constant pressure
Step 2-3	Compressor	Isentropic compression
Step 3-4:	Condenser	Heat rejected at constant pressure
Step 4-1:	Expansion Valve	Isenthalpic expansion

$$\mathbf{COP_R = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{\hat{H}_2 - \hat{H}_1}{\hat{H}_3 - \hat{H}_2}}$$



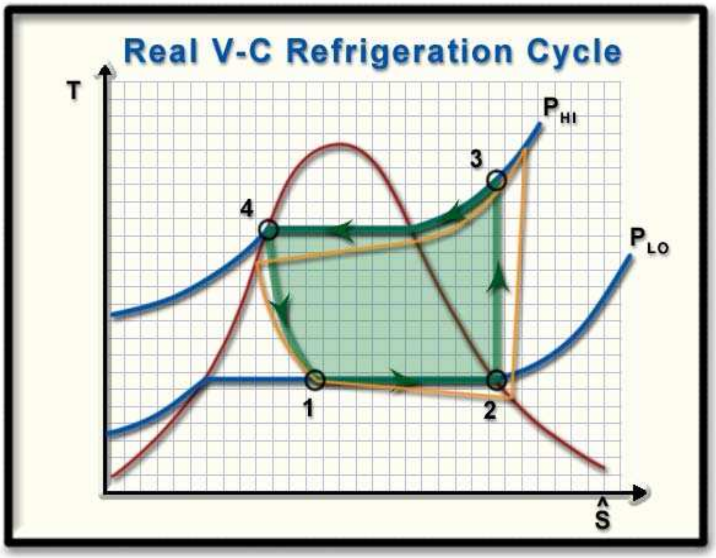
- The ideal VCR Cycle is <u>not</u> internally reversible because flow through the expansion valve has lots of friction and is therefore irreversible.

Ideal VCR Cycle: TS Diagram



- Notice that the expansion valve (step 4-1) is NOT isentropic !
  - ◊There is a great deal of friction in a throttling device.
  - ◊Lots of irreversibility.
  - ◊Lots of lost work !
  - ◊The expansion valve is isenthalpic.
- The evaporator effluent is saturated vapor.

Irreversibilities in the VCR Cycle



- The additional irreversibilities (above and beyond the expansion valve) come from the same sources as in the Rankine Vapor Power Cycle.
  - ◊ Fluid-solid friction in the Evaporator and condenser produce pressure drops in these device. They are not quite isobaric any longer.
  - ◊ Mechanical or solid-solid friction plus fluid-solid friction and even fluid-fluid friction due to turbulent flow contribute to the irreversibility of the compressor.
    - The result is that the entropy of the fluid increases through the real compressor.
    - As in power cycles, this is taken into account by the isentropic efficiency of the compressor.

Refrigerant Selection

- **Selection Criteria**
  - ◊ Cost
  - ◊ P* vs T Relationship
  - ◊ ΔH_{vap}
  - ◊ Chemical Stability
  - ◊ Corrosiveness
  - ◊ Toxicity
  - ◊ Flammability

- Cost
  - ◊ Cheap is always good.
- P* vs T Relationship.
  - ◊ It is not practical to use water as a refrigerant to chill food to **–5°C**. Water does not exist in VLE at **–5°C**.
- Heat of Vaporization.
  - ◊ The larger the heat of vaporization, the lower the required refrigerant flow rate.
- Chemical Stability
  - ◊ If the refrigerant degrades chemically in the system, it must be replaced.
  - ◊ If it degrades in the atmosphere due to interaction with UV light, it may lead to severe air pollution and/or global warming.
- Corrosiveness
  - ◊ If the refrigerant corrodes the devices and tubing in the system, it is not a good choice.
- Toxicity
  - ◊ Refrigerants always leak into the environment eventually.
  - ◊ This may cause a disaster if they are toxic at very low concentrations.
- Flammability
  - ◊ Non-flammable refrigerants are desirable.
  - ◊ A leak + spark = disaster, if the refrigerant is highly flammable.

Vapor Pressure - Temperature Relationship

- **Condenser**
  - ◊ $T_{sat}(P_{high}) > T_{hot\ res}$ because the working fluid must reject heat to the hot reservoir in the condenser.
  - ◊ Rule of thumb is at least: $\mathbf{T_{sat}(P_{high}) - T_{hot\ res} = 10^oC}$
- **Evaporator**
  - ◊ $T_{sat}(P_{low}) < T_{cold\ res}$ because the working fluid must absorb heat from the cold reservoir in the evaporator.
  - ◊ Rule of thumb is at least: $\mathbf{T_{cold\ res} - T_{sat}(P_{low}) = 10^oC}$
- **Vapor Pressure (Saturation Pressure)**
  - ◊ Not too high: makes equipment expensive and dangerous.
  - ◊ Not too low: must be greater than atmospheric pressure
    - Air leaks into the system drastically reduce COP_R.

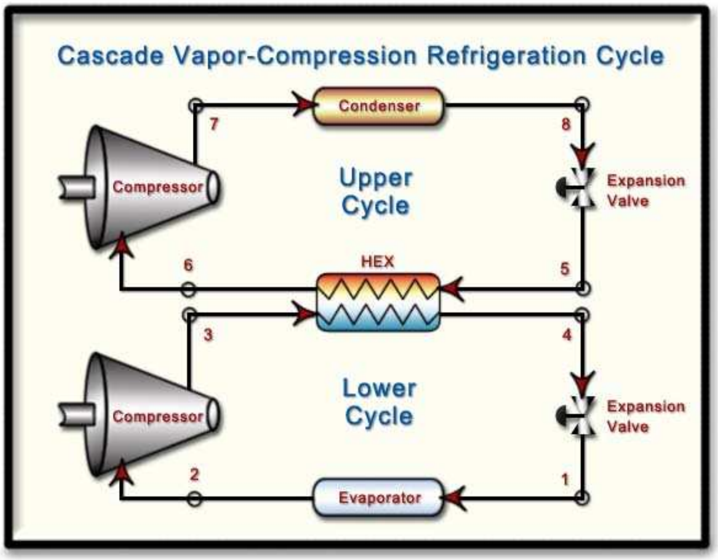
- This is usually the 1st criterion applied in refrigerant selection.
- Basically, this determines whether a refrigerant CAN be used to accomplish a given refrigeration task.
- Use this criterion to select a group of POSSIBLE refrigerants.
- THEN consider the criteria on the previous slide to choose the best refrigerant from the group.

Cascade VCR Cycles

- **Cascade V-C Refrigeration Cycles**
  - ◊ Two separate refrigeration cycles
  - ◊ Analogous to Binary Vapor Power Cycles
  - ◊ One provides cooling to the other
  - ◊ Usually two different refrigerants
  - ◊ High COPR
  - ◊ High cost
  - ◊ Can reach very low temperatures: $\mathbf{T_C \ll T_H}$

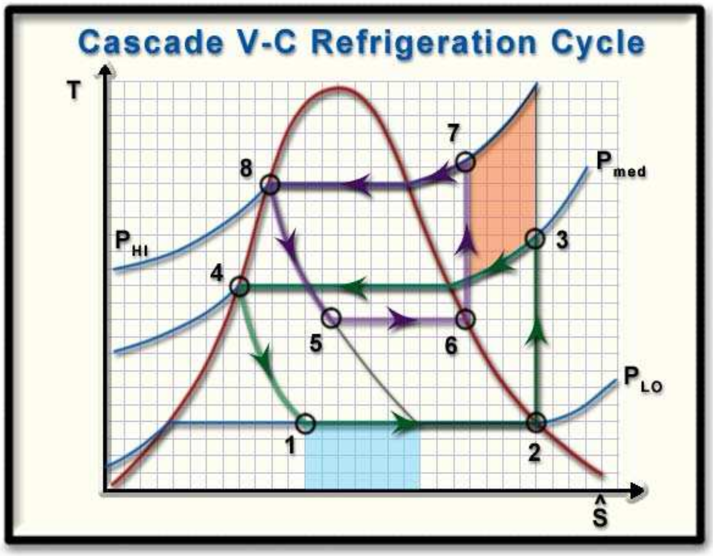
- Cascade VCR Cycles are comparable to Binary Vapor Power Cycles.
  - ◊ Two separate cycles only interact by exchanging heat.
  - ◊ The upper and lower cycles usually use different refrigerants.
    - This allows you to choose an appropriate refrigerant for a VERY low temperature range for the lower cycle and…
    - A different refrigerant that is appropriate for a higher temperature range for the upper cycle.
    - The result is a refrigeration system that makes the refrigerated space VERY cold using an ordinary high temperature reservoir, such as ambient air.
  - ◊ The lower cycle absorbs heat from the refrigerated space. And rejects heat into the upper cycle.
  - ◊ The upper cycle absorbs heat from the lower cycle and rejects heat into the high temperature reservoir.
  - ◊ Cascade refrigeration cycles can be used to reach temperatures as low as 100 K.
    - This is cold enough to liquefy many gases, such as light hydrocarbons.
  - ◊ The cascade cycle is more efficient than a single VCR cycle operating between the same two reservoirs, but is also more expensive to build and operate.
    - Therefore, it is generally only used for very low temperature applications.

Cascade VCR Flow Diagram



- Notice that streams 3 and 5 do NOT mix in the HEX. They just exchange heat.
  - ◊ This provides the flexibility to use different refrigerants in the lower and upper cycles.
  - ◊ It also results in lost work due to the irreversibility of heat transfer through a finite temperature difference.

Cascade VCR TS Diagram



- There is one little catch. This analysis and conclusion are only completely true if the MASS FLOW RATE of the refrigerant is the same in the lower and upper cycle.
- This point becomes critical in the next cycle, the Multi-Stage VCR Cycle.

- For simplicity, this **TS** Diagram is based on a cascade refrigeration cycle in which both cycles use the SAME refrigerant.
- Notice that $\mathbf{T_3 > T_5}$ in order to make the HEX work.
  - ◊ This seems odd, but remember that this is a refrigeration cycle.
  - ◊ The low temperature cycle must reject heat INTO the higher temperature cycle.
  - ◊ In order to make this happen, stream 3 must be hotter than stream 5.
  - ◊ This temperature difference is necessary to make a real cycle function, but this heat transfer is irreversible !
- The orange shaded area bounded by points 3 and 7 represents the DECREASE in $\mathbf{Q_H}$ when the cascade cycle is compared to a simple, single VCR Cycle operating between $\mathbf{P_{LO}}$ and **PHI**.
- Similarly, the light blue shaded area beneath the $\mathbf{P_{LO}}$ isobar (near point 1) is the INCREASE in $\mathbf{Q_C}$.
- The 1st Law tells us that if $\mathbf{Q_C}$ increases and $\mathbf{Q_H}$ decreases, then $\mathbf{W_{Sh}}$ must DECREASE. $\mathbf{Q_C + W_{Sh} = Q_H}$.
- From the definition of **COPR** ($\mathbf{COPR = Q_C / W_{Sh}}$) it is clear that the Cascade VCR Cycle is more efficient than the ordinary VCR Cycle operating between the same two thermal reservoirs.

Multi-Stage VCR Cycles

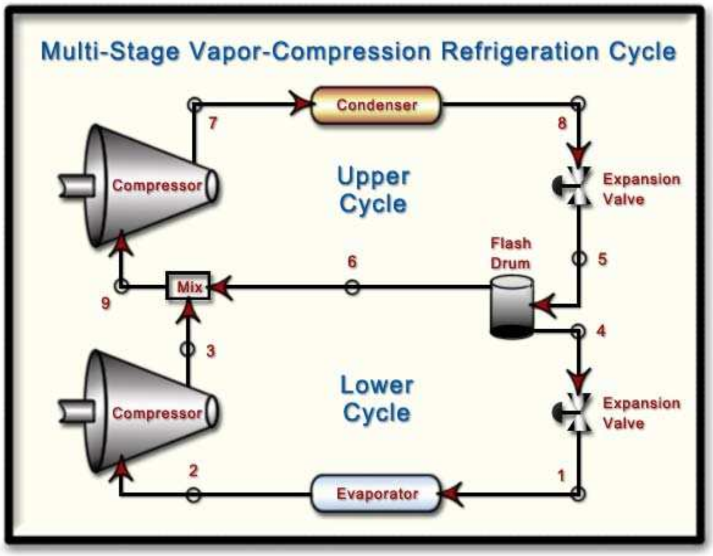
- **Similar to Cascade V-C Refrigeration**
- **Two cycles use the <u>same</u> refrigerant**
- **Instead of exchanging heat between two cycles, the refrigerant streams are mixed.**
- **This is more efficient than heat exchange.**
- **$\mathbf{COP_R}$ increases (slightly better than Cascade VCR)**
- **Cost increases (but not as expensive as Cascade VCR)**
  - ◊ Can reach very low temperatures

$$\mathbf{T_C \ll T_H}$$

  - ◊ Cannot reach as temperatures as cold as Cascade VCR

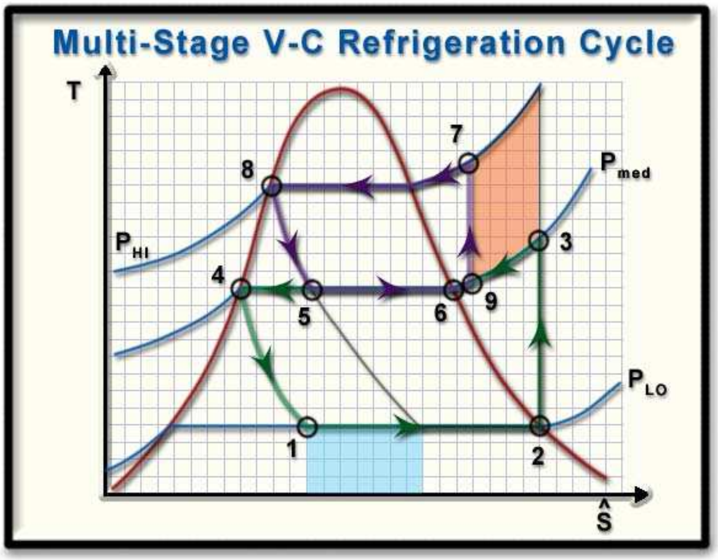
- In a Multi-Stage VCR Cycle, streams are simply MIXED instead of exchanging heat.
- This reduces the irreversibility of heat exchange, but requires that both cycles use the same refrigerant.
  - ◊ This has the fringe benefit of saving the cost of the HEX.
- For a given pair of thermal reservoirs, Multi-Stage is more efficient and less expensive than Cascade VCR.
  - ◊ The bad news is that Multi-Stage VCR cannot reach the very low temperatures that Cascade VCR can because just one refrigerant can be used.

Multi-Stage VCR Flow Diagram



- Stream 8 is a saturated liquid.
  - ◊ When the pressure drops isenthalpically in the expansion valve, some of the liquid vaporizes. So point 5 is in the 2-phase envelope.
  - ◊ The flash drum just separates the saturated liquid from the saturated vapor.
  - ◊ The saturated vapor, stream 6, comes out the top and flows to the mixer.
  - ◊ The saturated liquid, stream 4, drops out the bottom of the flash drum and proceeds to the second expansion valve.
  - ◊ Stream 6 is used to cool stream 3 by mixing with it (instead of exchanging heat as in the cascade cycle).
- One key point here is that the mass flow rate CANNOT be the same in all of the streams in this process !
  - ◊ Streams 1, 2, 3 & 4 all have the same mass flow rate.
  - ◊ Streams 5, 7, 8 & 9 all have the same mass flow rate.
  - ◊ BUT: $\mathbf{m_5 > m_4}$ because…
    - A mass balance on the flash drum tells us that: $\mathbf{m_5 = m_4 + m_{\wedge}}$.
- This makes it VERY tricky to interpret areas on the TS Diagram for a Multi-Stage VCR Cycle.

Multi-Stage VCR TS Diagram



- This **TS** Diagram is a bit different.
  - ◊ Stream 5 is split by the Flash Drum into streams 4 and 6, Saturated liquid and saturated vapor.
  - ◊ Streams 4 and 6 then flow to different places.
  - ◊ Stream 4 proceeds to the 2nd expansion valve and the lower cycle.
  - ◊ Stream 6 mixes with and thereby cools stream 3.
- The orange shaded area bounded by points 3 and 7 is proportional to the DECREASE in $\mathbf{Q_H}$ when the cascade cycle is compared to a simple, single VCR Cycle operating between $\mathbf{P_{LO}}$ and $\mathbf{P_{HI}}$.
- Similarly, the light blue shaded area beneath the $\mathbf{P_{LO}}$ isobar (near point 1) is proportional to the INCREASE in $\mathbf{Q_C}$.
- However, because the mass flow rates vary, we cannot visually estimate the MAGNITUDE of the decrease in $\mathbf{Q_H}$ or the increase in $\mathbf{Q_C}$.
  - ◊ We still know that $\mathbf{Q_H}$ decreases, $\mathbf{Q_C}$ increases and therefore $\mathbf{W_{Sh}}$ decreases and **COPR** increases.
  - ◊ We can calculate all of these changes.
  - ◊ It is just that the areas on the **TS** Diagram can be misleading because of the differences in the mass flow rates of the streams.
    - Areas on the **TS** Diagram are actually $\mathbf{Q_{hat}}$.
    - We must multiply by the mass flow rate to get $\mathbf{Q_{dot}}$.
    - $\mathbf{Q_{dot,C} = m_{dot,1} * Q_{hat,C}}$ and $\mathbf{Q_{dot,H} = m_{dot,7} * Q_{hat,H}}$ (watch the signs)
    - $\mathbf{m_{dot,7} > m_{dot,1}}$.
    - So, areas can be misleading if you interpret them as $\mathbf{Q_{dot}}$values

Heat Pumps - Review

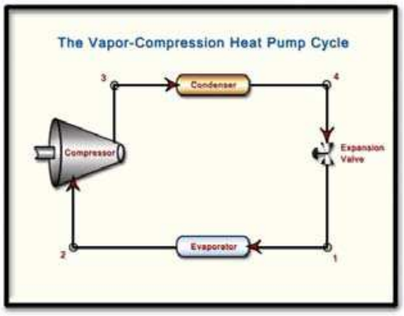
- **Goal: Transfer into a hot reservoir**
- **Requires work and must take in heat from cold a reservoir**

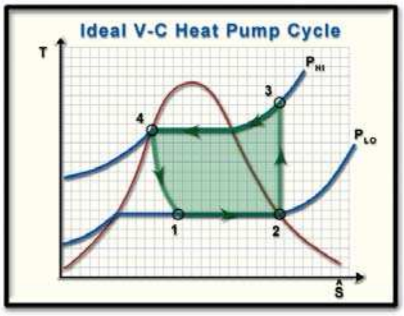
$$\mathbf{COP_{HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{1}{1 - Q_C / Q_H}}$$

- **HP's and Ref's operate with thermal reservoirs at very similar temperatures**
  - As a result, they use the same refrigerants. R-134a is popular.
- **Heat pump operate on essentially the same cycle as a V-C refrigerator, only in reverse**.

- R-134a is popular because…It does not damage the ozone layer in the Earth's atmosphere.
  - ◊ It has relatively low flammability and toxicity.
  - ◊ It is relatively inexpensive.It is chemically stabile.
  - ◊ It has a fairly large heat of vaporization.
  - ◊ Most important: it has the right vapor pressure over the range from **–20°C** to **40°C**.
    - This allows it to be used to freeze our food or cool our homes, but still reject heat to the surroundings at modest operating pressures.

Heat Pump: Flow Diagram & TS Diagram





- Just like the VCR Cycle !
- The only difference is the OBJECTIVE of the device.

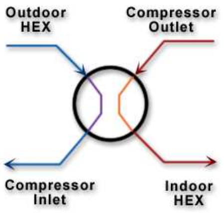
The Reversing Valve

- **Home heat pumps can function as both heating and air-conditioning systems**
- **Heat pump mode (HP)**
  - ◊ Indoor air is the hot reservoir
- **Air-conditioning mode (AC)**
  - ◊ Indoor air is the cold reservoir
- **Reversing Valve**
  - ◊ Clever way to switch from HP to AC mode without physically moving the HEX's !

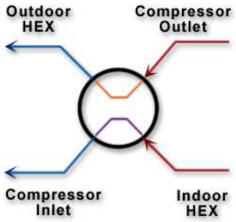
- The reversing valve is cool and clever.

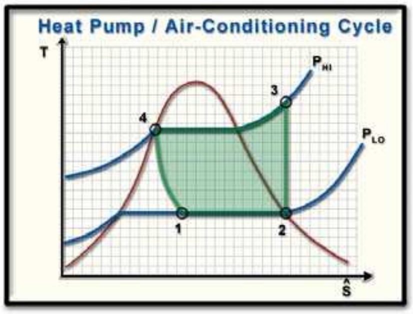
Reversing Valve & TS Diagram

- **HP Mode:**



- **AC Mode:**





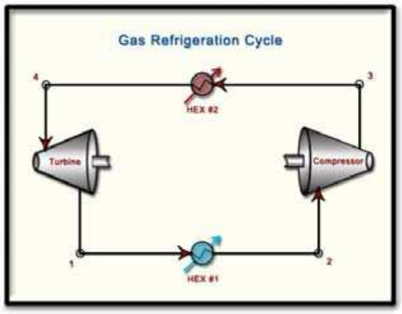
- Remember that the compressor outlet stream is the hottest stream.
  - ◊ It must flow to the HEX where the working fluid rejects heat.
- The compressor inlet stream comes FROM the HEX where the working fluid absorbs heat and is vaporized.
- Heat Pump Mode
  - ◊ The compressor outlet goes into a HEX inside your house to reject heat and keep you warm.
  - ◊ The compressor inlet receives the working fluid that has just been vaporized by absorbing heat from the air outside your house.
- Air-Conditioning Mode
  - ◊ The compressor outlet goes into a HEX outside your house to reject heat.
  - ◊ The compressor inlet receives the working fluid that has just been vaporized by absorbing heat from the air inside your house. This keeps you cool !
- The reversing valve is controlled electronically by your home thermostat and turned by a device called a solenoid.
  - ◊ When your home gets too cold, the thermostat sets the reversing valve to heat pump mode and heats your house.
  - ◊ When your home gets too hot, the thermostat sets the reversing valve to air-conditioning mode and cools your house.

Air-Standard Gas Refrigeration Cycle

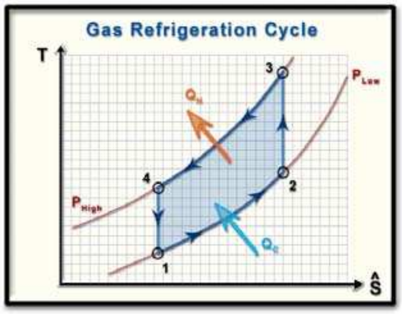
- **Less efficient than V-C Refrigeration Cycles**
- **Lightweight**
- **Capable of reaching temperatures below 100 K with a regenerator.**
- **Reverse Air-Standard Brayton Cycle**
- **Air-Standard Assumptions**
  - ◊ Air is the working fluid and it behaves as an ideal gas.
  - ◊ The GRC is modeled as a closed cycle.
  - ◊ All processes are internally reversible.
- **Cold Air-Standard Assumption**
  - ◊ The heat capacities of air are constant and always have the values determined at **25°C**.

- Because gas refrigeration cycles (GRC's) tend to be less efficient than VCR cycles, they are only used under special circumstances in which a VCR cycle would be difficult or impossible to use.
- GRC's are used to provide air-conditioning on passenger aircraft because they are lightweight.
- GRC's are used to help liquefy gases such as propane and methane.
  - ◊ No worries about vapor pressure – temperature relations because no phase change occurs in a GRC !
  - ◊ The limit is the temperature at which the working fluid (usually air) condenses.
  - ◊ At **100 kPa**, N_2 condenses at **77 K** and O_2 condenses at **90 K**.
  - ◊ So a GRC with air as the working fluid can chill a refrigerated space down to about **100 K** and meet the rule of thumb guideline for a **10°C** driving force to pull heat out of the refrigerated space.

Gas Refrigeration: Flow and TS Diagrams



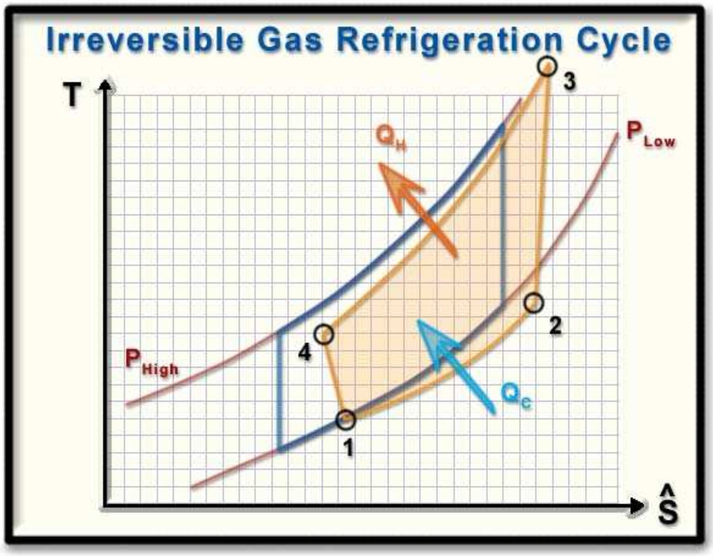
- **Reverse Brayton Cycle**
- **Heat exchange at constant pressure**



- **"Ideal" GRC**
  - ◊ Internally reversible
  - ◊ Pump & Compressor are adiabatic and inter-

- The flow diagram is similar to the one for the Brayton Cycle.
  - ◊ When we reverse the cycle, the compressor in the Brayton Cycle becomes a turbine and the turbine becomes a compressor.
  - ◊ In the GRC shown here, heat is absorbed from the cold reservoir in HEX #1 and rejected to the hot reservoir in HEX #2.
  - ◊ The turbine and compressor are isentropic in the "ideal" GRC.
    - The turbine and compressor are adiabatic and internally reversible.
- The TS Diagram looks just like the one for the Brayton Cycle, but the cycle proceeds counter-clockwise instead of clockwise.

Gas Refrigeration Cycle: Irreversibilities

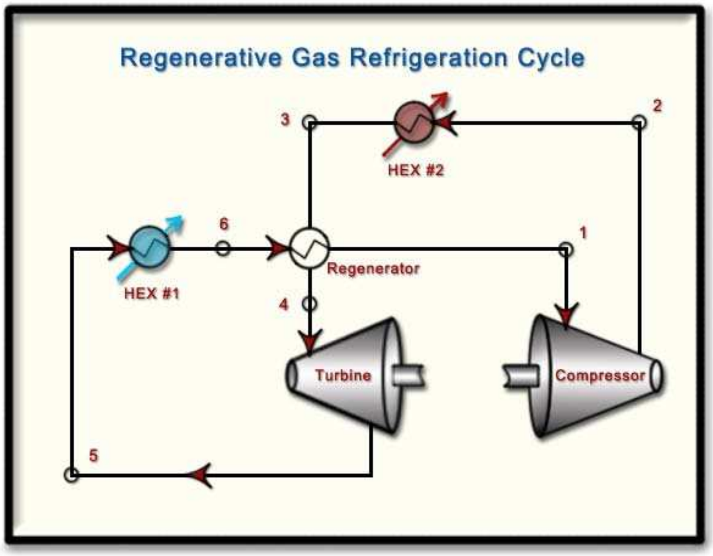


- Friction causes a pressure drop as the working fluid flows through each heat exchanger.
  - ◊ Friction means lost work.
- Mechanical and fluid friction within the compressor and turbine are sources of irreversibility.
  - ◊ The result is an increase in entropy along paths 2-3 and 4-1.Work is lost in these devices as a result of the irreversibilities.

Gas Refrigeration Cycle: Regeneration

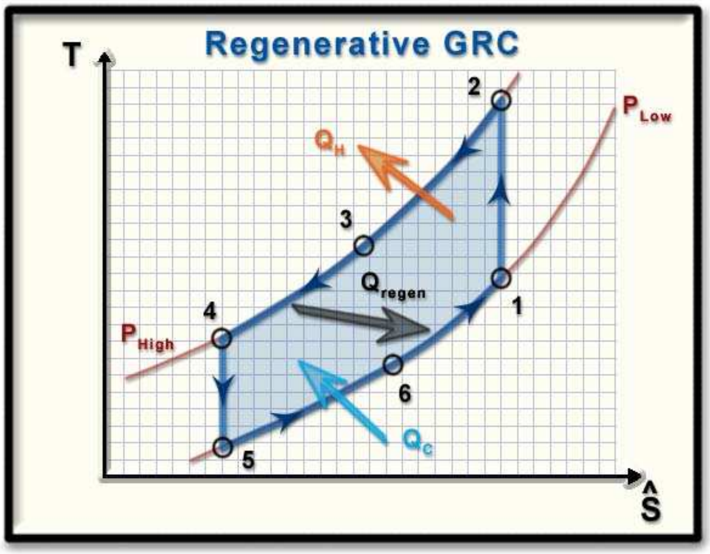
- **The purpose of regeneration is to allow the GRC to reach the lowest possible temperature.**
  - ◊ It does increase the COP_R of the cycle, but that is not the main objective.
- **Key: "pre-cool" the turbine feed so that when it expands through the turbine, it will reach a lower temperature than without pre-cooling.**
- **Multiple regenerative GRC's in series are used to reach temperatures below 100 K.**
- **Check out: http://www.stirling.nl**
  - They use the reverse Stirling Cycle, but it is similar to the reverse Brayton Cycle.

Regeneration Process Flow Diagram



- After the working fluid absorbs heat from the refrigerated space in HEX #1, it is still colder than the stream that just rejected heat to the hot reservoir.
  - ◊ That is: $\mathbf{T_6 < T_3}$.
- These two streams exchange heat in a COUNTERCURRENT heat exchanger called a Regenerator.
  - ◊ Countercurrent means that the two streams flow through the HEX in OPPOSITE directions.
  - ◊ This means that stream 3 is in direct contact with stream 1 at one end of the HEX and…
  - ◊ Stream 6 is in direct contact with stream 4 at one end of the HEX.
  - ◊ The 2nd Law tells us that heat will flow out of the turbine feed stream spontaneously at both ends of the HEX as long as $\mathbf{T_3 > T_1}$ AND $\mathbf{T_4 > T_6}$.
  - ◊ It is interesting to note that it is possible (and desirable) for $\mathbf{T_4 < T_1}$. Think about this carefully.

Regeneration TS Diagram



- The regenerator provides only a slight increase in the **COPR**, but it can dramatically lower $\mathbf{T_5}$.
- This is the goal because $\mathbf{T_5}$ (plus about **10°C** for our driving force for heat transfer) is the coldest that this cycle can maintain a refrigerated space.
- Very, VERY cool.

10B-1 Ideal Ammonia Vapor-Compression Refrigerator 7 pts

A cold-storage warehouse uses a refrigeration system to keep groceries at 2°C while the **temperature** outside the warehouse is 30°C. The groceries and the outside air act as thermal reservoirs in this process.
Although the warehouse is insulated, it absorbs heat from the surroundings at a rate of **775 kW**. Determine the power requirement and **COP** for a **Carnot** refrigeration cycle and for an **ideal ammonia** vapor-compression refrigeration cycle that will maintain the **temperature** of the groceries under these conditions. The condenser operates at **1.6 MPa** and the evaporator operates at **300 kPa**.

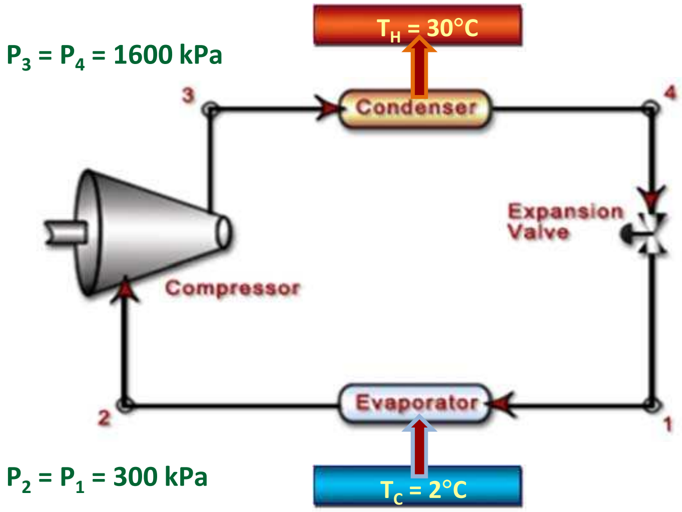
Read : In **Part (a)**, we can determine the **COP** of the **Carnot** refrigeration cycle directly from the [temperatures of the two thermal reservoirs. Then, we can use the definition of **COP** to evaluate W_{cycle}.

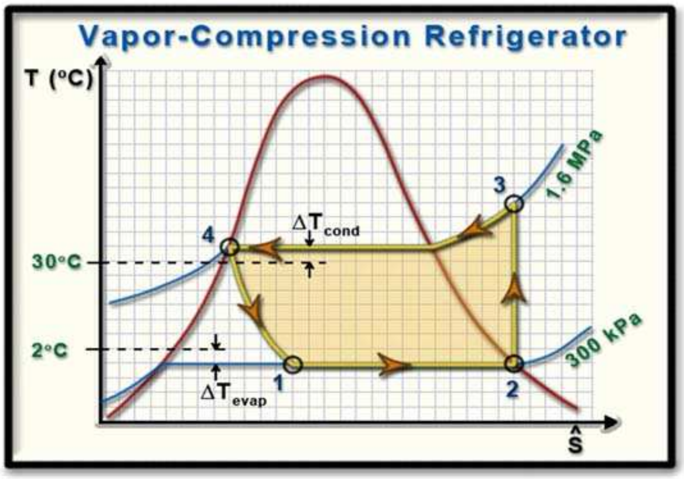
In **Part (b)**, we can work our way around the cycle in the **TS Diagram** and evaluate **H** for every **stream**. Then, use the **H** values determined to evaluate $W_{comp} = W_{cycle}$ and finally evaluate **COP** from its definition.

The approach illustrated in this solution is to start from an equation that includes the variable that is your objective, in this case the definition of **COP** is the equation and the objective is the **COP** itself. Then, you must proceed to determine the values of all the variables needed to evaluate the **COP**.

These two approaches to solving cycle problems often turn out to require the same amount of effort. So, you can choose whichever method appeals to you.

Diagram:





Given:	T_C	2	°C	Q_C	775	kW
		275.15	K			
	T_H	30	°C	$P_1 = P_2 =$	300	kPa
		303.15	K	$P_3 = P_4 =$	1600	kPa

Find:	a.)	Carnot :	$W_{cycle,rev}$	???	kW	$COP_{R,rev}$	???
	b.)	Ideal Ammonia VCR :	W_{cycle}	???	kW	COP_R	???

Assumptions:

1 - Each component is an open system operating at steady-state.
2 - All processes are internally reversible, except the expansion valve, which is an isenthalpic throttling processes.
3 - The compressor and valves operate adiabatically.
4 - Kinetic and potential energy changes are negligible.
5 - There are no **pressure** drops for flow through the heat exchangers.

Equations / Data / Solve:

Let's organize the **data** that we need to collect into a table. This will make it easier to keep track of the values we have looked-up and the values we have calculated.

Stream	T (°C)	P (kPa)	H (kJ/kg)	S (kJ/kg-K)	X (kg vap/kg)	Phase
1	-9.22	300	376.52	1.4582	0.1840	Two-Phase Mixture
2	-9.22	300	1432.5	5.4592	1	Saturated Vapor
3	113.53	1600	1680.1	5.4592	N/A	Superheated Vapor
4	41.02	1600	376.52	1.3737	0	Saturated Liquid

Additional **data** that may be useful.

State	T (°C)	P (kPa)	X	H (kJ/kg)	S (kJ/kg-K)
Sat Vap	-9.22	300	1	1432.5	5.4592
Sat Liquid	-9.22	300	0	138.39	0.55596
Sat Vap	41.02	1600	1	1471.0	4.8573
Sat Liquid	41.02	1600	0	376.5	1.3737

Part a.) Let's begin by determining the **COP** of a **Carnot** refrigeration cycle working between these two thermal reservoirs.

$$COP_{R,rev} = \frac{Q_C}{W_{cycle,rev}} = \frac{T_C}{T_H - T_C} \qquad \textbf{Eqn 1}$$

Plugging in values for T_H and T_C yields :

$COP_{R,rev}$ **9.83**

Solve **Eqn 1** for the unknown W_{cycle} :

$$W_{cycle,rev} = \frac{Q_C}{COP_{R,rev}} \qquad \textbf{Eqn 2}$$

Plugging values into **Eqn 2** yields :

$W_{cycle,rev}$ **78.87** **kW**

Part b.) **Eqn 2** can be modified slightly to apply to our **ideal** ammonia VCR cycle :

$$COP_R = \frac{Q_C}{W_{cycle}} \qquad \textbf{Eqn 3}$$

We were given the value of Q_C, so we need to determine W_{cycle} before we can use **Eqn 3** to evaluate COP_R.

Only the compressor produces or consumes **shaft work** in the **ideal** VCR cycle, so let's begin by applying the 1st Law to the compressor.

The 1st Law for a steady-state, single-inlet, single-outlet, adiabatic compressor with negligible kinetic and potential energy changes is:

$$\dot{W}_{cycle} = \dot{W}_{comp} = \dot{m}(\hat{H}_3 - \hat{H}_2) \qquad \textbf{Eqn 4}$$

We can immediately evaluate H_2 because we know it is a saturated vapor at **300 kPa**. Use the **Saturated Ammonia Tables** or the **NIST Webbook**.

H_2 **1432.5** **kJ/kg**

In order to determine H_3, we need to determine the value of a 2nd intensive variable, because we only know the **pressure**, P_3.

We can make use of the fact that an **ideal** VCR cycle is internally reversible and the compressor in an **ideal** VCR cycle is adiabatic. A device that is internally reversible and adiabatic is **isentropic**. **$S_3 = S_2$**.

We can evaluate **S_2** because we know it is a saturated vapor at **300 kPa**. Use the **Saturated Ammonia Tables** or the **NIST Webbook**.

$S_2 = S_3$ **5.4592** **kJ/kg**

Now, we can use **P_3** and **S_3** and the Superheated Ammonia Tables or the **NIST Webbook** to determine **H_3** by interpolation. This is much easier using the Isobaric Properties option in the **NIST Webbook**.

At **1600 kPa** :

T (°C)	H (kJ/kg)	S (kJ/kg-K)
110	1670.89	5.4354
T_3	H_3	5.4592
115	1683.89	5.4691

T_3 **113.54** **kJ/kg**

H_3 **1680.1** **kJ/kg**

We still need to determine the **mass flow rate** of the **Ammonia** through the cycle before we can use **Eqn 4** to determine **W_{cycle}**.

The key to determining the **mass flow rate** is the value of **Q_C** that was given in the problem statement.

We need to apply the 1st Law for a steady-state, single-inlet, single-outlet, evaporator with negligible kinetic and potential energy changes. No **shaft work** crosses the boundary of the evaporator.

$$\dot{Q}_C = \dot{m}(\hat{H}_2 - \hat{H}_1) \qquad \textbf{Eqn 5}$$

Solve **Eqn 5** for the **mass flow rate** :

$$\dot{m} = \frac{\dot{Q}_C}{(\hat{H}_2 - \hat{H}_1)} \qquad \textbf{Eqn 6}$$

We already determined **H_2**. The trouble is we don't know the value of **H_1**.

The key to determining the **H_1** is the assumption that the Expansion Valve is **isenthalpic**. This assumption is based on the application of the 1st Law to the Valve.

We need to apply the 1st Law for a steady-state, single-inlet, single-outlet, expansion valve with negligible kinetic and potential energy changes. No **shaft work** crosses the boundary of the expansion valve. We assume the valve is adiabatic because it is small and there is little opportunity for heat exchange.

$$\cancel{\dot{Q}} - \cancel{\dot{W}_s} = \dot{m}\left[\Delta\hat{H} + \cancel{\Delta\hat{E}_{kin}} + \cancel{\Delta\hat{E}_{pot}}\right] \qquad \textbf{Eqn 7}$$

Eqn 7 simplifies to : $\Delta\hat{H} = \hat{H}_1 - \hat{H}_4 = 0$ **Eqn 8** Or : $\hat{H}_1 = \hat{H}_4$ **Eqn 9**

Eqn 9 is helpful because we already have enough information to determine **H_4**. We know **stream 4** is a saturated liquid at **1600 kPa**. So, we can use the **Saturated Ammonia Tables** or the **NIST Webbook** to evaluate **H_4**.

$H_4 = H_1$ **376.52** **kJ/kg**

We can finally put values back into **Eqns 6**, **4**, & **3**, in that order, to complete the solution to **Part (b)**.

m **0.7339** **kg/s**

W_{cycle} **181.7** **kW**

COP_R **4.26**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

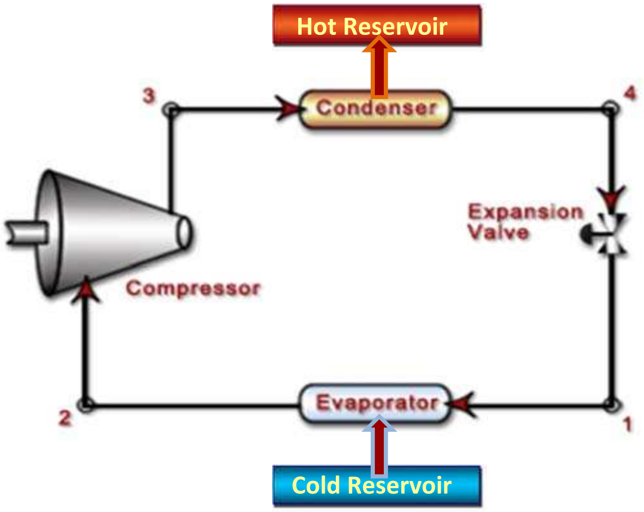
a.)	**$W_{cycle,rev}$**	**78.9**	**kW**	**$COP_{R,rev}$**	**9.83**
b.)	**W_{cycle}**	**182**	**kW**	**COP_R**	**4.26**

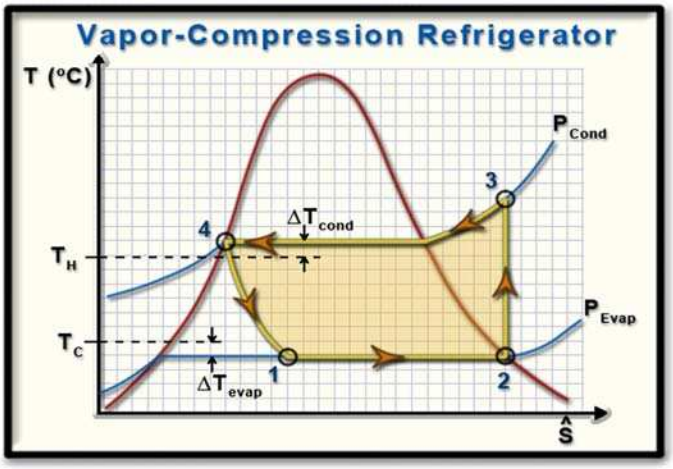
10B-2 Refrigerant Selection for a Home Refrigerator 4 pts

Why do modern home refrigerators use **R-134a** instead of **ammonia** or **Freon-12** (**R-12**)?

Read : Compare the performance of these three refrigerants in a **vapor-compression refrigeration cycle** using the **7 criteria** presented in **Lesson 10B**. Some calculations are necessary to assess the **P*-T Relationship** and the **Heat of Vaporization**.

Diagram:





Given: Consider three refrigerants : **R134a** **Ammonia** **R12**

Find: Why is **R-134a** the most common choice for home refrigeration systems?

Assumptions: None.

Equations / Data / Solve:

Here is a list of criteria used to choose the best refrigerant for a given application.

1 - **Cost**
2 - **The P*-T Relationship**
3 - **ΔH_{vap}**
4 - **Chemical Stability**
5 - **Corrosiveness**
6 - **Toxicity**
7 - **Flammability**

Cost : **R-134a** is much more expensive than **ammonia**, while **Freon** is not easy to buy since it was banned in 1990 because it damages the ozone layer in the upper atmosphere. **Ammonia** can cost as little as **$0.40/kg** while **R-134a** can cost several US dollars per kg.

The P*-T Relationship

A home rerigerator must keep food in the freezer colder than **-10°C**. Cool air from the freezer compartment is used to keep the refrigerator compartment colder than **3°C**. As a result, the freezer serves as the **Cold Reservoir**.

The air surrounding the refrigerator serves as the **Hot Reservoir**. A high value of T_H results in the lowest **COP** for the refrigerator and it also requires the highest **pressure** in the condenser. So, let's use a rather high **temperature** for the surroundings.

T_C **-10** °C

T_H **30** °C

The temperature of the refrigerant in the evaporator must be greater than T_C and in the condenser it must be less than T_H.

A **rule of thumb** in heat exchanger design is that you would like to have a ΔT of about **10°C** between two fluids entering the heat exchanger. In this case, since T_C is **-10°C**, assume that T_1 is **-20°C**. Since T_H is **30°C**, assume that T_4 is **40°C**. This gives our refrigerator the desired ΔT of **10°C**. in both the condenser and the evaporator.

T_1 **-20** °C

T_4 **40** °C

Now, we need to use thermodynamic tables or the **NIST Webbook** to determine the operating **pressures** of the condenser and the evaporator. The operating **pressures** are the saturation **pressures** that correspond to T_1 and T_4, determined in the previous paragraph.

	P_{Evap} (kPa)	P_{Cond} (kPa)
Ammonia	190.1	1555.4
R134a	132.7	1016.6
R12	150.7	958.8

Ammonia generally requires a fairly high P_{Cond}.
R-134a and **R-12** allow the use of lower P_{Cond} and P_{Evap} than **Ammonia** does.

The **R-12** is the winner in this test, but the **R-134a** does almost as well.

ΔH_{vap}

A higher **latent heat of vaporization** results in a smaller **mass flow rate** for the refrigerant. This may allow the use of smaller, less expensive equipment.

Consider the values of ΔH_{vap} at the values of P_{Cond} and P_{Evap} that we determined above.

	ΔH @ P_{Evap} (kJ/kg)	ΔH @ P_{Cond} (kJ/kg)
Ammonia	1329.1	1099.3
R134a	212.9	163.0
R12	162.0	129.7

Ammonia has the largest ΔH_{vap}, by far.
R-134a takes 2nd place this criterion.
R-12 is the biggest loser by this criterion.

Chemical Stability

All three of these refrigerants are relatively stable. **Ammonia** is perhaps more reactive and slightly less stable than **R-134a** and **R-12**.

Corrosiveness

R-134a and **R-12** are non-corrosive to ferrous metals unless exposed to flame or very high **temperatures**. So, they score well by this criterion.

Ammonia is non-corrosive to ferrous metals unless **water** is present. In the presence of water, **ammonia** becomes highly corrosive. So, **Ammonia** does not score quite as well by this criterion, because **water** has a way of penetrating devices that are not rigorously maintained.

Toxicity

R-134a and **R-12** have very low acute inhalation toxicity, but are irritants to eyes and skin

Ammonia is a strong irritant to eyes, skin, mouth, nose, throat and respiratory tract, but has low inhalation toxicity.

Ammonia may be the loser by this criterion, but all three refrigerants have fairly low toxicity overall.

Flammability

There is some debate about the flammability of **R-134a** and **R-12**, but they are definitely less flammable than **Ammonia**. **Ammonia** is quite flammable and can present an explosion hazard.

Verify: None.

Answers : **R-134a** is the best of the three choices for home refrigeration because it...

… Allows the condenser and evaporator to operate at **pressures** that are greater than atmospheric **pressure**, but not dangerously high.

… It is stabile, non-corrosive, has low toxicity and low flammability.

… It does not damage the ozone layer.

Ammonia is a commonly used industrial refrigerant because…

… Allows the condenser and evaporator to operate at **pressures** that are greater than atmospheric **pressure**, but not dangerously high., by industrial safety standards.

… The **price** of **Ammonia** is very low.

… With proper industrial safety practices, the problems with **corrosion**, **toxicity** and **flammability** can be overcome.

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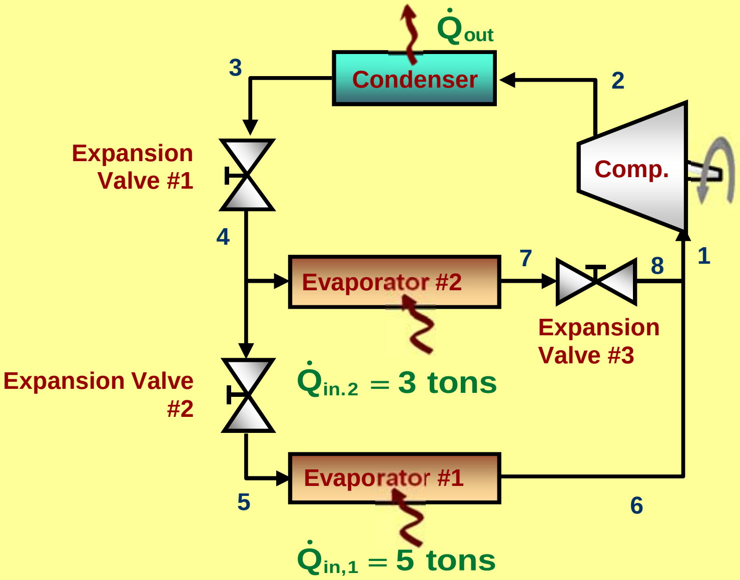
10C-1 Analysis of a Dual Evaporator V-C Refrigeration System 10 pts

The special **R-134a** refrigeration system with two **evaporators**, shown below, is used to cool both a refrigerator, with **evaporator 2**, and a freezer, with **evaporator 1**.



Evaporators 1 and **2** have refrigeration capacities of **3 tons** and **5 tons**, respectively. A **ton of refrigeration** is defined as the heat of fusion absorbed by melting **1 short ton** of pure ice at **0°C** in **24 hours**.

The key here is that one **ton of refrigeration** is equivalent to **211 kJ/min**. The **condenser** operates at **800 kPa**. **Evaporators 1** and **2** operate at **-15°C** and **250 kPa**, respectively.

The **R-134a** leaves each **evaporator** as a saturated vapor and it leaves the **condenser** as a saturated liquid.

Calculate...

a.) m_6 and m_7 in **kg/min**

b.) $W_{S,comp}$ in **kW**

c.) Q_{cond} in **kW**

Read : Don't let the **diagram** scare you. Analyze each unit or process in the cycle just as you would in an ordinary refrigeration cycle.

Assume the cycle operates at steady-state and that each process is internally reversible, except for the expansions through each valve. These are throttling processes. The compressor and valves operate **adiabatically**. Changes in kinetic and potential energies are negligible.

First, determine the **specific enthalpy** at **states 3** to **8**. These are the easy ones. To determine the **enthalpies** at **states 1** and **2**, you will need to know the **mass flow rates** through each evaporator. You can determine the **mass flow rates** by applying the 1st Law to each evaporator.

There are two **stealth** units on this **flow diagram**. A **stream splitter** where **steam** is divided before it enters Evaporator #2 or Valve #2 and a **Mixer** where **streams 6** and **8** combine to form **stream 1**. The Mixer is crucial to this problem. You can write mass and energy balances on the Mixer. The Mixer can be considered adiabatic. This will help you determine the **specific enthalpy** for **stream 1**. Then, because the compressor is isentropic, you can determine the **specific enthalpy** for **stream 2**.

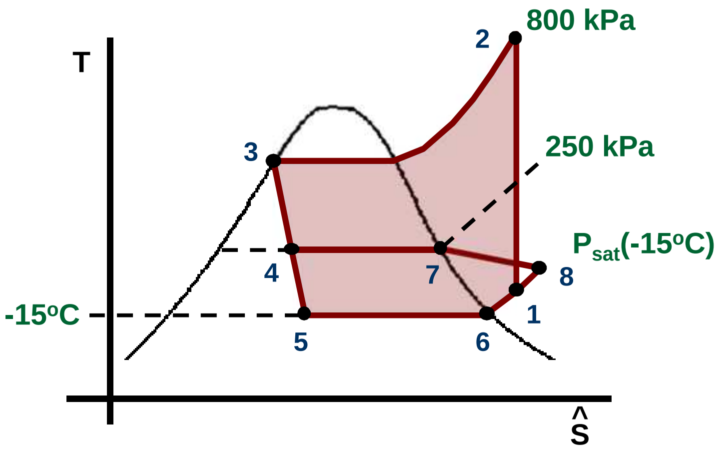
Given:

$Q_{in,1}$	5	tons	P_3	800	kPa
$Q_{in,2}$	3	tons	x_3	0	kg vap/kg
T_6	-15	°C	x_6	1	kg vap/kg
P_2	800	kPa	P_7	250	kPa
			x_7	1	kg vap/kg

Find:

Part (a)	m_6	???	kg/min	**Part (b)**	W_{comp}	???	kW
	m_8	???	kg/min	**Part (c)**	Q_{out}	???	kW

Diagram: The process flow diagram was provided in the problem statement.



Assumptions:

1 - Each component is an open system operating at steady-state.

2 - All processes are internally reversible, except the expansion valves, which are isenthalpic throttling processes.

3 - The compressor and valves operate adiabatically.

4 - Kinetic and potential energy changes are negligible.

Equations / Data / Solve:

Stream	T (oC)	P (kPa)	H (kJ/kg)	S (kJ/kg-K)	X (kg vap/kg)	Phase
1	-12.2	163.94	391.98	1.7462	N/A	Super. Vap.
2	40.77	800	425.39	1.7462	N/A	Super. Vap.
3	31.3	800	243.65	1.1497	0	Sat'd Liq.
4	-4.28	250	243.65	1.1626	0.245	VLE
5	-15	163.94	243.65	1.1716	0.303	VLE
6	-15	163.94	389.63	1.7371	1	Sat'd Vap.
7	-4.28	250	396.08	1.7296	1	Sat'd Vap.
8	-7.3	163.94	396.08	1.7617	N/A	Super. Vap.

Part a.) The key to determining $\mathbf{m_6}$ and $\mathbf{m_8}$ are the given values for $\mathbf{Q_{in,1}}$ and $\mathbf{Q_{in,2}}$. We can use these values when we apply the 1st Law to each evaporator to determine $\mathbf{m_6}$ and $\mathbf{m_8}$.

Each evaporator operates at steady-state, involves no **shaft work** and has negligible changes in kinetic and potential energies. The appropriate forms of the 1st Law are:

$$\dot{Q}_{in,1} = \dot{m}_6 \left(\hat{H}_6 - \hat{H}_5\right) \qquad \textbf{Eqn 1}$$

$$\dot{Q}_{in,2} = \dot{m}_8 \left(\hat{H}_7 - \hat{H}_4\right) \qquad \textbf{Eqn 2}$$

We can solve these equations for the unknown **mass flow rates**:

$$\dot{m}_6 = \frac{\dot{Q}_{in,1}}{\left(\hat{H}_6 - \hat{H}_5\right)} \qquad \textbf{Eqn 3} \qquad\qquad \dot{m}_8 = \frac{\dot{Q}_{in,2}}{\left(\hat{H}_7 - \hat{H}_4\right)} \qquad \textbf{Eqn 4}$$

Now, we need to determine the values of the **H**'s to use in **Eqns 3** & **4**.

The throttling valves are **isenthalpic** because they are adiabatic, have no **shaft work** interactions and changes in kinetic and potential energies are negligible. Therefore:

$$\hat{H}_3 = \hat{H}_4 = \hat{H}_5 \qquad \textbf{Eqn 5} \qquad\qquad \hat{H}_7 = \hat{H}_8 \qquad \textbf{Eqn 6}$$

Fortunately, we were given enough information to lookup the **specific enthalpy** of **states 3**, **6** and **7** in the **Saturated R-134a Tables** or the **NIST Webbook**. Once we have these values, we can use **Eqns 5** & **6** to evaluate the **H**'s at **states 4**, **5** and **8** as well !

H_3	**243.65**	**kJ/kg**	**H_6**	**389.63**	**kJ/kg**
H_4	**243.65**	**kJ/kg**	**H_7**	**396.08**	**kJ/kg**
H_5	**243.65**	**kJ/kg**	**H_8**	**396.08**	**kJ/kg**

Now, we can plug values into Eqns 3 & 4 to determine the two unknown mass flow rates.

1 ton =	**211**	**kJ/min**	**m_6**	**7.227**	**kg/min**
			m_8	**4.152**	**kg/min**

Part b.) We need to determine **W_S** for the compressor. We can accomplish this by applying the 1st Law to the compressor. The compressor operates at steady-state, is adiabatic and reversible and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{W}_S = \dot{m}_1\left(\hat{H}_1 - \hat{H}_2\right) \qquad \textbf{Eqn 7}$$

We know the **mass flow rates** from **part (a)**, but we don't know either of the **H**'s in **Eqn 7** yet.

We can determine **H_1** by applying the 1st Law to the mixer where **streams 6** and **8** combine to form **stream 1**. The mixer is a **MIMO** process that operates at steady-state, is adiabatic and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{m}_1 \hat{H}_1 = \dot{m}_6 \hat{H}_6 + \dot{m}_8 \hat{H}_8 \qquad \textbf{Eqn 8}$$

A mass balance on the mixer tells us that:

$$\dot{m}_1 = \dot{m}_6 + \dot{m}_8 \qquad \textbf{Eqn 9}$$

Solve **Eqn 8** for the only unknown in the equation: **H_1**.

$$\hat{H}_1 = \frac{\dot{m}_6 \hat{H}_6 + \dot{m}_8 \hat{H}_8}{\dot{m}_1} \qquad \textbf{Eqn 10}$$

Now, we can plug values into **Eqns 9** & **10** to evaluate **m_1** and **H_1** :

m_1	**11.38**	**kg/min**	**H_1**	**391.98**	**kJ/kg**

Now, we need to work on evaluating **H_2**. The key to determining **H_2** is the fact that the compressor is both adiabatic and internally reversible, so it is **isentropic**. **$S_2 = S_1$**. We can lookup **S_1** because we know **H_1** and we know that:

$$P_1 = P_6 = P_8 = P_{sat}(-18^oC) \qquad \textbf{Eqn 11}$$

The **Saturated R-134a Tables** or the **NIST Webbook** tells us:

P_{sat}(-18°C) = **163.94** **kPa**
P_1 **163.94** **kPa**

In evaluating **S_1**, we must first determine whether **stream 1** is a superheated vapor or a saturated mixture.

This is easier using the **NIST Webbook** than the **R-134a Tables** because no interpolation is required.

At P = 163.94 kPa :

$H_{sat\ liq}$	**180.14**	**kJ/kg**
$H_{sat\ vap}$	**389.63**	**kJ/kg**

Since **H_1 > $H_{sat\ vap}$**, **state 1** is a superheated vapor.

T (°C)	H (kJ/kg)	S (kJ/kg-K)
-13	391.30	1.7435
T_1	391.98	S_1
-10	393.80	1.7531

Interpolation yields :

T_1	**-12.17**	**°C**
S_1	**1.7462**	**kJ/kg-K**
S_2	**1.7462**	**kJ/kg-K**

Be careful with this interpolation. Use a narrow **temperature** range because the answer is very sensitive to this result.

Now, we know the values of two intensive variables at **state 2** (**P_2 & S_2**), so we can go back to the **R-134a Tables** or **NIST Webbook** and determine **H_2** by interpolation.

At **P = 800 kPa** :

T (°C)	H (kJ/kg)	S (kJ/kg-K)
40	424.59	1.7436
T_2	H_2	1.7462
45	429.74	1.7599

Interpolation yields :

T_2	**40.78**	**°C**
H_2	**425.40**	**kJ/kg-K**

Finally, we can plug values back into **Eqn 7** :

W_S **-380.20** **kJ/min**

W_S **-6.337** **kW**

Part c.) We can determine the **heat transfer rate** in the condenser by applying the 1st Law to it.

The condenser operates at steady-state, involves no **shaft work** and has negligible changes in kinetic and potential energies. The appropriate form of the 1st Law is:

$$\dot{Q}_{23} = \dot{m}_1 \left(\hat{H}_3 - \hat{H}_2 \right) \qquad \textbf{Eqn 12}$$

In **parts (a)** & **(b)** we evaluated **m_1**, **H_2** and **H_3**, so we cannow plug values into **Eqn 12**:

Q_{23} **-2068.2** **kJ/min**

Q_{23} **-34.47** **kW**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

a.)

m_6	**7.23**	**kg/min**
m_8	**4.15**	**kg/min**

b.)

W_S **-6.34** **kW**

(The "**-**" sign indicates that **shaft work** is done on the **working fluid** in the compressor.)

c.)

Q_{23} **-34.5** **kW**

(The "**-**" sign indicates that **heat** is transferred **out** of the **working fluid** in the condenser.)

10D-1 COP of a Heat Pump Used for Home Heating 8 pts

At steady-state, a house loses **10 kW** of heat to the surroundings when the outdoor **temperature** is **0°C** and the indoor **temperature** is **22°C**. This energy must be replaced by a heat pump that uses **R-134a** as the working fluid.
Assume **evaporator** effluent is a saturated vapor and the **condenser effluent** is a saturated liquid.
Assume the compressor has an **isentropic efficiency** of **85%**.

a.) Determine the operating **pressures** for the **evaporator** and **condenser** that provide at least a **10°C** driving force for heat exchange at the inlet and outlet of the evaporator and the condenser.

b.) Calculate the **mass flow rate** of the **R-134a**, the **power** requirement of the **compressor** and the **COP** of the heat pump.

Read : The operating **pressures** of the evaporator and condenser are governed by the saturation considerations. A **rule of thumb** in heat exchanger design is that you would like to have a ΔT of about **10°C** between two fluids entering the heat exchanger. In this case, since the outside **temperature** is **0°C**, assume that T_1 is **-10°C**. Since the inside **temperature** is **22°C**, the **temperature** of the **R-134a** entering the condenser must be **32°C** to give us the desired ΔT of **10°C**. This leads to some messy interpolation that you can avoid by using the **NIST Webbook**. With these assumptions, there are really no tricks to this problem!

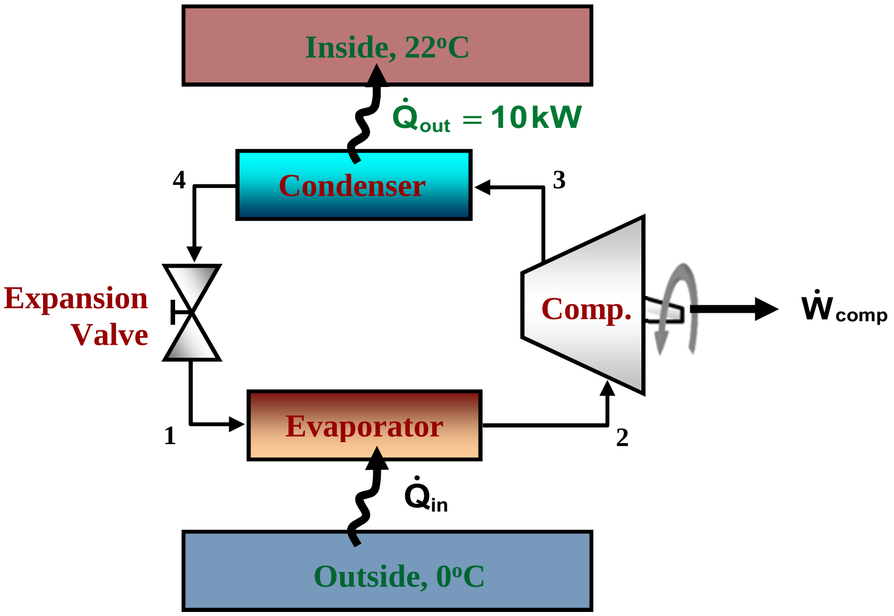
Given:

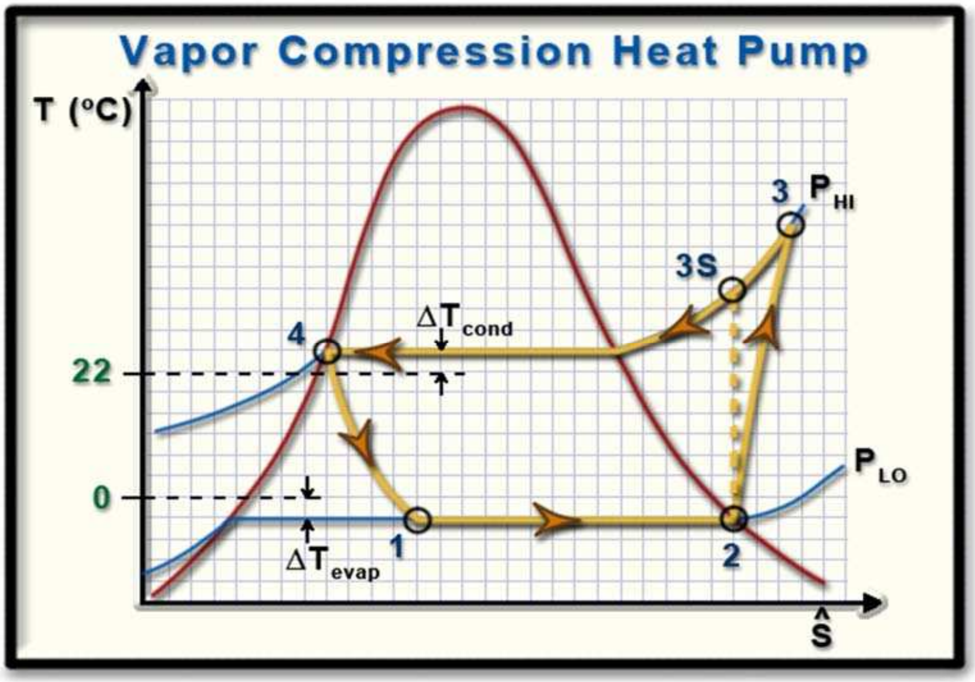
$T_{outside}$	0	°C	Q_{out}	10	kW
T_{inside}	22	°C	$\eta_{S,\,comp}$	0.85	

Find:

a.)	P_1	???	kPa	P_3	???	kPa
b.)	m_{dot}	???	kg/min			
	W_{comp}	???	kW	COP_{HP}	???	

Diagram:





Assumptions:

1 - Each component is an open system, operating at steady-state.
2 - There are no **pressure** drops through the evaporator or condenser.
3 - The compressor operates adiabatically with an **isentropic efficiency** of **80%.**
4 - The expansion through the valve is an isenthalpic throttling process.
5 - Kinetic and potential energy changes are negligible.
6 - The evaporator and condenser **pressures** must be chosen to allow for sufficient **ΔT**'s to avoid excessive heat exchanger sizes (**surface area**). For a preliminary design assume:

$\Delta T_{evap} = \Delta T_{cond} =$ **10** °C
$T_1 = T_2 =$ **-10** °C
$T_4 =$ **32** °C

Equations / Data / Solve:

Stream	T (°C)	P (kPa)	H (kJ/kg)	S (kJ/kg-K)	X (kg vap/kg)	Phase
1	-10.0	200.60	201.10	1.0054	0.0699	VLE
2	-10.0	200.60	392.66	1.7334	1	Sat'd Vap.
3	42.6	815.43	426.94	1.5485	N/A	Super. Vap.
3S	37.7	815.43	421.80	1.7334	N/A	Super. Vap.
4	32	815.43	201.10	1.0040	0	Sat'd Liq.

Part a.) The **mass flow rate** is the <u>same</u> through <u>each</u> piece of equipment. Therefore, we can determine the **mass flow rate** by applying the 1st Law to <u>any</u> <u>one</u> process. Because we are given the **heat transfer rate** at the condenser, it is the device where we will have the <u>fewest</u> <u>unknowns</u> in the 1st Law so we are <u>most</u> <u>likely</u> to be able to determine the **mass flow rate** of the **working fluid**.

The 1st Law for a steady-state, single-inlet, single-outlet condenser with <u>no</u> **shaft work** and <u>negligible</u> kinetic and potential energy <u>changes</u> is:

$$\dot{Q}_{out} = -\dot{Q}_{34} = \dot{m}\left(\hat{H}_3 - \hat{H}_4\right) \qquad \text{Eqn 1}$$

Solve **Eqn 1** for the **mass flow rate**:

$$\dot{m} = \frac{\dot{Q}_{out}}{\hat{H}_3 - \hat{H}_4} \qquad \text{Eqn 2}$$

We can look up H_4 in the **R-134a Tables** or in the **NIST Webbook** because it is saturated liquid at **32 °C**.

H_4 **201.10** kJ/kg

In order to determine H_3, we must use the **isentropic efficiency** of the compressor.

$$\eta_{S,comp} = \frac{-\dot{W}_{S,isen}}{-\dot{W}_{S,act}} = \frac{\hat{H}_{3S} - \hat{H}_2}{\hat{H}_3 - \hat{H}_2} \qquad \text{Eqn 3}$$

We can solve **Eqn 3** for H_3 :

$$\hat{H}_3 = \hat{H}_2 + \frac{\left(\hat{H}_{3S} - \hat{H}_2\right)}{\eta_{S,comp}} \qquad \text{Eqn 4}$$

Next, we need to determine H_{3S}. For an **isentropic** compressor, $S_{3S} = S_2$ and we can look-up S_2 in the **NIST Webbook** because **stream 2** is a saturated vapor at **-10°C**.

S_2 **1.7334** kJ/kg-K $\qquad$ S_{3S} **1.7334** kJ/kg-K
H_2 **392.66** kJ/kg

Now, we know the value of <u>two</u> intensive properties at **state 3S**: S_{3S} and P_3 (because the condenser is isobaric: $P_3 = P_4 = P_{sat}(32°C)$)

$P_3 = P_4 = P_{sat}(32°C) =$ **815.43** kPa

This involves some <u>painful</u> <u>interpolation</u>. I suggest you use the **NIST Webbook** to avoid the pain of

<u>At **800 kPa**:</u>

T (°C)	H (kJ/kg)	S (kJ/kg-K)			
31.33	**415.46**	**1.7140**			
$T_{3S,800}$	$H_{3S,800}$	1.7334	$H_{3S,800}$	36.99	kJ/kg
40	**424.59**	**1.7436**	$H_{3S,800}$	421.42	kJ/kg

<u>At **900 kPa**:</u>

T (°C)	H (kJ/kg)	S (kJ/kg-K)			
40	**422.32**	**1.7283**			
$T_{3S,900}$	$H_{3S,900}$	1.7334	$H_{3S,900}$	41.51	kJ/kg
50	**432.92**	**1.7616**	$H_{3S,900}$	423.92	kJ/kg

Finally, at **815.43 kPa** :

T (°C)	P (kPa)	H (kJ/kg)
36.99	800	421.42
T_{3S}	815.43	H_{3S}
41.51	900	423.92

T_{3S}	37.69	
H_{3S}	421.81	kJ/kg

Now ,we can plug values into **Eqn 4** to determine H_3 :

H_3	426.94	kJ/kg

Now, we can plug H_3 back into **Eqn 2** :

m_{dot}	0.04428	kg/s
m_{dot}	2.657	kg/min

Part b.) In order to determine the **shaft work** requirement for the compressor, we must apply the 1st Law to it. The 1st Law for a steady-state, single-inlet, single-outlet, adiabatic compressor with negligible kinetic and potential energy changes is:

$$\dot{W}_{comp} = \dot{m}(\hat{H}_3 - \hat{H}_2) \qquad \text{Eqn 5}$$

We already determined H_2 and H_3, so all we need to do is plug numbers into **Eqn 5**.

W_{comp}	1.518	kW

Part c.) We can determine the COP_R from its definition:

$$COP_{HP} = \frac{\dot{Q}_{out}}{\dot{W}_{comp}} \qquad \text{Eqn 6}$$

COP_R	6.589

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers :

a.)	m_{dot}	2.66	kg/min
b.)	W_{comp}	1.52	kW
c.)	COP_R	6.59	

10E-1 Brayton Refrigeration Cycle 8 pts

A Brayton refrigeration cycle uses **air** as the working fluid and operates between a high **pressure** of **800 kPa** and a low **pressure** of **120 kPa**. The **compressor** and **turbine** inlet **temperatures** are **540 K** and **510 K**, respectively.

The **turbine** is isentropic and the **compressor** has an **isentropic efficiency** of **88%**. Calculate $\mathbf{W_{net}}$, in **kJ/kg**, and the **coefficient of performance** for the cycle.

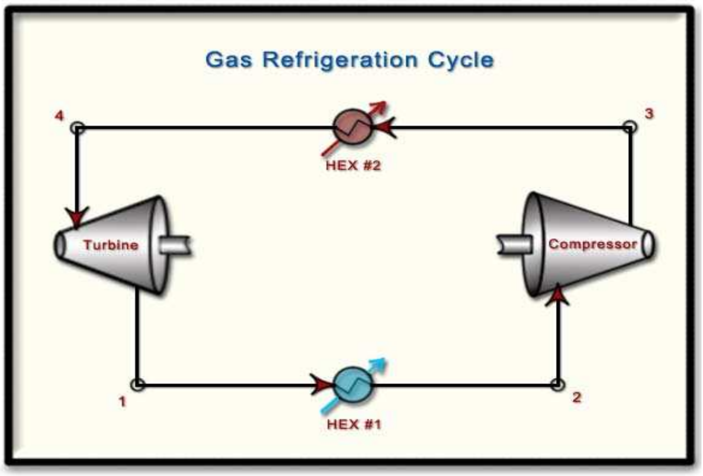
Read : The key here is that this is a **Brayton Cycle**. Because **air** behaves as an ideal gas, we can use the **Ideal Gas Property Tables** for **air**. Other key points include the fact that both the compressor and turbine are adiabatic, the compressor has an **isentropic efficiency** of **88%** and the turbine is **isentropic**.

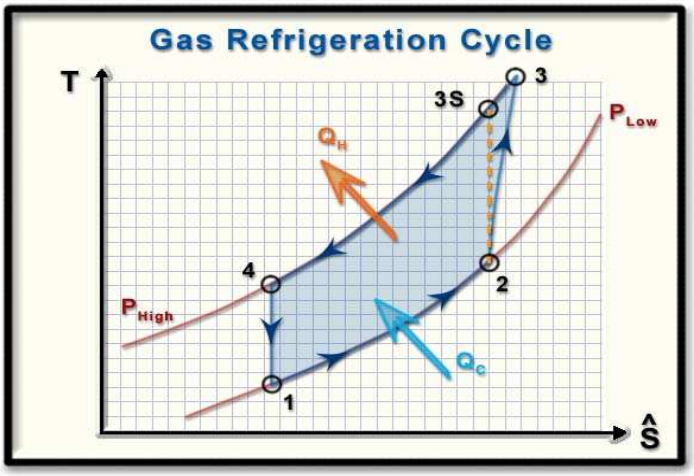
Given:

$P_1 = P_2$	120	kPa	$P_4 = P_3$	800	kPa
T_2	540	K	T_4	510	K
$\eta_{s,comp}$	88%				

Find: a.) W_{cycle} ? kJ/kg b.) COP_R ?

Diagram:





Assumptions:

1 - Each component is analyzed as an open system operating at steady-state.
2 - The turbine is **isentropic**.
3 - There are no **pressure** drops for flow through the heat exchangers.
4 - Kinetic and potential energy changes are negligible.
5 - The **working fluid** is **air** modeled as an ideal gas.
6 - There is no **heat** exchanged with the surroundings.

Equations / Data / Solve:

Stream	T (°R)	P (psia)	H° (Btu/lb$_m$)	S° (Btu/lb$_m$-°R)	P$_r$
1	299.12	120	86.535	0.0032422	1.0114
2	540	120	333.66	0.60768	8.3101
3	938.62	800	768.78		
3s	892.14	800	716.57	1.1521	55.400
4	510	800	302.17	0.54769	6.742

Part a.) There is no **shaft work** occurring in the HEX's, so **W$_{cycle}$** is :

$$\hat{W}_{cycle} = \hat{W}_{comp} + \hat{W}_{turb} \qquad \text{Eqn 1}$$

We can determine **W$_{comp}$** and **W$_{turb}$** by applying the 1st Law to each device.
The 1st Law equations for a steady-state, single-inlet, single-outlet adiabatic turbine and compressor with negligible kinetic and potential energy changes are:

$$\hat{W}_{turb} = \hat{W}_{S,41} = \hat{H}_4 - \hat{H}_1 \qquad \text{Eqn 2}$$

$$\hat{W}_{comp} = \hat{W}_{S,23} = \hat{H}_2 - \hat{H}_3 \qquad \text{Eqn 3}$$

So, in order to evaluate **W$_{cycle}$**, we must first determine the **specific enthalpy** of all four **streams** in the cycle. We can immediately find **H$_2$** and **H$_4$** in the **Ideal Gas Property Table** for **air** because both **T$_2$** and **T$_4$** are given.

H°$_{T2}$	333.66	kJ/kg	**H°$_{T4}$**	302.17	kJ/kg

Next, let's make use of the fact that the turbine is **isentropic** (**S$_2$ = S$_1$**) to evaluate **H$_1$**.
We can either use the **Ideal Gas Entropy Function** and the 2nd Gibbs Equation or we can use Relative Properties. Both methods are presented here.

Method 1: Use the Ideal Gas Entropy Function and the 2nd Gibbs Equation.

Apply the 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function:

$$\Delta\hat{S} = \hat{S}_1 - \hat{S}_4 = \hat{S}^o_{T1} - \hat{S}^o_{T4} - \frac{R}{MW}\,\mathrm{Ln}\frac{P_1}{P_4} = 0 \qquad \text{Eqn 4}$$

We can solve **Eqn 4** for **S°$_{T1}$** :

$$\hat{S}^o_{T1} = \hat{S}^o_{T4} + \frac{R}{MW}\,\mathrm{Ln}\frac{P_1}{P_4} \qquad \text{Eqn 5}$$

We can look-up **S°$_{T4}$** in the **Ideal Gas Property Table** for **air** and use it with the known **pressures** in **Eqn 5** to determine **S°$_{T1}$**. We can do this because the HEX's are isobaric. **P$_1$ = P$_2$** and **P$_3$ = P$_4$**.

R	8.314	J/mol-K	**MW**	28.97	g/mol

T (K)	H° (kJ/kg)	S° (kJ/kg-K)
298.15	85.565	0.00000
T$_1$	H°$_{T1}$	0.0032422
300	87.410	0.0061681

Interpolation yields :

S°$_{T4}$	0.54769	kJ/kg-K
S°$_{T1}$	0.0032422	kJ/kg-K
T$_1$	299.12	K
H°$_{T1}$	86.535	kJ/kg

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_1)}{P_r(T_4)} = \frac{P_1}{P_4} \qquad \text{Eqn 6}$$

Where **P$_r$** is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqn 6** For $\mathbf{P_r(T_1)}$, as follows :

$$P_r(T_1) = \frac{P_1}{P_4} P_r(T_4) \qquad \textbf{Eqn 7}$$

Look-up $\mathbf{P_r(T_4)}$ and use it in **Eqn 7** to determine $\mathbf{P_r(T_1)}$:

$\mathbf{P_r(T_4)}$	**6.7424**
$\mathbf{P_r(T_1)}$	**1.0114**

We can now determine $\mathbf{T_5}$ and $\mathbf{H_5}$ by interpolation on the **Ideal Gas Property Table** for **air**.

T (K)	P_r	H^o (kJ/kg)
298	**1.0000**	**85.565**
$\mathbf{T_1}$	**1.0114**	$\mathbf{H^o_{T1}}$
300	**1.0217**	**87.410**

Interpolation yields :

$\mathbf{T_1}$	**299.12**	**K**
$\mathbf{H^o_{T1}}$	**86.530**	**kJ/kg**

Since the two methods differ by less than **0.01%**, I will use the results from **Method 1** in the remaining calculations of this problem.

Next, we need to evaluate $\mathbf{H_3}$. To do this, we need to use the **isentropic efficiency** of the compressor.

$$\eta_{S,comp} = \frac{\hat{H}_2 - \hat{H}_{3S}}{\hat{H}_2 - \hat{H}_3} \qquad \textbf{Eqn 8}$$

Solving **Eqn 8** for $\mathbf{H_3}$ gives us:

$$\hat{H}_3 = \hat{H}_2 - \frac{\left(\hat{H}_2 - \hat{H}_{3S}\right)}{\eta_{S,comp}} \qquad \textbf{Eqn 9}$$

So, in order to determine $\mathbf{H_3}$, we must first determine $\mathbf{H_{3S}}$, the **enthalpy** of **stream 3** IF the turbine were **isentropic**. We can determine $\mathbf{T_{3S}}$ using either the **Ideal Gas Entropy Function** and the **2nd Gibbs Equation** or we can use **Relative Properties**. Both methods are presented here.

Method 1: Use the Ideal Gas Entropy Function and the 2nd Gibbs Equation.

Apply the 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function:

$$\Delta\hat{S} = \hat{S}_{3S} - \hat{S}_2 = \hat{S}^o_{T3S} - \hat{S}^o_{T2} - \frac{R}{MW} \operatorname{Ln}\frac{P_3}{P_2} = 0 \qquad \textbf{Eqn 10}$$

We can solve **Eqn 10** for the unknown $\mathbf{S^o_{T3S}}$:

$$\hat{S}^o_{T3S} = \hat{S}^o_{T2} + \frac{R}{MW} \operatorname{Ln}\frac{P_3}{P_2} \qquad \textbf{Eqn 11}$$

We can look up $\mathbf{S^o_{T2}}$ in the **Ideal Gas Property Table** for **air** and use it with the known **compression ratio** in **Eqn 11** to determine $\mathbf{S^o_{T3}}$. We can do this because the HEX's are isobaric. $\mathbf{P_1 = P_2}$ and $\mathbf{P_3 = P_{3S} = P_4}$.

R	**8.314**	**J/mol-K**	**MW**	**28.97**	**g/mol**
$\mathbf{S^o_{T2}}$	**0.60768**	**kJ/kg-K**	$\mathbf{S^o_{T3S}}$	**1.1521**	**kJ/kg-K**

Now, we can use $\mathbf{S^o_{T3S}}$ and the **Ideal Gas Property Table** for **air** to determine $\mathbf{T_{3S}}$ and then $\mathbf{H_{3S}}$ by interpolation :

T (K)	H^o (kJ/kg)	S^o (kJ/kg-K)
880	**702.98**	**1.1369**
$\mathbf{T_{3S}}$	$\mathbf{H_{3S}}$	**1.1521**
900	**725.37**	**1.1620**

Interpolation yields :

$\mathbf{T_{3S}}$	**892.14**	**K**
$\mathbf{H_{3S}}$	**716.57**	**kJ/kg**

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_{3S})}{P_r(T_2)} = \frac{P_3}{P_2} \qquad \text{Eqn 12}$$

Where P_r is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqn 12** For $P_r(T_3)$, as follows :

$$P_r(T_{3S}) = \frac{P_3}{P_2} P_r(T_2) \qquad \text{Eqn 13}$$

Look-up $P_r(T_2)$ and use it in **Eqn 13** to determine $P_r(T_{3S})$:

$P_r(T_2)$	8.3101
$P_r(T_{3S})$	55.400

We can now determine T_{3S} by interpolation on the the **Ideal Gas Property Table** for **air**.
Then, we use T_3 to determine H_3 from the **Ideal Gas Property Table** for **air**.

T (K)	P_r	H^o (kJ/kg)
880	52.530	702.98
T_{3S}	55.400	H_{3S}
900	57.342	725.37

Interpolation yields :

T_{3S}	891.93	K
H_{3S}	716.33	kJ/kg

Since the two methods differ by less than **0.05%**, I will use the results from **Method 1** in the remaining calculations of this problem.

Next, we use **Eqn 9** to evaluate H_3 : H_3 768.78 kJ/kg

T (K)	H^o (kJ/kg)
920	747.82
T_3	768.78
940	770.33

Interpolation yields : T_3 938.62 K

Now, we go back to **Eqns 2** & **3** to evaluate W_{turb} and W_{comp} :

W_{turb}	215.64	kJ/kg
W_{comp}	-435.12	kJ/kg

Finally, we plug values into **Eqn 1** to evaluate W_{cycle} : **W_{cycle} -219.48 kJ/kg**

Part b.) We can determine the **coefficient of performance** from its definition.

$$COP_R = \frac{\hat{Q}_C}{-\hat{W}_{cycle}} \qquad \text{Eqn 14}$$

We can evaluate Q_C by applying the 1st Law to **HEX #2** because $Q_C = Q_{12}$.

HEX #2 operates at steady-state, has no **shaft work** interaction and changes in kinetic and potential energies are negligible. So, the appropriate form of the 1st Law is:

$$\hat{Q}_{12} = \hat{H}_2 - \hat{H}_1 \qquad \text{Eqn 16}$$

Plugging values into **Eqn 16** gives us: Q_{12} 247.13 kJ/kg

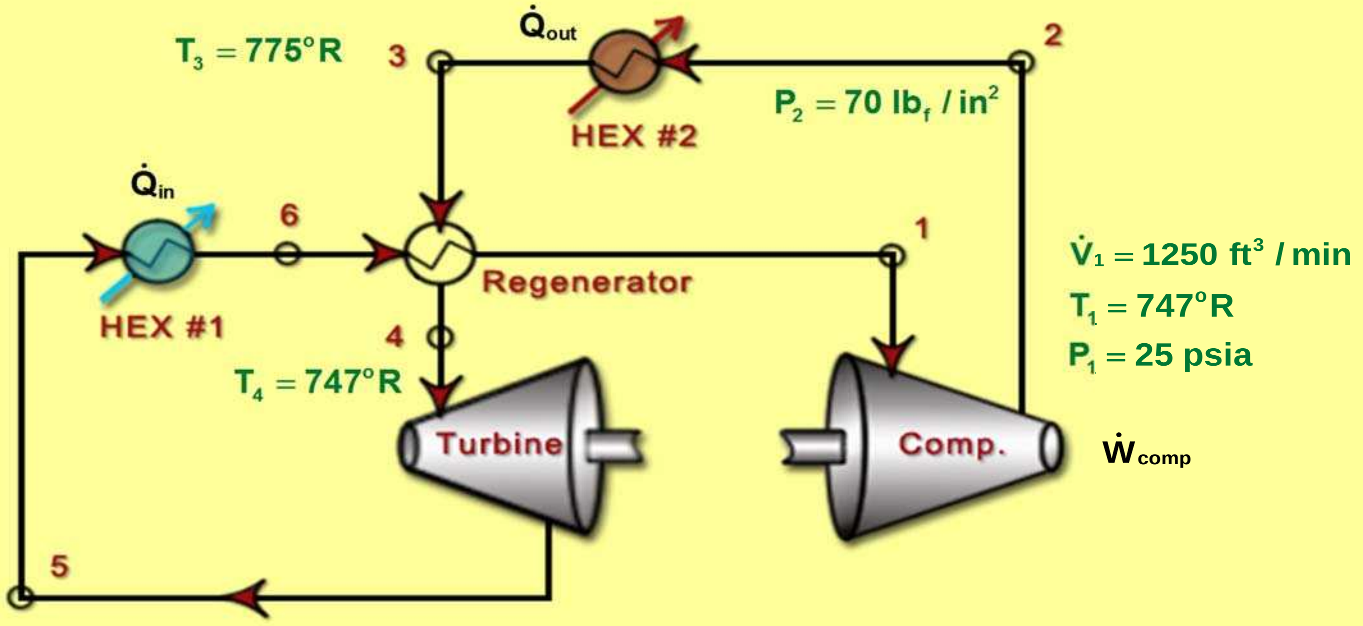
Finally, we can plug values back into **Eqn 14** : **COP_R 1.126**

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : **W_{cycle} -219 kJ/kg** **COP_R 1.13**

10E-2 **Ideal Regenerative Brayton Refrigeration Cycle** **9 pts**

Calculate the refrigeration capacity, Q_C, in **tons of refrigeration**, and the **coefficient of performance** of the regenerative Brayton Cycle, shown below.



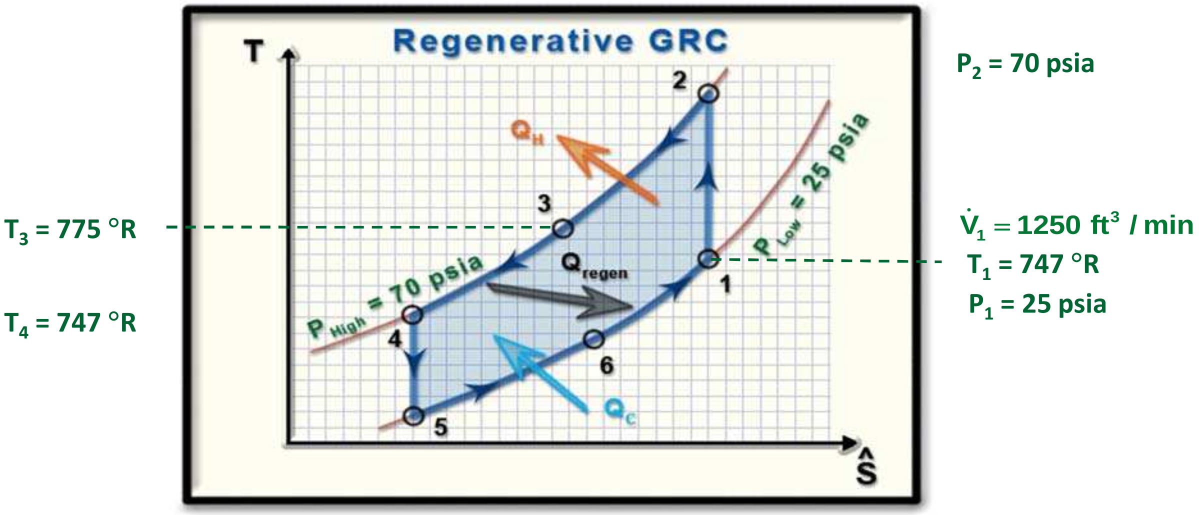
Assume that both the **compressor** and the **turbine** are isentropic.

Read : Make all the usual assumptions for the standard Brayton cycle. Use the ideal gas EOS to convert the **volumetric flow rate** to a **mass flow rate**. Determine the **specific enthalpy** for each **stream** and then use the 1st Law and the definition of the **COP** to answer the questions.

Given:	V_1	1250	ft^3/min	P_2	70	psia
	T_1	747	^oR	T_3	775	^oR
	P_1	25	psia	T_4	747	^oR

Find:	a.)	Q_{in}	???	tons
	b.)	COP_R	???	

Diagram:



Assumptions:

1 - Each process is analyzed as an open system operating at steady-state.
2 - The turbine and compressor are **isentropic**.
3 - There are no **pressure** drops for flow through the heat exchangers.
4 - Kinetic and potential energy changes are negligible.
5 - The **working fluid** is **air** modeled as an ideal gas.
6 - There is no **heat transfer** from the heat exchanger to its surroundings.

Equations / Data / Solve:

Stream	T (°R)	P (psia)	H^o (Btu/lb$_m$)	S^o (Btu/lb$_m$-°R)	P_r
1	747	25	87.598		3.2050
2	992.7	70			8.9740
3	775	70	94.467		
4	747	70	87.598	0.079819	3.2050
5	557.8	25	41.824	0.0091996	1.1446
6		25	80.728		

Part a.) The **refrigeration capacity** is how much **heat** the refrigerator can remove from the cold reservoir. So, we need to determine Q_{in} (from the diagram above). We can accomplish this by applying the 1st Law to HEX #1. The 1st Law for a steady-state, single-inlet, single-outlet, HEX with no **shaft work** and negligible kinetic and potential energy changes is:

$$\dot{Q}_{in} = \dot{Q}_{56} = \dot{m}\left(\hat{H}_6 - \hat{H}_5\right) \qquad \textbf{Eqn 1}$$

First, let's determine the **mass flow rate** of the **working fluid**. The key is that we know the **volumetric flow rate** and the **T** & **P** at the compressor inlet. And, remember that we have assumed that the **air** behaves as an ideal gas.

$$\dot{m} = \frac{\dot{V}_1}{\hat{V}_1} = \frac{P_1 \dot{V}_1}{(R\,/\,MW)T_1} \qquad \textbf{Eqn 2}$$

Plugging values into **Eqn 2** yields:

R	1545	ft lb$_f$ /lb$_m$-°R			
MW	29.00	lb$_m$/lbmole	m_{dot}	113.07	lb$_m$/min

Next, let's determine H_5. We can accomplish this because we assumed the turbine is **isentropic**. We can either use the Ideal Gas Entropy Function and the 2nd Gibbs Equation or we can use Relative Properties. Both methods are presented here.

Method 1: Use the Ideal Gas Entropy Function and the 2nd Gibbs Equation.

Apply the 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function:

$$\Delta\hat{S} = \hat{S}_5 - \hat{S}_4 = \hat{S}^o_{T5} - \hat{S}^o_{T4} - \frac{R}{MW}\,Ln\frac{P_5}{P_4} = 0 \qquad \textbf{Eqn 3}$$

We can solve **Eqn 3** for S^o_{T5} :

$$\hat{S}^o_{T5} = \hat{S}^o_{T4} + \frac{R}{MW}\,Ln\frac{P_5}{P_4} \qquad \textbf{Eqn 4}$$

We can look-up S^o_{T4} in the **Ideal Gas Property Table** for **air** and use it with the known **pressures** in **Eqn 4** to determine S^o_{T5}. We can do this because the HEX's are isobaric. $P_1 = P_5 = P_6$ and $P_2 = P_3 = P_4$.

R 1.987 **Btu/lbmole-°R**

MW 28.97 **lb_m/lbmole**

T (°R)	H^o (Btu/lb_m)	S^o (Btu/lb_m°R)
740.00	85.884	0.077519
747	H^o_{T4}	S^o_{T4}
750	88.332	0.080805

H^o_{T4} 87.598 Btu/lb_m

S^o_{T4} 0.079819 Btu/lb_m-°R

T (°R)	H^o (Btu/lb_m)	S^o (Btu/lb_m°R)
550	39.963	0.005848
T_5	H^o_{T5}	0.009200
560	42.351	0.010149

S^o_{T5} 0.0091996 Btu/lb_m-°R

T_5 557.79 °R

H^o_{T5} 41.824 Btu/lb_m

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_5)}{P_r(T_4)} = \frac{P_5}{P_4} \qquad \textbf{Eqn 5}$$

Where P_r is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqn 5** For $P_r(T_5)$, as follows :

$$P_r(T_5) = \frac{P_5}{P_4} P_r(T_4) \qquad \textbf{Eqn 6}$$

Look-up $P_r(T_4)$ and use it in Eqn 6 to determine $P_r(T_5)$:

T (°R)	P_r	H^o (Btu/lb_m)
740	3.0985	85.884
747	$P_r(T_4)$	H^o_{T4}
750	3.2506	88.332

H^o_{T4} 87.598 Btu/lb_m

$P_r(T_4)$ 3.2050

$P_r(T_5)$ 1.1446

We can now determine T_5 and H_5 by interpolation on the the **Ideal Gas Property Table** for air.

T (°R)	P_r	H^o (Btu/lb_m)
550	1.0891	39.963
T_5	1.1446	H^o_{T5}
560	1.1596	42.351

T_5 557.88 °R

H^o_{T5} 41.845 Btu/lb_m

Since the two methods differ by less than **0.01%**, I will use the results from **Method 1** in the remaining calculations of this problem.

Next, we need to evaluate $\mathbf{H_6}$. To do that, we need to apply the 1st Law to the Regenerator.

The 1st Law for a steady-state, multiple-inlet, multiple-outlet, adiabatic **HEX** with no **shaft work** and negligible kinetic and potential energy changes is:

$$\left(\hat{H}_3+\hat{H}_6\right)-\left(\hat{H}_1+\hat{H}_4\right)=0 \qquad \textbf{Eqn 7}$$

Solve **Eqn 7** for $\mathbf{H_6}$:

$$\hat{H}_6=\hat{H}_1+\hat{H}_4-\hat{H}_3 \qquad \textbf{Eqn 8}$$

We already found $\mathbf{H_4}$, so we need to find $\mathbf{H_1}$ and $\mathbf{H_3}$. We can do this by interpolation in the **Ideal Gas Property Table** for **air** because we know both $\mathbf{T_1}$ and $\mathbf{T_3}$. Because $\mathbf{T_1 = T_4}$, $\mathbf{H_1 = H_4}$ and we already found $\mathbf{H_4}$. So, all we need to work on is $\mathbf{H_3}$.

T (°R)	**H^o (Btu/lb$_m$)**
770	**93.238**
775	**H^o_{T3}**
780	**95.696**

H^o_{T1}	**87.598**	**Btu/lb$_m$**
H^o_{T3}	**94.467**	**Btu/lb$_m$**

Now, we plug values back into **Eqn 8** :

At last, we can plug numbers back into **Eqn 1**:

Converting units to **tons of refrigeration** yields the answer to **part (a)** :

H^o_{T6}	**80.728**	**Btu/lb$_m$**
Q_{in}	**4399.1**	**Btu/min**
1 ton =	**200**	**Btu/min**
Q_{in}	**21.995**	**tons**

Part b.) We can determine the **coefficient of performance** from its definition.

$$COP_R=\frac{Q_{in}}{-W_{cycle}} \qquad \textbf{Eqn 9}$$

Where :

$$W_{cycle}=W_{comp}+W_{turb} \qquad \textbf{Eqn 10}$$

We can determine $\mathbf{W_{comp}}$ and $\mathbf{W_{turb}}$ by applying the 1st Law to the compressor and the turbine separately.

The 1st Law for a steady-state, single-inlet, single-outlet adiabatic turbine and compressor with negligible kinetic and potential energy changes are:

$$\dot{W}_{turb}=\dot{W}_{S,45}=\dot{m}\left(\hat{H}_4-\hat{H}_5\right) \qquad \textbf{Eqn 11}$$

$$\dot{W}_{comp}=\dot{W}_{S,12}=\dot{m}\left(\hat{H}_1-\hat{H}_2\right) \qquad \textbf{Eqn 12}$$

We know all of the values we need except $\mathbf{H_2}$. We can determine it because we know the compressor is **isentropic**. We can use either **Method 1** or **2** described above.

Method 1: Use the Ideal Gas Entropy Function and the 2nd Gibbs Equation.

Apply the 2nd Gibbs Equation for Ideal Gases in terms of the Ideal Gas Entropy Function:

$$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = \hat{S}^o_{T2} - \hat{S}^o_{T1} - \frac{R}{MW}\,Ln\frac{P_2}{P_1} = 0 \qquad \textbf{Eqn 13}$$

Solving **Eqn 13** for S^o_{T2} yields :

$$\hat{S}^o_{T2} = \hat{S}^o_{T1} + \frac{R}{MW}\,Ln\frac{P_2}{P_1} \qquad \textbf{Eqn 14}$$

Because $T_1 = T_4$, $S^o_{T1} = S^o_{T4}$. Then, we can use **Eqn 4** to determine S^o_{T2}. We can do this because the **HEX**'s are isobaric. $P_1 = P_5 = P_6$ and $P_2 = P_3 = P_4$.

S^o_{T1} **0.079819** **Btu/lb$_m$-^{o}R**

S^o_{T2} **0.15044** **Btu/lb$_m$-^{o}R**

T (^{o}R)	**H^o (Btu/lb$_m$)**	**S^o (Btu/lb$_m$^{o}R)**
990	**147.98**	**0.14976**
T_2	**H^o_{T2}**	**0.15044**
1000	**150.50**	**0.15230**

T_2 **992.67** **^{o}R**

H^o_{T2} **148.65** **Btu/lb$_m$**

Method 2: Use the Ideal Gas Relative Pressure.

When an ideal gas undergoes an **isentropic** process :

$$\frac{P_r(T_2)}{P_r(T_1)} = \frac{P_2}{P_1} \qquad \textbf{Eqn 15}$$

Where P_r is the Ideal Gas Relative Pressure, which is a function of **T** only and we can look-up in the **Ideal Gas Property Table** for **air**.

We can solve **Eqn 15** For $P_r(T_2)$, as follows :

$$P_r(T_2) = \frac{P_2}{P_1}P_r(T_1) \qquad \textbf{Eqn 16}$$

$P_r(T_1) = P_r(T_4)$ because $T_1 = T_4$. Use $P_r(T_1)$ in **Eqn 16** to determine $P_r(T_2)$:

$P_r(T_1)$ **3.2050**

$P_r(T_2)$ **8.9740**

We can now determine T_5 and H_5 by interpolation on the the **Ideal Gas Property Table** for **air**.

T (^{o}R)	**P$_r$**	**H^o (Btu/lb$_m$)**
990	**8.8893**	**147.981**
T_2	**8.9740**	**H^o_{T2}**
1000	**9.2240**	**150.502**

T_2 **992.53** **^{o}R**

H^o_{T2} **148.62** **Btu/lb$_m$**

Since the two methods differ by less than **0.01%**, I will use the results from **Method 1** in the remaining calculations of this problem.

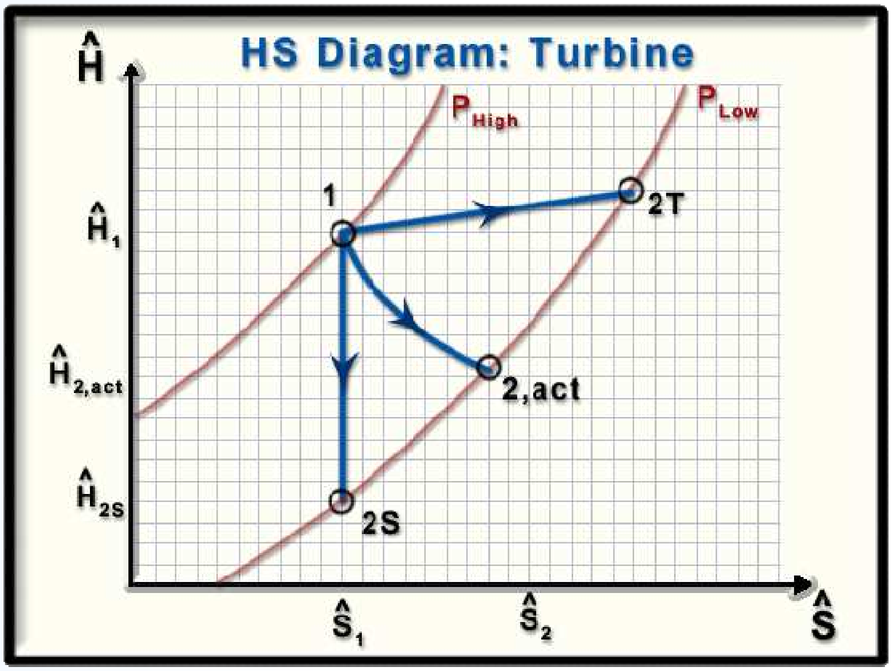
At last we return to **Eqns 11**, **12**, **10** & **9**, in that order:

W_{turb}	**5175.83**	**Btu/min**
W_{comp}	**-6903.91**	**Btu/min**
W_{cycle}	**-1728.08**	**Btu/min**
COP_R	**2.546**	

Verify: The assumptions made in the solution of this problem cannot be verified with the given information.

Answers : a.) **Q_{in}** **22.0** **tons** b.) **COP_R** **2.55**

Appendix



Tables of Thermodynamic Data

A. Steam Tables

B. Ammonia Tables

C. R-134a Tables

D. Ideal Gas Property Tables

E. Ideal Gas, Constant Pressure Heat Capacity

Saturation Temperature Table

Water
SI

Temp. (^{o}C)	Pressure (kPa)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
0.01	0.61165	0.0010002	205.99	0	2374.9	6.1178E-04	2500.9	0	9.1555
5	0.87258	0.0010001	147.01	21.019	2381.8	21.020	2510.1	0.076254	9.0248
10	1.2282	0.0010003	106.30	42.020	2388.6	42.021	2519.2	0.15109	8.8998
15	1.7058	0.0010009	77.875	62.980	2395.5	62.981	2528.3	0.22446	8.7803
20	2.3393	0.0010018	57.757	83.912	2402.3	83.914	2537.4	0.29648	8.6660
25	3.1699	0.0010030	43.337	104.83	2409.1	104.83	2546.5	0.36722	8.5566
30	4.2470	0.0010044	32.878	125.73	2415.9	125.73	2555.5	0.43675	8.4520
35	5.6290	0.0010060	25.205	146.63	2422.7	146.63	2564.5	0.50513	8.3517
40	7.3849	0.0010079	19.515	167.53	2429.4	167.53	2573.5	0.57240	8.2555
45	9.5950	0.0010099	15.252	188.43	2436.1	188.43	2582.4	0.63861	8.1633
50	12.352	0.0010121	12.027	209.33	2442.7	209.34	2591.3	0.70381	8.0748
55	15.762	0.0010146	9.5643	230.24	2449.3	230.26	2600.1	0.76802	7.9898
60	19.946	0.0010171	7.6672	251.16	2455.9	251.18	2608.8	0.83129	7.9081
65	25.042	0.0010199	6.1935	272.09	2462.4	272.12	2617.5	0.89365	7.8296
70	31.201	0.0010228	5.0395	293.03	2468.9	293.07	2626.1	0.95513	7.7540
75	38.595	0.0010258	4.1289	313.99	2475.2	314.03	2634.6	1.0158	7.6812
80	47.414	0.0010291	3.4052	334.96	2481.6	335.01	2643.0	1.0756	7.6111
85	57.867	0.0010324	2.8258	355.95	2487.8	356.01	2651.3	1.1346	7.5434
90	70.182	0.0010360	2.3591	376.97	2494.0	377.04	2659.5	1.1929	7.4781
95	84.608	0.0010396	1.9806	398.00	2500.0	398.09	2667.6	1.2504	7.4151
100	101.42	0.0010435	1.6718	419.06	2506.0	419.17	2675.6	1.3072	7.3541
105	120.90	0.0010474	1.4184	440.15	2511.9	440.27	2683.4	1.3633	7.2952
110	143.38	0.0010516	1.2093	461.26	2517.7	461.42	2691.1	1.4188	7.2381
115	169.18	0.0010559	1.0358	482.41	2523.3	482.59	2698.6	1.4737	7.1828
120	198.67	0.0010603	0.89121	503.60	2528.9	503.81	2705.9	1.5279	7.1291
125	232.24	0.0010649	0.77003	524.83	2534.3	525.07	2713.1	1.5816	7.0770
130	270.28	0.0010697	0.66800	546.09	2539.5	546.38	2720.1	1.6346	7.0264
135	313.23	0.0010746	0.58173	567.41	2544.7	567.74	2726.9	1.6872	6.9772
140	361.54	0.0010798	0.50845	588.77	2549.6	589.16	2733.4	1.7392	6.9293
145	415.68	0.0010850	0.44596	610.19	2554.4	610.64	2739.8	1.7907	6.8826
150	476.16	0.0010905	0.39245	631.66	2559.1	632.18	2745.9	1.8418	6.8371
155	543.50	0.0010962	0.34646	653.19	2563.5	653.79	2751.8	1.8924	6.7926
160	618.23	0.0011020	0.30678	674.79	2567.8	675.47	2757.4	1.9426	6.7491
165	700.93	0.0011080	0.27243	696.46	2571.9	697.24	2762.8	1.9923	6.7066
170	792.19	0.0011143	0.24259	718.20	2575.7	719.08	2767.9	2.0417	6.6650
175	892.60	0.0011207	0.21658	740.02	2579.4	741.02	2772.7	2.0906	6.6241
180	1002.8	0.0011274	0.19384	761.92	2582.8	763.05	2777.2	2.1392	6.5840
185	1123.5	0.0011343	0.17390	783.91	2586.0	785.19	2781.4	2.1875	6.5447
190	1255.2	0.0011415	0.15636	806.00	2589.0	807.43	2785.3	2.2355	6.5059
195	1398.8	0.0011489	0.14089	828.18	2591.7	829.79	2788.8	2.2832	6.4678
200	1554.9	0.0011565	0.12721	850.47	2594.2	852.27	2792.0	2.3305	6.4302
205	1724.3	0.0011645	0.11508	872.87	2596.4	874.88	2794.8	2.3777	6.3930
210	1907.7	0.0011727	0.10429	895.39	2598.3	897.63	2797.3	2.4245	6.3563
215	2105.8	0.0011813	0.094679	918.04	2599.9	920.53	2799.3	2.4712	6.3200
220	2319.6	0.0011902	0.086092	940.82	2601.2	943.58	2800.9	2.5177	6.2840
225	2549.7	0.0011994	0.078403	963.74	2602.2	966.80	2802.1	2.5640	6.2483
230	2797.1	0.0012090	0.071503	986.81	2602.9	990.19	2802.9	2.6101	6.2128
235	3062.5	0.0012190	0.065298	1010.0	2603.2	1013.8	2803.2	2.6561	6.1775

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Saturation Temperature Table

Water
SI

Temp. (^oC)	Pressure (kPa)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	**Sat. Vap**	**Sat. Liq**	**Sat. Vap**	**Sat. Liq**	**Sat. Vap**	**Sat. Liq**	**Sat. Vap**
240	3346.9	0.0012295	0.059705	1033.4	2603.1	1037.6	2803.0	2.7020	6.1423
245	3651.2	0.0012403	0.054654	1057.0	2602.7	1061.5	2802.2	2.7478	6.1072
250	3976.2	0.0012517	0.050083	1080.8	2601.8	1085.8	2800.9	2.7935	6.0721
255	4322.9	0.0012636	0.045938	1104.8	2600.5	1110.2	2799.1	2.8392	6.0369
260	4692.3	0.0012761	0.042173	1129.0	2598.7	1135.0	2796.6	2.8849	6.0016
265	5085.3	0.0012892	0.038746	1153.4	2596.5	1160.0	2793.5	2.9307	5.9661
270	5503.0	0.0013030	0.035621	1178.1	2593.7	1185.3	2789.7	2.9765	5.9304
275	5946.4	0.0013175	0.032766	1203.1	2590.3	1210.9	2785.2	3.0224	5.8944
280	6416.6	0.0013328	0.030153	1228.3	2586.4	1236.9	2779.9	3.0685	5.8579
285	6914.7	0.0013491	0.027756	1253.9	2581.8	1263.2	2773.7	3.1147	5.8209
290	7441.8	0.0013663	0.025555	1279.9	2576.5	1290.0	2766.7	3.1612	5.7834
295	7999.1	0.0013846	0.023529	1306.2	2570.5	1317.3	2758.7	3.2080	5.7451
300	8587.9	0.0014042	0.021660	1332.9	2563.6	1345.0	2749.6	3.2552	5.7059
305	9209.4	0.0014252	0.019933	1360.2	2555.9	1373.3	2739.4	3.3028	5.6657
310	9865.1	0.0014479	0.018335	1387.9	2547.1	1402.2	2727.9	3.3510	5.6244
315	10556	0.0014724	0.016851	1416.3	2537.2	1431.8	2715.1	3.3998	5.5816
320	11284	0.0014990	0.015471	1445.3	2526.0	1462.2	2700.6	3.4494	5.5372
325	12051	0.0015283	0.014183	1475.1	2513.4	1493.5	2684.3	3.5000	5.4908
330	12858	0.0015606	0.012979	1505.8	2499.2	1525.9	2666.0	3.5518	5.4422
335	13707	0.0015967	0.011847	1537.6	2483.0	1559.5	2645.4	3.6050	5.3906
340	14601	0.0016376	0.010781	1570.6	2464.4	1594.5	2621.8	3.6601	5.3356
345	15541	0.0016846	0.0097690	1605.3	2443.1	1631.5	2594.9	3.7176	5.2762
350	16529	0.0017400	0.0088024	1642.1	2418.1	1670.9	2563.6	3.7784	5.2110
355	17570	0.0018079	0.0078684	1682.0	2388.4	1713.7	2526.6	3.8439	5.1380
360	18666	0.0018954	0.0069493	1726.3	2351.8	1761.7	2481.5	3.9167	5.0536
365	19821	0.0020172	0.0060115	1777.8	2303.8	1817.8	2422.9	4.0014	4.9497
370	21044	0.0022152	0.0049544	1844.1	2230.3	1890.7	2334.5	4.1112	4.8012
373.95	22064	0.0031056	0.0031056	2015.7	2015.7	2084.3	2084.3	4.4070	4.4070

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Saturation Pressure Table

Water
SI

Pressure (kPa)	Temp. (^{o}C)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
1.0	6.970	0.0010001	129.18	29.298	2384.5	29.299	2513.7	0.10591	8.9749
1.5	13.02	0.0010007	87.958	54.681	2392.8	54.683	2524.7	0.19556	8.8270
2.0	17.49	0.0010014	66.987	73.426	2398.9	73.428	2532.9	0.26056	8.7226
2.5	21.08	0.0010021	54.240	88.417	2403.8	88.420	2539.4	0.31182	8.6420
3.0	24.08	0.0010028	45.653	100.97	2407.9	100.98	2544.8	0.35429	8.5764
4.0	28.96	0.0010041	34.791	121.38	2414.5	121.39	2553.7	0.42239	8.4734
5.0	32.87	0.0010053	28.185	137.74	2419.8	137.75	2560.7	0.47620	8.3938
7.5	40.29	0.0010080	19.233	168.74	2429.8	168.75	2574.0	0.57627	8.2501
10	45.81	0.0010103	14.670	191.80	2437.2	191.81	2583.9	0.64920	8.1488
15	53.97	0.0010140	10.020	225.93	2448.0	225.94	2598.3	0.75486	8.0071
20	60.06	0.0010172	7.6480	251.40	2456.0	251.42	2608.9	0.83202	7.9072
25	64.96	0.0010198	6.2032	271.93	2462.4	271.96	2617.4	0.89319	7.8302
30	69.10	0.0010222	5.2284	289.24	2467.7	289.27	2624.5	0.94407	7.7675
40	75.86	0.0010264	3.9930	317.58	2476.3	317.62	2636.1	1.0261	7.6690
50	81.32	0.0010299	3.2400	340.49	2483.2	340.54	2645.2	1.0912	7.5930
75	91.76	0.0010372	2.2170	384.36	2496.1	384.44	2662.4	1.2132	7.4557
100	99.61	0.0010432	1.6939	417.40	2505.6	417.50	2674.9	1.3028	7.3588
125	106.0	0.0010482	1.3749	444.22	2513.0	444.35	2684.9	1.3741	7.2840
150	111.3	0.0010527	1.1593	466.97	2519.2	467.13	2693.1	1.4337	7.2230
175	116.0	0.0010568	1.0036	486.82	2524.5	487.00	2700.1	1.4850	7.1715
200	120.2	0.0010605	0.88568	504.49	2529.1	504.70	2706.2	1.5302	7.1269
225	124.0	0.0010640	0.79320	520.47	2533.2	520.71	2711.6	1.5706	7.0876
250	127.4	0.0010672	0.71866	535.08	2536.8	535.34	2716.5	1.6072	7.0524
275	130.6	0.0010703	0.65725	548.56	2540.1	548.86	2720.9	1.6408	7.0207
300	133.5	0.0010732	0.60576	561.10	2543.2	561.43	2724.9	1.6717	6.9916
325	136.3	0.0010759	0.56194	572.84	2545.9	573.19	2728.6	1.7005	6.9649
350	138.9	0.0010786	0.52418	583.88	2548.5	584.26	2732.0	1.7274	6.9401
375	141.3	0.0010811	0.49129	594.32	2550.9	594.73	2735.1	1.7526	6.9171
400	143.6	0.0010836	0.46238	604.22	2553.1	604.65	2738.1	1.7765	6.8955
450	147.9	0.0010882	0.41390	622.65	2557.1	623.14	2743.4	1.8205	6.8560
500	151.8	0.0010925	0.37481	639.54	2560.7	640.09	2748.1	1.8604	6.8207
550	155.5	0.0010967	0.34260	655.16	2563.9	655.76	2752.3	1.8970	6.7886
600	158.8	0.0011006	0.31558	669.72	2566.8	670.38	2756.1	1.9308	6.7592
650	162.0	0.0011044	0.29259	683.36	2569.4	684.08	2759.6	1.9623	6.7322
700	164.9	0.0011080	0.27277	696.23	2571.8	697.00	2762.8	1.9918	6.7071
750	167.7	0.0011114	0.25551	708.40	2574.0	709.24	2765.6	2.0195	6.6836
800	170.4	0.0011148	0.24034	719.97	2576.0	720.86	2768.3	2.0457	6.6616
850	172.9	0.0011180	0.22689	731.00	2577.9	731.95	2770.8	2.0705	6.6409
900	175.4	0.0011212	0.21489	741.55	2579.6	742.56	2773.0	2.0940	6.6213
950	177.7	0.0011242	0.20410	751.67	2581.2	752.74	2775.1	2.1165	6.6027
1000	179.9	0.0011272	0.19436	761.39	2582.7	762.52	2777.1	2.1381	6.5850
1100	184.1	0.0011330	0.17745	779.78	2585.5	781.03	2780.6	2.1785	6.5520
1200	188.0	0.0011385	0.16326	796.96	2587.8	798.33	2783.7	2.2159	6.5217
1300	191.6	0.0011438	0.15119	813.11	2589.9	814.60	2786.5	2.2508	6.4936
1400	195.0	0.0011489	0.14078	828.36	2591.8	829.97	2788.8	2.2835	6.4675
1500	198.3	0.0011539	0.13171	842.83	2593.4	844.56	2791.0	2.3143	6.4430
1750	205.7	0.0011657	0.11343	876.13	2596.7	878.17	2795.2	2.3845	6.3877
2000	212.4	0.0011767	0.099585	906.14	2599.1	908.50	2798.3	2.4468	6.3390

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Saturation Pressure Table

Water
SI

Pressure (MPa)	Temp. (^{o}C)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
2250	218.41	0.0011873	0.088715	933.57	2600.9	936.24	2800.5	2.5029	6.2954
2500	223.95	0.0011974	0.079949	958.91	2602.1	961.91	2801.9	2.5543	6.2558
3000	233.85	0.0012167	0.066664	1004.7	2603.2	1008.3	2803.2	2.6455	6.1856
3500	242.56	0.0012350	0.057058	1045.5	2602.9	1049.8	2802.6	2.7254	6.1243
4000	250.35	0.0012526	0.049776	1082.5	2601.7	1087.5	2800.8	2.7968	6.0696
5000	263.94	0.0012864	0.039446	1148.2	2597.0	1154.6	2794.2	2.9210	5.9737
6000	275.58	0.0013193	0.032448	1206.0	2589.9	1213.9	2784.6	3.0278	5.8901
7000	285.83	0.0013519	0.027378	1258.2	2581.0	1267.7	2772.6	3.1224	5.8148
8000	295.01	0.0013847	0.023526	1306.2	2570.5	1317.3	2758.7	3.2081	5.7450
9000	303.34	0.0014181	0.020490	1351.1	2558.5	1363.9	2742.9	3.2870	5.6791
10000	311.00	0.0014526	0.018030	1393.5	2545.2	1408.1	2725.5	3.3606	5.6160
11000	318.08	0.0014885	0.015990	1434.1	2530.5	1450.4	2706.3	3.4303	5.5545
12000	324.68	0.0015263	0.014264	1473.1	2514.3	1491.5	2685.4	3.4967	5.4939
13000	330.85	0.0015665	0.012780	1511.1	2496.5	1531.5	2662.7	3.5608	5.4336
14000	336.67	0.0016097	0.011485	1548.4	2477.1	1571.0	2637.9	3.6232	5.3727
15000	342.16	0.0016570	0.010338	1585.3	2455.6	1610.2	2610.7	3.6846	5.3106
16000	347.35	0.0017094	0.0093088	1622.3	2431.8	1649.7	2580.8	3.7457	5.2463
17000	352.29	0.0017693	0.0083709	1659.9	2405.2	1690.0	2547.5	3.8077	5.1787
18000	356.99	0.0018398	0.0075017	1699.0	2374.8	1732.1	2509.8	3.8718	5.1061
19000	361.47	0.0019268	0.0066773	1740.5	2339.1	1777.2	2466.0	3.9401	5.0256
20000	365.75	0.0020400	0.0058652	1786.4	2295.0	1827.2	2412.3	4.0156	4.9314
21000	369.83	0.0022055	0.0049961	1841.2	2233.7	1887.6	2338.6	4.1064	4.8079
22000	373.71	0.0027044	0.0036475	1951.8	2092.8	2011.3	2173.1	4.2945	4.5446
22064	373.95	0.0031056	0.0031056	2015.7	2015.7	2084.3	2084.3	4.4070	4.4070

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Superheated Vapor

Water SI

Temp. (^{o}C)	P = 10 kPa (T_{sat} = 45.81^{o}C)				P = 50 kPa (T_{sat} = 81.32^{o}C)				P = 100 kPa (T_{sat} = 99.61^{o}C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	14.670	2437.16	2583.9	8.1488	3.2400	2483.2	2645.2	7.5930	1.6939	2505.6	2674.9	7.3588
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.42	7.6953				
150	19.513	2587.9	2783.0	8.6892	3.8897	2585.7	2780.21	7.9413	1.9367	2582.9	2776.6	7.6148
200	21.826	2661.3	2879.6	8.9049	4.3562	2660.0	2877.77	8.1592	2.1724	2658.2	2875.5	7.8356
250	24.136	2736.1	2977.4	9.1015	4.8206	2735.1	2976.15	8.3568	2.4062	2733.9	2974.5	8.0346
300	26.446	2812.3	3076.7	9.2827	5.2840	2811.6	3075.76	8.5386	2.6388	2810.6	3074.5	8.2172
400	31.063	2969.3	3279.9	9.6094	6.2094	2968.9	3279.33	8.8659	3.1027	2968.3	3278.6	8.5452
500	35.680	3132.9	3489.7	9.8998	7.1338	3132.6	3489.27	9.1566	3.5655	3132.2	3488.7	8.8361
600	40.296	3303.3	3706.3	10.163	8.0576	3303.1	3706.0	9.4201	4.0279	3302.8	3705.6	9.0998
700	44.911	3480.8	3929.9	10.406	8.9812	3480.6	3929.7	9.6625	4.4900	3480.4	3929.4	9.3424
800	49.527	3665.3	4160.6	10.631	9.9047	3665.2	4160.44	9.8882	4.9519	3665.0	4160.2	9.5681
900	54.142	3856.9	4398.3	10.843	10.828	3856.8	4398.19	10.100	5.4137	3856.6	4398.0	9.7800
1000	58.758	4055.2	4642.8	11.043	11.751	4055.1	4642.7	10.300	5.8754	4055.0	4642.6	9.9800

Temp. (^{o}C)	P = 200 kPa (T_{sat} = 120.2^{o}C)				P =300 kPa (T_{sat} = 133.5^{o}C)				P = 40 kPa (T_{sat} = 143.6^{o}C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.88568	2529.1	2706.23	7.1269	0.60576	2543.2	2724.9	6.9916	0.46238	2553.1	2738.1	6.8955
150	0.95986	2577.1	2769.1	7.2810	0.63401	2571.0	2761.2	7.0791	0.47088	2564.4	2752.8	6.9306
200	1.0805	2654.6	2870.7	7.5081	0.71642	2651.0	2865.9	7.3131	0.53433	2647.2	2860.9	7.1723
250	1.1989	2731.4	2971.2	7.7100	0.79644	2728.9	2967.9	7.5180	0.59520	2726.4	2964.5	7.3804
300	1.3162	2808.8	3072.1	7.8941	0.87534	2807.0	3069.6	7.7037	0.65489	2805.1	3067.1	7.5677
400	1.5493	2967.1	3277.0	8.2236	1.0315	2966.0	3275.5	8.0347	0.77264	2964.9	3273.9	7.9002
500	1.7814	3131.4	3487.7	8.5152	1.1867	3130.6	3486.6	8.3271	0.88936	3129.8	3485.5	8.1933
600	2.0130	3302.2	3704.8	8.7792	1.3414	3301.6	3704.0	8.5914	1.0056	3301.0	3703.2	8.4580
700	2.2443	3479.9	3928.8	9.0220	1.4958	3479.5	3928.2	8.8344	1.1215	3479.0	3927.6	8.7012
800	2.4755	3664.7	4159.8	9.2479	1.6500	3664.3	4159.3	9.0604	1.2373	3663.9	4158.8	8.9273
900	2.7066	3856.3	4397.6	9.4598	1.8042	3856.0	4397.3	9.2724	1.3530	3855.7	4396.9	9.1394
1000	2.9375	4054.8	4642.3	9.6599	1.9582	4054.5	4642.0	9.4726	1.4686	4054.3	4641.7	9.3396

Temp. (^{o}C)	P = 500 kPa (T_{sat} = 151.8^{o}C)				P = 600 kPa (T_{sat} = 158.8^{o}C)				P = 800 kPa (T_{sat} = 170.4^{o}C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.37481	2560.7	2748.1	6.8207	0.31558	2566.8	2756.1	6.7592	0.24034	2576.0	2768.3	6.6616
200	0.42503	2643.3	2855.8	7.0610	0.35212	2639.3	2850.6	6.9683	0.26088	2631.0	2839.7	6.8176
250	0.47443	2723.8	2961.0	7.2724	0.39390	2721.2	2957.6	7.1832	0.29320	2715.9	2950.4	7.0401
300	0.52261	2803.2	3064.6	7.4614	0.43442	2801.4	3062.0	7.3740	0.32416	2797.5	3056.9	7.2345
400	0.61730	2963.7	3272.3	7.7955	0.51374	2962.5	3270.8	7.7097	0.38428	2960.2	3267.6	7.5734
500	0.71094	3129.0	3484.5	8.0892	0.59200	3128.2	3483.4	8.0041	0.44332	3126.6	3481.3	7.8692
600	0.80409	3300.4	3702.5	8.3543	0.66976	3299.8	3701.7	8.2695	0.50185	3298.7	3700.1	8.1354
700	0.89696	3478.5	3927.0	8.5977	0.74725	3478.1	3926.4	8.5131	0.56011	3477.2	3925.3	8.3794
800	0.98966	3663.6	4158.4	8.8240	0.82457	3663.2	4157.9	8.7395	0.61820	3662.4	4157.0	8.6061
900	1.0823	3855.4	4396.6	9.0362	0.90178	3855.1	4396.2	8.9518	0.67619	3854.5	4395.5	8.8185
1000	1.1748	4054.0	4641.4	9.2364	0.97893	4053.7	4641.1	9.1521	0.73411	4053.2	4640.5	9.0189

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Superheated Vapor

Water
SI

Temp. (°C)	P = 1000 kPa (T_{sat} = 179.9°C)				P = 1200 kPa (T_{sat} = 188.0°C)				P = 1400 kPa (T_{sat} = 195.0°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.19436	2582.7	2777.1	6.5850	0.16326	2587.8	2783.7	6.5217	0.14078	2591.8	2788.8	6.4675
200	0.20602	2622.2	2828.3	6.6955	0.16934	2612.9	2816.1	6.5909	0.14303	2602.7	2803.0	6.4975
250	0.23275	2710.4	2977.8	6.9265	0.19241	2704.7	2935.6	6.8313	0.1635608	2698.9	2927.9	6.7488
300	0.25799	2793.6	3077.0	7.1246	0.21386	2789.7	3046.3	7.0335	0.18232	2785.7	3040.9	6.9552
400	0.30661	2957.9	3280.1	7.4669	0.25482	2955.5	3261.3	7.3793	0.21782	2953.1	3258.1	7.3046
500	0.35411	3125.0	3489.8	7.7641	0.29464	3123.4	3476.9	7.6779	0.25216	3121.8	3474.8	7.6047
600	0.40111	3297.5	3706.4	8.0310	0.33394	3296.3	3697.0	7.9455	0.28597	3295.1	3695.4	7.8730
700	0.44783	3476.2	3930.0	8.2755	0.37297	3475.3	3922.9	8.1904	0.31951	3474.4	3921.7	8.1183
800	0.49438	3661.7	4160.7	8.5024	0.41184	3661.0	4155.2	8.4176	0.35287	3660.2	4154.3	8.3457
900	0.54083	3853.9	4398.4	8.7150	0.45059	3853.3	4394.0	8.6303	0.38614	3852.7	4393.3	8.5587
1000	0.58721	4052.7	4642.8	8.9155	0.48928	4052.2	4639.4	8.8310	0.41933	4051.7	4638.8	8.7594

Temp. (°C)	P = 1600 kPa (T_{sat} = 201.4°C)				P = 1800 kPa (T_{sat} = 207.1°C)				P = 2000 kPa (T_{sat} = 212.4°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.12374	2594.8	2792.8	6.4199	0.11037	2597.2	2795.9	6.3775	0.0995851	2599.1	2798.3	6.3390
250	0.14190	2692.9	2919.9	6.6753	0.12502	2686.7	2911.7	6.6087	0.11150	2680.2	2903.2	6.5475
300	0.15866	2781.6	3035.4	6.8863	0.14025	2777.4	3029.9	6.8246	0.12551	2773.2	3024.2	6.7684
400	0.19007	2950.7	3254.9	7.2394	0.16849	2948.3	3251.6	7.1814	0.15121	2945.9	3248.3	7.1292
500	0.22029	3120.1	3472.6	7.5409	0.19551	3118.5	3470.4	7.4845	0.17568	3116.9	3468.2	7.4337
600	0.24999	3293.9	3693.9	7.8100	0.22200	3292.7	3692.3	7.7543	0.19961	3291.5	3690.7	7.7043
700	0.27940	3473.5	3920.5	8.0557	0.24821	3472.6	3919.4	8.0004	0.22326	3471.6	3918.2	7.9509
800	0.30865	3659.5	4153.3	8.2834	0.27426	3658.8	4152.4	8.2284	0.24674	3658.0	4151.5	8.1790
900	0.33780	3852.1	4392.6	8.4965	0.30020	3851.5	4391.9	8.4416	0.27012	3850.9	4391.1	8.3925
1000	0.36687	4051.2	4638.2	8.6974	0.32606	4050.7	4637.6	8.6426	0.29342	4050.2	4637.0	8.5936

Temp. (°C)	P = 2500 kPa (T_{sat} = 224.0°C)				P = 3000 kPa (T_{sat} = 233.9°C)				P = 3500 kPa (T_{sat} = 242.6°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.079949	2602.1	2801.9	6.25577	0.066664	2603.2	2803.2	6.1856	0.057058	2602.9	2802.6	6.1243
250	0.087053	2663.3	2880.9	6.41071	0.070627	2644.7	2856.5	6.2893	0.058757	2624.0	2829.7	6.1764
300	0.098937	2762.2	3009.6	6.64590	0.081179	2750.8	2994.3	6.5412	0.068453	2738.8	2978.4	6.4484
400	0.12012	2939.8	3240.1	7.01697	0.099379	2933.5	3231.7	6.9234	0.084556	2927.2	3223.2	6.8427
500	0.13999	3112.8	3462.7	7.32540	0.11620	3108.6	3457.2	7.2359	0.099195	3104.5	3451.6	7.1593
600	0.15931	3288.5	3686.8	7.59787	0.13245	3285.5	3682.8	7.5103	0.11325	3282.5	3678.9	7.4356
700	0.17835	3469.3	3915.2	7.84549	0.14841	3467.0	3912.2	7.7590	0.12702	3464.7	3909.3	7.6854
800	0.19721	3656.2	4149.2	8.07434	0.16420	3654.3	4146.9	7.9885	0.14061	3652.5	4144.6	7.9156
900	0.21597	3849.4	4389.3	8.28822	0.17988	3847.9	4387.5	8.2028	0.15410	3846.4	4385.7	8.1303
1000	0.23466	4048.9	4635.6	8.48964	0.19549	4047.7	4634.1	8.4045	0.16751	4046.4	4632.7	8.3324

Temp. (°C)	P = 4000 kPa (T_{sat} = 250.4°C)				P = 4500 kPa (T_{sat} = 257.4°C)				P = 5000 kPa (T_{sat} = 263.9°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.049776	2601.7	2800.8	6.0696	0.044059	2599.7	2797.9	6.0197	0.039446	2597.0	2794.2	5.9737
300	0.058870	2726.2	2961.7	6.3639	0.051378	2713.0	2944.2	6.2854	0.045346	2699.0	2925.7	6.2110
400	0.073431	2920.7	3214.5	6.7714	0.064772	2914.2	3205.6	6.7070	0.057837	2907.5	3196.7	6.6483
500	0.086442	3100.3	3446.0	7.0922	0.076521	3096.0	3440.4	7.0323	0.068583	3091.7	3434.7	6.9781
600	0.098859	3279.4	3674.9	7.3705	0.087662	3276.4	3670.9	7.3127	0.078704	3273.3	3666.8	7.2605
700	0.11098	3462.4	3906.3	7.6214	0.098500	3460.0	3903.3	7.5646	0.088518	3457.7	3900.3	7.5136
800	0.12292	3650.6	4142.3	7.8523	0.10916	3648.8	4140.0	7.7962	0.098158	3646.9	4137.7	7.7458
900	0.13476	3844.8	4383.9	8.0674	0.11972	3843.3	4382.1	8.0118	0.10769	3841.8	4380.2	7.9618
1000	0.14652	4045.1	4631.2	8.2697	0.13020	4043.9	4629.8	8.2144	0.11715	4042.6	4628.3	8.1648

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01°C and 0.61165 kPa).

Superheated Vapor

Water
SI

P = 6000 kPa (T_{sat} = 275.6°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.032448	2589.9	2784.6	5.8901
300	0.036189	2668.4	2885.5	6.0703
400	0.047419	2893.7	3178.2	6.5432
500	0.056671	3083.1	3423.1	6.8826
600	0.065265	3267.2	3658.7	7.1693
700	0.073545	3453.0	3894.3	7.4246
800	0.081648	3643.2	4133.1	7.6582
900	0.089641	3838.8	4376.6	7.8751
1000	0.097560	4040.1	4625.4	8.0786

P = 7000 kPa (T_{sat} = 285.8°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.027378	2581.0	2772.6	5.8148
300	0.029492	2633.5	2839.9	5.9337
400	0.039958	2879.5	3159.2	6.4502
500	0.048157	3074.3	3411.4	6.8000
600	0.055665	3260.9	3650.6	7.0910
700	0.062850	3448.3	3888.2	7.3486
800	0.069855	3639.5	4128.4	7.5836
900	0.076750	3835.7	4373.0	7.8014
1000	0.083571	4037.5	4622.5	8.0055

P = 8000 kPa (T_{sat} = 295.0°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.023526	2570.5	2758.7	5.7450
300	0.024279	2592.3	2786.5	5.7937
400	0.034344	2864.6	3139.4	6.3658
500	0.041767	3065.4	3399.5	6.7266
600	0.048463	3254.7	3642.4	7.0221
700	0.054828	3443.6	3882.2	7.2821
800	0.061011	3635.7	4123.8	7.5184
900	0.067082	3832.6	4369.3	7.7371
1000	0.073079	4035.0	4619.6	7.9419

P = 9000 kPa (T_{sat} = 303.3°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.020490	2558.5	2742.9	5.6791
350	0.025816	2724.9	2957.3	6.0380
400	0.029960	2849.2	3118.8	6.2876
500	0.036793	3056.3	3387.4	6.6603
600	0.042861	3248.4	3634.1	6.9605
700	0.048589	3438.8	3876.1	7.2229
800	0.054132	3632.0	4119.1	7.4606
900	0.059562	3829.6	4365.7	7.6802
1000	0.064918	4032.4	4616.7	7.8855

P = 10000 kPa (T_{sat} = 311.0°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.018030	2545.2	2725.5	5.6160
350	0.022440	2699.6	2924.0	5.9459
400	0.026436	2833.1	3097.4	6.2141
500	0.032811	3047.0	3375.1	6.5995
600	0.038378	3242.0	3625.8	6.9045
700	0.043597	3434.0	3870.0	7.1693
800	0.048629	3628.2	4114.5	7.4085
900	0.053547	3826.5	4362.0	7.6290
1000	0.058390	4029.9	4613.8	7.8349

P = 15000 kPa (T_{sat} = 342.2°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.0103384	2455.6	2610.7	5.3106
350				
400	0.015671	2740.6	2975.7	5.8819
500	0.020827	2998.4	3310.8	6.3480
600	0.024921	3209.3	3583.1	6.6796
700	0.028621	3409.8	3839.1	6.9572
800	0.032121	3609.2	4091.1	7.2037
900	0.035503	3811.2	4343.7	7.4288
1000	0.038808	4017.1	4599.2	7.6378

P = 20000 kPa (T_{sat} = 365.8°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.0058652	2295.0	2412.3	4.9314
400	0.0099503	2617.9	2816.9	5.5525
500	0.014793	2945.3	3241.2	6.1446
600	0.018185	3175.3	3539.0	6.5075
700	0.021133	3385.1	3807.8	6.7990
800	0.023869	3590.1	4067.5	7.0531
900	0.026483	3795.7	4325.4	7.2829
1000	0.029020	4004.3	4584.7	7.4950

P = 40000 kPa

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	Supercritical			
400	0.0019108	1854.9	1931.4	4.1145
500	0.0056231	2681.6	2906.5	5.4744
600	0.0080891	3026.8	3350.4	6.0170
700	0.0099297	3282.0	3679.1	6.3740
800	0.011521	3511.8	3972.6	6.6612
900	0.012980	3733.3	4252.5	6.9106
1000	0.014360	3952.9	4527.3	7.1355

P = 60000 kPa

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	Supercritical			
400	0.0016329	1745.2	1843.2	3.9317
500	0.0029522	2393.2	2570.3	4.9356
600	0.0048330	2866.8	3156.8	5.6527
700	0.0062649	3175.4	3551.3	6.0814
800	0.0074560	3432.6	3880.0	6.4033
900	0.0085188	3670.9	4182.0	6.6725
1000	0.0095043	3901.9	4472.2	6.9099

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01°C and 0.61165 kPa).

Subcooled Liquid

Water SI

P = 5000 kPa (263.9°C)

Temp. (°C)	ρ (kg/m³)	V (m³/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
10	1002.0	0.00099797	41.893	46.883	0.15062
20	1000.4	0.00099956	83.609	88.61	0.29543
40	994.36	0.0010057	166.92	171.95	0.57046
60	985.33	0.0010149	250.29	255.36	0.82865
80	973.97	0.0010267	333.82	338.95	1.0723
100	960.63	0.0010410	417.64	422.85	1.3034
120	945.49	0.0010576	501.90	507.19	1.5236
140	928.63	0.0010769	586.79	592.18	1.7344
160	910.05	0.0010988	672.55	678.04	1.9374
180	889.65	0.0011240	759.46	765.08	2.1338
200	867.26	0.0011531	847.91	853.68	2.3251
220	842.58	0.0011868	938.39	944.32	2.5127
240	815.10	0.0012268	1031.6	1037.7	2.6983
260	784.03	0.0012755	1128.5	1134.9	2.8841
280					
300					
Sat	777.369	0.0012864	1148.2	1154.64	2.9210

P = 10000 kPa (311.0°C)

Temp. (°C)	ρ (kg/m³)	V (m³/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
10	1004.4	0.00099564	41.762	51.719	0.15009
20	1002.7	0.00099731	83.308	93.281	0.29435
40	996.52	0.0010035	166.33	176.36	0.56851
60	987.48	0.0010127	249.42	259.55	0.82602
80	976.17	0.0010244	332.69	342.94	1.0691
100	962.93	0.0010385	416.23	426.62	1.2996
120	947.94	0.0010549	500.18	510.73	1.5191
140	931.28	0.0010738	584.71	595.45	1.7293
160	912.95	0.0010954	670.06	681.01	1.9315
180	892.88	0.0011200	756.48	767.68	2.1271
200	870.94	0.0011482	844.31	855.80	2.3174
220	846.84	0.0011809	934.00	945.81	2.5037
240	820.18	0.0012192	1026.1	1038.3	2.6876
260	790.30	0.0012653	1121.6	1134.3	2.8710
280	756.07	0.0013226	1221.8	1235.0	3.0565
300	715.29	0.0013980	1329.4	1343.3	3.2488
Sat	688.42	0.0014526	1393.5	1408.1	3.3606

P = 15000 kPa (342.2°C)

Temp. (°C)	ρ (kg/m³)	V (m³/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
10	1006.71	0.0009933	41.626	56.526	0.14951
20	1004.926	0.0009951	83.007	97.934	0.29323
40	998.65	0.0010013	165.75	180.77	0.56656
60	989.60	0.0010105	248.58	263.74	0.82340
80	978.35	0.0010221	331.59	346.92	1.0659
100	965.20	0.0010361	414.85	430.39	1.2958
120	950.35	0.0010522	498.49	514.28	1.5148
140	933.87	0.0010708	582.69	598.75	1.7243
160	915.79	0.0010920	667.63	684.01	1.9259
180	896.04	0.0011160	753.58	770.32	2.1206
200	874.50	0.0011435	840.84	857.99	2.3100
220	850.95	0.0011752	929.80	947.43	2.4951
240	825.03	0.0012121	1021.0	1039.2	2.6774
260	796.20	0.0012560	1115.1	1134.0	2.8586
280	763.58	0.0013096	1213.4	1233.0	3.0409
300	725.55	0.0013783	1317.6	1338.3	3.2279
320	678.77	0.0014733	1431.9	1454.0	3.4263
340	613.07	0.0016311	1567.9	1592.4	3.6555
360					
Sat	603.52	0.0016570	1585.3	1610.2	3.6846

P = 20000 kPa (365.8°C)

Temp. (°C)	ρ (kg/m³)	V (m³/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
10	1009.0	0.000991069	41.486	61.307	0.14888
20	1007.1	0.000992916	82.708	102.57	0.29207
40	1000.8	0.000999233	165.17	185.16	0.56461
60	991.71	0.0010084	247.75	267.92	0.82080
80	980.49	0.0010199	330.50	350.90	1.0627
100	967.44	0.0010337	413.50	434.17	1.2920
120	952.72	0.0010496	496.85	517.84	1.5105
140	936.42	0.0010679	580.71	602.07	1.7194
160	918.57	0.0010886	665.27	687.05	1.9203
180	899.12	0.0011122	750.77	773.02	2.1143
200	877.97	0.0011390	837.49	860.27	2.3027
220	854.91	0.0011697	925.77	949.16	2.4867
240	829.67	0.0012053	1016.1	1040.2	2.6676
260	801.78	0.0012472	1109.0	1134.0	2.8469
280	770.52	0.0012978	1205.5	1231.5	3.0265
300	734.71	0.0013611	1307.1	1334.4	3.2091
320	692.06	0.0014450	1416.6	1445.5	3.3996
340	637.23	0.0015693	1540.2	1571.6	3.6086
360	548.01	0.0018248	1703.6	1740.1	3.8787
Sat	490.19	0.0020400	1786.4	1827.2	4.0156

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01°C and 0.61165 kPa).

Subcooled Liquid

Water
SI

	P = 30000 kPa					P = 50000 kPa				
Temp. (^{o}C)	**ρ (kg/m^3)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**	**ρ (kg/m^3)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**
10	1013.5	9.8664E-04	41.193	70.792	0.14745	1022.3	9.7816E-04	40.559	89.467	0.14402
20	1011.5	9.8865E-04	82.112	111.77	0.28968	1019.9	9.8047E-04	80.931	129.95	0.28454
40	1004.9	9.9509E-04	164.05	193.90	0.56069	1013.0	9.8715E-04	161.90	211.25	0.55281
60	995.84	0.0010042	246.14	276.26	0.81564	1003.9	9.9616E-04	243.08	292.88	0.80549
80	984.71	0.0010155	328.40	358.86	1.0564	992.86	0.0010072	324.42	374.78	1.0442
100	971.82	0.0010290	410.87	441.74	1.2847	980.27	0.0010201	405.93	456.94	1.2705
120	957.35	0.0010445	493.66	525.00	1.5020	966.24	0.0010349	487.69	539.43	1.4859
140	941.39	0.0010623	576.89	608.76	1.7098	950.87	0.0010517	569.77	622.36	1.6916
160	923.97	0.0010823	660.74	693.21	1.9094	934.20	0.0010704	652.32	705.84	1.8889
180	905.07	0.0011049	745.40	778.54	2.1020	916.25	0.0010914	735.49	790.06	2.0790
200	884.62	0.0011304	831.10	865.02	2.2888	896.97	0.0011149	819.45	875.19	2.2628
220	862.46	0.0011595	918.14	952.93	2.4707	876.30	0.0011412	904.39	961.45	2.4414
240	838.40	0.0011927	1006.9	1042.7	2.6491	854.12	0.0011708	990.55	1049.09	2.6156
260	812.10	0.0012314	1097.8	1134.7	2.8250	830.26	0.0012044	1078.2	1138.4	2.7864
280	783.10	0.0012770	1191.5	1229.8	3.0001	804.48	0.0012430	1167.7	1229.9	2.9547
300	750.66	0.0013322	1288.9	1328.9	3.1760	776.48	0.0012879	1259.6	1324.0	3.1218
320	713.58	0.0014014	1391.6	1433.7	3.3557	745.79	0.0013409	1354.3	1421.4	3.2888
340	669.70	0.0014932	1502.3	1547.1	3.5438	711.78	0.0014049	1452.9	1523.1	3.4575
360	614.39	0.0016276	1626.7	1675.6	3.7498	673.51	0.0014848	1556.5	1630.7	3.6301
380	533.93	0.0018729	1782.0	1838.2	4.0025	629.57	0.0015884	1667.1	1746.5	3.8101
Sat	Supercritical					Supercritical				

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (0.01^{o}C and 0.61165 kPa).

Saturation Temperature Table

Water
AE

Temp. (^{o}F)	Pressure (psia)	V (ft^3/lb_m)		U (Btu/lb_m)		H (Btu/lb_m)		S ($\text{Btu/lb}_m{}^o\text{R}$)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
32.018	0.088713	0.016022	3299.7	0	1021.7	2.6319E-04	1075.9	0	2.1882
35	0.099980	0.016020	2945.4	3.0056	1022.7	3.0059	1077.2	0.0060944	2.1777
40	0.12173	0.016020	2443.3	8.0373	1024.3	8.0377	1079.4	0.016215	2.1604
45	0.14757	0.016021	2035.5	13.061	1026.0	13.061	1081.6	0.026219	2.1435
50	0.17814	0.016024	1702.8	18.077	1027.6	18.078	1083.8	0.036110	2.1271
55	0.21414	0.016029	1430.2	23.089	1029.3	23.089	1086.0	0.045895	2.1111
60	0.25640	0.016035	1206.0	28.096	1030.9	28.096	1088.2	0.055576	2.0954
65	0.30580	0.016043	1020.7	33.099	1032.5	33.100	1090.3	0.065158	2.0802
70	0.36336	0.016052	867.11	38.100	1034.2	38.101	1092.5	0.074644	2.0653
75	0.43017	0.016062	739.23	43.098	1035.8	43.100	1094.7	0.084037	2.0508
80	0.50747	0.016074	632.38	48.095	1037.4	48.097	1096.8	0.093340	2.0366
85	0.59660	0.016086	542.79	53.091	1039.0	53.093	1099.0	0.10255	2.0228
90	0.69904	0.016100	467.40	58.086	1040.6	58.088	1101.1	0.11168	2.0093
95	0.81642	0.016115	403.75	63.081	1042.3	63.083	1103.3	0.12073	1.9961
100	0.95051	0.016131	349.83	68.076	1043.9	68.078	1105.4	0.12969	1.9832
110	1.2767	0.016166	264.97	78.065	1047.1	78.069	1109.7	0.14738	1.9583
120	1.6950	0.016205	202.95	88.057	1050.3	88.062	1114.0	0.16477	1.9346
130	2.2259	0.016247	157.09	98.052	1053.4	98.059	1118.2	0.18187	1.9118
140	2.8930	0.016293	122.82	108.05	1056.6	108.06	1122.3	0.19868	1.8901
150	3.7232	0.016342	96.930	118.06	1059.7	118.07	1126.5	0.21523	1.8693
160	4.7472	0.016394	77.184	128.07	1062.7	128.08	1130.6	0.23152	1.8493
170	5.9998	0.016449	61.980	138.09	1065.8	138.11	1134.6	0.24756	1.8302
180	7.5195	0.016508	50.169	148.12	1068.8	148.14	1138.6	0.26336	1.8118
190	9.3496	0.016569	40.916	158.16	1071.8	158.18	1142.6	0.27893	1.7942
200	11.538	0.016633	33.609	168.21	1074.7	168.24	1146.5	0.29429	1.7772
210	14.136	0.016701	27.794	178.27	1077.5	178.31	1150.3	0.30943	1.7609
212	14.709	0.016715	26.779	180.28	1078.1	180.33	1151.1	0.31243	1.7577
220	17.201	0.016771	23.133	188.35	1080.4	188.40	1154.1	0.32436	1.7451
230	20.795	0.016845	19.371	198.44	1083.1	198.51	1157.7	0.33911	1.7299
240	24.986	0.016921	16.314	208.55	1085.8	208.63	1161.3	0.35366	1.7153
250	29.844	0.017001	13.815	218.68	1088.5	218.78	1164.8	0.36804	1.7011
260	35.447	0.017084	11.759	228.83	1091.0	228.95	1168.2	0.38225	1.6874
270	41.878	0.017170	10.058	239.01	1093.5	239.14	1171.5	0.39629	1.6741
280	49.222	0.017259	8.6431	249.21	1095.9	249.37	1174.7	0.41017	1.6612
290	57.574	0.017352	7.4600	259.44	1098.3	259.62	1177.8	0.42391	1.6487
300	67.029	0.017448	6.4658	269.69	1100.5	269.91	1180.7	0.43750	1.6365
310	77.691	0.017548	5.6263	279.98	1102.6	280.23	1183.6	0.45096	1.6246
320	89.667	0.017652	4.9142	290.30	1104.7	290.60	1186.3	0.46428	1.6131
330	103.07	0.017760	4.3075	300.66	1106.6	301.00	1188.8	0.47749	1.6018
340	118.02	0.017872	3.7884	311.06	1108.5	311.45	1191.3	0.49058	1.5908
350	134.63	0.017987	3.3425	321.50	1110.2	321.95	1193.5	0.50355	1.5800
360	153.03	0.018108	2.9580	331.99	1111.8	332.50	1195.6	0.51643	1.5694
370	173.36	0.018233	2.6252	342.52	1113.3	343.11	1197.5	0.52921	1.5591
380	195.74	0.018363	2.3361	353.11	1114.6	353.77	1199.3	0.54190	1.5489
390	220.33	0.018498	2.0841	363.75	1115.8	364.50	1200.9	0.55450	1.5388
400	247.26	0.018639	1.8638	374.45	1116.9	375.30	1202.2	0.56703	1.5289
410	276.68	0.018785	1.6706	385.21	1117.8	386.17	1203.4	0.57948	1.5192
420	308.76	0.018938	1.5006	396.04	1118.6	397.12	1204.4	0.59187	1.5096

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018^{o}F and 0.088713 psia).

Saturation Temperature Table

Water
AE

Temp. (^{o}F)	Pressure (psia)	V (ft^3/lb$_m$)		U (Btu/lb$_m$)		H (Btu/lb$_m$)		S (Btu/lb$_m$ ^{o}R)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
430	343.64	0.019097	1.3505	406.94	1119.2	408.15	1205.1	0.60420	1.5000
440	381.48	0.019263	1.2177	417.91	1119.6	419.27	1205.6	0.61648	1.4905
450	422.46	0.019437	1.0999	428.97	1119.9	430.49	1205.9	0.62872	1.4811
450	422.46	0.019437	1.0999	428.97	1119.9	430.49	1205.9	0.62872	1.4811
460	466.75	0.019619	0.99506	440.11	1119.9	441.81	1205.9	0.64092	1.4718
470	514.52	0.019810	0.90154	451.35	1119.8	453.24	1205.7	0.65309	1.4624
480	565.95	0.020010	0.81791	462.69	1119.4	464.78	1205.1	0.66524	1.4531
490	621.23	0.020220	0.74293	474.13	1118.8	476.46	1204.3	0.67738	1.4438
500	680.55	0.020441	0.67555	485.69	1118.0	488.27	1203.1	0.68952	1.4344
510	744.11	0.020675	0.61486	497.38	1116.9	500.23	1201.6	0.70167	1.4250
520	812.10	0.020923	0.56007	509.21	1115.5	512.35	1199.8	0.71384	1.4155
530	884.74	0.021185	0.51049	521.18	1113.9	524.65	1197.5	0.72604	1.4059
540	962.24	0.021464	0.46553	533.32	1111.9	537.14	1194.8	0.73829	1.3962
550	1044.8	0.021761	0.42466	545.63	1109.5	549.84	1191.7	0.75060	1.3863
560	1132.7	0.022080	0.38742	558.14	1106.8	562.77	1188.0	0.76299	1.3762
570	1226.2	0.022422	0.35341	570.87	1103.6	575.96	1183.8	0.77549	1.3658
580	1325.5	0.022791	0.32227	583.85	1099.9	589.44	1179.0	0.78812	1.3552
590	1430.8	0.023193	0.29369	597.10	1095.8	603.25	1173.6	0.80090	1.3443
600	1542.5	0.023631	0.26739	610.67	1091.0	617.42	1167.4	0.81388	1.3329
610	1660.9	0.024113	0.24311	624.60	1085.6	632.02	1160.3	0.82710	1.3210
620	1786.2	0.024647	0.22062	638.96	1079.3	647.11	1152.3	0.84062	1.3085
630	1918.9	0.025247	0.19971	653.82	1072.2	662.79	1143.1	0.85451	1.2953
640	2059.2	0.025930	0.18017	669.30	1063.9	679.19	1132.6	0.86888	1.2812
650	2207.8	0.026720	0.16180	685.55	1054.3	696.48	1120.4	0.88387	1.2659
660	2364.9	0.027661	0.14439	702.84	1042.8	714.96	1106.1	0.89974	1.2491
670	2531.2	0.028826	0.12768	721.61	1029.1	735.12	1088.9	0.91690	1.2301
680	2707.3	0.030361	0.11132	742.67	1011.8	757.89	1067.6	0.93611	1.2078
705.10	3200.1	0.049747	0.049747	867.19	867.19	896.67	896.67	1.0533	1.0533

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018^{o}F and 0.088713 psia).

Saturation Pressure Table

Water
AE

Pressure (psia)	Temp. (^oF)	V (ft^3/lb_m)		U (Btu/lb_m)		H (Btu/lb_m)		S ($Btu/lb_m{}^oR$)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
1	101.70	0.016137	333.50	69.766	1044.4	69.769	1106.2	0.13271	1.9789
2	126.03	0.016230	173.71	94.079	1052.2	94.085	1116.5	0.17511	1.9207
3	141.42	0.016300	118.70	109.47	1057.0	109.48	1122.9	0.20104	1.8871
4	152.91	0.016357	90.626	120.97	1060.6	120.98	1127.7	0.22000	1.8634
5	162.18	0.016406	73.521	130.26	1063.4	130.27	1131.5	0.23504	1.8451
6	170.00	0.016449	61.977	138.09	1065.8	138.11	1134.6	0.24756	1.8302
7	176.79	0.016489	53.647	144.90	1067.8	144.92	1137.4	0.25832	1.8176
8	182.81	0.016525	47.343	150.93	1069.6	150.96	1139.8	0.26776	1.8068
9	188.22	0.016558	42.402	156.37	1071.2	156.40	1141.9	0.27619	1.7973
10	193.16	0.016589	38.421	161.33	1072.7	161.36	1143.8	0.28381	1.7887
14.696	211.95	0.016714	26.802	180.24	1078.1	180.28	1151.0	0.31236	1.7578
15	212.99	0.016722	26.294	181.28	1078.4	181.33	1151.4	0.31391	1.7561
20	227.92	0.016829	20.091	196.34	1082.6	196.40	1157.0	0.33605	1.7331
25	240.03	0.016922	16.305	208.58	1085.8	208.66	1161.3	0.35371	1.7152
30	250.30	0.017003	13.747	218.98	1088.5	219.08	1164.9	0.36847	1.7007
35	259.25	0.017078	11.899	228.07	1090.8	228.18	1168.0	0.38119	1.6884
40	267.22	0.017146	10.500	236.17	1092.8	236.30	1170.6	0.39240	1.6777
45	274.41	0.017209	9.4019	243.51	1094.6	243.65	1172.9	0.40243	1.6684
50	280.99	0.017268	8.5168	250.22	1096.2	250.38	1175.0	0.41153	1.6599
55	287.05	0.017324	7.7876	256.42	1097.6	256.59	1176.9	0.41987	1.6523
60	292.68	0.017378	7.1761	262.19	1098.9	262.38	1178.6	0.42757	1.6454
65	297.95	0.017428	6.6556	267.59	1100.0	267.80	1180.2	0.43473	1.6390
70	302.91	0.017477	6.2071	272.68	1101.1	272.91	1181.6	0.44143	1.6330
75	307.58	0.017524	5.8165	277.49	1102.1	277.74	1182.9	0.44772	1.6275
80	312.02	0.017569	5.4730	282.06	1103.1	282.32	1184.1	0.45366	1.6223
85	316.24	0.017613	5.1687	286.41	1103.9	286.69	1185.3	0.45928	1.6174
90	320.26	0.017655	4.8970	290.57	1104.7	290.87	1186.3	0.46463	1.6128
95	324.11	0.017696	4.6530	294.56	1105.5	294.87	1187.3	0.46973	1.6084
100	327.81	0.017736	4.4326	298.38	1106.2	298.71	1188.3	0.47460	1.6043
105	331.35	0.017775	4.2324	302.07	1106.9	302.41	1189.2	0.47927	1.6003
110	334.77	0.017813	4.0498	305.62	1107.5	305.98	1190.0	0.48375	1.5965
115	338.07	0.017850	3.8826	309.05	1108.1	309.43	1190.8	0.48805	1.5929
120	341.25	0.017886	3.7288	312.36	1108.7	312.76	1191.5	0.49221	1.5894
125	344.33	0.017921	3.5870	315.58	1109.2	315.99	1192.2	0.49621	1.5861
130	347.32	0.017956	3.4556	318.70	1109.7	319.13	1192.9	0.50008	1.5829
135	350.21	0.017990	3.3337	321.72	1110.2	322.17	1193.6	0.50383	1.5798
140	353.03	0.018023	3.2201	324.67	1110.7	325.14	1194.2	0.50746	1.5768
145	355.76	0.018056	3.1142	327.54	1111.1	328.02	1194.7	0.51098	1.5739
150	358.42	0.018089	3.0150	330.33	1111.5	330.83	1195.3	0.51440	1.5711
155	361.01	0.018120	2.9220	333.05	1111.9	333.57	1195.8	0.51773	1.5684
160	363.54	0.018152	2.8347	335.71	1112.3	336.25	1196.3	0.52097	1.5657
165	366.01	0.018182	2.7524	338.31	1112.7	338.86	1196.8	0.52412	1.5632
170	368.41	0.018213	2.6748	340.85	1113.0	341.42	1197.2	0.52719	1.5607
175	370.77	0.018243	2.6015	343.33	1113.4	343.92	1197.7	0.53019	1.5583
180	373.07	0.018272	2.5321	345.76	1113.7	346.37	1198.1	0.53311	1.5559
185	375.32	0.018301	2.4664	348.15	1114.0	348.77	1198.5	0.53597	1.5536
190	377.52	0.018330	2.4040	350.48	1114.3	351.13	1198.9	0.53876	1.5514
195	379.68	0.018359	2.3447	352.77	1114.6	353.43	1199.2	0.54150	1.5492

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018^oF and 0.088667 psia).

Saturation Pressure Table

Water
AE

Pressure (psia)	Temp. (^{o}F)	V (ft^3/lb$_m$)		U (Btu/lb$_m$)		H (Btu/lb$_m$)		S (Btu/lb$_m$ ^{o}R)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
200	381.80	0.018387	2.2882	355.02	1114.8	355.70	1199.6	0.54417	1.5470
220	389.87	0.018496	2.0872	363.61	1115.8	364.36	1200.8	0.55434	1.5390
240	397.39	0.018602	1.9185	371.65	1116.6	372.48	1201.9	0.56376	1.5315
260	404.44	0.018703	1.7750	379.214	1117.3	380.114	1202.8	0.57256	1.5246
280	411.07	0.018801	1.6513	386.368	1117.9	387.342	1203.5	0.58081	1.5182
300	417.35	0.018897	1.5435	393.16	1118.4	394.21	1204.1	0.58859	1.5121
320	423.31	0.018990	1.4488	399.64	1118.8	400.77	1204.7	0.59596	1.5064
340	429.00	0.019081	1.3648	405.84	1119.1	407.04	1205.1	0.60296	1.5010
360	434.42	0.019170	1.2898	411.78	1119.4	413.06	1205.4	0.60964	1.4958
380	439.62	0.019257	1.2225	417.50	1119.6	418.85	1205.6	0.61602	1.4909
400	444.62	0.019342	1.1616	423.00	1119.8	424.44	1205.8	0.62213	1.4862
420	449.42	0.019427	1.1064	428.32	1119.9	429.84	1205.9	0.62801	1.4817
440	454.05	0.019510	1.0560	433.47	1119.9	435.06	1206.0	0.63366	1.4773
460	458.52	0.019592	1.0098	438.46	1119.9	440.13	1205.9	0.63912	1.4732
480	462.85	0.019672	0.96733	443.30	1119.9	445.05	1205.9	0.64439	1.4691
500	467.04	0.019752	0.92815	448.01	1119.8	449.84	1205.8	0.64949	1.4652
550	476.98	0.019948	0.84223	459.25	1119.6	461.28	1205.3	0.66157	1.4559
600	486.24	0.020140	0.77015	469.82	1119.1	472.06	1204.6	0.67282	1.4473
650	494.94	0.020328	0.70878	479.82	1118.4	482.27	1203.8	0.68337	1.4392
700	503.13	0.020513	0.65586	489.34	1117.7	492.00	1202.7	0.69332	1.4315
750	510.89	0.020697	0.60974	498.43	1116.8	501.31	1201.5	0.70275	1.4242
800	518.27	0.020879	0.56917	507.15	1115.8	510.24	1200.1	0.71173	1.4171
850	525.30	0.021060	0.53319	515.53	1114.7	518.85	1198.6	0.72030	1.4104
900	532.02	0.021240	0.50106	523.62	1113.5	527.16	1197.0	0.72851	1.4039
950	538.46	0.021420	0.47217	531.44	1112.2	535.21	1195.2	0.73640	1.3977
1000	544.65	0.021600	0.44605	539.02	1110.8	543.02	1193.4	0.74400	1.3916
1100	556.35	0.021961	0.40062	553.55	1107.8	558.02	1189.4	0.75846	1.3799
1200	567.26	0.022326	0.36243	567.36	1104.5	572.32	1185.0	0.77205	1.3687
1300	577.49	0.022696	0.32983	580.57	1100.9	586.03	1180.3	0.78493	1.3579
1400	587.14	0.023074	0.30163	593.28	1097.0	599.26	1175.2	0.79722	1.3474
1500	596.26	0.023462	0.27697	605.56	1092.9	612.08	1169.8	0.80900	1.3372
1600	604.93	0.023862	0.25518	617.49	1088.4	624.56	1164.0	0.82036	1.3271
1700	613.18	0.024277	0.23577	629.13	1083.7	636.77	1157.9	0.83137	1.3171
1800	621.07	0.024708	0.21832	640.52	1078.6	648.76	1151.4	0.84208	1.3072
1900	628.61	0.025160	0.20252	651.73	1073.2	660.58	1144.5	0.85255	1.2972
2000	635.85	0.025635	0.18813	662.79	1067.5	672.28	1137.2	0.86284	1.2872
2100	642.80	0.026139	0.17491	673.77	1061.4	683.93	1129.4	0.87300	1.2770
2200	649.49	0.026677	0.16271	684.71	1054.8	695.57	1121.1	0.88309	1.2667
2300	655.94	0.027257	0.15136	695.68	1047.7	707.28	1112.2	0.89317	1.2561
2400	662.16	0.027890	0.14073	706.75	1040.1	719.15	1102.6	0.90332	1.2452
2500	668.17	0.028590	0.13071	718.03	1031.8	731.27	1092.3	0.91363	1.2338
2600	673.98	0.029380	0.12116	729.65	1022.7	743.79	1081.0	0.92423	1.2217
2700	679.60	0.030290	0.11198	741.76	1012.5	756.90	1068.5	0.93528	1.2088
2800	685.04	0.031363	0.10300	754.58	1001.0	770.85	1054.4	0.94699	1.1947
3200.1	705.10	0.049747	0.049747	867.19	867.19	896.67	896.67	1.0533	1.0533

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018^{o}F and 0.088713 psia).

Superheated Vapor

Water AE

P = 1 psia (T_{sat} = 101.70°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	333.50	1044.4	1106.2	1.9789
150	362.57	1061.1	1128.2	2.0166
200	392.52	1078.2	1150.9	2.0523
250	422.41	1095.4	1173.6	2.0856
300	452.27	1112.7	1196.5	2.1167
350	482.10	1130.2	1219.5	2.1460
400	511.92	1147.8	1242.6	2.1737
500	571.54	1183.6	1289.5	2.2253
600	631.14	1220.2	1337.1	2.2725
700	690.73	1257.6	1385.6	2.3162
800	750.31	1295.9	1434.9	2.3570
1000	869.47	1375.2	1536.2	2.4316
1200	988.62	1458.1	1641.1	2.4989
1400	1107.8	1544.8	1749.9	2.5608

P = 5 psia (T_{sat} = 162.2°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	73.52	1063.4	1131.5	1.8451
150				
200	78.15	1076.9	1149.3	1.8729
250	84.22	1094.5	1172.5	1.9068
300	90.25	1112.1	1195.6	1.9383
350	96.25	1129.7	1218.8	1.9678
400	102.25	1147.4	1242.1	1.9957
500	114.21	1183.4	1289.1	2.0475
600	126.15	1220.0	1336.8	2.0948
700	138.09	1257.5	1385.3	2.1385
800	150.02	1295.8	1434.7	2.1793
1000	173.86	1375.1	1536.1	2.2540
1200	197.70	1458.0	1641.1	2.3214
1400	221.54	1544.7	1749.8	2.3832

P = 10 psia (T_{sat} = 193.2°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	38.42	1072.7	1143.8	1.7887
150				
200	38.85	1075.2	1147.2	1.7938
250	41.94	1093.4	1171.0	1.8287
300	44.99	1111.2	1194.6	1.8607
350	48.02	1129.1	1218.0	1.8906
400	51.04	1146.9	1241.4	1.9187
500	57.04	1183.0	1288.6	1.9707
600	63.03	1219.8	1336.5	2.0181
700	69.01	1257.3	1385.1	2.0619
800	74.98	1295.7	1434.5	2.1028
1000	86.91	1375.0	1535.9	2.1774
1200	98.84	1457.9	1641.0	2.2449
1400	110.8	1544.6	1749.7	2.3067

P = 14.696 psia (T_{sat} = 212.0°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	26.802	1078.1	1151.0	1.7578
250	28.428	1092.3	1169.6	1.7847
300	30.529	1110.5	1193.5	1.8172
350	32.607	1128.5	1217.2	1.8474
400	34.671	1146.5	1240.8	1.8757
500	38.774	1182.7	1288.2	1.9279
600	42.858	1219.5	1336.2	1.9754
800	51.002	1295.5	1434.3	2.0602
1000	59.129	1374.9	1535.8	2.1349
1200	67.248	1457.9	1640.9	2.2024
1400	75.363	1544.6	1749.7	2.2642
1600	83.476	1635.0	1862.2	2.3217

P = 20 psia (T_{sat} = 227.9°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	20.091	1082.6	1157.0	1.7331
250	20.795	1091.0	1168.0	1.7489
300	22.361	1109.5	1192.4	1.7820
350	23.903	1127.8	1216.3	1.8125
400	25.429	1145.9	1240.1	1.8411
500	28.458	1182.3	1287.7	1.8935
600	31.467	1219.3	1335.8	1.9412
800	37.461	1295.4	1434.1	2.0261
1000	43.438	1374.8	1535.6	2.1008
1200	49.407	1457.8	1640.8	2.1683
1400	55.372	1544.5	1749.6	2.2302
1600	61.335	1635.0	1862.1	2.2877

P = 40 psia (T_{sat} = 267.2°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	10.500	1092.8	1170.6	1.6777
250				
300	11.040	1105.9	1187.7	1.7008
350	11.841	1125.1	1212.8	1.7328
400	12.625	1143.8	1237.3	1.7622
500	14.165	1181.0	1285.9	1.8156
600	15.686	1218.3	1334.5	1.8637
800	18.702	1294.7	1433.3	1.9491
1000	21.700	1374.3	1535.1	2.0241
1200	24.691	1457.5	1640.4	2.0917
1400	27.678	1544.3	1749.3	2.1536
1600	30.662	1634.8	1861.9	2.2111

P = 60 psia (T_{sat} = 292.7°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	7.1761	1098.9	1178.6	1.6454
300	7.2603	1102.0	1182.7	1.6507
350	7.8172	1122.3	1209.2	1.6845
400	8.3548	1141.7	1234.5	1.7149
600	10.426	1217.3	1333.1	1.8181
800	12.448	1294.1	1432.4	1.9039
1000	14.454	1373.9	1534.5	1.9791
1200	16.452	1457.2	1640.0	2.0468
1400	18.446	1544.1	1749.0	2.1088
1600	20.438	1634.6	1861.7	2.1663
1800	22.428	1728.8	1978.0	2.2202

P = 80 psia (T_{sat} = 312.0°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	5.4730	1103.1	1184.1	1.6223
300				
350	5.8031	1119.3	1205.3	1.6491
400	6.2186	1139.5	1231.6	1.6806
600	7.7951	1216.2	1331.7	1.7854
800	9.3217	1293.5	1431.6	1.8717
1000	10.831	1373.5	1534.0	1.9471
1200	12.333	1456.9	1639.6	2.0148
1400	13.831	1543.8	1748.7	2.0769
1600	15.326	1634.4	1861.5	2.1345
1800	16.819	1728.6	1977.8	2.1884

P = 100 psia (T_{sat} = 327.8°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	4.4326	1106.2	1188.3	1.6043
300				
350	4.5926	1116.2	1201.2	1.6205
400	4.9359	1137.2	1228.6	1.6532
600	6.2166	1215.2	1330.3	1.7598
800	7.4457	1292.9	1430.8	1.8466
1000	8.6575	1373.1	1533.4	1.9222
1200	9.8614	1456.5	1639.2	1.9900
1400	11.061	1543.6	1748.4	2.0522
1600	12.258	1634.2	1861.2	2.1098
1800	13.454	1728.5	1977.6	2.1637

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

Superheated Vapor

Water
AE

	P = 120 psia (T_{sat} = 341.3°F)				P = 140 psia (T_{sat} = 353.0°F)				P = 160 psia (T_{sat} = 363.5°F)			
T °F	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R
Sat	3.7288	1108.7	1191.5	1.5894	3.2201	1110.7	1194.2	1.5768	2.8347	1112.3	1196.3	1.5657
350	3.7834	1112.8	1196.9	1.5961								
400	4.0799	1134.8	1225.4	1.6303	3.4676	1132.3	1222.2	1.6103	3.0076	1129.7	1218.8	1.5925
450	4.3612	1155.3	1252.2	1.6606	3.7147	1153.4	1249.7	1.6415	3.2293	1151.5	1247.2	1.6245
500	4.6340	1175.2	1278.2	1.6883	3.9525	1173.7	1276.1	1.6697	3.4412	1172.1	1274.1	1.6534
600	5.1642	1214.2	1328.9	1.7387	4.4124	1213.1	1327.5	1.7207	3.8484	1212.1	1326.1	1.7049
700	5.6829	1253.0	1379.3	1.7841	4.8603	1252.2	1378.2	1.7664	4.2434	1251.5	1377.2	1.7510
800	6.1950	1292.3	1429.9	1.8260	5.3016	1291.6	1429.1	1.8085	4.6316	1291.0	1428.2	1.7932
900	6.7030	1332.1	1481.1	1.8650	5.7388	1331.6	1480.4	1.8476	5.0156	1331.1	1479.7	1.8325
1000	7.2083	1372.7	1532.8	1.9018	6.1732	1372.2	1532.3	1.8845	5.3968	1371.8	1531.7	1.8695
1200	8.2137	1456.2	1638.8	1.9697	7.0367	1455.9	1638.4	1.9526	6.1540	1455.6	1638.0	1.9376
1400	9.2148	1543.3	1748.1	2.0319	7.8960	1543.1	1747.8	2.0148	6.9069	1542.9	1747.5	1.9999
1600	10.213	1634.0	1861.0	2.0896	8.7529	1633.9	1860.8	2.0725	7.6574	1633.7	1860.5	2.0577
1800	11.211	1728.3	1977.4	2.1435	9.6082	1728.2	1977.2	2.1264	8.4063	1728.0	1977.1	2.1116

	P = 180 psia (T_{sat} = 373.1°F)				P = 200 psia (T_{sat} = 381.8°F)				P = 250 psia (T_{sat} = 401.0°F)			
T °F	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R
Sat	2.5321	1113.7	1198.1	1.5559	2.2882	1114.8	1199.6	1.5470	1.8440	1117.0	1202.4	1.5280
400	2.6490	1127.0	1215.3	1.5763	2.3615	1124.2	1211.7	1.5613				
500	3.0433	1170.6	1272.007	1.6387	2.7247	1169.0	1269.887	1.6254	2.1506	1164.9	1264.419	1.5964
600	3.4097	1211.0	1324.6	1.6909	3.0586	1209.9	1323.2	1.6782	2.4264	1207.1	1319.5	1.6510
700	3.7635	1250.7	1376.1	1.7373	3.3795	1249.8	1375.0	1.7250	2.6883	1247.8	1372.3	1.6986
800	4.1104	1290.4	1427.4	1.7798	3.6934	1289.8	1426.6	1.7676	2.9428	1288.2	1424.4	1.7417
900	4.4531	1330.6	1479.0	1.8192	4.0030	1330.1	1478.3	1.8072	3.1930	1328.8	1476.6	1.7816
1000	4.7929	1371.4	1531.1	1.8562	4.3098	1371.0	1530.6	1.8443	3.4403	1369.9	1529.2	1.8189
1200	5.4674	1455.3	1637.5	1.9245	4.9182	1455.0	1637.1	1.9127	3.9295	1454.2	1636.1	1.8876
1400	6.1377	1542.6	1747.2	1.9868	5.5222	1542.4	1746.9	1.9751	4.4144	1541.8	1746.2	1.9501
1600	6.80539	1633.5	1860.3	2.0446	6.12377	1633.3	1860.1	2.0329	4.8968	1632.8	1859.5	2.0080
1800	7.47155	1727.8	1976.9	2.0986	6.72375	1727.7	1976.7	2.0869	5.37769	1727.3	1976.3	2.0621

	P = 300 psia (T_{sat} = 417.4°F)				P = 350 psia (T_{sat} = 431.7°F)				P = 400 psia (T_{sat} = 444.6°F)			
T °F	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R	**V** ft^3/lb$_m$	**U** Btu/lb$_m$	**H** Btu/lb$_m$	**S** Btu/lb$_m$°R
Sat	1.5435	1118.4	1204.1	1.5121	1.3263	1119.3	1205.2	1.4984	1.1616	1119.8	1205.8	1.4862
500	1.7670	1160.5	1258.7	1.5716	1.4921	1156.0	1252.7	1.5497	1.2851	1151.2	1246.4	1.5298
600	2.00461	1204.3	1315.7	1.6282	1.70301	1201.4	1311.8	1.6084	1.47653	1198.4	1307.8	1.5907
700	2.22731	1245.8	1369.5	1.6767	1.89791	1243.7	1366.7	1.6579	1.65073	1241.5	1363.8	1.6413
800	2.44237	1286.6	1422.3	1.7204	2.08483	1285.0	1420.1	1.7021	1.81661	1283.3	1417.9	1.6860
900	2.65291	1327.5	1474.9	1.7605	2.26711	1326.2	1473.1	1.7426	1.97772	1324.9	1471.4	1.7269
1000	2.86054	1368.8	1527.7	1.7981	2.44642	1367.8	1526.3	1.7803	2.13582	1366.7	1524.9	1.7649
1200	3.27038	1453.5	1635.1	1.8670	2.79958	1452.7	1634.1	1.8495	2.44647	1451.9	1633.1	1.8343
1400	3.6759	1541.2	1745.4	1.9297	3.14838	1540.6	1744.6	1.9124	2.75274	1540.0	1743.9	1.8973
1600	4.0789	1632.3	1858.9	1.9877	3.49466	1631.9	1858.4	1.9704	3.05647	1631.4	1857.8	1.9555
1800	4.48032	1726.9	1975.8	2.0418	3.83935	1726.5	1975.4	2.0247	3.35861	1726.1	1974.9	2.0098

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

Superheated Vapor

Water AE

P = 450 psia (T_{sat} = 456.3°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	1.0324	1119.9	1206.0	1.4752
500	1.1233	1146.2	1239.8	1.5113
550	1.2152	1171.8	1273.1	1.5452
600	1.3001	1195.4	1303.7	1.5748
650	1.3807	1217.7	1332.8	1.6016
700	1.4584	1239.4	1360.9	1.6264
750	1.5339	1260.6	1388.5	1.6497
800	1.6079	1281.7	1415.7	1.6717
900	1.7526	1323.6	1469.6	1.7129
1000	1.8942	1365.6	1523.4	1.7511
1200	2.1718	1451.1	1632.1	1.8209
1400	2.4450	1539.4	1743.1	1.8840
1600	2.7157	1630.9	1857.2	1.9423
1800	2.9847	1725.8	1974.5	1.9966

P = 500 psia (T_{sat} = 467.0°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.92815	1119.8	1205.8	1.4652
500	0.99303	1140.8	1232.8	1.4938
550	1.0797	1167.9	1267.8	1.5295
600	1.1588	1192.2	1299.5	1.5601
650	1.2331	1215.1	1329.3	1.5876
700	1.3044	1237.2	1357.9	1.6128
750	1.3735	1258.7	1385.9	1.6365
800	1.4410	1280.0	1413.4	1.6588
900	1.5725	1322.3	1467.9	1.7003
1000	1.7009	1364.5	1522.0	1.7387
1200	1.9521	1450.3	1631.1	1.8088
1400	2.1988	1538.8	1742.4	1.8721
1600	2.4430	1630.4	1856.6	1.9304
1800	2.6856	1725.4	1974.0	1.9848

P = 550 psia (T_{sat} = 477.0°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.8422	1119.6	1205.3	1.4559
500	0.8856	1135.1	1225.3	1.4770
550	0.9685	1163.7	1262.4	1.5147
600	1.0429	1189.0	1295.2	1.5464
650	1.1122	1212.5	1325.7	1.5746
700	1.1783	1235.0	1355.0	1.6004
750	1.2421	1256.8	1383.3	1.6243
800	1.3043	1278.4	1411.2	1.6469
900	1.4251	1320.9	1466.1	1.6888
1000	1.5428	1363.4	1520.6	1.7275
1200	1.7723	1449.5	1630.0	1.7978
1400	1.9974	1538.2	1741.6	1.8612
1600	2.2199	1630.0	1856.1	1.9197
1800	2.4409	1725.0	1973.6	1.9741

P = 600 psia (T_{sat} = 486.3°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.77015	1119.1	1204.6	1.4473
500	0.79526	1129.0	1217.3	1.4606
550	0.87542	1159.4	1256.7	1.5006
600	0.94605	1185.7	1290.8	1.5336
700	1.0732	1232.7	1351.9	1.5888
800	1.1904	1276.7	1408.9	1.6359
900	1.3023	1319.6	1464.3	1.6782
1000	1.4110	1362.3	1519.1	1.7171
1200	1.6225	1448.8	1629.0	1.7877
1400	1.8296	1537.6	1740.9	1.8513
1600	2.0340	1629.5	1855.5	1.9098
1800	2.2369	1724.6	1973.1	1.9643

P = 650 psia (T_{sat} = 494.9°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.70878	1118.4	1203.8	1.4392
500	0.71781	1122.4	1208.8	1.4444
550	0.79625	1154.9	1250.8	1.4871
600	0.86392	1182.2	1286.2	1.5214
700	0.98410	1230.4	1348.8	1.5779
800	1.0939	1275.0	1406.6	1.6257
900	1.1983	1318.2	1462.5	1.6684
1000	1.2994	1361.2	1517.6	1.7075
1200	1.4957	1448.0	1628.0	1.7784
1400	1.6875	1537.0	1740.1	1.8422
1600	1.8767	1629.0	1854.9	1.9008
1800	2.0643	1724.2	1972.7	1.9553

P = 700 psia (T_{sat} = 503.1°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.65586	1117.7	1202.7	1.4315
500	0.02044	485.6	488.3	0.6894
550	0.72799	1150.2	1244.6	1.4740
600	0.79332	1178.7	1281.5	1.5097
700	0.90769	1228.1	1345.7	1.5677
800	1.0112	1273.2	1404.3	1.6162
900	1.1092	1316.9	1460.7	1.6592
1000	1.2038	1360.1	1516.2	1.6986
1200	1.3871	1447.2	1627.0	1.7697
1400	1.5658	1536.4	1739.4	1.8337
1600	1.7419	1628.5	1854.3	1.8924
1800	1.9164	1723.8	1972.2	1.9470

P = 750 psia (T_{sat} = 510.9°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.60974	1116.8	1201.5	1.4242
550	0.66842	1145.3	1238.1	1.4612
600	0.73192	1175.0	1276.7	1.4985
700	0.84139	1225.7	1342.5	1.5579
800	0.93955	1271.5	1402.0	1.6071
900	1.0319	1315.5	1458.8	1.6506
1000	1.1209	1359.0	1514.7	1.6902
1200	1.2929	1446.4	1625.9	1.7617
1400	1.4603	1535.8	1738.6	1.8257
1600	1.6251	1628.1	1853.8	1.8845
1800	1.7882	1723.4	1971.8	1.9392

P = 800 psia (T_{sat} = 518.3°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.56917	1115.8	1200.1	1.4171
550				
600	0.67799	1171.3	1271.7	1.4877
700	0.78330	1223.3	1339.3	1.5487
800	0.87678	1269.7	1399.6	1.5986
900	0.96433	1314.2	1457.0	1.6425
1000	1.0484	1357.9	1513.2	1.6823
1200	1.2105	1445.6	1624.9	1.7541
1400	1.3680	1535.2	1737.8	1.8183
1600	1.5228	1627.6	1853.2	1.8772
1800	1.6761	1723.0	1971.3	1.9319

P = 900 psia (T_{sat} = 532.0°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.50106	1113.5	1197.0	1.4039
550				
600	0.58750	1163.4	1261.3	1.4668
700	0.68629	1218.3	1332.7	1.5313
800	0.77207	1266.1	1394.8	1.5827
900	0.85160	1311.4	1453.3	1.6274
1000	0.92751	1355.7	1510.2	1.6678
1200	1.0732	1444.0	1622.8	1.7401
1400	1.2141	1534.0	1736.3	1.8047
1600	1.3524	1626.63	1852.0	1.8637
1800	1.4891	1722.3	1970.4	1.9186

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

Superheated Vapor

Water AE

P = 1000 psia (T_{sat} = 544.7°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.44605	1110.8	1193.4	1.3916
600	0.51431	1154.9	1250.1	1.4466
650	0.56410	1185.9	1290.3	1.4837
700	0.60844	1213.2	1325.8	1.5151
750	0.64944	1238.4	1358.7	1.5428
800	0.68821	1262.5	1389.9	1.5681
900	0.76136	1308.6	1449.6	1.6137
1000	0.83077	1353.4	1507.2	1.6546
1100	0.89782	1397.8	1564.1	1.6923
1200	0.96327	1442.4	1620.8	1.7275
1300	1.0276	1487.3	1677.6	1.7608
1400	1.0910	1532.8	1734.8	1.7924
1600	1.2161	1625.7	1850.9	1.8516
1800	1.3396	1721.5	1969.5	1.9066

P = 1500 psia (T_{sat} = 596.3°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.27697	1092.9	1169.8	1.3372
600	0.28189	1097.9	1176.2	1.3432
650	0.33310	1148.0	1240.5	1.4026
700	0.37198	1184.4	1287.7	1.4443
750	0.40534	1215.2	1327.8	1.4781
800	0.43550	1243.0	1364.0	1.5075
900	0.49014	1294.0	1430.1	1.5580
1000	0.54031	1341.8	1491.8	1.6018
1100	0.58780	1388.3	1551.5	1.6414
1200	0.63355	1434.3	1610.3	1.6779
1300	0.67808	1480.3	1668.7	1.7121
1400	0.72172	1526.7	1727.1	1.7444
1600	0.80714	1620.9	1845.1	1.8046
1800	0.89090	1717.6	1965.0	1.8602

P = 2000 psia (T_{sat} = 635.9°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.18813	1067.5	1137.2	1.2872
600				
650	0.20586	1092.1	1168.4	1.3155
700	0.24894	1148.4	1240.6	1.3793
750	0.28074	1188.2	1292.1	1.4228
800	0.30763	1221.3	1335.2	1.4577
900	0.35390	1278.3	1409.4	1.5145
1000	0.39479	1329.6	1475.8	1.5616
1100	0.43266	1378.4	1538.6	1.6032
1200	0.46864	1426.0	1599.6	1.6411
1300	0.50332	1473.3	1659.7	1.6763
1400	0.53708	1520.5	1719.4	1.7093
1600	0.60269	1616.0	1839.3	1.7705
1800	0.66660	1713.7	1960.6	1.8267

P = 2500 psia (T_{sat} = 668.2°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.13071	1031.8	1092.3	1.2338
700	0.16849	1099.1	1177.1	1.3081
750	0.20327	1155.7	1249.8	1.3695
800	0.22949	1196.7	1302.9	1.4125
850	0.25174	1231.0	1347.5	1.4473
900	0.27165	1261.6	1387.3	1.4772
950	0.29001	1290.0	1424.2	1.5038
1000	0.30726	1316.9	1459.2	1.5282
1200	0.36966	1417.6	1588.7	1.6114
1400	0.42631	1514.3	1711.7	1.6814
1600	0.48004	1611.2	1833.4	1.7436
1800	0.53204	1709.8	1956.1	1.8004

P = 3000 psia (T_{sat} = 695.4°F)

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.08466	970.58	1017.6	1.1597
700	0.09837	1006.0	1060.6	1.1969
750	0.14840	1114.8	1197.2	1.3127
800	0.17601	1168.3	1266.1	1.3685
850	0.19771	1209.0	1318.8	1.4096
900	0.21639	1243.6	1363.8	1.4433
950	0.23321	1274.7	1404.3	1.4726
1000	0.24876	1303.7	1441.9	1.4988
1200	0.30367	1409.0	1577.7	1.5861
1400	0.35249	1508.0	1703.8	1.6579
1600	0.39830	1606.3	1827.6	1.7211
1800	0.44236	1705.8	1951.6	1.7785

P = 3500 psia

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
700	0.030653	760.51	780.37	0.95177
750	0.10460	1058.3	1126.1	1.2443
800	0.13639	1135.0	1223.4	1.3233
850	0.15847	1184.6	1287.3	1.3730
900	0.17659	1224.2	1338.7	1.4115
950	0.19245	1258.6	1383.3	1.4438
1000	0.20687	1289.9	1423.9	1.4721
1200	0.25654	1400.2	1566.5	1.5638
1400	0.29978	1501.7	1696.0	1.6375
1600	0.33994	1601.4	1821.7	1.7017
1800	0.37833	1701.9	1947.1	1.7598

P = 4000 psia

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
700	0.028713	742.81	764.08	0.93536
750	0.063703	962.70	1009.9	1.1417
800	0.10520	1095.0	1172.9	1.2743
850	0.12848	1157.4	1252.6	1.3364
900	0.14647	1203.3	1311.8	1.3808
1000	0.17538	1275.4	1405.3	1.4473
1200	0.22121	1391.3	1555.1	1.5437
1400	0.26028	1495.3	1688.1	1.6193
1600	0.29620	1596.5	1815.9	1.6846
1800	0.33033	1698.0	1942.6	1.7434

P = 5000 psia

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
700	0.026777	722.26	747.05	0.91627
750	0.033730	822.38	853.61	1.0061
800	0.059365	987.53	1042.5	1.1589
850	0.085508	1093.1	1172.3	1.2602
900	0.10390	1156.7	1252.9	1.3207
1000	0.13128	1244.8	1366.4	1.4014
1200	0.17185	1373.0	1532.1	1.5081
1400	0.20508	1482.4	1672.3	1.5879
1600	0.23505	1586.7	1804.3	1.6553
1800	0.26320	1690.1	1933.8	1.7153

P = 6000 psia

T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
700	0.025635	708.54	737.03	0.90344
750	0.029815	789.22	822.34	0.97541
800	0.039491	897.71	941.58	1.0719
850	0.058151	1019.2	1083.8	1.1827
900	0.075844	1104.3	1188.5	1.2612
1000	0.10208	1212.2	1325.6	1.3587
1200	0.13911	1354.3	1508.8	1.4768
1400	0.16841	1469.3	1656.5	1.5609
1600	0.19438	1576.8	1792.7	1.6305
1800	0.21853	1682.2	1925.0	1.6918

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

Subcooled Liquid

Water
AE

P = 500 psia (467.0°F)

T °F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
32.02	62.524	0.015994	0.028146	1.5090	5.4617E-05
50	62.509	0.015998	18.040	19.521	0.036035
100	62.086	0.016107	67.906	69.397	0.12939
150	61.286	0.016317	117.78	119.29	0.21477
200	60.217	0.016607	167.82	169.35	0.29369
250	58.921	0.016972	218.18	219.75	0.36733
300	57.416	0.017417	269.09	270.71	0.43671
350	55.697	0.017954	320.85	322.51	0.50275
400	53.737	0.018609	373.86	375.58	0.56634
450	51.481	0.019425	428.73	430.53	0.62845
500					
Sat	50.628	0.019752	448.01	449.84	0.64949

P = 1000 psia (544.7°F)

T °F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
32.02	62.632	0.015966	0.051537	3.0081	9.4391E-05
50	62.611	0.015972	18.002	20.959	0.035953
100	62.179	0.016083	67.738	70.716	0.12908
150	61.380	0.016292	117.50	120.51	0.21430
200	60.315	0.016580	167.42	170.49	0.29309
250	59.028	0.016941	217.66	220.80	0.36659
300	57.536	0.017380	268.42	271.64	0.43581
350	55.835	0.017910	319.98	323.30	0.50167
400	53.903	0.018552	372.73	376.16	0.56501
450	51.689	0.019347	427.22	430.80	0.62678
500	49.097	0.020368	484.35	488.12	0.68811
Sat	46.297	0.021600	539.02	543.019	0.74400

P = 2000 psia (635.9°F)

T °F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
32.02	62.846	0.015912	0.090413	5.9833	1.4281E-04
50	62.812	0.015921	17.922	23.819	0.03577
100	62.365	0.016035	67.407	73.345	0.12847
150	61.564	0.016243	116.95	122.96	0.21338
200	60.508	0.016527	166.65	172.77	0.29190
250	59.237	0.016881	216.64	222.89	0.36513
300	57.770	0.017310	267.10	273.51	0.43405
350	56.105	0.017824	318.29	324.89	0.49955
400	54.223	0.018442	370.55	377.38	0.56244
450	52.086	0.019199	424.34	431.45	0.62357
500	49.618	0.020154	480.40	487.86	0.68393
550	46.671	0.021427	540.02	547.96	0.74495
600	42.888	0.023317	606.17	614.81	0.80953
640					
680					
Sat	39.009	0.025635	662.79	672.28	0.86284

P = 3000 psia (695.4°F)

T °F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
32.02	63.056	0.015859	0.11927	8.9292	1.5155E-04
50	63.010	0.015870	17.839	26.656	0.03556
100	62.547	0.015988	67.083	75.964	0.12785
150	61.746	0.016195	116.41	125.41	0.21247
200	60.698	0.016475	165.90	175.05	0.29073
250	59.442	0.016823	215.65	225.00	0.36370
300	57.998	0.017242	265.83	275.41	0.43234
350	56.365	0.017741	316.67	326.53	0.49750
400	54.530	0.018338	368.47	378.66	0.55997
450	52.461	0.019062	421.64	432.22	0.62053
500	50.099	0.019960	476.76	487.85	0.68005
550	47.335	0.021126	534.84	546.58	0.73968
600	43.940	0.022759	597.82	610.46	0.80141
640	40.379	0.024765	654.96	668.72	0.85535
680	34.697	0.028821	729.12	745.13	0.92351
Sat	29.121	0.034339	783.94	803.02	0.97391

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

AE

T °F	P = 4000 psia ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	P = 5000 psia ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
32.02	62.846	0.015912	0.09041	5.983	1.4281E-04	63.056	0.015859	0.11927	8.929	1.5155E-04
50	62.812	0.015921	17.922	23.819	0.035769	63.010	0.015870	17.839	26.656	0.03556
100	62.365	0.016035	67.407	73.345	0.12847	62.547	0.015988	67.083	75.964	0.12785
150	61.564	0.016243	116.95	122.96	0.21338	61.746	0.016195	116.41	125.41	0.21247
200	60.508	0.016527	166.65	172.77	0.29190	60.698	0.016475	165.90	175.05	0.29073
250	59.237	0.016881	216.64	222.89	0.36513	59.442	0.016823	215.65	225.00	0.36370
300	57.770	0.017310	267.10	273.51	0.43405	57.998	0.017242	265.83	275.41	0.43234
350	56.105	0.017824	318.29	324.89	0.49955	56.365	0.017741	316.67	326.53	0.49750
400	54.223	0.018442	370.55	377.38	0.56244	54.530	0.018338	368.47	378.66	0.55997
450	52.086	0.019199	424.34	431.45	0.62357	52.461	0.019062	421.64	432.22	0.62053
500	49.618	0.020154	480.40	487.86	0.68393	50.099	0.019960	476.76	487.85	0.68005
550	46.671	0.021427	540.02	547.96	0.74495	47.335	0.021126	534.84	546.58	0.73968
600	42.888	0.023317	606.17	614.81	0.80953	43.940	0.022759	597.82	610.46	0.80141
650	4.8576	0.205862	1092.1	1168.4	1.3155	39.272	0.025463	670.99	685.13	0.87021
700	4.0171	0.248935	1148.4	1240.6	1.3793	10.165	0.098374	1006.0	1060.6	1.1969
Sat	Supercritical					Supercritical				

Reference States: U = 0 and S = 0 at for saturated liquid at the triple point (32.018°F and 0.088713 psia).

Saturation Temperature Table

NH_3
SI

Temp. (^oC)	Pressure (kPa)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
-60	21.893	0.0014013	4.7057	-87.262	1251.5	-87.232	1354.6	-0.39078	6.3735
-55	30.145	0.0014126	3.4895	-65.679	1258.2	-65.637	1363.4	-0.29069	6.2600
-50	40.836	0.0014243	2.6277	-43.956	1264.7	-43.897	1372.0	-0.19223	6.1529
-45	54.489	0.0014364	2.0071	-22.096	1271.1	-22.017	1380.4	-0.095353	6.0517
-40	71.692	0.0014490	1.5533	-0.10388	1277.2	0	1388.6	0	5.9558
-35	93.098	0.0014619	1.2168	22.015	1283.2	22.151	1396.5	0.093870	5.8648
-30	119.43	0.0014753	0.96396	44.257	1289.0	44.433	1404.1	0.18630	5.7784
-25	151.47	0.0014891	0.77167	66.617	1294.6	66.843	1411.5	0.27734	5.6960
-20	190.08	0.0015035	0.62373	89.095	1299.9	89.380	1418.5	0.36703	5.6174
-15	236.17	0.0015183	0.50868	111.69	1305.1	112.05	1425.2	0.45541	5.5422
-10	290.71	0.0015336	0.41830	134.39	1309.9	134.84	1431.5	0.54255	5.4701
-5	354.76	0.0015495	0.34664	157.22	1314.5	157.77	1437.5	0.62848	5.4009
0	429.38	0.0015660	0.28930	180.16	1318.8	180.83	1443.1	0.71327	5.3343
5	515.75	0.0015831	0.24304	203.22	1322.9	204.04	1448.2	0.79697	5.2700
10	615.05	0.0016009	0.20543	226.41	1326.6	227.40	1452.9	0.87964	5.2079
15	728.52	0.0016195	0.17461	249.74	1330.0	250.92	1457.2	0.96134	5.1477
20	857.48	0.0016388	0.14920	273.21	1333.1	274.61	1461.0	1.0421	5.0891
25	1003.2	0.0016590	0.12809	296.83	1335.8	298.50	1464.3	1.1221	5.0321
30	1167.2	0.0016802	0.11046	320.63	1338.1	322.59	1467.0	1.2013	4.9764
35	1350.8	0.0017024	0.095632	344.60	1340.0	346.90	1469.2	1.2799	4.9218
40	1555.4	0.0017258	0.083101	368.78	1341.5	371.47	1470.7	1.3578	4.8682
45	1782.7	0.0017505	0.072450	393.18	1342.5	396.31	1471.7	1.4353	4.8153
50	2034.0	0.0017766	0.063350	417.84	1343.0	421.45	1471.9	1.5123	4.7630
55	2311.1	0.0018044	0.055537	442.76	1343.0	446.93	1471.4	1.5890	4.7110
60	2615.6	0.0018340	0.048797	468.00	1342.5	472.80	1470.1	1.6656	4.6591
65	2949.1	0.0018658	0.042955	493.59	1341.2	499.09	1467.9	1.7421	4.6071
70	3313.5	0.0019000	0.037868	519.58	1339.3	525.87	1464.8	1.8187	4.5548
75	3710.5	0.0019371	0.033419	546.02	1336.6	553.20	1460.6	1.8955	4.5017
80	4142.0	0.0019776	0.029509	572.98	1332.9	581.17	1455.1	1.9729	4.4477
85	4610.0	0.0020221	0.026058	600.55	1328.2	609.87	1448.4	2.0510	4.3921
90	5116.7	0.0020714	0.022997	628.84	1322.3	639.44	1440.0	2.1301	4.3346
95	5664.3	0.0021269	0.020268	657.97	1315.0	670.02	1429.8	2.2105	4.2744
100	6255.3	0.0021899	0.017820	688.13	1306.0	701.83	1417.5	2.2929	4.2107
105	6892.3	0.0022630	0.015610	719.58	1294.8	735.18	1402.4	2.3779	4.1424
110	7578.3	0.0023496	0.013596	752.71	1280.9	770.51	1383.9	2.4666	4.0675
115	8317.0	0.0024559	0.011740	788.14	1263.2	808.57	1360.8	2.5607	3.9835
120	9112.5	0.0025941	0.0099932	827.11	1240.0	850.75	1331.1	2.6635	3.8852
132.25	11333	0.0044444	0.0044444	1049.7	1049.7	1100.1	1100.1	3.2674	3.2674

Reference States:

H = 0 kJ/kg for saturated liquid at-40°C.
S = 0 kJ/kg-K for saturated liquid at -40 °C.

Saturation Pressure Table

NH_3
SI

Pressure (kPa)	Temp. (^oC)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
10	-71.22	0.0013773	9.8000	-135.17	1236.1	-135.16	1334.1	-0.62165	6.6545
30	-55.08	0.0014124	3.5053	-66.014	1258.1	-65.972	1363.3	-0.29223	6.2617
50	-46.52	0.0014327	2.1749	-28.743	1269.1	-28.671	1377.9	-0.12459	6.0818
100	-33.59	0.0014656	1.1381	28.282	1284.9	28.429	1398.7	0.12011	5.8400
200	-18.85	0.0015068	0.59465	94.290	1301.1	94.591	1420.1	0.38750	5.5998
250	-13.65	0.0015224	0.48216	117.80	1306.4	118.18	1426.9	0.47903	5.5225
500	4.14	0.0015801	0.25032	199.24	1322.2	200.03	1447.4	0.78264	5.2809
750	15.88	0.0016228	0.16980	253.85	1330.6	255.06	1457.9	0.97559	5.1373
1000	24.90	0.0016586	0.12850	296.33	1335.7	297.99	1464.2	1.1204	5.0333
1250	32.32	0.0016904	0.10325	331.75	1339.0	333.86	1468.1	1.2379	4.9509
1500	38.70	0.0017196	0.086169	362.46	1341.1	365.04	1470.4	1.3376	4.8821
1750	44.31	0.0017470	0.073816	389.81	1342.4	392.87	1471.6	1.4246	4.8225
2000	49.35	0.0017732	0.064453	414.62	1343.0	418.17	1471.9	1.5023	4.7697
2250	53.94	0.0017984	0.057102	437.44	1343.1	441.49	1471.6	1.5727	4.7220
2500	58.16	0.0018229	0.051170	458.65	1342.7	463.21	1470.7	1.6374	4.6782
2750	62.07	0.0018469	0.046279	478.55	1342.0	483.62	1469.3	1.6972	4.6376
3000	65.72	0.0018706	0.042174	497.33	1341.0	502.94	1467.5	1.7532	4.5996
3250	69.16	0.0018941	0.038676	515.18	1339.7	521.33	1465.4	1.8058	4.5636
3500	72.40	0.0019175	0.035657	532.22	1338.1	538.93	1462.9	1.8556	4.5294
3750	75.48	0.0019408	0.033025	548.56	1336.2	555.84	1460.1	1.9029	4.4967
4000	78.40	0.0019642	0.030707	564.29	1334.2	572.15	1457.0	1.9481	4.4651
4250	81.19	0.0019878	0.028649	579.49	1331.9	587.93	1453.7	1.9914	4.4346
4500	83.86	0.0020115	0.026808	594.21	1329.4	603.26	1450.0	2.0331	4.4049
4750	86.42	0.0020356	0.025151	608.51	1326.7	618.18	1446.2	2.0733	4.3760
5000	88.88	0.0020599	0.023651	622.45	1323.8	632.75	1442.0	2.1123	4.3477
5250	91.25	0.0020847	0.022284	636.05	1320.7	647.00	1437.6	2.1501	4.3198
5500	93.54	0.0021100	0.021034	649.36	1317.3	660.97	1433.0	2.1869	4.2923
5750	95.75	0.0021358	0.019884	662.42	1313.8	674.70	1428.1	2.2228	4.2651
6000	97.89	0.0021622	0.018823	675.24	1310.0	688.22	1423.0	2.2578	4.2382
6250	99.96	0.0021894	0.017840	687.87	1306.1	701.55	1417.6	2.2922	4.2113
6500	101.97	0.0022173	0.016925	700.33	1301.9	714.74	1411.9	2.3260	4.1845
6750	103.92	0.0022461	0.016071	712.63	1297.4	727.80	1405.9	2.3592	4.1577
7000	105.81	0.0022760	0.015271	724.82	1292.8	740.76	1399.7	2.3920	4.1307
7250	107.65	0.0023069	0.014519	736.92	1287.8	753.65	1393.1	2.4244	4.1036
7500	109.45	0.0023392	0.013810	748.95	1282.6	766.49	1386.1	2.4566	4.0762
7750	111.20	0.0023729	0.013140	760.94	1277.0	779.33	1378.9	2.4885	4.0484
8000	112.90	0.0024083	0.012503	772.91	1271.2	792.18	1371.2	2.5203	4.0201
8500	116.18	0.0024851	0.011318	796.98	1258.3	818.10	1354.5	2.5840	3.9617
9000	119.32	0.0025725	0.010228	821.49	1243.6	844.64	1335.7	2.6487	3.8998
9500	122.31	0.0026755	0.0092067	846.97	1226.5	872.39	1314.0	2.7158	3.8325
11333	132.25	0.0044444	0.0044444	1049.7	1049.7	1100.1	1100.1	3.2674	3.2674

Reference States:

H = 0 kJ/kg for saturated liquid at-40°C.
S = 0 kJ/kg-K for saturated liquid at -40 °C.

Superheated Vapor

NH_3
SI

Temp. (°C)	P = 50 kPa (-46.52°C)				P = 100 kPa (-33.59°C)				P = 200 kPa (-18.85°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	2.1749	1269.1	1377.9	6.0818	1.1381	1284.9	1398.7	5.8400	0.59465	1301.1	1420.1	5.5998
-40	2.2425	1280.0	1392.1	6.1438								
-30	2.3450	1296.5	1413.7	6.2344	1.1573	1291.2	1406.9	5.8739				
-20	2.4465	1312.8	1435.1	6.3205	1.2102	1308.3	1429.4	5.9645				
-10	2.5472	1329.0	1456.3	6.4028	1.2622	1325.2	1451.5	6.0502	0.61929	1317.4	1441.3	5.6818
0	2.6474	1345.1	1477.5	6.4818	1.3136	1342.0	1473.3	6.1317	0.64649	1335.4	1464.7	5.7689
10	2.7471	1361.3	1498.7	6.5580	1.3646	1358.6	1495.1	6.2098	0.67317	1352.9	1487.6	5.8512
20	2.8466	1377.6	1519.9	6.6316	1.4153	1375.2	1516.7	6.2850	0.69947	1370.2	1510.1	5.9296
30	2.9457	1393.9	1541.2	6.7030	1.4656	1391.8	1538.3	6.3575	0.72546	1387.4	1532.5	6.0048
40	3.0447	1410.3	1562.6	6.7723	1.5158	1408.4	1560.0	6.4278	0.75122	1404.6	1554.8	6.0771
50	3.1434	1426.8	1584.0	6.8398	1.5657	1425.2	1581.7	6.4961	0.77679	1421.7	1577.1	6.1470
60	3.2421	1443.5	1605.6	6.9056	1.6155	1442.0	1603.5	6.5626	0.80221	1438.9	1599.3	6.2148
70	3.3406	1460.3	1627.3	6.9698	1.6652	1458.9	1625.4	6.6274	0.82750	1456.1	1621.6	6.2807

Temp. (°C)	P = 250 kPa (-13.65°C)				P = 500 kPa (4.14°C)				P = 750 kPa (15.88°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.48216	1306.4	1426.9	5.5225	0.25032	1322.2	1447.4	5.2809	0.16980	1330.6	1457.9	5.1373
0	0.51293	1331.9	1460.2	5.6473								
25	0.56685	1376.5	1518.2	5.8507	0.27526	1364.1	1501.8	5.4702	0.17770	1350.7	1484.0	5.2261
50	0.61898	1420.0	1574.7	6.0327	0.30323	1411.0	1562.6	5.6663	0.19781	1401.6	1550.0	5.4388
75	0.67012	1463.4	1630.9	6.2002	0.33005	1456.5	1621.5	5.8419	0.21661	1449.4	1611.9	5.6233
100	0.72063	1507.3	1687.5	6.3570	0.35621	1501.8	1679.9	6.0038	0.23469	1496.1	1672.2	5.7906
125	0.77075	1552.0	1744.7	6.5054	0.38194	1547.4	1738.4	6.1555	0.25231	1542.8	1732.0	5.9459
150	0.82058	1597.7	1802.8	6.6470	0.40738	1593.8	1797.5	6.2995	0.26963	1589.9	1792.1	6.0923
175	0.87022	1644.4	1862.0	6.7829	0.43261	1641.1	1857.4	6.4371	0.28673	1637.8	1852.8	6.2317
200	0.91971	1692.4	1922.3	6.9139	0.45770	1689.5	1918.4	6.5694	0.30369	1686.6	1914.4	6.3653
225	0.96909	1741.6	1983.9	7.0407	0.48267	1739.1	1980.4	6.6972	0.32053	1736.5	1976.9	6.4941
250	1.0184	1792.2	2046.8	7.1639	0.50755	1789.9	2043.7	6.8211	0.33728	1787.7	2040.6	6.6188
275	1.0676	1844.1	2111.0	7.2838	0.53237	1842.1	2108.3	6.9417	0.35395	1840.0	2105.5	6.7400
300	1.1168	1897.4	2176.6	7.4008	0.55713	1895.6	2174.1	7.0591	0.37058	1893.7	2171.6	6.8580

Reference States: H = 0 kJ/kg for saturated liquid at-40°C.
S = 0 kJ/kg-K for saturated liquid at -40 °C.

Superheated Vapor

NH_3
SI

Temp. (°C)	P = 1000 kPa (24.9°C)				P = 1500 kPa (38.7°C)				P = 2000 kPa (49.35°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.12850	1335.7	1464.2	5.0333	0.086169	1341.1	1470.4	4.8821	0.064453	1343.0	1471.9	4.7697
25	0.12857	1336.0	1464.5	5.0344								
50	0.14496	1391.7	1536.6	5.2669	0.091774	1370.0	1507.6	4.9994	0.064729	1344.9	1474.4	4.7773
75	0.15983	1442.1	1601.9	5.4614	0.10288	1426.6	1580.9	5.2180	0.074231	1409.8	1558.3	5.0277
100	0.17389	1490.4	1664.3	5.6345	0.11301	1478.5	1648.0	5.4041	0.082479	1465.9	1630.9	5.2293
125	0.18747	1538.1	1725.6	5.7936	0.12259	1528.5	1712.4	5.5712	0.090095	1518.6	1698.7	5.4054
150	0.20074	1586.0	1786.7	5.9425	0.13182	1578.0	1775.7	5.7255	0.097332	1569.8	1764.5	5.5654
175	0.21379	1634.4	1848.2	6.0837	0.14082	1627.6	1838.8	5.8704	0.10432	1620.7	1829.3	5.7143
200	0.22668	1683.7	1910.4	6.2187	0.14966	1677.8	1902.3	6.0082	0.11114	1671.8	1894.1	5.8550
225	0.23945	1734.0	1973.4	6.3485	0.15837	1728.8	1966.3	6.1401	0.11782	1723.5	1959.2	5.9891
250	0.25214	1785.4	2037.5	6.4740	0.16699	1780.8	2031.2	6.2672	0.12441	1776.1	2024.9	6.1178
275	0.26475	1838.0	2102.7	6.5958	0.17554	1833.8	2097.2	6.3903	0.13093	1829.7	2091.5	6.2422
300	0.27730	1891.9	2169.2	6.7143	0.18403	1888.1	2164.2	6.5098	0.13739	1884.4	2159.1	6.3628
325	0.28981	1947.0	2236.9	6.8299	0.19247	1943.7	2232.4	6.6263	0.14380	1940.3	2227.8	6.4801

Temp. (°C)	P = 5000 kPa (88.88°C)				P = 7500 kPa (109.45°C)				P = 10000 kPa (125.17°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.023651	1323.8	1442.0	4.3477	0.013810	1282.6	1386.1	4.0762	0.00823	1206.0	1288.3	3.7567
100	0.026362	1370.1	1501.9	4.5107								
125	0.031082	1449.6	1605.0	4.7785	0.017196	1370.4	1499.3	4.3667				
150	0.034975	1515.6	1690.4	4.9867	0.020803	1461.2	1617.2	4.6543	0.013381	1392.3	1526.2	4.3400
175	0.038463	1576.0	1768.3	5.1656	0.023672	1533.9	1711.5	4.8709	0.016152	1485.8	1647.3	4.6186
200	0.041706	1633.9	1842.4	5.3266	0.026196	1599.4	1795.9	5.0543	0.018386	1561.8	1745.7	4.8323
225	0.044785	1690.7	1914.7	5.4754	0.028513	1661.6	1875.4	5.2181	0.020353	1630.5	1834.0	5.0144
250	0.047748	1747.3	1986.0	5.6151	0.030692	1722.0	1952.2	5.3686	0.022156	1695.6	1917.2	5.1772
275	0.050625	1804.1	2057.2	5.7480	0.032773	1781.9	2027.7	5.5094	0.023848	1758.8	1997.3	5.3269
300	0.053435	1861.4	2128.6	5.8753	0.034781	1841.6	2102.5	5.6429	0.025459	1821.3	2075.9	5.4670
325	0.056192	1919.5	2200.4	5.9980	0.036731	1901.7	2177.2	5.7704	0.027010	1883.5	2153.6	5.5998
350	0.058906	1978.5	2273.0	6.1169	0.038637	1962.4	2252.1	5.8932	0.028512	1946.0	2231.1	5.7267
375	0.061587	2038.5	2346.4	6.2324	0.040506	2023.8	2327.6	6.0119	0.029977	2008.9	2308.7	5.8488
400	0.064238	2099.6	2420.8	6.3450	0.042346	2086.2	2403.8	6.1272	0.031411	2072.6	2386.7	5.9668
425	0.066865	2162.0	2496.3	6.4551	0.044161	2149.5	2480.8	6.2395	0.032820	2137.0	2465.2	6.0814

Reference States: H = 0 kJ/kg for saturated liquid at-40°C.
S = 0 kJ/kg-K for saturated liquid at -40 °C.

Subcooled Liquid

NH_3
SI

	P = 1 MPa (24.9°C)				P = 1500 kPa (38.7°C)				P = 2000 kPa (49.35°C)			
Temp. (°C)	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**
Sat	0.0016586	296.33	297.99	1.1204	0.0017196	362.46	365.04	1.3376	0.0017732	414.62	418.17	1.5023
40									0.0017242	368.10	371.55	1.3556
30					0.0016792	320.18	322.70	1.1999	0.0016778	319.52	322.87	1.1977
20	0.0016385	273.04	274.68	1.0416	0.0016373	272.45	274.91	1.0396	0.0016361	271.87	275.15	1.0376
10	0.0016001	226.01	227.61	0.87822	0.0015991	225.50	227.89	0.87640	0.0015981	224.98	228.18	0.87458
0	0.0015650	179.63	181.20	0.71134	0.0015642	179.17	181.52	0.70966	0.0015633	178.72	181.84	0.70798
-10	0.0015326	133.81	135.34	0.54033	0.0015318	133.40	135.70	0.53877	0.0015311	132.99	136.05	0.53721
-20	0.0015024	88.497	90.000	0.36466	0.0015018	88.13	90.38	0.36321	0.0015011	87.77	90.77	0.36177
-30	0.0014743	43.674	45.149	0.18390	0.0014738	43.345	45.556	0.18255	0.0014732	43.018	45.964	0.18120
-40	0.0014481	-0.65410	0.79396	-0.0023620	0.0014476	-0.94903	1.22232	-0.0036296	0.0014471	-1.2430	1.6512	-0.0048940
-50	0.0014235	-44.464	-43.041	-0.19451	0.0014231	-44.728	-42.593	-0.19570	0.0014227	-44.991	-42.146	-0.19688
-60	0.0014006	-87.724	-86.324	-0.39295	0.0014002	-87.960	-85.859	-0.39406	0.0013998	-88.194	-85.395	-0.39516

	P = 5000 kPa (88.88°C)				P = 7500 kPa (109.45°C)				P = 10000 kPa (125.17°C)			
Temp. (°C)	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**	**V (m^3/kg)**	**U (kJ/kg)**	**H (kJ/kg)**	**S (kJ/kg*K)**
Sat	0.0020599	622.45	632.75	2.1123	0.0023392	748.95	766.49	2.4566	0.0028035	874.36	902.40	2.7879
110									0.0022636	733.84	756.48	2.4154
110									0.0022636	733.84	756.48	2.4154
100					0.0021621	681.11	697.33	2.2736	0.0021166	669.05	690.22	2.2402
90					0.0020388	619.34	634.63	2.1033	0.0020101	610.62	630.72	2.0786
80	0.0019690	570.23	580.07	1.9650	0.0019462	562.76	577.35	1.9434	0.0019261	555.96	575.22	1.9237
70	0.0018880	515.30	524.74	1.8061	0.0018715	509.36	523.40	1.7885	0.0018566	503.83	522.40	1.7720
60	0.0018212	463.04	472.14	1.6505	0.0018087	458.14	471.71	1.6356	0.0017971	453.51	471.48	1.6214
50	0.0017641	412.64	421.46	1.4961	0.0017543	408.50	421.65	1.4831	0.0017450	404.54	421.99	1.4706
40	0.0017141	363.61	372.18	1.3412	0.0017061	360.04	372.84	1.3296	0.0016985	356.60	373.59	1.3184
30	0.0016695	315.63	323.97	1.1847	0.0016629	312.51	324.98	1.1743	0.0016566	309.49	326.05	1.1642
20	0.0016292	268.46	276.61	1.0259	0.0016236	265.71	277.88	1.0163	0.0016183	263.03	279.21	1.0071
10	0.0015923	221.96	229.92	0.86383	0.0015876	219.51	231.42	0.85509	0.0015830	217.13	232.96	0.84652
0	0.0015583	176.02	183.81	0.69806	0.0015542	173.84	185.49	0.68996	0.0015503	171.70	187.20	0.68200
-10	0.0015268	130.58	138.22	0.52800	0.0015233	128.62	140.04	0.52045	0.0015198	126.70	141.90	0.51303
-20	0.0014974	85.603	93.090	0.35317	0.0014943	83.838	95.046	0.34612	0.0014913	82.106	97.019	0.33918
-30	0.0014699	41.075	48.425	0.17316	0.0014672	39.488	50.492	0.16656	0.0014646	37.927	52.573	0.16005
-40	0.0014442	-2.9863	4.2348	-0.012414	0.0014418	-4.4130	6.4009	-0.018597	0.0014395	-5.8167	8.5785	-0.024705
-50	0.0014201	-46.552	-39.452	-0.20392	0.0014181	-47.831	-37.196	-0.20971	0.0014160	-49.090	-34.930	-0.21543
-60	0.0013976	-89.587	-82.599	-0.40173	0.0013958	-90.728	-80.260	-0.40714	0.0013940	-91.853	-77.913	-0.41249

Reference States: H = 0 kJ/kg for saturated liquid at-40°C.
S = 0 kJ/kg-K for saturated liquid at -40 °C.

Saturation Temperature Table

NH_3
AE

Temp. (^{o}F)	Pressure (psia)	V (ft^3/lb$_m$)		U (Btu/lb$_m$)		H (Btu/lb$_m$)		S (Btu/lb$_m$^{o}R)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
-100	1.2374	0.021994	182.19	-61.999	530.52	-61.994	572.26	-0.15922	1.6042
-95	1.5234	0.022085	149.92	-56.940	532.19	-56.934	574.48	-0.14525	1.5862
-90	1.8637	0.022177	124.12	-51.862	533.86	-51.854	576.69	-0.13142	1.5689
-85	2.2661	0.022272	103.35	-46.765	535.50	-46.755	578.87	-0.11773	1.5521
-80	2.7394	0.022368	86.546	-41.648	537.13	-41.637	581.03	-0.10416	1.5359
-75	3.2930	0.022467	72.857	-36.512	538.75	-36.499	583.17	-0.090721	1.5202
-70	3.9374	0.022567	61.646	-31.358	540.34	-31.341	585.29	-0.077407	1.5050
-65	4.6837	0.022669	52.415	-26.184	541.92	-26.165	587.38	-0.064214	1.4904
-60	5.5439	0.022773	44.774	-20.992	543.47	-20.969	589.44	-0.051141	1.4761
-55	6.5312	0.022879	38.418	-15.782	545.01	-15.754	591.47	-0.038185	1.4624
-50	7.6592	0.022988	33.105	-10.554	546.52	-10.521	593.48	-0.025344	1.4490
-45	8.9429	0.023098	28.644	-5.3077	548.01	-5.2695	595.45	-0.012617	1.4360
-40	10.398	0.023210	24.881	-0.044690	549.48	0	597.39	0	1.4235
-35	12.041	0.023324	21.694	5.2352	550.92	5.2872	599.29	0.012507	1.4113
-30	13.890	0.023441	18.983	10.532	552.34	10.592	601.16	0.024907	1.3994
-25	15.962	0.023560	16.668	15.844	553.73	15.914	602.99	0.037201	1.3878
-20	18.279	0.023681	14.683	21.173	555.09	21.253	604.79	0.049391	1.3766
-15	20.859	0.023804	12.976	26.517	556.42	26.609	606.54	0.061479	1.3657
-10	23.723	0.023929	11.502	31.877	557.73	31.982	608.26	0.073466	1.3550
-5	26.895	0.024057	10.226	37.253	559.00	37.372	609.93	0.085355	1.3446
0	30.397	0.024187	9.1159	42.643	560.24	42.779	611.55	0.097148	1.3345
5	34.253	0.024320	8.1483	48.049	561.45	48.203	613.13	0.10885	1.3246
10	38.487	0.024456	7.3020	53.470	562.63	53.644	614.67	0.12045	1.3150
15	43.126	0.024594	6.5597	58.906	563.77	59.103	616.15	0.13197	1.3055
20	48.194	0.024735	5.9067	64.358	564.88	64.579	617.59	0.14340	1.2963
25	53.720	0.024879	5.3307	69.825	565.95	70.072	618.97	0.15474	1.2873
30	59.731	0.025025	4.8213	75.308	566.98	75.585	620.31	0.16600	1.2784
35	66.255	0.025175	4.3695	80.807	567.97	81.116	621.58	0.17717	1.2698
40	73.322	0.025328	3.9680	86.323	568.93	86.666	622.80	0.18827	1.2613
45	80.962	0.025484	3.6102	91.855	569.84	92.237	623.97	0.19929	1.2529
50	89.205	0.025644	3.2906	97.405	570.72	97.828	625.07	0.21024	1.2447
55	98.083	0.025808	3.0045	102.97	571.55	103.44	626.12	0.22111	1.2367
60	107.63	0.025975	2.7479	108.56	572.33	109.08	627.10	0.23192	1.2287
65	117.87	0.026146	2.5172	114.16	573.07	114.73	628.01	0.24266	1.2210
70	128.85	0.026322	2.3094	119.79	573.77	120.42	628.86	0.25334	1.2133
75	140.59	0.026502	2.1217	125.44	574.41	126.13	629.65	0.26396	1.2057
80	153.13	0.026686	1.9521	131.10	575.01	131.86	630.36	0.27452	1.1982
85	166.51	0.026876	1.7983	136.80	575.55	137.62	631.00	0.28503	1.1909
90	180.76	0.027070	1.6588	142.51	576.04	143.42	631.56	0.29549	1.1836
100	212.01	0.027476	1.4163	154.02	576.86	155.10	632.46	0.31626	1.1692
120	286.60	0.028364	1.0452	177.39	577.76	178.90	633.23	0.35736	1.1411
140	379.36	0.029379	0.7817	201.34	577.54	203.40	632.45	0.39808	1.1136
160	492.95	0.030563	0.58993	226.04	575.95	228.83	629.80	0.43875	1.0858
180	630.24	0.031986	0.44729	251.74	572.60	255.47	624.80	0.47980	1.0572
200	794.38	0.033761	0.33869	278.84	566.86	283.81	616.69	0.52188	1.0265
220	989.03	0.036110	0.25381	308.04	557.63	314.65	604.11	0.56605	0.99193
240	1218.7	0.039556	0.18487	340.84	542.46	349.77	584.18	0.61462	0.94966
270.05	1643.7	0.071193	0.071193	451.57	451.57	473.25	473.25	0.78093	0.78093

Reference States: H = 0 Btu/lb$_m$ for saturated liquid at -40°F.

S = 0 Btu/lb$_m$°R for saturated liquid at -40°F.

Saturation Pressure Table

NH_3
AE

Pressure (psia)	Temp. (^oF)	V (ft^3/lb_m)		U (Btu/lb_m)		H (Btu/lb_m)		S ($Btu/lb_m{}^oR$)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
1	-104.96	0.021906	222.48	-67.002	528.84	-66.998	570.04	-0.17323	1.6227
5	-63.08	0.022709	49.310	-24.192	542.52	-24.171	588.17	-0.059178	1.4848
10	-41.31	0.023181	25.805	-1.4228	549.10	-1.3799	596.88	-0.0032891	1.4267
15	-27.99	0.023488	18.007	12.668	552.90	12.732	601.90	0.029867	1.3947
20	-16.61	0.023764	13.497	24.799	556.00	24.887	605.98	0.057608	1.3691
30	-0.54	0.024173	9.2292	42.056	560.11	42.191	611.38	0.095871	1.3356
40	11.68	0.024502	7.0417	55.295	563.02	55.477	615.17	0.12433	1.3118
50	21.68	0.024783	5.7050	66.195	565.24	66.424	618.06	0.14722	1.2932
60	30.21	0.025032	4.8007	75.544	567.02	75.822	620.36	0.16648	1.2780
70	37.70	0.025257	4.1470	83.782	568.49	84.109	622.25	0.18317	1.2651
80	44.39	0.025465	3.6516	91.180	569.73	91.558	623.83	0.19795	1.2539
90	50.46	0.025659	3.2628	97.919	570.79	98.347	625.17	0.21125	1.2440
100	56.03	0.025842	2.9492	104.13	571.71	104.60	626.32	0.22335	1.2350
120	66.00	0.026181	2.4740	115.28	573.21	115.87	628.19	0.24480	1.2194
140	74.76	0.026493	2.1304	125.16	574.38	125.85	629.61	0.26345	1.2061
160	82.61	0.026784	1.8700	134.07	575.30	134.86	630.70	0.28001	1.1944
180	89.74	0.027060	1.6657	142.22	576.02	143.12	631.54	0.29495	1.1839
200	96.30	0.027323	1.5008	149.75	576.58	150.76	632.17	0.30859	1.1745
220	102.38	0.027576	1.3650	156.77	577.02	157.89	632.62	0.32117	1.1658
240	108.05	0.027820	1.2510	163.36	577.34	164.60	632.94	0.33286	1.1578
260	113.37	0.028057	1.1540	169.59	577.58	170.94	633.14	0.34380	1.1504
280	118.40	0.028289	1.0704	175.50	577.73	176.97	633.22	0.35410	1.1434
300	123.17	0.028515	0.99749	181.14	577.81	182.72	633.22	0.36382	1.1368
350	134.11	0.029064	0.85053	194.22	577.74	196.10	632.86	0.38611	1.1217
400	143.94	0.029597	0.73907	206.14	577.35	208.33	632.09	0.40608	1.1081
450	152.89	0.030119	0.65151	217.15	576.69	219.66	630.98	0.42427	1.0957
500	161.12	0.030636	0.58080	227.45	575.81	230.29	629.58	0.44104	1.0842
550	168.76	0.031152	0.52243	237.15	574.73	240.33	627.93	0.45665	1.0734
600	175.90	0.031670	0.47336	246.37	573.45	249.89	626.05	0.47132	1.0632
650	182.60	0.032193	0.43149	255.17	572.00	259.05	623.94	0.48520	1.0533
700	188.93	0.032725	0.39530	263.63	570.38	267.88	621.62	0.49841	1.0438
750	194.92	0.033268	0.36365	271.80	568.59	276.42	619.10	0.51106	1.0345
800	200.63	0.033824	0.33572	279.72	566.64	284.73	616.37	0.52322	1.0255
900	211.26	0.034992	0.28850	294.96	562.20	300.79	610.28	0.54640	1.0077
1000	221.03	0.036254	0.24992	309.63	557.02	316.34	603.30	0.56842	0.9900
1200	238.49	0.039233	0.18968	338.17	543.92	346.89	586.07	0.61069	0.9533
1300	246.36	0.041098	0.16508	352.63	535.51	362.52	575.25	0.63190	0.9332
1400	253.75	0.043431	0.14263	367.86	525.16	379.12	562.13	0.65418	0.91072
1500	260.70	0.046709	0.12101	385.12	511.50	398.09	545.11	0.67948	0.88357
1600	267.24	0.053180	0.096827	409.74	489.23	425.50	517.92	0.71607	0.84322
1643.7	270.05	0.071193	0.071193	451.57	451.57	473.25	473.25	0.78093	0.78093

Reference States:

H = 0 Btu/lb_m for saturated liquid at -40°F.
S = 0 $Btu/lb_m{}°R$ for saturated liquid at -40°F.

Superheated Vapor

NH_3
AE

P = 1 psia (-104.96°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	222.48	528.84	570.04	1.6227
-90	232.07	534.43	577.40	1.6431
-80	238.46	538.15	582.30	1.6561
-70	244.84	541.86	587.20	1.6689
-60	251.21	545.56	592.08	1.6812
-50	257.57	549.27	596.96	1.6933
-40	263.92	552.97	601.84	1.7051
-20	276.60	560.39	611.61	1.7278
0	289.27	567.84	621.41	1.7496
20	301.92	575.34	631.24	1.7705
40	314.57	582.88	641.13	1.7907
60	327.20	590.47	651.06	1.8102
80	339.84	598.14	661.07	1.8291
100	352.46	605.87	671.14	1.8474

P = 5 psia (-63.08°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	49.310	542.52	588.17	1.4848
-90				
-80				
-70				
-60	49.718	543.73	589.76	1.4888
-50	51.037	547.64	594.89	1.5015
-40	52.349	551.52	599.98	1.5138
-20	54.953	559.22	610.09	1.5373
0	57.539	566.87	620.15	1.5597
20	60.111	574.52	630.18	1.5810
40	62.674	582.18	640.21	1.6015
60	65.228	589.88	650.27	1.6213
80	67.777	597.62	660.37	1.6404
100	70.322	605.42	670.53	1.6588

P = 10 psia (-41.31°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	25.805	549.10	596.88	1.4267
-90				
-80				
-70				
-60				
-50				
-40	25.894	549.63	597.58	1.4284
-20	27.242	557.70	608.15	1.4530
0	28.569	565.63	618.53	1.4761
20	29.882	573.49	628.82	1.4980
40	31.185	581.31	639.06	1.5189
60	32.480	589.13	649.27	1.5389
80	33.769	596.97	659.50	1.5583
100	35.054	604.84	669.76	1.5769

P = 14.696 psia (-27.99°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	18.007	552.90	601.90	1.3947
-20	18.383	556.24	606.26	1.4047
-10	18.849	560.36	611.65	1.4168
0	19.310	564.44	616.99	1.4286
10	19.767	568.48	622.27	1.4399
20	20.221	572.50	627.53	1.4510
30	20.673	576.49	632.75	1.4618
40	21.122	580.47	637.95	1.4723
50	21.570	584.45	643.14	1.4826
60	22.015	588.41	648.32	1.4926
70	22.459	592.38	653.50	1.5025
90	23.343	600.32	663.85	1.5217

P = 20 psia (-16.61°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	13.497	556.00	605.98	1.3691
-20				
-10	13.730	558.83	609.68	1.3774
0	14.078	563.06	615.20	1.3896
10	14.423	567.23	620.65	1.4013
20	14.764	571.36	626.04	1.4126
30	15.103	575.45	631.38	1.4237
40	15.439	579.52	636.69	1.4344
50	15.772	583.56	641.98	1.4449
60	16.104	587.60	647.24	1.4551
70	16.435	591.62	652.49	1.4651
90	17.092	599.66	662.96	1.4845

P = 25 psia (-7.93°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	10.951	558.26	608.95	1.3507
-20				
-10				
0	11.178	561.72	613.47	1.3606
10	11.460	566.03	619.08	1.3727
20	11.739	570.26	624.61	1.3843
30	12.015	574.45	630.07	1.3956
40	12.288	578.60	635.49	1.4065
50	12.559	582.72	640.86	1.4172
60	12.829	586.82	646.21	1.4276
70	13.096	590.90	651.53	1.4377
90	13.628	599.04	662.12	1.4573

P = 30 psia (-0.54°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	9.2292	560.11	611.38	1.3356
0	9.2424	560.35	611.70	1.3363
10	9.4836	564.79	617.47	1.3487
20	9.7211	569.15	623.15	1.3607
30	9.9554	573.44	628.74	1.3722
40	10.187	577.67	634.26	1.3834
60	10.644	586.03	645.16	1.4048
100	11.540	602.51	666.62	1.4445
150	12.637	623.04	693.25	1.4901
200	13.721	643.82	720.04	1.5323
300	15.864	686.73	774.86	1.6097
400	17.990	732.03	831.97	1.6803

P = 40 psia (11.68°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	7.0417	563.02	615.17	1.3118
0				
10				
20	7.1965	566.84	620.15	1.3222
30	7.3794	571.35	626.00	1.3343
40	7.5594	575.77	631.76	1.3459
60	7.9126	584.43	643.04	1.3681
100	8.5994	601.32	665.01	1.4088
150	9.4352	622.16	692.05	1.4551
200	10.256	643.13	719.10	1.4977
300	11.874	686.27	774.23	1.5755
400	13.476	731.69	831.51	1.6463

P = 50 psia (21.68°F)

T °F	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^\circ R$
Sat	5.7050	565.24	618.06	1.2932
0				
10				
20				
30	5.8317	569.18	623.17	1.3038
40	5.9811	573.80	629.18	1.3159
60	6.2726	582.78	640.86	1.3388
100	6.8347	600.10	663.38	1.3806
150	7.5136	621.27	690.84	1.4276
200	8.1774	642.44	718.16	1.4707
300	9.4808	685.82	773.60	1.5489
400	10.767	731.36	831.05	1.6199

Reference States:

$H = 0\ Btu/lb_m$ for saturated liquid at -40°F.

$S = 0\ Btu/lb_m{}^\circ R$ for saturated liquid at -40°F.

Superheated Vapor

NH$_3$ AE

	P = 75 psia (41.13°F)				P = 100 psia (56.03°F)				P = 150 psia (78.78°F)			
T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	3.8834	569.14	623.07	1.2593	2.9492	571.71	626.32	1.2350	1.9919	574.87	630.19	1.2000
60	4.0822	578.46	635.16	1.2830	2.9826	573.82	629.05	1.2403				
80	4.2843	587.88	647.38	1.3061	3.1454	584.03	642.28	1.2653	1.9993	575.58	631.11	1.2017
100	4.4800	596.98	659.20	1.3276	3.3007	593.72	654.84	1.2881	2.1167	586.71	645.50	1.2279
120	4.6709	605.88	670.75	1.3479	3.4506	603.06	666.96	1.3094	2.2271	597.10	658.97	1.2516
140	4.8582	614.66	682.13	1.3672	3.5966	612.19	678.79	1.3295	2.3327	607.03	671.82	1.2734
160	5.0426	623.36	693.39	1.3857	3.7395	621.17	690.42	1.3485	2.4348	616.64	684.27	1.2938
180	5.2249	632.03	704.60	1.4035	3.8801	630.08	701.93	1.3668	2.5341	626.04	696.43	1.3131
200	5.4052	640.71	715.78	1.4207	4.0187	638.94	713.36	1.3844	2.6312	635.32	708.40	1.3315
240	5.7618	658.13	738.15	1.4536	4.2916	656.66	736.13	1.4179	2.8208	653.66	732.02	1.3663
280	6.1142	675.75	760.67	1.4849	4.5601	674.50	758.94	1.4496	3.0057	671.97	755.46	1.3989
320	6.4635	693.66	783.42	1.5149	4.8255	692.58	781.93	1.4799	3.1873	690.39	778.92	1.4298
360	6.8105	711.90	806.49	1.5437	5.0886	710.95	805.18	1.5090	3.3666	709.04	802.55	1.4593
400	7.1557	730.52	829.90	1.5716	5.3499	729.68	828.75	1.5371	3.5440	727.99	826.43	1.4878

	P = 200 psia (96.3°F)				P = 250 psia (110.75°F)				P = 300 psia (123.17°F)			
T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	1.5008	576.58	632.17	1.1745	2.9492	892.95	633.05	1.1540	0.99749	577.81	633.22	1.1368
120	1.6116	590.63	650.31	1.2065	1.2385	583.50	640.84	1.1676				
140	1.6982	601.52	664.41	1.2304	1.3150	595.60	656.47	1.1941	1.0570	589.16	647.88	1.1616
160	1.7805	611.86	677.80	1.2523	1.3862	606.80	670.97	1.2179	1.1217	601.41	663.73	1.1876
200	1.9365	631.56	703.28	1.2922	1.5187	627.66	697.97	1.2601	1.2393	623.61	692.46	1.2325
240	2.0848	650.59	727.80	1.3283	1.6426	647.44	723.49	1.2977	1.3474	644.20	719.05	1.2717
280	2.2281	669.38	751.90	1.3618	1.7612	666.75	748.29	1.3321	1.4497	664.07	744.60	1.3072
320	2.3680	688.17	775.87	1.3933	1.8762	685.92	772.78	1.3644	1.5482	683.64	769.64	1.3402
360	2.5054	707.11	799.89	1.4234	1.9885	705.15	797.21	1.3950	1.6439	703.17	794.49	1.3712
400	2.6409	726.28	824.08	1.4522	2.0989	724.55	821.72	1.4242	1.7376	722.81	819.34	1.4008
440	2.7749	745.76	848.52	1.4800	2.2078	744.22	846.43	1.4522	1.8297	742.67	844.32	1.4292
500	2.9738	775.64	885.77	1.5201	2.3690	774.32	884.00	1.4927	1.9658	773.00	882.21	1.4700
600	3.3015	827.46	949.73	1.5834	2.6337	826.41	948.34	1.5564	2.1885	825.36	946.94	1.5342

	P = 400 psia (143.94°F)				P = 500 psia (161.12°F)				P = 1000 psia (221.03°F)			
T °F	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.73907	577.35	632.09	1.1081	0.58080	575.81	629.58	1.0842	0.24992	557.02	603.30	0.98998
150	0.75752	582.06	638.17	1.1181								
200	0.88799	614.94	680.71	1.1853	0.67434	605.33	667.77	1.1440				
250	0.99835	642.80	716.74	1.2380	0.77209	635.99	707.48	1.2021	0.30570	592.49	649.10	1.0560
300	1.0991	668.80	750.20	1.2836	0.85835	663.52	743.00	1.2505	0.37138	633.30	702.07	1.1282
350	1.1942	694.10	782.56	1.3248	0.93835	689.81	776.69	1.2934	0.42399	666.37	744.89	1.1829
400	1.2857	719.28	814.51	1.3631	1.0144	715.68	809.60	1.3329	0.47051	696.51	783.64	1.2293
450	1.3747	744.63	846.45	1.3992	1.0878	741.54	842.26	1.3698	0.51344	725.36	820.44	1.2709
500	1.4617	770.34	878.61	1.4337	1.1593	767.64	874.97	1.4048	0.55401	753.69	856.27	1.3093
600	1.6321	823.25	944.14	1.4986	1.2982	821.13	941.33	1.4706	0.63052	810.28	927.04	1.3794
800	1.9635	936.62	1082.1	1.6177	1.5667	935.17	1080.2	1.5905	0.77330	927.88	1071.1	1.5039

Reference States: H = 0 Btu/lb$_m$ for saturated liquid at -40°F.

S = 0 Btu/lb$_m$°R for saturated liquid at -40°F.

Subcooled Liquid

NH_3
AE

T	P = 50 psia (21.68°F)					P = 100 psia (56.03°F)				
°F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	40.351	0.024783	66.195	66.424	0.14722	38.697	0.025842	104.13	104.604	0.22335
50						38.999	0.025642	97.371	97.85	0.21017
40						39.490	0.025323	86.246	86.714	0.18812
30						39.972	0.025018	75.199	75.662	0.16577
20	40.429	0.024734	64.353	64.582	0.14339	40.443	0.024726	64.226	64.684	0.14312
10	40.893	0.024454	53.442	53.669	0.12039	40.906	0.024446	53.323	53.776	0.12014
0	41.349	0.024185	42.599	42.8230	0.097052	41.361	0.024177	42.487	42.93463	0.096808
-10	41.796	0.023926	31.822	32.0432	0.073342	41.808	0.023919	31.716	32.159	0.073108
-20	42.236	0.023677	21.110	21.329	0.049247	42.247	0.023670	21.011	21.449	0.049022
-30	42.668	0.023437	10.464	10.681	0.024749	42.679	0.023431	10.371	10.804	0.024532
-40	43.093	0.023206	-0.11447	0.10039	-1.6633E-04	43.103	0.023200	-0.20238	0.2272	-3.7599E-04
-50	43.510	0.022983	-10.6237	-10.411	-0.025516	43.519	0.022978	-10.706	-10.281	-0.025718
-60	43.919	0.022769	-21.061	-20.850	-0.051314	43.928	0.022764	-21.139	-20.717	-0.051509
-70	44.321	0.022563	-31.425	-31.216	-0.077579	44.329	0.022559	-31.498	-31.080	-0.077767
-80	44.714	0.022364	-41.713	-41.506	-0.10433	44.722	0.022360	-41.781	-41.367	-0.10451
-90	45.098	0.022174	-51.924	-51.718	-0.13159	45.106	0.022170	-51.987	-51.577	-0.13176
-100	45.474	0.021990	-62.058	-61.854	-0.15938	45.481	0.021987	-62.117	-61.710	-0.15955

T	P = 150 psia (78.78°F)					P = 200 psia (96.3°F)				
°F	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	ρ lb_m/ft_3	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	37.536	0.026641	129.72	130.46	0.27196	36.599	0.027323	149.75	150.76	0.30859
90						36.950	0.027064	142.43	143.43	0.29534
80						37.492	0.026672	130.93	131.91	0.27419
70	37.999	0.026316	119.71	120.45	0.25320	38.019	0.026303	119.54	120.51	0.25287
60	38.514	0.025965	108.42	109.14	0.23165	38.532	0.025953	108.25	109.22	0.23134
50	39.016	0.025631	97.218	97.929	0.20987	39.032	0.025620	97.064	98.013	0.20957
40	39.506	0.025312	86.102	86.805	0.18783	39.522	0.025302	85.958	86.895	0.18754
30	39.987	0.025008	75.064	75.759	0.16550	40.001	0.024999	74.930	75.856	0.16522
20	40.458	0.024717	64.100	64.786	0.14286	40.471	0.024709	63.974	64.889	0.14259
10	40.920	0.024438	53.204	53.883	0.11989	40.933	0.024430	53.086	53.990	0.11963
0	41.374	0.024170	42.375	43.046	0.096564	41.386	0.024163	42.264	43.158	0.096321
-10	41.820	0.023912	31.611	32.275	0.072874	41.831	0.023905	31.506	32.391	0.072639
-20	42.258	0.023664	20.912	21.569	0.048795	42.269	0.023658	20.813	21.689	0.048570
-30	42.689	0.023425	10.277	10.928	0.024314	42.700	0.023419	10.184	11.052	0.024098
-40	43.113	0.023195	-0.29009	0.35417	-5.8529E-04	43.123	0.023190	-0.37760	0.48122	-7.9422E-04
-50	43.529	0.022973	-10.789	-10.151	-0.025919	43.538	0.022968	-10.871	-10.020	-0.026121
-60	43.937	0.022760	-21.216	-20.584	-0.051703	43.946	0.022755	-21.294	-20.451	-0.051897
-70	44.338	0.022554	-31.571	-30.944	-0.077953	44.346	0.022550	-31.643	-30.808	-0.078140
-80	44.730	0.022356	-41.849	-41.228	-0.10469	44.738	0.022352	-41.917	-41.089	-0.10487
-90	45.114	0.022166	-52.051	-51.436	-0.13193	45.121	0.022163	-52.115	-51.294	-0.13211
-100	45.489	0.021984	-62.177	-61.566	-0.15972	45.496	0.021980	-62.236	-61.422	-0.15988

Subcooled Liquid

Ammonia AE

	P = 300 psia (123.17°F)					**P = 400 psia (143.94°F)**				
T °F	**ρ lb_m/ft_3**	**V ft^3/lb_m**	**U Btu/lb_m**	**H Btu/lb_m**	**S $Btu/lb_m{}^oR$**	**ρ lb_m/ft_3**	**V ft^3/lb_m**	**U Btu/lb_m**	**H Btu/lb_m**	**S $Btu/lb_m{}^oR$**
Sat	35.069	0.028515	181.14	182.72	0.36382	33.787	0.029597	206.14	208.33	0.40608
140						34.053	0.029366	201.21	203.38	0.39786
120	35.264	0.028357	177.32	178.90	0.35724	35.322	0.028311	176.80	178.90	0.35634
100	36.439	0.027443	153.63	155.16	0.31557	36.487	0.027407	153.19	155.22	0.31478
80	37.534	0.026643	130.55	132.03	0.27349	37.575	0.026614	130.18	132.15	0.27280
60	38.568	0.025928	107.93	109.37	0.23071	38.603	0.025904	107.61	109.52	0.23008
40	39.553	0.025282	85.673	87.078	0.18697	39.585	0.025262	85.390	87.261	0.18640
20	40.499	0.024692	63.723	65.094	0.14207	40.527	0.024675	63.473	65.301	0.14155
0	41.411	0.024148	42.042	43.383	0.095837	41.436	0.024134	41.821	43.608	0.095354
-20	42.291	0.023645	20.616	21.929	0.048121	42.314	0.023633	20.420	22.170	0.047674
-40	43.143	0.023179	-0.55201	0.73563	-0.0012110	43.162	0.023168	-0.72562	0.99044	-0.0016263
-60	43.964	0.022746	-21.448	-20.184	-0.052284	43.982	0.022737	-21.601	-19.917	-0.052669
-80	44.754	0.022344	-42.053	-40.812	-0.10523	44.770	0.022336	-42.188	-40.533	-0.10558
-100	45.510	0.021973	-62.355	-61.134	-0.16021	45.524	0.021966	-62.473	-60.846	-0.16054

	P = 500 psia (161.12°F)					**P = 1000 psia (221.03°F)**				
T °F	**ρ lb_m/ft_3**	**V ft^3/lb_m**	**U Btu/lb_m**	**H Btu/lb_m**	**S $Btu/lb_m{}^oR$**	**ρ lb_m/ft_3**	**V ft^3/lb_m**	**U Btu/lb_m**	**H Btu/lb_m**	**S $Btu/lb_m{}^oR$**
Sat	32.641	0.030636	227.45	230.29	0.44104	27.583	0.036254	309.63	316.34	0.56842
220						27.722	0.036072	307.80	314.48	0.56569
200						29.949	0.033390	276.09	282.27	0.51761
180						31.679	0.031566	248.16	254.01	0.47411
160	32.725	0.030557	225.98	228.81	0.43866	33.151	0.030165	222.25	227.83	0.43254
140	34.124	0.029305	200.58	203.29	0.39680	34.459	0.029020	197.60	202.97	0.39176
120	35.380	0.028265	176.28	178.90	0.35544	35.654	0.028047	173.82	179.01	0.35113
100	36.535	0.027371	152.76	155.30	0.31400	36.766	0.027199	150.67	155.71	0.31023
80	37.616	0.026585	129.81	132.27	0.27211	37.814	0.026445	128.01	132.90	0.26874
60	38.639	0.025881	107.28	109.68	0.22946	38.812	0.025765	105.72	110.49	0.22641
40	39.616	0.025242	85.109	87.446	0.18583	39.768	0.025146	83.731	88.388	0.18305
20	40.555	0.024658	63.225	65.508	0.14103	40.690	0.024576	62.007	66.558	0.13846
0	41.460	0.024120	41.601	43.834	0.094874	41.581	0.024049	40.520	44.973	0.092501
-20	42.336	0.023621	20.225	22.412	0.047228	42.444	0.023560	19.265	23.628	0.045025
-40	43.182	0.023158	-0.8984	1.2457	-0.0020402	43.280	0.023105	-1.7508	2.5277	-4.0886E-03
-60	44.000	0.022728	-21.754	-19.650	-0.053053	44.088	0.022682	-22.508	-18.308	-0.054956
-80	44.786	0.022329	-42.322	-40.255	-0.105940	44.865	0.022289	-42.986	-38.859	-0.10770
-100	45.538	0.021959	-62.591	-60.558	-0.160871	45.609	0.021926	-63.172	-59.112	-0.16250

Reference States:

H = 0 Btu/lb_m for saturated liquid at -40°F.
S = 0 Btu/lb_m°R for saturated liquid at -40°F.

Saturation Temperature Table

R-134a
SI

Temp. (°C)	Pressure (kPa)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
-100	0.55940	0.00063195	25.193	75.362	322.76	75.362	336.85	0.43540	1.9456
-95	0.93899	0.00063729	15.435	81.287	325.29	81.288	339.78	0.46913	1.9201
-90	1.5241	0.00064274	9.7698	87.225	327.87	87.226	342.76	0.50201	1.8972
-85	2.3990	0.00064831	6.3707	93.180	330.49	93.182	345.77	0.53409	1.8766
-80	3.6719	0.00065401	4.2682	99.158	333.15	99.161	348.83	1	1.8580
-75	5.4777	0.00065985	2.9312	105.16	335.85	105.17	351.91	0.59613	1.8414
-70	7.9814	0.00066583	2.0590	111.19	338.59	111.20	355.02	0.62619	1.8264
-65	11.380	0.00067197	1.4765	117.26	341.35	117.26	358.16	0.65568	1.8130
-60	15.906	0.00067827	1.0790	123.35	344.15	123.36	361.31	0.68462	1.8010
-55	21.828	0.00068475	0.80236	129.48	346.96	129.50	364.48	0.71305	1.7902
-50	29.451	0.00069142	0.60620	135.65	349.80	135.67	367.65	0.74101	1.7806
-45	39.117	0.00069828	0.46473	141.86	352.65	141.89	370.83	0.76852	1.7720
-40	51.209	0.00070537	0.36108	148.11	355.51	148.14	374.00	0.79561	1.7643
-35	66.144	0.00071268	0.28402	154.40	358.38	154.44	377.17	0.82230	1.7575
-30	84.378	0.00072025	0.22594	160.73	361.25	160.79	380.32	0.84863	1.7515
-25	106.40	0.00072809	0.18162	167.11	364.12	167.19	383.45	0.87460	1.7461
-20	132.73	0.00073623	0.14739	173.54	366.99	173.64	386.55	0.90025	1.7413
-15	163.94	0.00074469	0.12067	180.02	369.85	180.14	389.63	0.92559	1.7371
-10	200.60	0.00075351	0.099590	186.55	372.69	186.70	392.66	0.95065	1.7334
-5	243.34	0.00076271	0.082801	193.13	375.51	193.32	395.66	0.97544	1.7300
0	292.80	0.00077233	0.069309	199.77	378.31	200	398.60	1	1.7271
5	349.66	0.00078243	0.058374	206.48	381.08	206.75	401.49	1.0243	1.7245
10	414.61	0.00079305	0.049442	213.25	383.82	213.58	404.32	1.0485	1.7221
15	488.37	0.00080425	0.042090	220.09	386.52	220.48	407.07	1.0724	1.7200
20	571.71	0.00081610	0.035997	227.00	389.17	227.47	409.75	1.0962	1.7180
25	665.38	0.00082870	0.030912	233.99	391.77	234.55	412.33	1.1199	1.7162
30	770.20	0.00084213	0.026642	241.07	394.30	241.72	414.82	1.1435	1.7145
35	886.98	0.00085653	0.023033	248.25	396.76	249.01	417.19	1.1670	1.7128
40	1016.6	0.00087204	0.019966	255.52	399.13	256.41	419.43	1.1905	1.7111
45	1159.9	0.00088885	0.017344	262.91	401.40	263.94	421.52	1.2139	1.7092
50	1317.9	0.00090719	0.015089	270.43	403.55	271.62	423.44	1.2375	1.7072
55	1491.5	0.00092737	0.013140	278.09	405.55	279.47	425.15	1.2611	1.7050
60	1681.8	0.00094979	0.011444	285.91	407.38	287.50	426.63	1.2848	1.7024
65	1889.8	0.00097500	0.0099604	293.92	408.99	295.76	427.82	1.3088	1.6993
70	2116.8	0.0010038	0.0086527	302.16	410.33	304.28	428.65	1.3332	1.6956
75	2364.1	0.0010372	0.0074910	310.68	411.32	313.13	429.03	1.3580	1.6909
80	2633.2	0.0010773	0.0064483	319.55	411.83	322.39	428.81	1.3836	1.6850
85	2925.8	0.0011272	0.0054990	328.93	411.67	332.22	427.76	1.4104	1.6771
90	3244.2	0.0011936	0.0046134	339.06	410.45	342.93	425.42	1.4390	1.6662
95	3591.2	0.0012942	0.0037434	350.60	407.23	355.25	420.67	1.4715	1.6492
100	3972.4	0.0015357	0.0026809	367.20	397.03	373.30	407.68	1.5188	1.6109
101.06	4059.3	0.0019535	0.0019535	381.71	381.71	389.64	389.64	1.5621	1.5621

Reference States:

H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Saturation Pressure Table

R-134a
SI

Pressure (kPa)	Temp. (^{o}C)	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)		S (kJ/kg*K)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
50	-40.45	0.00070471	0.36925	147.54	355.25	147.57	373.71	0.79317	1.7650
100	-26.36	0.00072593	0.19256	165.37	363.34	165.44	382.60	0.86756	1.7475
150	-17.13	0.00074104	0.13128	177.25	368.63	177.36	388.32	0.91482	1.7388
200	-10.08	0.00075337	0.099877	186.45	372.64	186.60	392.62	0.95027	1.7334
250	-4.28	0.00076406	0.080685	194.08	375.91	194.27	396.08	0.97898	1.7296
300	0.67	0.00077366	0.067704	200.67	378.68	200.90	399.00	1.00328	1.7267
350	5.03	0.00078249	0.058319	206.52	381.10	206.79	401.51	1.02447	1.7244
400	8.93	0.00079073	0.051207	211.79	383.24	212.11	403.72	1.04333	1.7226
450	12.48	0.00079852	0.045625	216.63	385.16	216.99	405.69	1.06036	1.7210
500	15.73	0.00080595	0.041123	221.10	386.91	221.50	407.47	1.07594	1.7197
550	18.75	0.00081308	0.037413	225.27	388.51	225.72	409.09	1.09032	1.7185
600	21.57	0.00081998	0.034300	229.19	389.99	229.68	410.57	1.10370	1.7175
650	24.22	0.00082667	0.031650	232.89	391.36	233.43	411.94	1.11622	1.7165
700	26.71	0.00083320	0.029365	236.41	392.64	236.99	413.20	1.12801	1.7156
750	29.08	0.00083959	0.027375	239.76	393.84	240.39	414.37	1.13916	1.7148
800	31.33	0.00084585	0.025625	242.97	394.96	243.65	415.46	1.14975	1.7140
850	33.47	0.00085202	0.024073	246.05	396.02	246.77	416.48	1.15984	1.7133
900	35.53	0.00085811	0.022687	249.01	397.01	249.78	417.43	1.16948	1.7126
950	37.50	0.00086412	0.021442	251.86	397.95	252.69	418.32	1.17872	1.7119
1000	39.39	0.00087007	0.020316	254.63	398.85	255.50	419.16	1.18760	1.7113
1100	42.97	0.00088185	0.018360	259.90	400.49	260.87	420.69	1.20441	1.7100
1200	46.31	0.00089351	0.016718	264.88	401.98	265.95	422.04	1.22012	1.7087
1300	49.46	0.00090511	0.015319	269.60	403.32	270.78	423.24	1.23490	1.7075
1400	52.42	0.00091671	0.014110	274.12	404.54	275.40	424.30	1.24887	1.7062
1500	55.23	0.00092836	0.013056	278.45	405.64	279.84	425.23	1.26216	1.7049
1600	57.91	0.00094010	0.012126	282.61	406.64	284.11	426.04	1.27484	1.7036
1700	60.46	0.00095197	0.011301	286.63	407.54	288.25	426.75	1.28700	1.7022
1800	62.90	0.00096400	0.010562	290.52	408.35	292.26	427.36	1.29868	1.7007
1900	65.23	0.00097626	0.0098956	294.30	409.06	296.15	427.87	1.30995	1.6992
2000	67.48	0.00098877	0.0092915	297.98	409.70	299.95	428.28	1.32085	1.6976
2200	71.73	0.00101474	0.0082357	305.07	410.72	307.30	428.84	1.34171	1.6941
2400	75.69	0.00104234	0.0073402	311.88	411.42	314.38	429.04	1.36153	1.6902
2600	79.41	0.00107211	0.0065670	318.47	411.80	321.26	428.88	1.38054	1.6858
2800	82.90	0.00110474	0.0058879	324.92	411.84	328.01	428.33	1.39898	1.6807
3000	86.20	0.00114128	0.0052813	331.28	411.49	334.70	427.34	1.4171	1.6748
3200	89.33	0.00118330	0.0047295	337.64	410.70	341.43	425.83	1.4350	1.6679
3400	92.30	0.00123354	0.0042160	344.12	409.32	348.31	423.65	1.4533	1.6595
3600	95.12	0.00129737	0.0037218	350.91	407.11	355.58	420.50	1.4724	1.6487
3800	97.80	0.00138860	0.0032143	358.50	403.41	363.77	415.63	1.4939	1.6336
4000	100.34	0.00158011	0.0025626	369.25	395.13	375.57	405.38	1.5247	1.6046
4059.3	101.06	0.0019535	0.0019535	381.71	381.71	389.64	389.64	1.5621	1.5621

Reference States:

H = 200 kJ/kg for saturated liquid at 0°C.

S = 1 kJ/kg-K for saturated liquid at 0°C.

Superheated Vapor

R-134a
SI

P = 60 kPa (-36.93°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.31123	357.27	375.94	1.7601
-30	0.32149	361.93	381.22	1.7821
-20	0.33608	368.74	388.91	1.8131
-10	0.35049	375.70	396.73	1.8434
0	0.36476	382.80	404.69	1.8730
10	0.37893	390.07	412.81	1.9022
20	0.39303	397.50	421.08	1.9310
30	0.40705	405.10	429.52	1.9593
40	0.42102	412.86	438.12	1.9872
50	0.43495	420.79	446.88	2.0147
60	0.44884	428.88	455.81	2.0419
70	0.46269	437.14	464.90	2.0688
80	0.47652	445.56	474.15	2.0954
90	0.49032	454.14	483.56	2.1217
100	0.50410	462.89	493.14	2.1477

P = 100 kPa (-26.36°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.19256	363.34	382.60	1.7475
-30				
-20	0.19841	367.81	387.65	1.7677
-10	0.20743	374.89	395.64	1.7986
0	0.21630	382.10	403.73	1.8288
10	0.22506	389.45	411.95	1.8584
20	0.23373	396.94	420.31	1.8874
30	0.24233	404.59	428.82	1.9160
40	0.25088	412.40	437.49	1.9441
50	0.25938	420.37	446.30	1.9718
60	0.26784	428.49	455.28	1.9991
70	0.27626	436.78	464.41	2.0261
80	0.28466	445.23	473.70	2.0528
90	0.29303	453.84	483.14	2.0792
100	0.30138	462.61	492.74	2.1053

P = 140 kPa (-18.76°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.14015	367.70	387.32	1.7402
-30				
-20				
-10	0.14606	374.05	394.50	1.7680
0	0.15263	381.37	402.74	1.7987
10	0.15908	388.81	411.08	1.8287
20	0.16544	396.37	419.53	1.8580
30	0.17172	404.08	428.12	1.8868
40	0.17795	411.93	436.84	1.9151
50	0.18412	419.94	445.72	1.9430
60	0.19025	428.10	454.74	1.9705
70	0.19635	436.42	463.91	1.9977
80	0.20243	444.90	473.24	2.0244
90	0.20847	453.53	482.72	2.0509
100	0.21450	462.32	492.35	2.0771

P = 180 kPa (-12.71°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.11042	371.15	391.02	1.7353
-10	0.11190	373.17	393.31	1.7441
0	0.11722	380.62	401.72	1.7754
10	0.12240	388.15	410.18	1.8059
20	0.12748	395.79	418.73	1.8355
30	0.13248	403.55	427.40	1.8646
40	0.13742	411.46	436.19	1.8931
50	0.14230	419.51	445.13	1.9212
60	0.14715	427.71	454.20	1.9489
70	0.15196	436.06	463.41	1.9761
80	0.15674	444.56	472.78	2.0030
90	0.16149	453.22	482.29	2.0296
100	0.16622	462.03	491.95	2.0558
110	0.17094	470.99	501.76	2.0818

P = 200 kPa (-10.08°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.099877	372.64	392.62	1.7334
-10	0.099915	372.70	392.68	1.7337
0	0.10481	380.23	401.20	1.7654
10	0.10955	387.82	409.73	1.7961
20	0.11419	395.49	418.33	1.8259
30	0.11874	403.29	427.04	1.8551
40	0.12323	411.22	435.87	1.8838
50	0.12766	419.29	444.83	1.9120
60	0.13206	427.51	453.92	1.9397
70	0.13642	435.88	463.16	1.9670
80	0.14074	444.39	472.54	1.9939
90	0.14505	453.06	482.07	2.0206
100	0.14933	461.88	491.75	2.0468
110	0.15359	470.86	501.57	2.0728

P = 240 kPa (-5.37°C)

Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.083906	375.30	395.44	1.7303
-10				
0	0.086170	379.43	400.11	1.7475
10	0.090262	387.13	408.79	1.7787
20	0.094233	394.89	417.51	1.8090
30	0.098118	402.75	426.30	1.8385
40	0.10193	410.74	435.20	1.8674
50	0.10570	418.86	444.22	1.8957
60	0.10942	427.11	453.37	1.9236
70	0.11310	435.51	462.65	1.9511
80	0.11675	444.06	472.08	1.9781
90	0.12038	452.75	481.64	2.0048
100	0.12398	461.59	491.35	2.0312
110	0.12756	470.58	501.20	2.0573

Reference States: H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Superheated Vapor

R-134a
SI

	P = 280 kPa (-1.23°C)				P = 320 kPa (2.48°C)				P = 400 kPa (8.93°C)			
Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.072360	377.62	397.89	1.7278	0.063611	379.69	400.04	1.7257	0.051207	383.24	403.72	1.7226
0	0.072819	378.59	398.98	1.7318								
10	0.076460	386.42	407.83	1.7636	0.066088	385.69	406.83	1.7501	0.051506	384.12	404.72	1.7261
20	0.079966	394.27	416.67	1.7943	0.069252	393.64	415.80	1.7812	0.054214	392.32	414.01	1.7584
30	0.083378	402.21	425.55	1.8241	0.072313	401.65	424.79	1.8113	0.056797	400.50	423.22	1.7893
40	0.086719	410.25	434.53	1.8532	0.075299	409.75	433.85	1.8407	0.059293	408.73	432.45	1.8192
50	0.090003	418.41	443.61	1.8818	0.078226	417.96	443.00	1.8695	0.061724	417.05	441.74	1.8484
60	0.093242	426.71	452.82	1.9098	0.081106	426.30	452.25	1.8977	0.064104	425.47	451.11	1.8770
70	0.096443	435.14	462.14	1.9374	0.083947	434.77	461.63	1.9254	0.066443	434.01	460.58	1.9050
80	0.099612	443.71	471.61	1.9646	0.086755	443.37	471.13	1.9527	0.068748	442.67	470.17	1.9325
90	0.10275	452.43	481.20	1.9914	0.089536	452.11	480.77	1.9796	0.071023	451.47	479.88	1.9597
100	0.10587	461.30	490.94	2.0178	0.092293	461.00	490.54	2.0062	0.073275	460.41	489.72	1.9864
110	0.10897	470.31	500.82	2.0440	0.095030	470.03	500.44	2.0324	0.075505	469.48	499.68	2.0127
120	0.11206	479.47	510.85	2.0698	0.097749	479.21	510.49	2.0583	0.077717	478.69	509.78	2.0387
130	0.11512	488.78	521.01	2.0953	0.10045	488.54	520.68	2.0838	0.079914	488.05	520.02	2.0645
140	0.11818	498.24	531.33	2.1206	0.10314	498.01	531.02	2.1092	0.082097	497.55	530.39	2.0899

	P = 500 kPa (15.73°C)				P = 600 kPa (21.57°C)				P = 700 kPa (26.71°C)			
Temp. (°C)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.041123	386.91	407.47	1.7197	0.034300	389.99	410.57	1.7175	0.02937	392.64	413.20	1.7156
20	0.042116	390.55	411.61	1.7339								
30	0.044338	398.99	421.16	1.7659	0.035984	397.37	418.96	1.7455	0.029966	395.62	416.60	1.7269
40	0.046456	407.40	430.63	1.7967	0.037865	406.01	428.73	1.7772	0.031696	404.53	426.72	1.7598
50	0.048499	415.86	440.11	1.8265	0.039659	414.63	438.43	1.8077	0.033322	413.35	436.67	1.7910
60	0.050486	424.40	449.64	1.8555	0.041389	423.30	448.13	1.8373	0.034875	422.16	446.57	1.8212
70	0.052427	433.04	459.25	1.8839	0.043070	432.04	457.88	1.8661	0.036374	431.01	456.47	1.8505
80	0.054331	441.78	468.95	1.9118	0.044710	440.87	467.70	1.8943	0.037829	439.94	466.42	1.8791
90	0.056205	450.65	478.76	1.9392	0.046319	449.82	477.61	1.9220	0.039250	448.97	476.44	1.9070
100	0.058054	459.65	488.68	1.9661	0.047900	458.88	487.62	1.9492	0.040642	458.09	486.54	1.9345
110	0.059880	468.78	498.72	1.9927	0.049459	468.06	497.74	1.9759	0.042010	467.34	496.74	1.9615
120	0.061688	478.04	508.88	2.0189	0.050998	477.37	507.97	2.0023	0.043358	476.70	507.05	1.9880
130	0.063479	487.44	519.18	2.0447	0.052520	486.82	518.33	2.0283	0.044689	486.19	517.47	2.0142
140	0.065257	496.98	529.60	2.0703	0.054027	496.39	528.81	2.0540	0.046004	495.80	528.01	2.0400
150	0.067022	506.65	540.17	2.0955	0.055522	506.11	539.42	2.0794	0.047307	505.55	538.67	2.0655
160	0.068776	516.47	550.86	2.1205	0.057006	515.96	550.16	2.1045	0.048598	515.44	549.45	2.0907

Reference States: H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Superheated Vapor

R-134a
SI

Temp. (°C)	P = 800 kPa (31.33°C)				P = 900 kPa (35.53°C)				P = 1000 kPa (39.39°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.025625	394.96	415.46	1.7140	0.022687	397.01	417.43	1.7126	0.020316	398.85	419.16	1.7113
40	0.027036	402.97	424.59	1.7436	0.023375	401.28	422.32	1.7283	0.020407	399.45	419.86	1.7135
50	0.028547	412.00	434.84	1.7758	0.024810	410.59	432.92	1.7616	0.021796	409.09	430.88	1.7482
60	0.029974	420.97	444.95	1.8067	0.026146	419.75	443.28	1.7932	0.023068	418.46	441.53	1.7806
70	0.031340	429.96	455.03	1.8365	0.027414	428.87	453.54	1.8236	0.024262	427.74	452.00	1.8116
80	0.032659	438.99	465.12	1.8654	0.028630	438.01	463.78	1.8530	0.025399	437.00	462.40	1.8414
90	0.033942	448.10	475.25	1.8937	0.029807	447.21	474.03	1.8816	0.026493	446.30	472.79	1.8705
100	0.035193	457.30	485.45	1.9214	0.030951	456.48	484.34	1.9096	0.027552	455.65	483.21	1.8988
110	0.036420	466.60	495.74	1.9486	0.032069	465.85	494.71	1.9370	0.028584	465.09	493.67	1.9264
120	0.037626	476.02	506.12	1.9754	0.033164	475.32	505.17	1.9640	0.029593	474.62	504.21	1.9536
130	0.038813	485.55	516.60	2.0017	0.034241	484.91	515.72	1.9905	0.030582	484.25	514.83	1.9803
140	0.039985	495.21	527.20	2.0277	0.035302	494.61	526.38	2.0166	0.031554	494.00	525.55	2.0065
150	0.041144	504.99	537.91	2.0533	0.036349	504.43	537.14	2.0423	0.032512	503.86	536.37	2.0324
160	0.042291	514.91	548.74	2.0786	0.037384	514.38	548.02	2.0678	0.033458	513.84	547.30	2.0579
170	0.043427	524.96	559.70	2.1036	0.038408	524.46	559.02	2.0929	0.034392	523.95	558.35	2.0831
180	0.044554	535.14	570.78	2.1283	0.039423	534.67	570.15	2.1177	0.035318	534.19	569.51	2.1080

Temp. (°C)	P = 1200 kPa (46.31°C)				P = 1400 kPa (52.42°C)				P = 1600 kPa (57.91°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.016718	401.98	422.04	1.7087	0.014110	404.54	424.30	1.7062	0.012126	406.64	426.04	1.7036
50	0.017201	405.77	426.41	1.7223								
60	0.018404	415.70	437.79	1.7570	0.015005	412.61	433.62	1.7345	0.012373	409.04	428.84	1.7120
70	0.019502	425.36	448.76	1.7895	0.016060	422.77	445.25	1.7689	0.013430	419.91	441.40	1.7491
80	0.020530	434.90	459.53	1.8204	0.017023	432.65	456.48	1.8012	0.014362	430.24	453.22	1.7831
90	0.021506	444.41	470.22	1.8502	0.017923	442.42	467.52	1.8320	0.015216	440.32	464.66	1.8150
100	0.022443	453.94	480.87	1.8792	0.018778	452.16	478.45	1.8617	0.016015	450.29	475.91	1.8456
110	0.023348	463.53	491.54	1.9074	0.019597	461.90	489.34	1.8905	0.016773	460.22	487.06	1.8751
120	0.024228	473.18	502.25	1.9350	0.020388	471.70	500.24	1.9186	0.017500	470.16	498.16	1.9037
130	0.025087	482.92	513.03	1.9621	0.021156	481.55	511.17	1.9460	0.018201	480.15	509.27	1.9316
140	0.025928	492.76	523.87	1.9886	0.021904	491.49	522.16	1.9730	0.018882	490.19	520.40	1.9589
150	0.026753	502.70	534.81	2.0148	0.022636	501.52	533.21	1.9994	0.019546	500.32	531.59	1.9856
160	0.027566	512.76	545.84	2.0405	0.023355	511.65	544.35	2.0254	0.020194	510.53	542.84	2.0119
170	0.028367	522.93	556.97	2.0660	0.024061	521.89	555.58	2.0510	0.020830	520.84	554.17	2.0378
180	0.029159	533.23	568.22	2.0911	0.024758	532.25	566.91	2.0763	0.021456	531.26	565.59	2.0632
190	0.029941	543.64	579.57	2.1158	0.025445	542.72	578.34	2.1013	0.022072	541.78	577.10	2.0884

Reference States: H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Subcooled Liquid

R-134a
SI

Temp. (°C)	P = 800 kPa (31.33°C)				P = 900 kPa (35.53°C)				P = 1000 kPa (39.39°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.00084585	242.97	243.65	1.1497	0.00085811	249.01	249.78	1.1695	0.00087007	254.63	255.50	1.1876
35					0.00085646	248.23	249.01	1.1670	0.00085591	248.14	249.00	1.1667
30	0.00084199	241.05	241.72	1.1434	0.00084151	240.96	241.72	1.1431	0.00084104	240.87	241.72	1.1428
25	0.00082814	233.89	234.55	1.1196	0.00082772	233.81	234.55	1.1193	0.00082731	233.73	234.56	1.1190
20	0.00081527	226.83	227.49	1.0957	0.00081491	226.76	227.50	1.0954	0.00081454	226.69	227.50	1.0952
15	0.00080324	219.88	220.52	1.0717	0.00080292	219.81	220.53	1.0715	0.00080260	219.74	220.55	1.0712
10	0.00079193	213.01	213.64	1.0476	0.00079165	212.95	213.66	1.0474	0.00079136	212.88	213.68	1.0472
0	0.00077115	199.50	200.12	0.99899	0.00077092	199.45	200.14	0.99880	0.00077069	199.39	200.16	0.99860
-10	0.00075236	186.26	186.86	0.94957	0.00075217	186.21	186.89	0.94939	0.00075198	186.17	186.92	0.94921
-20	0.00073517	173.26	173.85	0.89914	0.00073501	173.22	173.88	0.89897	0.00073485	173.18	173.91	0.89881
-30	0.00071929	160.46	161.04	0.84752	0.00071916	160.43	161.07	0.84737	0.00071903	160.39	161.11	0.84722
-40	0.00070451	147.86	148.42	0.79454	0.00070440	147.82	148.46	0.79439	0.00070428	147.79	148.50	0.79425

Temp. (°C)	P = 1200 kPa (46.31°C)				P = 1400 kPa (52.42°C)				P = 1600 kPa (57.91°C)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)
Sat	0.00089351	264.88	265.95	1.2201	0.00091671	274.12	275.40	1.2489	0.00094010	282.61	284.11	1.2748
55									0.00092620	277.92	279.41	1.2606
50					0.00090646	270.32	271.59	1.2371	0.00090470	270.06	271.51	1.2363
45	0.00088854	262.86	263.93	1.2138	0.00088706	262.63	263.87	1.2131	0.00088561	262.40	263.82	1.2123
40	0.00087087	255.33	256.37	1.1899	0.00086962	255.12	256.34	1.1892	0.00086840	254.92	256.31	1.1885
35	0.00085483	247.95	248.98	1.1660	0.00085376	247.76	248.96	1.1654	0.00085271	247.58	248.94	1.1648
30	0.00084011	240.70	241.71	1.1423	0.00083919	240.53	241.71	1.1417	0.00083828	240.37	241.71	1.1412
25	0.00082650	233.57	234.57	1.1185	0.00082569	233.42	234.57	1.1180	0.00082490	233.27	234.59	1.1175
20	0.00081383	226.55	227.52	1.0947	0.00081312	226.40	227.54	1.0942	0.00081242	226.26	227.56	1.0937
15	0.00080196	219.61	220.57	1.0708	0.00080134	219.48	220.60	1.0703	0.00080071	219.35	220.63	1.0699
10	0.00079080	212.76	213.71	1.0467	0.00079024	212.64	213.75	1.0463	0.00078968	212.52	213.78	1.0459
5	0.00078024	205.99	206.93	1.0226	0.00077974	205.88	206.97	1.0222	0.00077924	205.76	207.01	1.0218
0	0.00077023	199.29	200.21	0.99821	0.00076978	199.18	200.26	0.99782	0.00076933	199.08	200.31	0.99743
-5	0.00076070	192.65	193.56	0.97365	0.00076029	192.55	193.61	0.97327	0.00075988	192.45	193.67	0.97290
-10	0.00075161	186.07	186.98	0.94885	0.00075123	185.98	187.03	0.94850	0.00075086	185.89	187.09	0.94814
-15	0.00074290	179.56	180.45	0.92380	0.00074256	179.47	180.51	0.92346	0.00074222	179.38	180.57	0.92312
-20	0.00073454	173.09	173.97	0.89848	0.00073423	173.01	174.04	0.89815	0.00073392	172.93	174.10	0.89783
-25	0.00072650	166.68	167.55	0.87286	0.00072622	166.60	167.62	0.87254	0.00072593	166.52	167.69	0.87223
-30	0.00071876	160.32	161.18	0.84691	0.00071850	160.24	161.25	0.84661	0.00071824	160.17	161.32	0.84631
-40	0.00070406	147.73	148.57	0.79397	0.00070383	147.66	148.64	0.79368	0.00070361	147.59	148.72	0.79340

Reference States: H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Saturation Temperature Table

R-134a
AE

Temp. (°F)	Pressure (psia)	V (ft^3/lb$_m$)		U (Btu/lb$_m$)		H (Btu/lb$_m$)		S (Btu/lb$_m$°R)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
-40	7.4272	0.011299	5.7839	63.718	152.95	63.733	160.90	0.19016	0.42169
-35	8.5756	0.011364	5.0544	65.219	153.63	65.237	161.66	0.19371	0.42076
-30	9.8624	0.011429	4.4330	66.725	154.32	66.746	162.41	0.19724	0.41989
-25	11.299	0.011496	3.9014	68.238	155.00	68.262	163.17	0.20074	0.41907
-20	12.898	0.011565	3.4449	69.756	155.69	69.784	163.92	0	0.41831
-15	14.671	0.011635	3.0514	71.281	156.38	71.313	164.67	0.20766	0.41760
-10	16.632	0.011706	2.7109	72.812	157.06	72.848	165.41	0.21109	0.41693
-5	18.794	0.011779	2.4154	74.350	157.75	74.391	166.15	0.21449	0.41631
0	21.171	0.011853	2.1579	75.894	158.43	75.940	166.89	0.21786	0.41572
5	23.777	0.011929	1.9330	77.445	159.11	77.497	167.62	0.22122	0.41518
10	26.628	0.012007	1.7357	79.002	159.79	79.062	168.35	0.22456	0.41467
15	29.739	0.012086	1.5623	80.567	160.47	80.634	169.07	0.22787	0.41419
20	33.124	0.012168	1.4094	82.140	161.14	82.214	169.79	0.23117	0.41374
25	36.800	0.012251	1.2742	83.720	161.82	83.803	170.50	0.23445	0.41332
32	42.468	0.012372	1.1102	85.945	162.75	86.042	171.48	0.23901	0.41278
40	49.741	0.012515	0.95280	88.507	163.81	88.623	172.59	0.24418	0.41222
45	54.749	0.012608	0.86796	90.120	164.47	90.248	173.27	0.24739	0.41190
50	60.134	0.012703	0.79198	91.742	165.12	91.883	173.94	0.25059	0.41159
55	65.913	0.012802	0.72380	93.372	165.77	93.529	174.60	0.25378	0.41131
60	72.105	0.012903	0.66246	95.013	166.41	95.185	175.26	0.25695	0.41103
65	78.729	0.013008	0.60718	96.663	167.05	96.853	175.90	0.26011	0.41077
70	85.805	0.013116	0.55724	98.324	167.67	98.532	176.53	0.26327	0.41052
75	93.351	0.013229	0.51204	100.00	168.30	100.22	177.15	0.26641	0.41028
80	101.39	0.013345	0.47104	101.68	168.91	101.93	177.75	0.26955	0.41005
85	109.93	0.013465	0.43379	103.37	169.51	103.65	178.34	0.27268	0.40982
90	119.01	0.013590	0.39988	105.08	170.11	105.38	178.92	0.27580	0.40959
95	128.65	0.013720	0.36896	106.80	170.69	107.13	179.48	0.27892	0.40937
100	138.85	0.013856	0.34070	108.53	171.26	108.89	180.02	0.28204	0.40914
105	149.65	0.013998	0.31483	110.28	171.82	110.67	180.55	0.28515	0.40891
110	161.07	0.014146	0.29111	112.04	172.37	112.46	181.05	0.28827	0.40867
115	173.14	0.014301	0.26933	113.82	172.90	114.28	181.53	0.29139	0.40842
120	185.86	0.014464	0.24928	115.62	173.41	116.12	181.99	0.29451	0.40815
125	199.28	0.014636	0.23080	117.43	173.91	117.97	182.42	0.29764	0.40787
130	213.41	0.014817	0.21374	119.27	174.38	119.85	182.83	0.30077	0.40757
135	228.28	0.015010	0.19795	121.12	174.83	121.76	183.20	0.30392	0.40725
140	243.92	0.015214	0.18332	123.00	175.26	123.69	183.54	0.30708	0.40689
145	260.36	0.015432	0.16973	124.90	175.66	125.65	183.84	0.31026	0.40650
150	277.61	0.015666	0.15709	126.84	176.02	127.64	184.10	0.31346	0.40606
160	314.73	0.016191	0.13428	130.79	176.64	131.74	184.46	0.31995	0.40503
170	355.53	0.016815	0.11424	134.91	177.05	136.02	184.57	0.32659	0.40371
180	400.34	0.017588	0.096375	139.24	177.19	140.54	184.33	0.33350	0.40196
190	588.75	0.031292	0.031292	164.22	164.22	167.63	167.63	0.37335	0.37335
200	503.59	0.020096	0.064663	149.07	175.80	150.95	181.83	0.34896	0.39577
213.91	588.72	0.031292	0.031292	164.22	164.22	167.63	167.63	0.37335	0.37335

Reference States:

H = 85.985 Btu/lb$_m$ for Saturated Liquid at 32°F.
S = 0.23885 Btu/lb$_m$-°R for Saturated Liquid at 32°F.

Saturation Pressure Table

R-134a
AE

Pressure (psia)	Temp. (^{o}F)	V (ft^3/lb$_m$)		U (Btu/lb$_m$)		H (Btu/lb$_m$)		S (Btu/lb$_m$^{o}R)	
		Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap	Sat. Liq	Sat. Vap
5	-53.07	0.011135	8.3793	59.821	151.16	59.832	158.92	0.18072	0.42441
10	-29.50	0.011436	4.3756	66.877	154.39	66.898	162.49	0.19759	0.41980
15	-14.13	0.011647	2.9882	71.548	156.50	71.581	164.80	0.20826	0.41748
20	-2.40	0.011817	2.2774	75.150	158.10	75.194	166.54	0.21624	0.41600
25	7.20	0.011963	1.8431	78.129	159.41	78.184	167.94	0.22269	0.41495
30	15.40	0.012093	1.5493	80.693	160.52	80.760	169.13	0.22814	0.41415
35	22.60	0.012211	1.3370	82.961	161.49	83.040	170.16	0.23288	0.41352
40	29.05	0.012320	1.1761	85.004	162.36	85.095	171.07	0.23709	0.41301
45	34.90	0.012423	1.0498	86.870	163.14	86.974	171.89	0.24088	0.41257
50	40.27	0.012520	0.94802	88.593	163.85	88.709	172.63	0.24435	0.41220
55	45.24	0.012612	0.86410	90.198	164.50	90.326	173.30	0.24755	0.41188
60	49.88	0.012701	0.79371	91.703	165.11	91.844	173.93	0.25051	0.41160
65	54.23	0.012787	0.73379	93.122	165.67	93.276	174.50	0.25329	0.41135
70	58.34	0.012869	0.68214	94.467	166.20	94.633	175.04	0.25590	0.41112
80	65.92	0.013028	0.59758	96.969	167.16	97.162	176.02	0.26070	0.41073
85	69.45	0.013104	0.56251	98.140	167.61	98.346	176.46	0.26292	0.41055
90	72.82	0.013179	0.53121	99.265	168.03	99.484	176.88	0.26504	0.41039
95	76.05	0.013253	0.50308	100.35	168.43	100.58	177.28	0.26707	0.41023
100	79.16	0.013325	0.47767	101.39	168.81	101.64	177.65	0.26902	0.41009
110	85.04	0.013466	0.43353	103.38	169.52	103.66	178.35	0.27270	0.40982
120	90.53	0.013604	0.39650	105.26	170.17	105.56	178.98	0.27613	0.40957
130	95.68	0.013738	0.36496	107.03	170.77	107.36	179.55	0.27935	0.40934
140	100.55	0.013871	0.33777	108.72	171.32	109.08	180.08	0.28238	0.40911
150	105.16	0.014002	0.31406	110.33	171.84	110.72	180.56	0.28525	0.40890
160	109.54	0.014132	0.29321	111.88	172.32	112.30	181.00	0.28798	0.40869
170	113.73	0.014261	0.27471	113.37	172.76	113.82	181.41	0.29059	0.40848
180	117.73	0.014389	0.25818	114.80	173.18	115.28	181.79	0.29309	0.40827
190	121.57	0.014517	0.24332	116.18	173.57	116.70	182.13	0.29549	0.40807
200	125.26	0.014645	0.22988	117.53	173.93	118.07	182.45	0.29780	0.40786
220	132.25	0.014902	0.20649	120.10	174.59	120.70	183.00	0.30218	0.40743
240	138.77	0.015163	0.18682	122.54	175.16	123.21	183.46	0.30630	0.40698
260	144.89	0.015428	0.17001	124.86	175.65	125.61	183.84	0.31019	0.40651
280	150.67	0.015699	0.15546	127.10	176.07	127.91	184.13	0.31389	0.40600
300	156.15	0.015978	0.14271	129.25	176.42	130.14	184.35	0.31743	0.40546
320	161.35	0.016268	0.13143	131.34	176.71	132.30	184.50	0.32083	0.40487
340	166.30	0.016570	0.12136	133.37	176.93	134.41	184.57	0.32411	0.40424
360	171.04	0.016888	0.11229	135.35	177.08	136.47	184.57	0.32730	0.40355
380	175.58	0.017223	0.10404	137.29	177.17	138.50	184.49	0.33040	0.40280
400	179.93	0.017582	0.096497	139.20	177.19	140.51	184.33	0.33345	0.40197
450	190.09	0.018614	0.079969	143.92	176.88	145.47	183.54	0.34089	0.39948
475	194.83	0.019238	0.072617	146.29	176.50	147.98	182.88	0.34461	0.39793
500	199.37	0.019977	0.065644	148.72	175.91	150.57	181.98	0.34840	0.39607
588.75	213.91	0.031292	0.031292	164.22	164.22	167.63	167.63	0.37335	0.37335

Reference States:

H = 85.985 Btu/lb$_m$ for Saturated Liquid at 32°F.
S = 0.23885 Btu/lb$_m$-°R for Saturated Liquid at 32°F.

Superheated Vapor

R-134a
AE

Temp.	P = 1 psia (-97.56°F)				P = 10 psia (-29.5°F)				P = 14.696 psia (-14.93°F)			
(^{o}C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R
Sat	37.834	145.20	152.20	0.43789	4.3756	154.39	162.49	0.41980	3.0465	156.39	164.68	0.41759
-20	46.109	156.61	165.14	0.47022	4.4856	155.93	164.23	0.42382				
0	48.229	159.78	168.72	0.47816	4.7136	159.21	167.94	0.43206	3.1669	158.90	167.51	0.42387
20	50.346	163.06	172.38	0.48597	4.9380	162.57	171.71	0.44009	3.3248	162.30	171.35	0.43203
40	52.461	166.44	176.15	0.49366	5.1601	166.00	175.56	0.44795	3.4799	165.77	175.24	0.43998
60	54.574	169.91	180.01	0.50124	5.3802	169.52	179.49	0.45565	3.6331	169.32	179.21	0.44776
80	56.685	173.47	183.97	0.50871	5.5990	173.13	183.50	0.46323	3.7848	172.95	183.25	0.45539
100	58.796	177.13	188.02	0.51608	5.8166	176.82	187.59	0.47068	3.9352	176.66	187.37	0.46289
120	60.906	180.88	192.16	0.52335	6.0332	180.60	191.78	0.47802	4.0847	180.46	191.57	0.47027
140	63.015	184.73	196.40	0.53053	6.2490	184.47	196.04	0.48526	4.2334	184.34	195.86	0.47754
160	65.123	188.66	200.72	0.53762	6.4643	188.43	200.40	0.49240	4.3814	188.30	200.23	0.48470
180	67.231	192.68	205.13	0.54463	6.6789	192.47	204.84	0.49945	4.5289	192.35	204.68	0.49178
200	69.338	196.79	209.63	0.55156	6.8931	196.60	209.36	0.50641	4.6759	196.49	209.22	0.49876
220	71.445	200.99	214.22	0.55841	7.1069	200.81	213.97	0.51330	4.8225	200.71	213.84	0.50566
240	73.552	205.28	218.90	0.56519	7.3204	205.11	218.66	0.52010	4.9688	205.02	218.54	0.51248

Temp.	P = 20 psia (-2.4°F)				P = 30 psia (15.4°F)				P = 40 psia (29.05°F)			
(^{o}C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$^{o}R
Sat	2.2774	158.10	166.54	0.41600	1.5493	160.52	169.13	0.41415	1.1761	162.36	171.07	0.41301
0	2.2922	158.52	167.01	0.41702								
20	2.4130	161.99	170.92	0.42536	1.5691	161.35	170.07	0.41612				
40	2.5306	165.50	174.87	0.43343	1.6528	164.97	174.15	0.42446	1.2126	164.41	173.39	0.41770
60	2.6461	169.08	178.88	0.44130	1.7338	168.62	178.26	0.43251	1.2768	168.15	177.60	0.42597
80	2.7600	172.74	182.96	0.44900	1.8130	172.33	182.41	0.44035	1.3389	171.92	181.83	0.43396
100	2.8726	176.47	187.11	0.45655	1.8908	176.11	186.62	0.44801	1.3995	175.74	186.11	0.44173
120	2.9842	180.29	191.34	0.46397	1.9675	179.96	190.89	0.45551	1.4589	179.63	190.44	0.44933
140	3.0950	184.18	195.65	0.47128	2.0434	183.89	195.24	0.46289	1.5173	183.59	194.83	0.45678
160	3.2051	188.16	200.03	0.47848	2.1185	187.90	199.67	0.47014	1.5751	187.63	199.29	0.46410
180	3.3147	192.23	204.50	0.48557	2.1931	191.98	204.16	0.47729	1.6322	191.73	203.82	0.47130
200	3.4237	196.37	209.05	0.49258	2.2672	196.15	208.74	0.48434	1.6888	195.92	208.43	0.47839
220	3.5324	200.60	213.68	0.49949	2.3408	200.39	213.40	0.49129	1.7449	200.18	213.11	0.48537
240	3.6407	204.91	218.40	0.50633	2.4141	204.72	218.13	0.49815	1.8007	204.53	217.86	0.49227

Reference States:

H = 85.985 Btu/lb$_m$ for Saturated Liquid at 32°F.
S = 0.23885 Btu/lb$_m$-°R for Saturated Liquid at 32°F.

Superheated Vapor

R-134a
AE

Temp. (^{o}C)	P = 50 psia (40.27°F)				P = 60 psia (49.88°F)				P = 70 psia (58.34°F)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$ ^{o}R
Sat	0.94802	163.85	172.63	0.41220	0.79371	165.11	173.93	0.41160	0.68214	166.20	175.04	0.41112
60	1.0019	167.64	176.92	0.42063	0.81790	167.11	176.20	0.41601	0.81790	167.11	176.20	0.41601
80	1.0540	171.49	181.24	0.42879	0.86356	171.03	180.63	0.42438	0.72710	170.56	179.99	0.42048
100	1.1043	175.36	185.59	0.43670	0.90725	174.97	185.05	0.43243	0.76618	174.57	184.50	0.42869
120	1.1534	179.29	189.97	0.44440	0.94955	178.95	189.50	0.44024	0.80372	178.59	189.01	0.43661
140	1.2015	183.29	194.41	0.45192	0.99079	182.98	193.99	0.44785	0.84013	182.66	193.55	0.44431
160	1.2488	187.35	198.91	0.45931	1.0312	187.07	198.53	0.45530	0.87565	186.78	198.13	0.45183
180	1.2955	191.48	203.48	0.46656	1.0709	191.23	203.13	0.46260	0.91047	190.97	202.77	0.45919
200	1.3416	195.69	208.11	0.47369	1.1101	195.46	207.79	0.46978	0.94472	195.22	207.46	0.46642
220	1.3873	199.97	212.82	0.48071	1.1489	199.76	212.52	0.47684	0.97849	199.54	212.22	0.47352
240	1.4326	204.33	217.59	0.48764	1.1872	204.13	217.32	0.48380	1.0119	203.93	217.05	0.48051
260	1.4776	208.76	222.44	0.49448	1.2252	208.58	222.19	0.49067	1.0449	208.39	221.94	0.48741
280	1.5223	213.28	227.37	0.50123	1.2629	213.10	227.14	0.49745	1.0776	212.93	226.90	0.49421
300	1.5668	217.87	232.37	0.50790	1.3004	217.71	232.15	0.50414	1.1101	217.54	231.93	0.50092
320	1.6111	222.53	237.45	0.51450	1.3377	222.38	237.25	0.51075	1.1424	222.23	237.04	0.50756

Temp. (^{o}C)	P = 80 psia (65.92°F)				P = 90 psia (72.82°F)				P = 100 psia (79.16°F)			
	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V (m^3/kg)	U (kJ/kg)	H (kJ/kg)	S (kJ/kg*K)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$ ^{o}R
Sat	0.59758	167.16	176.02	0.41073	0.53121	168.03	176.88	0.41039	0.47767	168.81	177.65	0.41009
80	0.62430	170.07	179.32	0.41693	0.54389	169.55	178.61	0.41362	0.47907	168.99	177.86	0.41047
100	0.66009	174.15	183.93	0.42531	0.57729	173.71	183.33	0.42221	0.51077	173.26	182.72	0.41931
120	0.69416	178.23	188.51	0.43336	0.60874	177.85	188.00	0.43040	0.54023	177.47	187.47	0.42766
140	0.72699	182.34	193.11	0.44116	0.63885	182.01	192.65	0.43830	0.56822	181.67	192.19	0.43566
160	0.75888	186.49	197.74	0.44875	0.66796	186.20	197.33	0.44597	0.59513	185.90	196.92	0.44342
180	0.79004	190.71	202.41	0.45617	0.69629	190.44	202.04	0.45345	0.62123	190.17	201.67	0.45097
200	0.82060	194.98	207.14	0.46345	0.72400	194.74	206.80	0.46078	0.64667	194.49	206.47	0.45835
220	0.85066	199.32	211.92	0.47059	0.75120	199.10	211.62	0.46797	0.67158	198.87	211.31	0.46558
240	0.88031	203.73	216.77	0.47762	0.77797	203.52	216.49	0.47503	0.69606	203.32	216.20	0.47268
260	0.90962	208.21	221.68	0.48454	0.80438	208.02	221.42	0.48198	0.72017	207.83	221.16	0.47966
280	0.93862	212.76	226.66	0.49137	0.83049	212.58	226.42	0.48884	0.74397	212.40	226.18	0.48654
300	0.96737	217.38	231.71	0.49811	0.85634	217.22	231.49	0.49559	0.76750	217.05	231.26	0.49332
320	0.99590	222.08	236.83	0.50476	0.88196	221.92	236.62	0.50227	0.79080	221.77	236.41	0.50001

Reference States: H = 85.985 Btu/lb$_m$ for Saturated Liquid at 32°F.
S = 0.23885 Btu/lb$_m$-°R for Saturated Liquid at 32°F.

Superheated Vapor

R-134a AE

	P = 150 psia (105.16°F)				P = 200 psia (125.26°F)				P = 250 psia (141.88°F)			
Temp. (°C)	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.31406	171.84	180.56	0.40890	0.22988	173.93	182.45	0.40786	0.17810	175.41	183.66	0.40675
120	0.33233	175.31	184.54	0.41585								
140	0.35477	179.83	189.69	0.42458	0.24541	177.66	186.75	0.41512				
160	0.37558	184.29	194.73	0.43285	0.26412	182.48	192.26	0.42416	0.19527	180.33	189.37	0.41611
180	0.39527	188.74	199.72	0.44078	0.28115	187.17	197.58	0.43262	0.21146	185.40	195.19	0.42535
200	0.41413	193.21	204.71	0.44846	0.29705	191.82	202.82	0.44068	0.22600	190.30	200.76	0.43393
220	0.43234	197.71	209.72	0.45594	0.31213	196.46	208.02	0.44845	0.23945	195.13	206.21	0.44206
240	0.45003	202.25	214.75	0.46324	0.32658	201.12	213.22	0.45599	0.25213	199.93	211.60	0.44988
260	0.46730	206.84	219.82	0.47039	0.34055	205.82	218.43	0.46333	0.26421	204.74	216.97	0.45744
280	0.48422	211.50	224.94	0.47740	0.35411	210.55	223.66	0.47050	0.27584	209.56	222.33	0.46480
300	0.50084	216.21	230.12	0.48430	0.36734	215.33	228.94	0.47754	0.28709	214.43	227.72	0.47198
320	0.51721	220.98	235.35	0.49110	0.38029	220.17	234.25	0.48444	0.29803	219.33	233.13	0.47901
340	0.53337	225.82	240.64	0.49780	0.39301	225.06	239.62	0.49124	0.30872	224.28	238.58	0.48591
360	0.54934	230.73	245.99	0.50441	0.40553	230.02	245.04	0.49793	0.31919	229.29	244.07	0.49269
380	0.56514	235.71	251.40	0.51094	0.41787	235.04	250.51	0.50453	0.32947	234.36	249.61	0.49937

	P = 300 psia (156.15°F)				P = 400 psia (179.93°F)				P = 500 psia (199.37°F)			
Temp. (°C)	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$	V ft^3/lb_m	U Btu/lb_m	H Btu/lb_m	S $Btu/lb_m{}^oR$
Sat	0.14271	176.42	184.35	0.40546	0.096497	177.19	184.33	0.40197	0.065644	175.91	181.98	0.39607
160	0.14656	177.63	185.77	0.40776								
180	0.16356	183.34	192.42	0.41833	0.096581	177.22	184.37	0.40203	0.017125	138.11	139.70	0.33168
200	0.17777	188.60	198.48	0.42765	0.11440	184.33	192.81	0.41502	0.066753	176.38	182.56	0.39695
220	0.19044	193.67	204.25	0.43627	0.12746	190.26	199.70	0.42532	0.086577	185.65	193.66	0.41355
240	0.20211	198.65	209.88	0.44443	0.13853	195.77	206.03	0.43450	0.098822	192.26	201.41	0.42479
260	0.21306	203.59	215.43	0.45226	0.14844	201.09	212.08	0.44303	0.10876	198.19	208.26	0.43444
280	0.22347	208.53	220.95	0.45982	0.15756	206.31	217.98	0.45111	0.11745	203.81	214.69	0.44326
300	0.23346	213.48	226.45	0.46716	0.16611	211.48	223.78	0.45886	0.12535	209.28	220.89	0.45153
320	0.24310	218.46	231.97	0.47433	0.17423	216.64	229.54	0.46634	0.13269	214.67	226.96	0.45942
340	0.25247	223.48	237.51	0.48134	0.18201	221.81	235.29	0.47361	0.13960	220.02	232.95	0.46700
360	0.26159	228.54	243.08	0.48822	0.18951	226.99	241.03	0.48071	0.14618	225.36	238.90	0.47435
380	0.27052	233.66	248.69	0.49498	0.19678	232.22	246.79	0.48765	0.15250	230.71	244.83	0.48150
400	0.27928	238.82	254.34	0.50164	0.20386	237.48	252.58	0.49446	0.15860	236.08	250.76	0.48848

Reference States:

H = 200 kJ/kg for saturated liquid at 0°C.
S = 1 kJ/kg-K for saturated liquid at 0°C.

Subcooled Liquid

R-134a
AE

P = 100 psia (79.16°F)

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.013325	101.39	101.64	0.26902
150				
120				
100				
80				
60				
40				
32	0.012357	85.852	86.081	0.23882
10	0.011992	78.902	79.124	0.22434
0	0.011838	75.793	76.012	0.21765
-10	0.011692	72.712	72.929	0.21086
-20	0.011552	69.659	69.873	0.20399
-30	0.011417	66.630	66.842	0.19702
-40	0.011287	63.626	63.835	0.18994
-50	0.011162	60.644	60.851	0.18274

P = 200 psia (125.26°F)

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.014645	117.53	118.07	0.29780
150				
120				
100	0.013816	108.35	108.86	0.28171
80	0.013298	101.44	101.93	0.26911
60	0.012858	94.758	95.234	0.25646
40	0.012473	88.253	88.715	0.24367
32	0.012331	85.694	86.151	0.23849
10	0.011972	78.767	79.210	0.22405
0	0.011820	75.667	76.104	0.21737
-10	0.011676	72.594	73.027	0.21060
-20	0.011537	69.548	69.975	0.20374
-30	0.011403	66.526	66.948	0.19677
-40	0.011275	63.527	63.945	0.18970
-50	0.011151	60.551	60.964	0.18251

P = 300 psia (156.15°F)

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.015978	129.25	130.14	0.31743
150	0.015624	126.70	127.57	0.31323
120	0.014360	115.20	115.99	0.29377
100	0.013754	108.06	108.83	0.28120
80	0.013253	101.21	101.95	0.26868
60	0.012823	94.565	95.277	0.25608
40	0.012446	88.089	88.780	0.24334
32	0.012307	85.539	86.223	0.23818
10	0.011952	78.634	79.298	0.22377
0	0.011803	75.542	76.198	0.21710
-10	0.011660	72.478	73.125	0.21034
-20	0.011522	69.438	70.078	0.20349
-30	0.011390	66.423	67.056	0.19653
-40	0.011262	63.430	64.056	0.18947
-50	0.011139	60.460	61.079	0.18229

P = 500 psia (199.37°F)

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	0.019977	148.72	150.57	0.34840
350				
300				
250				
200				
180	0.017125	138.11	139.70	0.33168
160	0.015788	129.57	131.03	0.31791
140	0.014886	121.81	123.19	0.30507
120	0.014198	114.52	115.83	0.29258
100	0.013640	107.53	108.79	0.28022
80	0.013168	100.77	101.99	0.26784
60	0.012758	94.192	95.373	0.25536
40	0.012394	87.769	88.916	0.24269
32	0.012259	85.237	86.372	0.23756
20	0.012067	81.477	82.594	0.22978
0	0.011768	75.299	76.388	0.21656
-20	0.011493	69.223	70.287	0.20299
-30	0.011364	66.220	67.272	0.19606
-40	0.011238	63.239	64.280	0.18901
-50	0.011117	60.279	61.308	0.18184

P = 1000 psia

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
350	0.057233	212.59	223.19	0.44466
300	0.042185	194.14	201.95	0.41754
250	0.023076	166.29	170.56	0.37474
200	0.016985	142.57	145.72	0.33851
180	0.015906	134.65	137.60	0.32602
160	0.015084	127.17	129.96	0.31389
140	0.014423	120.00	122.67	0.30193
120	0.013870	113.07	115.63	0.29000
100	0.013396	106.33	108.81	0.27803
80	0.012980	99.760	102.163	0.26593
60	0.012609	93.328	95.663	0.25366
40	0.012274	87.019	89.292	0.24115
32	0.012149	84.527	86.776	0.23608
20	0.011969	80.819	83.04	0.22838
0	0.011687	74.718	76.882	0.21527
-20	0.011425	68.706	70.822	0.20180
-30	0.011301	65.732	67.825	0.19490
-40	0.011181	62.778	64.848	0.18789
-50	0.011064	59.843	61.891	0.18076

P = 5000 psia

Temp. (°C)	V ft^3/lb$_m$	U Btu/lb$_m$	H Btu/lb$_m$	S Btu/lb$_m$°R
Sat	Supercritical			
350	0.016647	179.06	194.48	0.38805
300	0.015546	162.68	177.08	0.36587
250	0.014574	146.58	160.07	0.34272
200	0.013722	130.79	143.50	0.31850
180	0.013411	124.57	136.99	0.30848
160	0.013116	118.40	130.55	0.29825
140	0.012836	112.29	124.18	0.28780
120	0.012569	106.24	117.87	0.27711
100	0.012314	100.23	111.64	0.26616
80	0.012071	94.289	105.47	0.25494
60	0.011839	88.398	99.359	0.24341
40	0.011617	82.559	93.315	0.23155
32	0.011531	80.239	90.915	0.22670
20	0.011404	76.774	87.332	0.21933
0	0.011199	71.041	81.409	0.20672
-20	0.011002	65.359	75.545	0.19367
-30	0.010906	62.536	72.634	0.18697
-40	0.010811	59.726	69.736	0.18015
-50	0.010719	56.929	66.853	0.17320

Reference States: H = 200 kJ/kg = 85.985 Btu/lbm for Saturated Liquid at 0°C = 32°F.
S = 1 kJ/kg = 0.23885 Btu/lbm-°R for Saturated Liquid at 0°C = 32°F.

Ideal Gas Property Table: AIR **MW = 28.970 g/mole** **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	85.570	0	1	1
300	1.3137	87.415	0.0061681	1.0217	0.98481
310	8.4305	97.401	0.038914	1.1452	0.90791
320	15.573	107.41	0.070702	1.2793	0.83893
330	22.741	117.45	0.10159	1.4247	0.77688
340	29.933	127.51	0.13163	1.5819	0.72088
350	37.151	137.60	0.16087	1.7516	0.67020
360	44.393	147.71	0.18936	1.9344	0.62421
370	51.659	157.85	0.21713	2.1309	0.58238
380	58.949	168.01	0.24422	2.3419	0.54423
390	66.263	178.19	0.27068	2.5680	0.50938
400	73.600	188.40	0.29652	2.8099	0.47745
410	80.961	198.63	0.32178	3.0685	0.44815
420	88.345	208.89	0.34649	3.3444	0.42121
430	95.751	219.16	0.37067	3.6383	0.39640
440	103.18	229.46	0.39435	3.9512	0.37349
450	110.63	239.78	0.41755	4.2839	0.35232
460	118.11	250.13	0.44028	4.6370	0.33272
470	125.60	260.49	0.46258	5.0116	0.31455
480	133.12	270.88	0.48445	5.4084	0.29767
490	140.66	281.29	0.50591	5.8284	0.28197
500	148.22	291.73	0.52699	6.2725	0.26736
510	155.81	302.18	0.54769	6.7417	0.25373
520	163.41	312.65	0.56803	7.2368	0.24100
530	171.04	323.15	0.58802	7.7590	0.22911
540	178.69	333.67	0.60768	8.3091	0.21797
550	186.36	344.21	0.62702	8.8882	0.20755
560	194.05	354.77	0.64605	9.4975	0.19776
570	201.76	365.35	0.66478	10.138	0.18858
580	209.49	375.95	0.68321	10.810	0.17995
590	217.24	386.57	0.70137	11.516	0.17183
600	225.01	397.21	0.71926	12.257	0.16418
610	232.80	407.88	0.73688	13.033	0.15698
620	240.62	418.56	0.75425	13.846	0.15018
630	248.45	429.26	0.77137	14.698	0.14377
640	256.30	439.98	0.78826	15.588	0.13770
650	264.17	450.72	0.80492	16.520	0.13197
660	272.06	461.49	0.82135	17.493	0.12654
670	279.98	472.27	0.83756	18.510	0.12141
680	287.91	483.07	0.85356	19.571	0.11654
690	295.86	493.89	0.86936	20.678	0.11192
700	303.82	504.73	0.88495	21.833	0.10753
710	311.81	515.59	0.90035	23.037	0.10337
720	319.82	526.46	0.91557	24.291	0.099416
730	327.85	537.36	0.93060	25.597	0.095654
740	335.89	548.27	0.94545	26.956	0.092075
750	343.95	559.21	0.96012	28.370	0.088667
760	352.04	570.16	0.97463	29.841	0.085421
770	360.14	581.13	0.98897	31.370	0.082327
780	368.26	592.12	1.0031	32.959	0.079376
790	376.39	603.12	1.0172	34.609	0.076560
800	384.55	614.15	1.0310	36.322	0.073872

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
820	400.91	636.25	1.0583	39.946	0.068851
840	417.35	658.43	1.0850	43.843	0.064261
860	433.85	680.67	1.1112	48.029	0.060057
880	450.42	702.99	1.1369	52.519	0.056200
900	467.07	725.37	1.1620	57.329	0.052654
920	483.78	747.82	1.1867	62.475	0.049391
940	500.56	770.34	1.2109	67.974	0.046382
960	517.40	792.92	1.2347	73.845	0.043603
980	534.31	815.57	1.2580	80.104	0.041033
1000	551.28	838.29	1.2810	86.771	0.038654
1020	568.32	861.06	1.3035	93.864	0.036447
1040	585.42	883.90	1.3257	101.40	0.034399
1060	602.58	906.81	1.3475	109.41	0.032494
1080	619.81	929.77	1.3690	117.91	0.030722
1100	637.09	952.80	1.3901	126.91	0.029070
1150	680.57	1010.6	1.4415	151.81	0.025408
1200	724.40	1068.8	1.4910	180.40	0.022310
1250	768.59	1127.3	1.5388	213.09	0.019675
1300	813.12	1186.2	1.5850	250.29	0.017420
1350	857.98	1245.4	1.6297	292.47	0.015482
1400	903.16	1305.0	1.6730	340.09	0.013807
1450	948.65	1364.8	1.7150	393.68	0.012353
1500	994.44	1424.9	1.7558	453.78	0.011087
1550	1040.5	1485.4	1.7954	520.96	9.9790E-03
1600	1086.9	1546.1	1.8339	595.85	9.0064E-03
1650	1133.5	1607.0	1.8715	679.07	8.1495E-03
1700	1180.4	1668.3	1.9080	771.33	7.3922E-03
1750	1227.5	1729.8	1.9437	873.32	6.7209E-03
1800	1274.9	1791.5	1.9784	985.82	6.1241E-03
1850	1322.5	1853.4	2.0124	1109.6	5.5920E-03
1900	1370.3	1915.6	2.0456	1245.5	5.1164E-03
1950	1418.4	1978.0	2.0780	1394.4	4.6903E-03
2000	1466.6	2040.6	2.1097	1557.3	4.3075E-03
2100	1563.7	2166.4	2.1710	1928.5	3.6522E-03
2200	1661.5	2292.9	2.2299	2367.5	3.1168E-03
2300	1760.0	2420.1	2.2864	2882.9	2.6758E-03
2400	1859.1	2547.9	2.3408	3484.4	2.3102E-03
2500	1958.7	2676.2	2.3932	4182.3	2.0049E-03
2600	2058.9	2805.1	2.4437	4987.6	1.7484E-03
2700	2159.5	2934.4	2.4925	5912.1	1.5317E-03
2800	2260.5	3064.1	2.5397	6968.5	1.3477E-03
2900	2361.9	3194.2	2.5854	8170.0	1.1905E-03
3000	2463.7	3324.7	2.6296	9530.9	1.0557E-03
3500	2976.2	3980.7	2.8318	19282	6.0880E-04
4000	3493.0	4641.0	3.0082	35643	3.7640E-04
4500	4012.5	5304.1	3.1644	61422	2.4573E-04
5000	4535.3	5970.3	3.3047	100177	1.6740E-04
5500	5063.7	6642.2	3.4328	156509	1.1787E-04
6000	5601.7	7323.8	3.5514	236591	8.5059E-05

Ideal Gas Property Table: CO MW = 28.013 g/mole SI

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	88.493	0	1	1
300	1.3758	90.418	0.0064361	1.0219	0.98462
310	8.8090	100.82	0.040542	1.1464	0.90700
320	16.239	111.22	0.073554	1.2812	0.83770
330	23.668	121.61	0.10555	1.4271	0.77560
340	31.100	132.01	0.13659	1.5844	0.71974
350	38.535	142.42	0.16675	1.7539	0.66933
360	45.978	152.83	0.19608	1.9360	0.62368
370	53.429	163.25	0.22463	2.1315	0.58222
380	60.890	173.68	0.25244	2.3408	0.54447
390	68.362	184.12	0.27956	2.5648	0.51000
400	75.848	194.57	0.30602	2.8040	0.47846
410	83.347	205.04	0.33187	3.0592	0.44952
420	90.862	215.52	0.35713	3.3309	0.42291
430	98.393	226.02	0.38184	3.6200	0.39840
440	105.94	236.54	0.40601	3.9272	0.37578
450	113.51	247.07	0.42969	4.2533	0.35485
460	121.09	257.62	0.45288	4.5990	0.33547
470	128.70	268.20	0.47562	4.9652	0.31749
480	136.32	278.79	0.49792	5.3526	0.30077
490	143.97	289.40	0.51980	5.7622	0.28522
500	151.63	300.04	0.54129	6.1947	0.27072
510	159.32	310.69	0.56239	6.6512	0.25718
520	167.03	321.37	0.58312	7.1324	0.24453
530	174.76	332.07	0.60350	7.6394	0.23269
540	182.52	342.79	0.62355	8.1731	0.22160
550	190.29	353.54	0.64327	8.7345	0.21120
560	198.10	364.31	0.66267	9.3247	0.20143
570	205.92	375.10	0.68177	9.9446	0.19224
580	213.77	385.92	0.70059	10.595	0.18360
590	221.64	396.76	0.71912	11.278	0.17546
600	229.54	407.63	0.73738	11.994	0.16779
610	237.46	418.52	0.75538	12.744	0.16055
620	245.41	429.43	0.77313	13.529	0.15371
630	253.38	440.37	0.79063	14.351	0.14724
640	261.38	451.33	0.80790	15.210	0.14113
650	269.40	462.32	0.82494	16.109	0.13534
660	277.44	473.34	0.84175	17.048	0.12985
670	285.52	484.38	0.85836	18.029	0.12465
680	293.61	495.44	0.87475	19.052	0.11971
690	301.74	506.53	0.89094	20.120	0.11502
700	309.88	517.65	0.90693	21.234	0.11057
710	318.05	528.79	0.92273	22.396	0.10633
720	326.25	539.95	0.93835	23.606	0.10230
730	334.48	551.15	0.95379	24.866	0.098466
740	342.72	562.36	0.96905	26.178	0.094813
750	351.00	573.60	0.98414	27.543	0.091331
760	359.30	584.87	0.99906	28.963	0.088010
770	367.62	596.16	1.0138	30.440	0.084842
780	375.97	607.48	1.0284	31.975	0.081818
790	384.34	618.82	1.0429	33.570	0.078930
800	392.74	630.19	1.0572	35.227	0.076170
820	409.61	653.00	1.0853	38.733	0.071007
840	426.58	675.90	1.1129	42.507	0.066281
860	443.65	698.90	1.1400	46.565	0.061945
880	460.81	722.00	1.1665	50.923	0.057961
900	478.07	745.20	1.1926	55.596	0.054295
920	495.42	768.49	1.2182	60.603	0.050917
940	512.87	791.87	1.2433	65.959	0.047799
960	530.40	815.34	1.2680	71.685	0.044917
980	548.02	838.89	1.2923	77.797	0.042250
1000	565.73	862.54	1.3162	84.317	0.039779
1020	583.52	886.27	1.3397	91.262	0.037486
1040	601.40	910.08	1.3628	98.655	0.035357
1060	619.36	933.97	1.3856	106.52	0.033378
1080	637.39	957.94	1.4080	114.87	0.031535
1100	655.50	981.99	1.4300	123.73	0.029818
1150	701.09	1042.4	1.4838	148.28	0.026012
1200	747.10	1103.3	1.5356	176.55	0.022797
1250	793.47	1164.5	1.5855	208.93	0.020067
1300	840.16	1226.0	1.6338	245.82	0.017738
1350	887.21	1287.9	1.6805	287.72	0.015737
1400	934.57	1350.1	1.7258	335.09	0.014013
1450	982.21	1412.6	1.7696	388.44	0.012520
1500	1030.1	1475.3	1.8122	448.31	0.011222
1550	1078.3	1538.3	1.8535	515.25	0.010090
1600	1126.6	1601.5	1.8936	589.86	0.0090977
1650	1175.2	1665.0	1.9326	672.77	0.0082259
1700	1224.0	1728.6	1.9706	764.62	0.0074571
1750	1273.0	1792.4	2.0076	866.11	0.0067769
1800	1322.1	1856.4	2.0437	977.94	0.0061734
1850	1371.4	1920.5	2.0788	1100.9	0.0056363
1900	1420.9	1984.8	2.1131	1235.7	0.0051571
1950	1470.4	2049.2	2.1466	1383.2	0.0047283
2000	1520.2	2113.8	2.1793	1544.3	0.0043437
2100	1620.0	2243.3	2.2425	1910.8	0.0036862
2200	1720.4	2373.3	2.3029	2342.6	0.0031498
2300	1821.1	2503.8	2.3609	2848.0	0.0027087
2400	1922.3	2634.6	2.4166	3435.7	0.0023430
2500	2023.8	2765.8	2.4702	4115.1	0.0020376
2600	2125.6	2897.3	2.5217	4896.2	0.0017811
2700	2227.8	3029.2	2.5715	5789.9	0.0015641
2800	2330.3	3161.3	2.6196	6807.5	0.0013795
2900	2433.0	3293.7	2.6660	7961.1	0.0012218
3000	2536.0	3426.4	2.7110	9263.7	0.0010862
3500	3054.3	4093.1	2.9165	18514	6.3405E-04
4000	3577.1	4764.4	3.0958	33869	3.9611E-04
4500	4103.7	5439.3	3.2548	57868	2.6082E-04
5000	4633.3	6117.3	3.3976	93644	1.7908E-04
5500	5165.7	6798.1	3.5274	144997	1.2722E-04
6000	5700.8	7481.6	3.6463	216472	9.2964E-05

Ideal Gas Property Table: CO_2

MW = 44.010 g/mole **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	56.327	0	1	1
300	1.2131	57.890	0.0052249	1.0280	0.97876
310	7.8336	66.400	0.033127	1.1917	0.87252
320	14.558	75.013	0.060474	1.3773	0.77929
330	21.384	83.728	0.087289	1.5873	0.69730
340	28.307	92.541	0.11360	1.8245	0.62504
350	35.326	101.45	0.13942	2.0917	0.56122
360	42.438	110.45	0.16478	2.3921	0.50476
370	49.640	119.54	0.18969	2.7293	0.45469
380	56.931	128.72	0.21417	3.1069	0.41023
390	64.308	137.99	0.23824	3.5290	0.37066
400	71.770	147.34	0.26191	4.0002	0.33539
410	79.314	156.77	0.28520	4.5251	0.30389
420	86.939	166.29	0.30813	5.1089	0.27573
430	94.643	175.88	0.33070	5.7573	0.25050
440	102.42	185.55	0.35293	6.4763	0.22787
450	110.28	195.30	0.37484	7.2724	0.20754
460	118.21	205.12	0.39642	8.1526	0.18925
470	126.22	215.01	0.41770	9.1244	0.17277
480	134.29	224.98	0.43868	10.196	0.15790
490	142.44	235.01	0.45937	11.376	0.14447
500	150.65	245.11	0.47978	12.674	0.13232
510	158.94	255.29	0.49992	14.100	0.12132
520	167.28	265.52	0.51980	15.664	0.11134
530	175.70	275.82	0.53942	17.379	0.10229
540	184.17	286.19	0.55880	19.256	0.094058
550	192.71	296.62	0.57793	21.308	0.086572
560	201.31	307.11	0.59683	23.550	0.079754
570	209.97	317.66	0.61550	25.997	0.073539
580	218.69	328.27	0.63395	28.664	0.067867
590	227.47	338.93	0.65219	31.568	0.062686
600	236.30	349.66	0.67021	34.728	0.057948
610	245.19	360.43	0.68803	38.163	0.053611
620	254.14	371.27	0.70565	41.893	0.049638
630	263.14	382.16	0.72307	45.940	0.045996
640	272.19	393.10	0.74030	50.326	0.042653
650	281.29	404.09	0.75734	55.078	0.039582
660	290.45	415.14	0.77420	60.220	0.036760
670	299.65	426.23	0.79089	65.779	0.034163
680	308.91	437.37	0.80739	71.786	0.031771
690	318.21	448.56	0.82373	78.271	0.029568
700	327.56	459.80	0.83990	85.266	0.027535
710	336.96	471.09	0.85591	92.806	0.025660
720	346.40	482.42	0.87176	100.93	0.023927
730	355.88	493.80	0.88745	109.67	0.022326
740	365.42	505.22	0.90299	119.07	0.020845
750	374.99	516.68	0.91838	129.17	0.019474
760	384.61	528.19	0.93362	140.03	0.018204
770	394.27	539.74	0.94872	151.67	0.017027
780	403.97	551.33	0.96367	164.17	0.015936
790	413.71	562.96	0.97848	177.56	0.014923
800	423.49	574.62	0.99316	191.90	0.013982

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
820	443.16	598.08	1.0221	223.69	0.012295
840	462.99	621.68	1.0506	260.03	0.010835
860	482.95	645.43	1.0785	301.47	0.0095679
880	503.06	669.31	1.1060	348.63	0.0084661
900	523.30	693.33	1.1329	402.17	0.0075058
920	543.68	717.48	1.1595	462.83	0.0066671
940	564.17	741.76	1.1856	531.40	0.0059329
960	584.79	766.15	1.2113	608.77	0.0052891
980	605.52	790.67	1.2365	695.90	0.0047233
1000	626.37	815.29	1.2614	793.82	0.0042251
1020	647.32	840.02	1.2859	903.68	0.0037857
1040	668.38	864.86	1.3100	1026.7	0.0033974
1060	689.54	889.80	1.3338	1164.3	0.0030537
1080	710.80	914.83	1.3572	1317.8	0.0027489
1100	732.15	939.96	1.3802	1488.8	0.0024781
1150	785.93	1003.2	1.4364	2004.7	0.0019240
1200	840.24	1066.9	1.4907	2671.8	0.0015064
1250	895.02	1131.2	1.5431	3526.5	0.0011889
1300	950.27	1195.9	1.5939	4613.2	0.00094516
1350	1006.0	1261.0	1.6430	5984.2	0.00075665
1400	1062.0	1326.5	1.6907	7700.9	0.00060975
1450	1118.4	1392.4	1.7369	9835.9	0.00049445
1500	1175.2	1458.6	1.7818	12474	0.00040333
1550	1232.3	1525.1	1.8254	15713	0.00033086
1600	1289.6	1591.9	1.8678	19666	2.7287E-04
1650	1347.2	1658.9	1.9091	24467	2.2619E-04
1700	1405.0	1726.2	1.9492	30264	1.8840E-04
1750	1463.1	1793.7	1.9884	37231	1.5765E-04
1800	1521.4	1861.4	2.0265	45564	1.3250E-04
1850	1579.9	1929.4	2.0638	55487	1.1183E-04
1900	1638.5	1997.5	2.1001	67253	9.4757E-05
1950	1697.4	2065.8	2.1356	81147	8.0598E-05
2000	1756.4	2134.2	2.1702	97492	6.8806E-05
2100	1874.9	2271.7	2.2373	1.3902E+05	5.0664E-05
2200	1994.0	2409.6	2.3015	1.9527E+05	3.7787E-05
2300	2113.6	2548.2	2.3631	2.7051E+05	2.8517E-05
2400	2233.8	2687.2	2.4222	3.6999E+05	2.1757E-05
2500	2354.3	2826.6	2.4791	5.0009E+05	1.6767E-05
2600	2475.3	2966.5	2.5340	6.6857E+05	1.3043E-05
2700	2596.6	3106.7	2.5869	8.8475E+05	1.0236E-05
2800	2718.3	3247.3	2.6381	1.1597E+06	8.0977E-06
2900	2840.4	3388.3	2.6875	1.5068E+06	6.4552E-06
3000	2962.7	3529.5	2.7354	1.9415E+06	5.1827E-06
3500	3578.6	4239.8	2.9544	6.1873E+06	1.8973E-06
4000	4200.2	4955.9	3.1456	1.7025E+07	7.8801E-07
4500	4826.7	5676.9	3.3154	4.1829E+07	3.6083E-07
5000	5457.8	6402.4	3.4683	9.3949E+07	1.7850E-07
5500	6093.4	7132.5	3.6075	1.9625E+08	9.3996E-08
6000	6734.3	7867.8	3.7354	3.8634E+08	5.2089E-08

Ideal Gas Property Table: H_2 **MW = 2.016 g/mole** **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	1229.6	0	1	1
300	18.838	1256.1	0.088501	1.0217	0.98484
310	120.85	1399.4	0.55823	1.1449	0.90812
320	223.14	1542.9	1.0139	1.2787	0.83935
330	325.67	1686.7	1.4563	1.4235	0.77754
340	428.41	1830.6	1.8861	1.5799	0.72182
350	531.31	1974.8	2.3040	1.7483	0.67145
360	634.37	2119.1	2.7105	1.9294	0.62581
370	737.56	2263.5	3.1062	2.1237	0.58435
380	840.86	2408.1	3.4917	2.3318	0.54659
390	944.25	2552.7	3.8674	2.5542	0.51213
400	1047.7	2697.4	4.2338	2.7915	0.48061
410	1151.3	2842.2	4.5913	3.0442	0.45172
420	1254.9	2987.1	4.9403	3.3131	0.42519
430	1358.5	3132.0	5.2813	3.5986	0.40077
440	1462.2	3276.9	5.6145	3.9014	0.37826
450	1566.0	3421.9	5.9403	4.2222	0.35747
460	1669.7	3566.9	6.2591	4.5614	0.33824
470	1773.6	3711.9	6.5710	4.9198	0.32042
480	1877.4	3857.0	6.8765	5.2980	0.30387
490	1981.3	4002.1	7.1757	5.6966	0.28850
500	2085.2	4147.3	7.4689	6.1164	0.27418
510	2189.1	4292.4	7.7563	6.5579	0.26084
520	2293.0	4437.6	8.0383	7.0219	0.24838
530	2397.0	4582.8	8.3149	7.5090	0.23673
540	2501.0	4728.1	8.5864	8.0199	0.22583
550	2605.0	4873.3	8.8529	8.5553	0.21562
560	2709.1	5018.6	9.1147	9.1161	0.20604
570	2813.2	5164.0	9.3719	9.7027	0.19704
580	2917.3	5309.3	9.6248	10.316	0.18857
590	3021.4	5454.7	9.8733	10.957	0.18060
600	3125.6	5600.2	10.118	11.626	0.17310
610	3229.9	5745.7	10.358	12.324	0.16601
620	3334.2	5891.2	10.595	13.052	0.15933
630	3438.5	6036.8	10.828	13.810	0.15301
640	3542.9	6182.4	11.057	14.600	0.14703
650	3647.3	6328.0	11.283	15.422	0.14137
660	3751.8	6473.8	11.505	16.276	0.13600
670	3856.3	6619.6	11.725	17.165	0.13092
680	3960.9	6765.4	11.941	18.088	0.12609
690	4065.6	6911.3	12.154	19.047	0.12150
700	4170.4	7057.3	12.364	20.042	0.11714
710	4275.2	7203.4	12.571	21.075	0.11299
720	4380.1	7349.5	12.775	22.146	0.10904
730	4485.1	7495.8	12.977	23.256	0.10528
740	4590.1	7642.1	13.176	24.406	0.10170
750	4695.3	7788.5	13.373	25.597	0.098274
760	4800.5	7935.0	13.567	26.830	0.095008
770	4905.9	8081.6	13.758	28.106	0.091888
780	5011.3	8228.2	13.948	29.426	0.088906
790	5116.9	8375.0	14.135	30.791	0.086054
800	5222.5	8521.9	14.319	32.202	0.083326
820	5434.2	8816.1	14.683	35.166	0.078210
840	5646.3	9110.7	15.038	38.326	0.073511
860	5858.9	9405.8	15.385	41.692	0.069184
880	6072.1	9701.4	15.725	45.273	0.065194
900	6285.7	9997.6	16.057	49.078	0.061507
920	6500.0	10294	16.383	53.116	0.058094
940	6714.9	10592	16.703	57.398	0.054929
960	6930.4	10890	17.017	61.934	0.051989
980	7146.5	11188	17.325	66.734	0.049254
1000	7363.4	11488	17.627	71.810	0.046707
1020	7580.9	11788	17.924	77.173	0.044330
1040	7799.1	12088	18.216	82.834	0.042110
1060	8018.1	12390	18.503	88.806	0.040034
1080	8237.8	12692	18.786	95.101	0.038089
1100	8458.3	12995	19.064	101.73	0.036266
1150	9013.0	13756	19.740	119.86	0.032179
1200	9572.7	14522	20.392	140.39	0.028669
1250	10138	15293	21.022	163.55	0.025635
1300	10708	16070	21.631	189.58	0.022999
1350	11284	16852	22.221	218.75	0.020699
1400	11865	17639	22.794	251.35	0.018682
1450	12452	18432	23.351	287.66	0.016906
1500	13044	19231	23.892	328.00	0.015338
1550	13636	20028	24.415	372.37	0.013961
1600	14235	20833	24.926	421.49	0.012732
1650	14840	21645	25.426	475.75	0.011632
1700	15451	22463	25.914	535.54	0.010647
1750	16069	23286	26.391	601.27	0.0097618
1800	16692	24115	26.858	673.39	0.0089655
1850	17320	24950	27.316	752.33	0.0082476
1900	17953	25789	27.763	838.60	0.0075991
1950	18591	26633	28.202	932.69	0.0070123
2000	19234	27482	28.632	1035.1	0.0064804
2100	20532	29193	29.466	1267.3	0.0055579
2200	21846	30920	30.269	1539.8	0.0047920
2300	23176	32662	31.044	1857.9	0.0041521
2400	24521	34419	31.792	2227.2	0.0036143
2500	25879	36190	32.514	2653.8	0.0031596
2600	27251	37974	33.214	3144.5	0.0027732
2700	28635	39770	33.892	3706.4	0.0024433
2800	30032	41579	34.550	4347.5	0.0021602
2900	31440	43401	35.189	5076.1	0.0019162
3000	32861	45233	35.811	5901.4	0.0017050
3500	40128	54563	38.686	11850	9.9061E-04
4000	47652.0	64149	41.245	22041	6.0868E-04
4500	55409	73968	43.558	38615	3.9086E-04
5000	63375	83996	45.670	64451	2.6020E-04
5500	71521	94204	47.616	103302	1.7857E-04
6000	79814	104559	49.418	159903	1.2585E-04

Ideal Gas Property Table: H_2O **MW = 18.016 g/mole** **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	137.60	0	1	1
500	291.18	521.93	0.98239	8.4037	0.19956
510	306.14	541.51	1.0212	9.1402	0.18715
520	321.16	561.14	1.0593	9.9274	0.17568
530	336.24	580.84	1.0968	10.768	0.16508
540	351.38	600.59	1.1337	11.665	0.15526
550	366.58	620.41	1.1701	12.621	0.14616
560	381.84	640.29	1.2059	13.640	0.13770
570	397.17	660.22	1.2412	14.724	0.12984
580	412.55	680.22	1.2760	15.876	0.12253
590	428.00	700.29	1.3103	17.101	0.11572
600	443.51	720.42	1.3441	18.402	0.10936
610	459.09	740.61	1.3775	19.782	0.10343
620	474.73	760.86	1.4104	21.245	0.097881
630	490.43	781.18	1.4429	22.796	0.092694
640	506.20	801.56	1.4750	24.438	0.087838
650	522.04	822.01	1.5067	26.176	0.083287
660	537.94	842.53	1.5381	28.014	0.079019
670	553.90	863.11	1.5690	29.957	0.075013
680	569.94	883.76	1.5996	32.010	0.071250
690	586.04	904.48	1.6298	34.178	0.067712
700	602.20	925.26	1.6597	36.466	0.064384
710	618.44	946.11	1.6893	38.879	0.061250
720	634.74	967.02	1.7186	41.424	0.058298
730	651.11	988.01	1.7475	44.105	0.055514
740	667.55	1009.1	1.7762	46.929	0.052888
750	684.06	1030.2	1.8045	49.902	0.050409
760	700.63	1051.4	1.8326	53.031	0.048067
770	717.28	1072.6	1.8604	56.323	0.045853
780	733.99	1094.0	1.8879	59.784	0.043760
790	750.77	1115.4	1.9151	63.421	0.041779
800	767.62	1136.8	1.9422	67.242	0.039904
810	784.54	1158.4	1.9689	71.256	0.038127
820	801.53	1180.0	1.9954	75.469	0.036443
830	818.59	1201.6	2.0217	79.890	0.034846
840	835.72	1223.4	2.0477	84.527	0.033331
850	852.92	1245.2	2.0735	89.391	0.031893
860	870.19	1267.1	2.0991	94.489	0.030527
870	887.53	1289.0	2.1245	99.831	0.029229
880	904.94	1311.1	2.1497	105.43	0.027996
890	922.42	1333.2	2.1747	111.29	0.026823
900	939.98	1355.3	2.1994	117.42	0.025707
910	957.60	1377.6	2.2240	123.84	0.024645
920	975.29	1399.9	2.2484	130.56	0.023634
930	993.05	1422.3	2.2726	137.59	0.022671
940	1010.9	1444.7	2.2966	144.94	0.021753
950	1028.8	1467.2	2.3204	152.62	0.020878
960	1046.8	1489.8	2.3441	160.64	0.020044
970	1064.8	1512.5	2.3675	169.03	0.019248
980	1082.9	1535.2	2.3909	177.78	0.018488
990	1101.1	1558.0	2.4140	186.93	0.017763
1000	1119.4	1580.9	2.4370	196.47	0.017071
1000	1119.4	1580.9	2.4370	196.47	0.017071
1020	1156.1	1626.8	2.4825	216.83	0.015778
1040	1193.1	1673.0	2.5274	238.98	0.014596
1060	1230.4	1719.6	2.5717	263.05	0.013515
1080	1267.9	1766.3	2.6154	289.20	0.012526
1100	1305.8	1813.4	2.6586	317.56	0.011618
1120	1343.9	1860.8	2.7012	348.31	0.010785
1140	1382.3	1908.4	2.7434	381.61	0.010020
1160	1420.9	1956.3	2.7850	417.65	0.0093156
1180	1459.9	2004.4	2.8262	456.62	0.0086675
1200	1499.1	2052.9	2.8669	498.72	0.0080703
1220	1538.5	2101.6	2.9071	544.17	0.0075196
1240	1578.3	2150.5	2.9469	593.20	0.0070111
1260	1618.3	2199.8	2.9863	646.05	0.0065414
1280	1658.5	2249.3	3.0253	702.97	0.0061071
1330	1760.3	2374.1	3.1210	864.95	0.0051574
1380	1863.7	2500.6	3.2143	1058.8	0.0043715
1430	1968.6	2628.6	3.3054	1289.8	0.0037185
1480	2075.0	2758.0	3.3944	1564.1	0.0031736
1500	2117.9	2810.2	3.4294	1687.4	0.0029814
1550	2226.3	2941.6	3.5156	2033.9	0.0025560
1600	2336.0	3074.4	3.5999	2441.5	0.0021980
1650	2446.9	3208.4	3.6824	2919.3	0.0018957
1700	2559.0	3343.6	3.7631	3477.2	0.0016398
1750	2672.3	3479.9	3.8421	4126.8	0.0014223
1800	2786.7	3617.5	3.9196	4881.2	0.0012368
1850	2902.3	3756.1	3.9956	5754.5	0.0010783
1900	3018.9	3895.8	4.0701	6762.7	9.4233E-04
1950	3136.5	4036.5	4.1432	7923.1	8.2548E-04
2000	3255.1	4178.1	4.2149	9255.3	7.2478E-04
2050	3374.6	4320.7	4.2853	10781	6.3779E-04
2100	3494.9	4464.1	4.3544	12523	5.6246E-04
2150	3616.1	4608.4	4.4223	14507	4.9707E-04
2250	3860.9	4899.3	4.5546	19321	3.9058E-04
2350	4108.6	5193.1	4.6824	25485	3.0927E-04
2450	4359.1	5489.8	4.8060	33312	2.4668E-04
2550	4612.1	5788.9	4.9256	43173	1.9810E-04
2650	4867.4	6090.4	5.0416	55507	1.6013E-04
2750	5124.9	6394.1	5.1541	70826	1.3023E-04
2850	5384.5	6699.8	5.2633	89733	1.0653E-04
2950	5645.9	7007.4	5.3694	112922	8.7621E-05
3000	5777.4	7161.9	5.4213	126371	7.9623E-05
3100	6041.4	7472.1	5.5230	157534	6.6001E-05
3500	7113.0	8728.2	5.9040	359705	3.2635E-05
4000	8480.1	10326	6.3307	906669	1.4797E-05
4500	9870.8	11948	6.7126	2074198	7.2766E-06
5000	11280	13588	7.0582	4385884	3.8236E-06
5500	12706	15244	7.3739	8693076	2.1220E-06
6000	14148	16917	7.6650	16334017	1.2320E-06

Ideal Gas Property Table: N_2 MW = 28.010 g/mole **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	88.503	0	1	1
300	1.3579	90.410	0.0063766	1.0217	0.98482
310	8.7063	100.73	0.040205	1.1450	0.90804
320	16.069	111.06	0.073005	1.2788	0.83927
330	23.447	121.40	0.10484	1.4236	0.77748
340	30.841	131.77	0.13578	1.5800	0.72176
350	38.252	142.15	0.16586	1.7485	0.67137
360	45.680	152.54	0.19515	1.9298	0.62567
370	53.126	162.96	0.22369	2.1245	0.58412
380	60.590	173.39	0.25151	2.3333	0.54623
390	68.072	183.84	0.27865	2.5567	0.51162
400	75.573	194.31	0.30516	2.7955	0.47991
410	83.093	204.80	0.33106	3.0504	0.45081
420	90.632	215.30	0.35638	3.3220	0.42404
430	98.191	225.83	0.38115	3.6111	0.39938
440	105.77	236.38	0.40539	3.9185	0.37662
450	113.37	246.94	0.42914	4.2448	0.35556
460	120.99	257.53	0.45241	4.5909	0.33606
470	128.62	268.14	0.47522	4.9576	0.31797
480	136.28	278.76	0.49759	5.3457	0.30116
490	143.96	289.41	0.51954	5.7560	0.28552
500	151.66	300.08	0.54109	6.1894	0.27095
510	159.37	310.76	0.56225	6.6468	0.25735
520	167.11	321.47	0.58304	7.1290	0.24465
530	174.87	332.20	0.60348	7.6370	0.23276
540	182.65	342.94	0.62356	8.1717	0.22164
550	190.45	353.71	0.64332	8.7342	0.21121
560	198.27	364.50	0.66276	9.3252	0.20142
570	206.11	375.31	0.68189	9.9460	0.19222
580	213.97	386.14	0.70072	10.597	0.18357
590	221.85	396.98	0.71926	11.281	0.17542
600	229.75	407.85	0.73753	11.997	0.16775
610	237.67	418.74	0.75553	12.746	0.16051
620	245.61	429.65	0.77327	13.531	0.15368
630	253.57	440.58	0.79075	14.352	0.14723
640	261.55	451.52	0.80799	15.210	0.14112
650	269.55	462.49	0.82500	16.107	0.13535
660	277.56	473.48	0.84177	17.044	0.12988
670	285.60	484.49	0.85832	18.021	0.12470
680	293.66	495.51	0.87466	19.041	0.11978
690	301.74	506.56	0.89079	20.104	0.11512
700	309.84	517.62	0.90671	21.212	0.11069
710	317.95	528.71	0.92243	22.365	0.10647
720	326.09	539.81	0.93796	23.567	0.10247
730	334.24	550.94	0.95331	24.817	0.098660
740	342.42	562.08	0.96847	26.117	0.095032
750	350.61	573.24	0.98345	27.469	0.091575
760	358.82	584.42	0.99826	28.874	0.088280
770	367.06	595.62	1.0129	30.334	0.085138
780	375.31	606.84	1.0274	31.850	0.082139
790	383.57	618.08	1.0417	33.424	0.079275
800	391.86	629.33	1.0558	35.057	0.076539

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
820	408.49	651.90	1.0837	38.507	0.071424
840	425.20	674.54	1.1110	42.213	0.066742
860	441.97	697.25	1.1377	46.189	0.062449
880	458.82	720.04	1.1639	50.450	0.058504
900	475.74	742.89	1.1896	55.009	0.054875
920	492.73	765.82	1.2148	59.882	0.051530
940	509.79	788.82	1.2395	65.084	0.048442
960	526.92	811.88	1.2638	70.631	0.045587
980	544.11	835.01	1.2876	76.540	0.042944
1000	561.37	858.21	1.3111	82.827	0.040494
1020	578.70	881.48	1.3341	89.511	0.038220
1040	596.10	904.81	1.3568	96.610	0.036106
1060	613.56	928.21	1.3790	104.14	0.034139
1080	631.08	951.67	1.4010	112.13	0.032306
1100	648.67	975.20	1.4226	120.58	0.030597
1150	692.91	1034.3	1.4751	143.92	0.026800
1200	737.54	1093.7	1.5257	170.68	0.023581
1250	782.52	1153.6	1.5745	201.21	0.020837
1300	827.87	1213.8	1.6217	235.89	0.018484
1350	873.55	1274.3	1.6674	275.14	0.016457
1400	919.57	1335.1	1.7117	319.39	0.014702
1450	965.91	1396.3	1.7546	369.10	0.013176
1500	1012.6	1457.8	1.7963	424.76	0.011844
1550	1059.5	1519.6	1.8368	486.88	0.010678
1600	1106.7	1581.7	1.8763	556.02	9.6515E-03
1650	1154.2	1644.0	1.9146	632.75	8.7461E-03
1700	1202.0	1706.6	1.9520	717.68	7.9448E-03
1750	1250.1	1769.5	1.9885	811.45	7.2334E-03
1800	1298.3	1832.6	2.0240	914.72	6.6001E-03
1850	1346.8	1896.0	2.0587	1028.2	6.0347E-03
1900	1395.6	1959.6	2.0927	1152.6	5.5287E-03
1950	1444.5	2023.4	2.1258	1288.8	5.0748E-03
2000	1493.7	2087.4	2.1582	1437.5	4.6666E-03
2100	1592.6	2216.0	2.2209	1775.8	3.9664E-03
2200	1692.2	2345.3	2.2811	2174.7	3.3931E-03
2300	1792.5	2475.2	2.3389	2641.9	2.9200E-03
2400	1893.4	2605.8	2.3944	3185.7	2.5268E-03
2500	1994.8	2736.9	2.4479	3815.0	2.1979E-03
2600	2096.6	2868.4	2.4995	4539.2	1.9211E-03
2700	2198.9	3000.4	2.5493	5368.4	1.6869E-03
2800	2301.6	3132.7	2.5975	6313.3	1.4875E-03
2900	2404.5	3265.4	2.6440	7385.2	1.3170E-03
3000	2507.8	3398.3	2.6891	8596.0	1.1706E-03
3500	3026.9	4065.8	2.8949	17193	6.8277E-04
4000	3548.2	4735.6	3.0737	31409	4.2714E-04
4500	4070.2	5406.0	3.2317	53472	2.8226E-04
5000	4593.4	6077.6	3.3732	86133	1.9470E-04
5500	5120.4	6753.0	3.5019	132894	1.3881E-04
6000	5655.8	7436.8	3.6209	198429	1.0142E-04

Ideal Gas Property Table: O_2 **MW = 32.000 g/mole** **SI**

T (K)	$\hat{U}^o$ (kJ/kg)	$\hat{H}^o$ (kJ/kg)	$\hat{S}^o$ (kJ/kg-K)	P_r	V_r
298.15	0	77.467	0	1	1
300	1.1921	79.140	0.0055933	1.0218	0.98478
310	7.6755	88.222	0.035371	1.1458	0.90741
320	14.222	97.367	0.064404	1.2813	0.83766
330	20.827	106.57	0.092725	1.4289	0.77463
340	27.488	115.83	0.12036	1.5892	0.71756
350	34.200	125.14	0.14735	1.7632	0.66579
360	40.961	134.50	0.17372	1.9515	0.61873
370	47.767	143.90	0.19949	2.1549	0.57588
380	54.618	153.35	0.22468	2.3744	0.53678
390	61.510	162.84	0.24934	2.6107	0.50104
400	68.442	172.37	0.27346	2.8648	0.46831
410	75.411	181.94	0.29709	3.1374	0.43830
420	82.417	191.54	0.32023	3.4297	0.41073
430	89.458	201.18	0.34291	3.7426	0.38536
440	96.531	210.86	0.36515	4.0770	0.36198
450	103.64	220.56	0.38696	4.4339	0.34040
460	110.77	230.30	0.40835	4.8145	0.32046
470	117.94	240.06	0.42936	5.2198	0.30200
480	125.14	249.86	0.44998	5.6510	0.28489
490	132.36	259.68	0.47023	6.1091	0.26902
500	139.62	269.53	0.49013	6.5954	0.25427
510	146.89	279.41	0.50969	7.1111	0.24055
520	154.20	289.31	0.52892	7.6573	0.22777
530	161.53	299.24	0.54783	8.2355	0.21585
540	168.88	309.19	0.56644	8.8468	0.20473
550	176.26	319.17	0.58474	9.4926	0.19433
560	183.67	329.17	0.60276	10.174	0.18461
570	191.09	339.19	0.62051	10.893	0.17550
580	198.54	349.24	0.63798	11.651	0.16697
590	206.01	359.31	0.65519	12.449	0.15896
600	213.50	369.40	0.67215	13.289	0.15144
610	221.02	379.51	0.68886	14.172	0.14437
620	228.55	389.64	0.70534	15.099	0.13772
630	236.11	399.80	0.72159	16.074	0.13146
640	243.68	409.97	0.73761	17.096	0.12556
650	251.28	420.17	0.75342	18.168	0.11999
660	258.89	430.38	0.76901	19.292	0.11474
670	266.53	440.61	0.78440	20.469	0.10978
680	274.18	450.87	0.79959	21.702	0.10509
690	281.86	461.14	0.81458	22.991	0.10066
700	289.55	471.43	0.82939	24.339	0.096462
710	297.26	481.74	0.84401	25.748	0.092486
720	304.99	492.06	0.85846	27.220	0.088718
730	312.73	502.41	0.87272	28.757	0.085143
740	320.50	512.77	0.88682	30.360	0.081751
750	328.28	523.15	0.90076	32.033	0.078530
760	336.08	533.55	0.91453	33.776	0.075469
770	343.90	543.96	0.92814	35.593	0.072558
780	351.73	554.40	0.94160	37.486	0.069790
790	359.58	564.84	0.95491	39.456	0.067155
800	367.45	575.31	0.96808	41.507	0.064645
820	383.23	596.29	0.99398	45.858	0.059974
840	399.08	617.33	1.0193	50.559	0.055725
860	414.99	638.44	1.0442	55.629	0.051851
880	430.96	659.61	1.0685	61.091	0.048313
900	447.00	680.84	1.0924	66.966	0.045077
920	463.10	702.14	1.1158	73.277	0.042110
940	479.25	723.49	1.1387	80.047	0.039386
960	495.47	744.90	1.1613	87.301	0.036882
980	511.74	766.37	1.1834	95.063	0.034576
1000	528.07	787.89	1.2051	103.36	0.032450
1020	544.45	809.48	1.2265	112.22	0.030485
1040	560.89	831.11	1.2475	121.67	0.028669
1060	577.39	852.80	1.2682	131.74	0.026987
1080	593.94	874.55	1.2885	142.46	0.025427
1100	610.54	896.35	1.3085	153.86	0.023980
1150	652.27	951.07	1.3571	185.54	0.020789
1200	694.33	1006.1	1.4040	222.20	0.018113
1250	736.69	1061.5	1.4492	264.42	0.015856
1300	779.35	1117.1	1.4928	312.79	0.013940
1350	822.30	1173.1	1.5351	367.99	0.012304
1400	865.54	1229.3	1.5760	430.72	0.010902
1450	909.06	1285.8	1.6156	501.75	9.6928E-03
1500	952.84	1342.6	1.6541	581.87	8.6463E-03
1550	996.89	1399.6	1.6915	671.97	7.7366E-03
1600	1041.2	1456.9	1.7279	772.95	6.9427E-03
1650	1085.7	1514.5	1.7633	885.82	6.2475E-03
1700	1130.5	1572.2	1.7978	1011.6	5.6364E-03
1750	1175.6	1630.3	1.8315	1151.4	5.0976E-03
1800	1220.8	1688.5	1.8643	1306.4	4.6211E-03
1850	1266.3	1747.0	1.8963	1477.9	4.1984E-03
1900	1312.0	1805.7	1.9276	1667.1	3.8225E-03
1950	1357.9	1864.6	1.9582	1875.5	3.4872E-03
2000	1404.0	1923.7	1.9882	2104.5	3.1874E-03
2100	1496.9	2042.5	2.0461	2630.7	2.6774E-03
2200	1590.5	2162.1	2.1018	3258.8	2.2643E-03
2300	1684.9	2282.5	2.1553	4003.8	1.9267E-03
2400	1779.9	2403.5	2.2068	4881.7	1.6489E-03
2500	1875.6	2525.2	2.2565	5910.3	1.4187E-03
2600	1972.0	2647.5	2.3044	7109.0	1.2267E-03
2700	2069.0	2770.5	2.3508	8498.9	1.0655E-03
2800	2166.5	2894.0	2.3958	10103	9.2956E-04
2900	2264.6	3018.1	2.4393	11946	8.1423E-04
3000	2363.2	3142.7	2.4815	14055	7.1592E-04
3500	2863.5	3772.9	2.6758	29683	3.9548E-04
4000	3374.5	4413.9	2.8469	57356	2.3391E-04
4500	3895.4	5064.7	3.0002	103464	1.4588E-04
5000	4426.5	5725.6	3.1395	176825	9.4840E-05
5500	4969.0	6398.0	3.2676	289571	6.3705E-05
6000	5525.5	7084.4	3.3871	458547	4.3887E-05

Ideal Gas Property Table: AIR MW = 28.970 lb_m/lbmole AE

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
536.67	0	36.789	0	1	1
540	0.56481	37.582	0.0014732	1.0217	0.98481
550	2.2632	39.966	0.0058475	1.0890	0.94104
560	3.9651	42.353	0.010149	1.1596	0.89987
570	5.6704	44.744	0.014381	1.2334	0.86111
580	7.3790	47.138	0.018545	1.3107	0.82458
590	9.0910	49.535	0.022643	1.3914	0.79012
600	10.806	51.936	0.026678	1.4758	0.75758
610	12.525	54.340	0.030652	1.5638	0.72682
620	14.247	56.748	0.034566	1.6558	0.69773
630	15.972	59.158	0.038424	1.7516	0.67020
640	17.701	61.572	0.042225	1.8515	0.64410
650	19.432	63.989	0.045973	1.9555	0.61937
660	21.167	66.410	0.049668	2.0638	0.59589
670	22.905	68.833	0.053312	2.1765	0.57360
680	24.646	71.260	0.056908	2.2937	0.55241
690	26.391	73.690	0.060455	2.4155	0.53227
700	28.138	76.123	0.063956	2.5421	0.51310
710	29.889	78.559	0.067411	2.6735	0.49484
720	31.643	80.998	0.070823	2.8099	0.47745
730	33.399	83.441	0.074192	2.9515	0.46087
740	35.159	85.886	0.077519	3.0983	0.44505
750	36.922	88.334	0.080805	3.2504	0.42995
760	38.688	90.786	0.084052	3.4081	0.41552
770	40.457	93.241	0.087261	3.5714	0.40174
780	42.229	95.698	0.090432	3.7405	0.38856
790	44.004	98.159	0.093567	3.9155	0.37595
800	45.782	100.62	0.096665	4.0966	0.36388
810	47.564	103.09	0.099730	4.2839	0.35232
820	49.348	105.56	0.10276	4.4775	0.34125
830	51.135	108.03	0.10576	4.6776	0.33064
840	52.924	110.51	0.10872	4.8843	0.32046
850	54.717	112.98	0.11165	5.0978	0.31069
860	56.513	115.47	0.11456	5.3183	0.30132
870	58.312	117.95	0.11743	5.5458	0.29231
880	60.113	120.44	0.12027	5.7806	0.28366
890	61.918	122.93	0.12308	6.0228	0.27535
900	63.725	125.42	0.12587	6.2725	0.26736
910	65.535	127.92	0.12863	6.5300	0.25967
920	67.348	130.41	0.13136	6.7954	0.25227
930	69.164	132.92	0.13406	7.0688	0.24515
940	70.983	135.42	0.13674	7.3505	0.23829
950	72.805	137.93	0.13939	7.6406	0.23168
960	74.629	140.44	0.14202	7.9392	0.22531
970	76.456	142.95	0.14463	8.2466	0.21918
980	78.286	145.46	0.14721	8.5628	0.21326
990	80.119	147.98	0.14976	8.8882	0.20755
1000	81.955	150.50	0.15230	9.2229	0.20203
1010	83.793	153.03	0.15481	9.5670	0.19671
1020	85.634	155.55	0.15730	9.9208	0.19158
1030	87.478	158.08	0.15976	10.284	0.18662
1040	89.324	160.62	0.16221	10.658	0.18182
1060	93.026	165.69	0.16704	11.436	0.172708
1080	96.738	170.77	0.17179	12.257	0.164184
1100	100.46	175.87	0.17647	13.122	0.156204
1120	104.19	180.97	0.18106	14.032	0.148725
1140	107.94	186.08	0.18559	14.990	0.141707
1160	111.69	191.21	0.19005	15.997	0.135115
1180	115.46	196.35	0.19444	17.055	0.128919
1200	119.23	201.49	0.19876	18.166	0.123088
1250	128.72	214.40	0.20930	21.186	0.109941
1300	138.26	227.38	0.21948	24.577	0.098563
1350	147.87	240.42	0.22932	28.370	0.088667
1400	157.55	253.51	0.23885	32.600	0.080020
1450	167.28	266.67	0.24808	37.302	0.072431
1500	177.07	279.89	0.25705	42.512	0.065746
1550	186.92	293.17	0.26575	48.270	0.059834
1600	196.82	306.50	0.27422	54.616	0.054587
1650	206.79	319.90	0.28246	61.593	0.049916
1700	216.81	333.34	0.29049	69.246	0.045745
1750	226.88	346.84	0.29832	77.622	0.042009
1800	237.01	360.40	0.30596	86.771	0.038654
1850	247.19	374.01	0.31341	96.742	0.035633
1900	257.42	387.67	0.32070	107.59	0.032906
1950	267.71	401.38	0.32782	119.37	0.030439
2000	278.04	415.14	0.33479	132.15	0.028201
2100	298.86	442.81	0.34829	160.91	0.024318
2200	319.86	470.67	0.36125	194.40	0.021087
2300	341.05	498.72	0.37372	233.17	0.018380
2400	362.42	526.94	0.38573	277.83	0.016096
2500	383.96	555.34	0.39732	329.01	0.014159
2600	405.67	583.90	0.40852	387.42	0.012505
2700	427.53	612.62	0.41936	453.78	0.011087
2800	449.55	641.49	0.42986	528.89	0.0098647
2900	471.71	670.51	0.44004	613.60	0.0088066
3000	494.02	699.67	0.44993	708.79	0.0078867
3100	516.46	728.97	0.45954	815.41	0.0070840
3200	539.04	758.39	0.46888	934.48	0.0063808
3300	561.73	787.95	0.47797	1067.0	0.0057627
3400	584.55	817.62	0.48683	1214.2	0.0052176
3500	607.48	847.41	0.49546	1377.2	0.0047354
3600	630.53	877.31	0.50389	1557.3	0.0043075
3700	653.67	907.31	0.51211	1755.7	0.0039269
3800	676.92	937.41	0.52014	1973.8	0.0035873
3900	700.27	967.61	0.52798	2213.1	0.0032836
4000	723.70	997.90	0.53565	2475.1	0.0030114
4500	842.10	1150.6	0.57161	4182.3	0.0020049
5000	962.19	1304.9	0.60413	6721.7	1.3861E-03
5500	1083.5	1460.6	0.63380	10361	9.8909E-04
6000	1205.8	1617.1	0.66104	15418	7.2513E-04
6500	1328.8	1774.4	0.68621	22258	5.4415E-04
7000	1452.2	1932.1	0.70959	31303	4.1668E-04
7500	1576.0	2090.2	0.73140	43032	3.2476E-04
8000	1700.2	2248.6	0.75185	57990	2.5706E-04

Ideal Gas Property Table: CO MW = 28.013 $lb_m/lbmole$ **AE**

T (°R)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
536.67	0	38.045	0	1	1
540	0.59148	38.873	0.0015372	1.0219	0.98462
550	2.3671	41.357	0.0060962	1.0898	0.94039
560	4.1422	43.841	0.010572	1.1608	0.89891
570	5.9168	46.325	0.014968	1.2351	0.85995
580	7.6912	48.808	0.019287	1.3127	0.82332
590	9.4656	51.292	0.023532	1.3937	0.78883
600	11.240	53.775	0.027706	1.4782	0.75633
610	13.015	56.259	0.031812	1.5663	0.72567
620	14.791	58.744	0.035852	1.6582	0.69671
630	16.567	61.229	0.039828	1.7539	0.66933
640	18.345	63.715	0.043743	1.8534	0.64342
650	20.123	66.202	0.047600	1.9571	0.61887
660	21.902	68.690	0.051399	2.0648	0.59561
670	23.683	71.180	0.055142	2.1768	0.57353
680	25.465	73.671	0.058833	2.2931	0.55256
690	27.248	76.163	0.062471	2.4139	0.53264
700	29.033	78.657	0.066060	2.5392	0.51368
710	30.820	81.153	0.069600	2.6692	0.49564
720	32.609	83.651	0.073093	2.8040	0.47846
730	34.399	86.150	0.076540	2.9438	0.46208
740	36.192	88.651	0.079944	3.0885	0.44645
750	37.986	91.155	0.083304	3.2384	0.43154
760	39.783	93.660	0.086623	3.3936	0.41729
770	41.581	96.168	0.089901	3.5543	0.40368
780	43.382	98.678	0.093139	3.7204	0.39066
790	45.186	101.19	0.096340	3.8922	0.37820
800	46.991	103.70	0.099503	4.0698	0.36628
810	48.800	106.22	0.10263	4.2533	0.35485
820	50.610	108.74	0.10572	4.4429	0.34391
830	52.423	111.26	0.10878	4.6387	0.33341
840	54.239	113.79	0.11180	4.8408	0.32334
850	56.057	116.32	0.11479	5.0494	0.31367
860	57.879	118.85	0.11775	5.2646	0.30438
870	59.702	121.38	0.12068	5.4866	0.29546
880	61.529	123.91	0.12358	5.7155	0.28689
890	63.358	126.45	0.12645	5.9515	0.27865
900	65.190	128.99	0.12928	6.1947	0.27072
910	67.025	131.54	0.13210	6.4453	0.26308
920	68.863	134.08	0.13488	6.7034	0.25573
930	70.704	136.63	0.13764	6.9692	0.24865
940	72.548	139.19	0.14037	7.2428	0.24183
950	74.395	141.74	0.14307	7.5245	0.23526
960	76.244	144.30	0.14575	7.8143	0.22892
970	78.097	146.86	0.14840	8.1125	0.22280
980	79.953	149.43	0.15103	8.4191	0.21690
990	81.812	151.99	0.15364	8.7345	0.21120
1000	83.674	154.57	0.15623	9.0588	0.20570
1010	85.539	157.14	0.15879	9.3920	0.20038
1020	87.408	159.72	0.16133	9.7345	0.19524
1030	89.279	162.30	0.16384	10.086	0.19028
1040	91.154	164.88	0.16634	10.448	0.18548

T (°R)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
1060	94.913	170.06	0.17127	11.200	0.17634
1080	98.685	175.25	0.17612	11.994	0.16779
1100	102.47	180.45	0.18089	12.829	0.15977
1120	106.27	185.67	0.18559	13.708	0.15224
1140	110.08	190.90	0.19022	14.633	0.14517
1160	113.90	196.14	0.19478	15.605	0.13851
1180	117.74	201.39	0.19927	16.626	0.13225
1200	121.59	206.66	0.20370	17.697	0.12635
1250	131.28	219.89	0.21450	20.610	0.11301
1300	141.05	233.21	0.22494	23.881	0.10143
1350	150.90	246.61	0.23506	27.543	0.091331
1400	160.84	260.09	0.24486	31.629	0.082478
1450	170.86	273.65	0.25438	36.175	0.074689
1500	180.96	287.30	0.26364	41.218	0.067811
1550	191.14	301.03	0.27264	46.799	0.061715
1600	201.41	314.83	0.28141	52.960	0.056295
1650	211.75	328.72	0.28995	59.744	0.051461
1700	222.17	342.68	0.29829	67.199	0.047139
1750	232.66	356.72	0.30642	75.373	0.043263
1800	243.22	370.83	0.31437	84.317	0.039779
1850	253.86	385.00	0.32214	94.083	0.036640
1900	264.56	399.25	0.32974	104.73	0.033805
1950	275.33	413.56	0.33718	116.31	0.031240
2000	286.15	427.94	0.34446	128.88	0.028915
2100	307.99	456.86	0.35857	157.27	0.024881
2200	330.04	486.00	0.37212	190.41	0.021529
2300	352.27	515.32	0.38515	228.84	0.018728
2400	374.67	544.81	0.39771	273.17	0.016371
2500	397.26	574.49	0.40982	324.06	0.014375
2600	419.99	604.31	0.42152	382.20	0.012676
2700	442.87	634.28	0.43283	448.31	0.011222
2800	465.88	664.37	0.44377	523.15	0.0099729
2900	489.00	694.59	0.45437	607.55	0.0088943
3000	512.24	724.91	0.46465	702.36	0.0079589
3100	535.57	755.34	0.47463	808.49	0.0071446
3200	559.01	785.86	0.48432	926.91	0.0064329
3300	582.53	816.47	0.49374	1058.6	0.0058085
3400	606.13	847.16	0.50290	1204.7	0.0052590
3500	629.81	877.93	0.51182	1366.2	0.0047736
3600	653.56	908.77	0.52051	1544.3	0.0043437
3700	677.38	939.68	0.52898	1740.3	0.0039616
3800	701.27	970.66	0.53724	1955.4	0.0036211
3900	725.22	1001.7	0.54530	2190.9	0.0033169
4000	749.23	1032.8	0.55318	2448.3	0.0030443
4500	870.07	1189.1	0.58999	4115.1	0.0020376
5000	992.03	1346.5	0.62315	6570.0	1.4181E-03
5500	1114.9	1504.8	0.65333	10057	1.0191E-03
6000	1238.6	1663.9	0.68102	14862	7.5227E-04
6500	1362.9	1823.7	0.70660	21318	5.6814E-04
7000	1487.8	1984.0	0.73036	29808	4.3757E-04
7500	1613.2	2144.9	0.75256	40767	3.4281E-04
8000	1739.0	2306.2	0.77338	54682	2.7261E-04

Ideal Gas Property Table: CO_2 MW = 44.010 $lb_m/lbmole$ **AE**

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
536.67	0	24.216	0	1	1
540	0.52155	24.888	0.0012480	1.0280	0.97876
550	2.0972	26.915	0.0049671	1.1164	0.91801
560	3.6869	28.956	0.0086445	1.2112	0.86155
570	5.2903	31.011	0.012281	1.3128	0.80903
580	6.9073	33.079	0.015878	1.4217	0.76015
590	8.5375	35.160	0.019436	1.5384	0.71463
600	10.181	37.255	0.022957	1.6632	0.67220
610	11.837	39.362	0.026440	1.7967	0.63263
620	13.506	41.483	0.029887	1.9393	0.59570
630	15.187	43.615	0.033300	2.0917	0.56122
640	16.881	45.760	0.036678	2.2543	0.52901
650	18.587	47.917	0.040022	2.4277	0.49889
660	20.305	50.087	0.043334	2.6126	0.47072
670	22.035	52.268	0.046614	2.8096	0.44435
680	23.776	54.460	0.049862	3.0193	0.41966
690	25.529	56.664	0.053080	3.2424	0.39652
700	27.294	58.880	0.056268	3.4798	0.37483
710	29.069	61.107	0.059426	3.7321	0.35448
720	30.856	63.345	0.062556	4.0002	0.33539
730	32.653	65.593	0.065658	4.2848	0.31746
740	34.462	67.853	0.068733	4.5869	0.30061
750	36.281	70.123	0.071780	4.9074	0.28477
760	38.110	72.404	0.074801	5.2472	0.26988
770	39.950	74.696	0.077796	5.6073	0.25587
780	41.801	76.997	0.080766	5.9888	0.24269
790	43.661	79.309	0.083711	6.3927	0.23027
800	45.532	81.631	0.086632	6.8202	0.21857
810	47.413	83.963	0.089529	7.2724	0.20754
820	49.303	86.305	0.092402	7.7505	0.19714
830	51.204	88.656	0.095252	8.2559	0.18733
840	53.113	91.017	0.098080	8.7898	0.17807
850	55.033	93.388	0.10089	9.3537	0.16933
860	56.962	95.768	0.10367	9.9489	0.16107
870	58.900	98.157	0.10643	10.577	0.15327
880	60.847	100.56	0.10917	11.239	0.14589
890	62.804	102.96	0.11189	11.938	0.13892
900	64.769	105.38	0.11459	12.674	0.13232
910	66.744	107.81	0.11727	13.450	0.12607
920	68.727	110.24	0.11994	14.267	0.12016
930	70.719	112.68	0.12258	15.127	0.11456
940	72.720	115.14	0.12520	16.032	0.10925
950	74.730	117.60	0.12780	16.984	0.10422
960	76.747	120.07	0.13039	17.986	0.099456
970	78.774	122.54	0.13296	19.039	0.094934
980	80.808	125.03	0.13551	20.146	0.090644
990	82.851	127.52	0.13804	21.308	0.086572
1000	84.902	130.03	0.14055	22.530	0.082706
1010	86.961	132.54	0.14305	23.812	0.079035
1020	89.028	135.05	0.14553	25.158	0.075548
1030	91.102	137.58	0.14799	26.570	0.072234
1040	93.185	140.11	0.15044	28.051	0.069083

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
1060	97.373	145.20	0.15529	31.233	0.063239
1080	101.59	150.33	0.16008	34.728	0.057948
1100	105.84	155.48	0.16480	38.562	0.053152
1120	110.12	160.66	0.16947	42.764	0.048802
1140	114.42	165.86	0.17408	47.363	0.044850
1160	118.76	171.10	0.17863	52.392	0.041256
1180	123.12	176.36	0.18313	57.884	0.037985
1200	127.51	181.65	0.18758	63.878	0.035004
1250	138.59	194.99	0.19847	81.315	0.028644
1300	149.83	208.49	0.20905	102.81	0.023560
1350	161.22	222.13	0.21935	129.17	0.019474
1400	172.75	235.92	0.22938	161.31	0.016171
1450	184.41	249.84	0.23915	200.31	0.013489
1500	196.20	263.89	0.24867	247.38	0.011299
1550	208.11	278.05	0.25796	303.94	0.0095026
1600	220.14	292.34	0.26703	371.60	0.0080231
1650	232.28	306.73	0.27589	452.19	0.0067992
1700	244.52	321.23	0.28454	547.80	0.0057825
1750	256.86	335.82	0.29301	660.79	0.0049347
1800	269.29	350.51	0.30128	793.82	0.0042251
1850	281.81	365.29	0.30938	949.89	0.0036290
1900	294.42	380.16	0.31731	1132.4	0.0031265
1950	307.12	395.11	0.32508	1345.0	0.0027015
2000	319.89	410.13	0.33268	1592.0	0.0023408
2100	345.65	440.41	0.34745	2208.6	1.7717E-03
2200	371.68	470.95	0.36166	3026.0	1.3547E-03
2300	397.96	501.75	0.37535	4098.5	1.0457E-03
2400	424.48	532.78	0.38856	5492.0	8.1428E-04
2500	451.22	564.03	0.40131	7286.0	6.3935E-04
2600	478.14	595.46	0.41364	9575.5	5.0594E-04
2700	505.24	627.08	0.42558	12474	4.0333E-04
2800	532.51	658.85	0.43713	16114	3.2377E-04
2900	559.92	690.78	0.44833	20655	2.6162E-04
3000	587.46	722.83	0.45920	26280	2.1271E-04
3100	615.14	755.02	0.46975	33205	1.7396E-04
3200	642.93	787.32	0.48001	41678	1.4307E-04
3300	670.83	819.73	0.48998	51988	1.1828E-04
3400	698.83	852.25	0.49969	64465	9.8276E-05
3500	726.93	884.86	0.50914	79489	8.2046E-05
3600	755.12	917.56	0.51836	97492	6.8806E-05
3700	783.39	950.35	0.52734	118968	5.7952E-05
3800	811.75	983.22	0.53610	144474	4.9010E-05
3900	840.18	1016.2	0.54466	174643	4.1611E-05
4000	868.68	1049.2	0.55302	210185	3.5461E-05
4500	1012.2	1215.2	0.59213	500088	1.6767E-05
5000	1157.0	1382.7	0.62741	1.0929E+06	8.5247E-06
5500	1303.0	1551.2	0.65954	2.2274E+06	4.6010E-06
6000	1450.0	1720.7	0.68904	4.2824E+06	2.6107E-06
6500	1597.7	1891.0	0.71630	7.8361E+06	1.5456E-06
7000	1746.2	2062.1	0.74165	1.3743E+07	9.4908E-07
7500	1895.3	2233.8	0.76534	2.3234E+07	6.0150E-07
8000	2045.1	2406.1	0.78759	3.8035E+07	3.9192E-07

Ideal Gas Property Table: H_2 MW = 2.016 $lb_m/lbmole$ AE

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
536.67	0	528.65	0	1	1
540	8.0991	540.03	0.021138	1.0217	0.98484
550	32.449	574.23	0.083893	1.0889	0.94117
560	56.838	608.47	0.14559	1.1593	0.90011
570	81.263	642.75	0.20625	1.2329	0.86146
580	105.72	677.06	0.26592	1.3099	0.82505
590	130.21	711.40	0.32463	1.3903	0.79072
600	154.73	745.76	0.38239	1.4743	0.75833
610	179.27	780.16	0.43924	1.5619	0.72773
620	203.84	814.57	0.49520	1.6532	0.69881
630	228.42	849.01	0.55030	1.7483	0.67145
640	253.03	883.47	0.60457	1.8473	0.64555
650	277.66	917.95	0.65802	1.9503	0.62101
660	302.30	952.44	0.71069	2.0574	0.59773
670	326.96	986.95	0.76258	2.1687	0.57565
680	351.63	1021.5	0.81373	2.2843	0.55468
690	376.32	1056.0	0.86415	2.4043	0.53476
700	401.02	1090.6	0.91386	2.5287	0.51581
710	425.72	1125.1	0.96288	2.6577	0.49778
720	450.44	1159.7	1.0112	2.7915	0.48061
730	475.17	1194.3	1.0589	2.9299	0.46426
740	499.91	1228.9	1.1060	3.0733	0.44866
750	524.65	1263.4	1.1524	3.2216	0.43379
760	549.40	1298.0	1.1982	3.3751	0.41959
770	574.16	1332.7	1.2435	3.5337	0.40603
780	598.92	1367.3	1.2881	3.6976	0.39307
790	623.69	1401.9	1.3323	3.8669	0.38068
800	648.47	1436.5	1.3758	4.0417	0.36882
810	673.25	1471.1	1.4188	4.2222	0.35747
820	698.03	1505.8	1.4613	4.4083	0.34661
830	722.82	1540.4	1.5033	4.6003	0.33619
840	747.62	1575.1	1.5448	4.7982	0.32621
850	772.41	1609.7	1.5858	5.0021	0.31663
860	797.22	1644.4	1.6263	5.2122	0.30745
870	822.02	1679.0	1.6664	5.4286	0.29862
880	846.83	1713.7	1.7060	5.6513	0.29015
890	871.64	1748.3	1.7452	5.8806	0.28201
900	896.46	1783.0	1.7839	6.1164	0.27418
910	921.28	1817.7	1.8222	6.3589	0.26665
920	946.10	1852.4	1.8601	6.6083	0.25941
930	970.93	1887.0	1.8976	6.8647	0.25244
940	995.76	1921.7	1.9347	7.1281	0.24572
950	1020.6	1956.4	1.9714	7.3987	0.23926
960	1045.4	1991.1	2.0077	7.6766	0.23302
970	1070.3	2025.8	2.0437	7.9619	0.22701
980	1095.1	2060.5	2.0793	8.2548	0.22121
990	1120.0	2095.2	2.1145	8.5553	0.21562
1000	1144.8	2129.9	2.1494	8.8637	0.21022
1010	1169.7	2164.6	2.1839	9.1799	0.20501
1020	1194.5	2199.3	2.2181	9.5042	0.19998
1030	1219.4	2234.0	2.2520	9.8367	0.19511
1040	1244.3	2268.7	2.2855	10.177	0.19041

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
1060	1294.0	2338.2	2.3517	10.884	0.18147
1080	1343.8	2407.7	2.4166	11.626	0.17310
1100	1393.6	2477.1	2.4803	12.403	0.16525
1120	1443.4	2546.7	2.5430	13.218	0.15789
1140	1493.2	2616.2	2.6045	14.070	0.15098
1160	1543.1	2685.8	2.6650	14.961	0.14447
1180	1593.0	2755.4	2.7245	15.892	0.13835
1200	1642.9	2825.0	2.7830	16.865	0.13258
1250	1767.9	2999.2	2.9253	19.485	0.11954
1300	1893.1	3173.7	3.0621	22.389	0.10819
1350	2018.6	3348.5	3.1940	25.597	0.098274
1400	2144.4	3523.5	3.3213	29.129	0.089557
1450	2270.5	3698.9	3.4444	33.006	0.081860
1500	2397.1	3874.7	3.5636	37.250	0.075034
1550	2524.0	4050.8	3.6791	41.885	0.068954
1600	2651.3	4227.4	3.7913	46.936	0.063520
1650	2779.1	4404.5	3.9003	52.426	0.058645
1700	2907.5	4582.1	4.0063	58.383	0.054257
1750	3036.3	4760.2	4.1095	64.835	0.050295
1800	3165.7	4938.8	4.2102	71.810	0.046707
1850	3295.7	5118.0	4.3084	79.338	0.043449
1900	3426.2	5297.8	4.4043	87.451	0.040484
1950	3557.4	5478.3	4.4980	96.182	0.037777
2000	3689.2	5659.3	4.5897	105.56	0.035302
2100	3954.9	6023.5	4.7674	126.43	0.030951
2200	4223.2	6390.4	4.9380	150.34	0.027266
2300	4494.4	6760.1	5.1024	177.64	0.024126
2400	4768.5	7132.6	5.2609	208.66	0.021432
2500	5045.5	7508.1	5.4142	243.79	0.019108
2600	5325.3	7886.5	5.5626	283.43	0.017093
2700	5608.1	8267.7	5.7065	328.00	0.015338
2800	5890.9	8649.0	5.8451	377.58	0.013818
2900	6177.4	9034.1	5.9803	433.09	0.012477
3000	6467.4	9422.6	6.1120	495.05	0.011292
3100	6760.6	9814.3	6.2404	564.00	0.010242
3200	7056.9	10209	6.3657	640.52	0.0093091
3300	7356.0	10607	6.4881	725.23	0.0084787
3400	7657.8	11007	6.6076	818.77	0.0077376
3500	7962.2	11410	6.7244	921.83	0.0070747
3600	8269.0	11815	6.8386	1035.1	0.0064804
3700	8578.1	12223	6.9503	1159.4	0.0059465
3800	8889.5	12633	7.0596	1295.5	0.0054657
3900	9203.1	13045	7.1666	1444.2	0.0050320
4000	9518.7	13459	7.2715	1606.4	0.0046399
4500	11126	15559	7.7660	2653.8	0.0031596
5000	12777	17703	8.2177	4197.8	0.0022194
5500	14469	19887	8.6339	6405.3	1.6000E-03
6000	16198	22108	9.0205	9483.1	1.1789E-03
6500	17962	24365	9.3816	13683	8.8514E-04
7000	19759	26654	9.7210	19311	6.7544E-04
7500	21588	28976	10.041	26731	5.2280E-04
8000	23446	31327	10.345	36376	4.0980E-04

Ideal Gas Property Table: H_2O MW = 18.016 lb_m/lbmole **AE**

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
536.67	0	59.157	0	1	1
900	125.18	224.39	0.23464	8.4037	0.19956
910	128.75	229.06	0.23980	8.8068	0.19254
920	132.33	233.74	0.24492	9.2251	0.18583
930	135.92	238.43	0.24999	9.6592	0.17941
940	139.51	243.13	0.25501	10.110	0.17326
950	143.11	247.83	0.25999	10.577	0.16737
960	146.72	252.54	0.26492	11.061	0.16173
970	150.34	257.26	0.26982	11.563	0.15632
980	153.97	261.99	0.27466	12.083	0.151133
990	157.60	266.73	0.27947	12.621	0.14616
1000	161.24	271.47	0.28424	13.179	0.14138
1010	164.89	276.23	0.28897	13.757	0.13680
1020	168.55	280.99	0.29366	14.355	0.13240
1030	172.22	285.75	0.29831	14.974	0.12817
1040	175.89	290.53	0.30293	15.614	0.12411
1050	179.58	295.32	0.30751	16.276	0.12021
1060	183.27	300.11	0.31205	16.961	0.11645
1070	186.97	304.91	0.31656	17.670	0.11284
1080	190.68	309.72	0.32104	18.402	0.10936
1090	194.39	314.54	0.32548	19.158	0.10601
1100	198.12	319.37	0.32989	19.940	0.10279
1110	201.85	324.21	0.33426	20.748	0.099688
1120	205.59	329.05	0.33861	21.582	0.096698
1130	209.35	333.90	0.34292	22.443	0.093817
1140	213.11	338.77	0.34720	23.333	0.091040
1150	216.87	343.64	0.35146	24.251	0.088362
1160	220.65	348.52	0.35568	25.198	0.085779
1170	224.44	353.40	0.35988	26.176	0.083287
1180	228.23	358.30	0.36405	27.184	0.080883
1190	232.03	363.21	0.36819	28.225	0.078562
1200	235.85	368.12	0.37230	29.297	0.076321
1210	239.67	373.04	0.37638	30.404	0.074157
1220	243.50	377.97	0.38044	31.544	0.072066
1230	247.33	382.92	0.38448	32.720	0.070047
1240	251.18	387.86	0.38848	33.931	0.068095
1250	255.04	392.82	0.39247	35.180	0.066208
1260	258.90	397.79	0.39642	36.466	0.064384
1270	262.78	402.77	0.40036	37.791	0.062620
1280	266.66	407.75	0.40427	39.155	0.060913
1290	270.55	412.75	0.40815	40.560	0.059262
1300	274.45	417.75	0.41202	42.007	0.057665
1310	278.36	422.76	0.41586	43.497	0.056119
1320	282.28	427.78	0.41968	45.030	0.054622
1330	286.21	432.81	0.42347	46.608	0.053172
1340	290.15	437.85	0.42725	48.232	0.051768
1350	294.09	442.90	0.43100	49.902	0.050409
1360	298.05	447.96	0.43473	51.621	0.049091
1370	302.01	453.03	0.43845	53.389	0.047815
1380	305.99	458.10	0.44214	55.207	0.046577
1390	309.97	463.19	0.44581	57.077	0.045378
1400	313.96	468.28	0.44946	59.000	0.044215

T (^oR)	$\hat{U}^o$ (Btu/lb_m)	$\hat{H}^o$ (Btu/lb_m)	$\hat{S}^o$ ($Btu/lb_m\ ^oR$)	P_r	V_r
1420	321.97	478.50	0.45671	63.008	0.041994
1440	330.02	488.75	0.46388	67.242	0.039904
1460	338.10	499.04	0.47097	71.714	0.037935
1480	346.23	509.36	0.47800	76.433	0.036081
1500	354.38	519.73	0.48495	81.411	0.034332
1520	362.58	530.13	0.49184	86.660	0.032683
1540	370.81	540.57	0.49866	92.193	0.031125
1560	379.08	551.04	0.50542	98.022	0.029655
1580	387.39	561.55	0.51212	104.16	0.028265
1600	395.74	572.10	0.51875	110.62	0.026950
1650	416.76	598.64	0.53508	128.29	0.023965
1700	438.02	625.41	0.55107	148.31	0.021359
1750	459.51	652.41	0.56672	170.94	0.019076
1800	481.24	679.65	0.58207	196.47	0.017071
1850	503.20	707.12	0.59712	225.22	0.015306
1900	525.39	734.82	0.61189	257.53	0.013747
1950	547.81	762.76	0.62641	293.77	0.012369
2000	570.46	790.92	0.64067	334.34	0.011146
2050	593.34	819.31	0.65469	379.69	0.010060
2100	616.45	847.93	0.66848	430.30	0.0090936
2150	639.79	876.78	0.68206	486.70	0.0082314
2200	663.34	905.85	0.69542	549.43	0.0074611
2250	687.12	935.13	0.70858	619.13	0.0067717
2300	711.12	964.64	0.72156	696.44	0.0061537
2400	759.75	1024.3	0.74694	876.82	0.0051003
2500	809.22	1084.8	0.77164	1097.0	0.0042465
2600	859.50	1146.1	0.79568	1364.4	0.0035509
2700	910.55	1208.2	0.81911	1687.4	0.0029814
2800	962.35	1271.0	0.84195	2076.0	0.0025131
2900	1014.8	1334.5	0.86424	2541.3	0.0021264
3000	1068.0	1398.7	0.88600	3095.8	0.0018057
3100	1121.8	1463.5	0.90724	3753.9	1.5388E-03
3200	1176.2	1528.9	0.92801	4532.1	1.3157E-03
3300	1231.2	1594.9	0.94832	5449.3	1.1284E-03
3400	1286.7	1661.5	0.96820	6526.1	9.7077E-04
3500	1342.8	1728.6	0.98766	7786.1	8.3761E-04
3600	1399.4	1796.3	1.0067	9255.3	7.2478E-04
3700	1456.5	1864.4	1.0254	10963	6.2888E-04
3800	1514.1	1933.0	1.0437	12942	5.4713E-04
3900	1572.1	2002.0	1.0616	15228	4.7723E-04
4000	1630.5	2071.4	1.0792	17861	4.1730E-04
4500	1928.3	2424.3	1.1623	37963	2.2088E-04
5000	2234.2	2785.4	1.2384	75690	1.2309E-04
5500	2546.8	3153.1	1.3084	142940	7.1697E-05
6000	2864.9	3526.3	1.3734	257641	4.3394E-05
6500	3187.6	3904.1	1.4339	445964	2.7159E-05
7000	3514.2	4285.8	1.4904	745055	1.7507E-05
7500	3844.1	4670.8	1.5435	1206399	1.1584E-05
8000	4176.8	5058.7	1.5936	1899904	7.8460E-06

Ideal Gas Property Table: N_2 MW = 28.010 lb$_m$/lbmole **AE**

T (°R)	$\hat{U}^o$ (Btu/lb$_m$)	$\hat{H}^o$ (Btu/lb$_m$)	$\hat{S}^o$ (Btu/lb$_m$ °R)	P_r	V_r
536.67	0	38.049	0	1	1
540	0.58381	38.869	0.0015230	1.0217	0.98482
550	2.3382	41.333	0.0060431	1.0890	0.94110
560	4.0944	43.798	0.010485	1.1594	0.90002
570	5.8526	46.265	0.014852	1.2330	0.86137
580	7.6128	48.734	0.019146	1.3100	0.82497
590	9.3750	51.206	0.023371	1.3905	0.79066
600	11.139	53.679	0.027528	1.4744	0.75827
610	12.906	56.154	0.031619	1.5620	0.72767
620	14.675	58.632	0.035648	1.6533	0.69875
630	16.446	61.112	0.039616	1.7485	0.67137
640	18.219	63.594	0.043525	1.8476	0.64544
650	19.994	66.079	0.047377	1.9508	0.62086
660	21.772	68.566	0.051174	2.0581	0.59754
670	23.553	71.055	0.054918	2.1697	0.57540
680	25.335	73.547	0.058609	2.2857	0.55436
690	27.120	76.041	0.062250	2.4061	0.53435
700	28.908	78.537	0.065842	2.5312	0.51531
710	30.698	81.036	0.069387	2.6609	0.49718
720	32.491	83.538	0.072886	2.7955	0.47991
730	34.286	86.042	0.076340	2.9351	0.46344
740	36.083	88.549	0.079751	3.0797	0.44772
750	37.884	91.058	0.083119	3.2296	0.43272
760	39.686	93.570	0.086445	3.3847	0.41839
770	41.492	96.084	0.089732	3.5453	0.40469
780	43.300	98.601	0.092980	3.7115	0.39159
790	45.110	101.12	0.096190	3.8834	0.37906
800	46.923	103.64	0.099362	4.0611	0.36706
810	48.739	106.17	0.10250	4.2448	0.35556
820	50.558	108.69	0.10560	4.4346	0.34455
830	52.379	111.23	0.10867	4.6306	0.33399
840	54.203	113.76	0.11170	4.8331	0.32385
850	56.029	116.29	0.11470	5.0420	0.31413
860	57.858	118.83	0.11767	5.2576	0.30479
870	59.690	121.37	0.12061	5.4800	0.29582
880	61.524	123.92	0.12351	5.7093	0.28721
890	63.361	126.46	0.12639	5.9457	0.27892
900	65.201	129.01	0.12924	6.1894	0.27095
910	67.043	131.56	0.13206	6.4405	0.26328
920	68.888	134.12	0.13485	6.6991	0.25590
930	70.736	136.67	0.13761	6.9654	0.24879
940	72.587	139.23	0.14035	7.2396	0.24194
950	74.440	141.79	0.14306	7.5218	0.23534
960	76.295	144.36	0.14575	7.8122	0.22898
970	78.154	146.93	0.14841	8.1110	0.22284
980	80.015	149.50	0.15104	8.4182	0.21692
990	81.879	152.07	0.15366	8.7342	0.21121
1000	83.746	154.64	0.15624	9.0589	0.20569
1010	85.615	157.22	0.15881	9.3927	0.20037
1020	87.487	159.80	0.16135	9.7357	0.19522
1030	89.361	162.39	0.16387	10.088	0.19025
1040	91.238	164.97	0.16637	10.450	0.18544
1060	95.001	170.15	0.17131	11.203	0.17630
1080	98.774	175.35	0.17616	11.997	0.16775
1100	102.56	180.55	0.18093	12.832	0.15973
1120	106.35	185.76	0.18563	13.710	0.15222
1140	110.16	190.98	0.19025	14.634	0.14515
1160	113.97	196.22	0.19480	15.604	0.13852
1180	117.80	201.46	0.19928	16.623	0.13227
1200	121.63	206.71	0.20370	17.691	0.12639
1250	131.27	219.90	0.21446	20.590	0.11312
1300	140.97	233.14	0.22485	23.840	0.10161
1350	150.74	246.45	0.23489	27.469	0.091575
1400	160.56	259.82	0.24462	31.508	0.082793
1450	170.45	273.26	0.25405	35.990	0.075072
1500	180.40	286.75	0.26320	40.948	0.068258
1550	190.42	300.31	0.27209	46.418	0.062221
1600	200.49	313.93	0.28073	52.438	0.056854
1650	210.62	327.60	0.28915	59.047	0.052069
1700	220.80	341.33	0.29735	66.286	0.047788
1750	231.05	355.12	0.30534	74.198	0.043948
1800	241.35	368.97	0.31314	82.827	0.040494
1850	251.70	382.87	0.32076	92.222	0.037379
1900	262.11	396.82	0.32820	102.43	0.034564
1950	272.58	410.83	0.33548	113.50	0.032013
2000	283.09	424.89	0.34260	125.49	0.029697
2100	304.28	453.17	0.35640	152.45	0.025668
2200	325.66	481.64	0.36964	183.76	0.022308
2300	347.24	510.30	0.38238	219.94	0.019486
2400	369.00	539.16	0.39466	261.53	0.017100
2500	390.94	568.18	0.40651	309.10	0.015071
2600	413.05	597.39	0.41796	363.29	0.013336
2700	435.32	626.75	0.42905	424.76	0.011844
2800	457.76	656.27	0.43978	494.21	0.010557
2900	480.34	685.95	0.45020	572.40	0.0094405
3000	503.07	715.77	0.46031	660.12	0.0084682
3100	525.94	745.73	0.47013	758.23	0.0076183
3200	548.95	775.82	0.47968	867.61	0.0068726
3300	572.08	806.04	0.48898	989.20	0.0062162
3400	595.33	836.39	0.49804	1124.0	0.0056364
3500	618.70	866.84	0.50687	1273.1	0.0051228
3600	642.17	897.41	0.51548	1437.5	0.0046666
3700	665.76	928.08	0.52389	1618.4	0.0042601
3800	689.44	958.86	0.53209	1817.0	0.0038970
3900	713.22	989.72	0.54011	2034.5	0.0035719
4000	737.08	1020.7	0.54795	2272.3	0.0032801
4500	857.60	1176.6	0.58468	3815.0	0.0021979
5000	979.68	1334.2	0.61787	6092.8	1.5291E-03
5500	1102.9	1492.8	0.64811	9333.2	1.0981E-03
6000	1226.8	1652.2	0.67584	13800	8.1012E-04
6500	1351.1	1811.9	0.70142	19795	6.1185E-04
7000	1475.6	1971.9	0.72513	27658	4.7160E-04
7500	1600.3	2132.0	0.74722	37768	3.7002E-04
8000	1724.9	2292.1	0.76789	50553	2.9488E-04

Ideal Gas Property Table: O_2 MW = 32.000 lb$_m$/lbmole AE

T (°R)	$\hat{U}^o$ (Btu/lb$_m$)	$\hat{H}^o$ (Btu/lb$_m$)	$\hat{S}^o$ (Btu/lb$_m$ °R)	P_r	V_r
536.67	0	33.305	0	1	1
540	0.51252	34.024	0.0013359	1.0218	0.98478
550	2.0576	36.190	0.0053097	1.0893	0.94080
560	3.6113	38.364	0.0092274	1.1603	0.89930
570	5.1733	40.547	0.013091	1.2348	0.86012
580	6.7433	42.737	0.016900	1.3130	0.82310
590	8.3210	44.936	0.020658	1.3950	0.78809
600	9.9061	47.141	0.024365	1.4808	0.75498
610	11.498	49.354	0.028023	1.5707	0.72363
620	13.097	51.574	0.031632	1.6648	0.69393
630	14.703	53.800	0.035195	1.7632	0.66579
640	16.316	56.033	0.038711	1.8660	0.63910
650	17.934	58.272	0.042183	1.9733	0.61377
660	19.559	60.518	0.045610	2.0854	0.58973
670	21.189	62.769	0.048996	2.2023	0.56688
680	22.826	65.026	0.052339	2.3242	0.54516
690	24.467	67.288	0.055642	2.4513	0.52451
700	26.115	69.556	0.058905	2.5836	0.50485
710	27.767	71.829	0.062129	2.7214	0.48614
720	29.425	74.107	0.065316	2.8648	0.46831
730	31.087	76.390	0.068465	3.0139	0.45132
740	32.755	78.679	0.071578	3.1689	0.43512
750	34.427	80.972	0.074656	3.3301	0.41966
760	36.104	83.269	0.077699	3.4974	0.40491
770	37.786	85.571	0.080709	3.6712	0.39082
780	39.472	87.878	0.083685	3.8516	0.37735
790	41.163	90.189	0.086629	4.0387	0.36448
800	42.857	92.504	0.089542	4.2328	0.35218
810	44.556	94.824	0.092423	4.4339	0.34040
820	46.259	97.148	0.095274	4.6424	0.32913
830	47.967	99.475	0.098096	4.8583	0.31834
840	49.678	101.81	0.10089	5.0819	0.30800
850	51.393	104.14	0.10365	5.3134	0.29809
860	53.111	106.48	0.10639	5.5529	0.28858
870	54.834	108.83	0.10910	5.8007	0.27947
880	56.560	111.17	0.11178	6.0569	0.27072
890	58.290	113.52	0.11444	6.3217	0.26233
900	60.024	115.88	0.11707	6.5954	0.25427
910	61.761	118.23	0.11967	6.8782	0.24652
920	63.502	120.60	0.12225	7.1702	0.23908
930	65.246	122.96	0.12481	7.4718	0.23193
940	66.993	125.33	0.12734	7.7830	0.22505
950	68.744	127.70	0.12985	8.1042	0.21843
960	70.498	130.07	0.13234	8.4355	0.21206
970	72.255	132.45	0.13480	8.7772	0.20593
980	74.016	134.83	0.13724	9.1295	0.20002
990	75.780	137.22	0.13966	9.4926	0.19433
1000	77.547	139.61	0.14206	9.8668	0.18885
1010	79.316	142.00	0.14444	10.252	0.18357
1020	81.089	144.39	0.14680	10.649	0.17847
1030	82.866	146.79	0.14914	11.058	0.17356
1040	84.645	149.19	0.15146	11.479	0.16882

T (°R)	$\hat{U}^o$ (Btu/lb$_m$)	$\hat{H}^o$ (Btu/lb$_m$)	$\hat{S}^o$ (Btu/lb$_m$ °R)	P_r	V_r
1060	88.211	153.99	0.15604	12.358	0.15982
1080	91.790	158.81	0.16054	13.289	0.15144
1100	95.380	163.64	0.16497	14.273	0.14361
1120	98.981	168.49	0.16934	15.312	0.13630
1140	102.59	173.34	0.17363	16.409	0.12945
1160	106.22	178.20	0.17786	17.566	0.12305
1180	109.85	183.08	0.18203	18.786	0.11704
1200	113.49	187.96	0.18613	20.071	0.11141
1250	122.65	200.22	0.19614	23.583	0.098766
1300	131.86	212.54	0.20580	27.556	0.087907
1350	141.14	224.92	0.21514	32.033	0.078530
1400	150.47	237.35	0.22419	37.059	0.070393
1450	159.86	249.84	0.23295	42.681	0.063303
1500	169.30	262.39	0.24146	48.952	0.057098
1550	178.80	274.99	0.24972	55.922	0.051646
1600	188.34	287.64	0.25776	63.650	0.046840
1650	197.94	300.34	0.26557	72.194	0.042587
1700	207.59	313.09	0.27318	81.616	0.038812
1750	217.29	325.89	0.28060	91.982	0.035451
1800	227.03	338.73	0.28784	103.36	0.032450
1850	236.82	351.63	0.29491	115.82	0.029762
1900	246.65	364.57	0.30181	129.45	0.027350
1950	256.54	377.55	0.30855	144.31	0.025179
2000	266.46	390.58	0.31515	160.49	0.023220
2100	286.44	416.76	0.32793	197.18	0.019845
2200	306.59	443.11	0.34018	240.24	0.017063
2300	326.89	469.63	0.35197	290.49	0.014753
2400	347.36	496.30	0.36332	348.79	0.012822
2500	367.98	523.12	0.37427	416.09	0.011195
2600	388.74	550.09	0.38485	493.42	0.0098186
2700	409.65	577.21	0.39508	581.87	0.0086463
2800	430.70	604.46	0.40499	682.63	0.0076430
2900	451.88	631.85	0.41460	796.97	0.0067803
3000	473.19	659.37	0.42393	926.26	0.0060351
3100	494.63	687.02	0.43300	1071.9	0.0053887
3200	516.20	714.79	0.44182	1235.6	0.0048258
3300	537.88	742.68	0.45040	1418.8	0.0043338
3400	559.69	770.69	0.45876	1623.5	0.0039023
3500	581.60	798.81	0.46691	1851.4	0.0035226
3600	603.63	827.04	0.47486	2104.5	0.0031874
3700	625.77	855.38	0.48263	2385.0	0.0028907
3800	648.01	883.83	0.49022	2695.2	0.0026272
3900	670.35	912.38	0.49763	3037.2	0.0023927
4000	692.79	941.02	0.50488	3413.7	0.0021833
4500	806.38	1085.6	0.53895	5910.3	1.4187E-03
5000	922.09	1232.4	0.56986	9726.7	9.5785E-04
5500	1039.6	1381.0	0.59818	15352	6.6757E-04
6000	1158.8	1531.2	0.62433	23394	4.7791E-04
6500	1279.5	1682.9	0.64861	34599	3.5006E-04
7000	1401.6	1836.0	0.67130	49871	2.6154E-04
7500	1525.0	1990.4	0.69261	70298	1.9880E-04
8000	1649.7	2146.1	0.71270	97181	1.5339E-04

Gas Phase Heat Capacity (Shomate Equation)

$$C_p^\circ = A + (B/1000)\,T + (C/1000^{2})\,T^{2} + (D/1000^{3})\,T^{3} + (E/1000^{-2})\,T^{-2}$$

C_p = heat capacity (J/mol*K)

T = temperature (K)

Compound	Symbol	Temperature Range (K)	A	B	C	D	E
Air *	N_2 (g) / O_2 (g)	298 - 6000	26.84107	7.7816776	-1.8103208	0.14594026	-0.01102637
Ammonia	NH_3 (g)	298 - 1400	19.99563	49.77119	-15.37599	1.921168	0.189174
Ammonia	NH_3 (g)	1400 - 6000	52.02427	18.48801	-3.765128	0.248541	-12.45799
Oxygen	O_2 (g)	298 - 6000	29.659	6.137261	-1.186521	0.09578	-0.219663
Carbon Dioxide	CO_2(g)	298 - 1200	24.99735	55.18696	-33.69137	7.948387	-0.136638
Carbon Dioxide	CO_2(g)	1200 - 6000	58.16639	2.720074	-0.492289	0.038844	-6.447293
Carbon Monoxide	CO (g)	298 - 1300	25.56759	6.09613	4.054656	-2.671301	0.131021
Carbon Monoxide	CO (g)	1300 - 6000	35.1507	1.300095	-0.205921	0.01355	-3.28278
Methane	CH_4 (g)	298 - 1300	-0.703029	108.4773	-42.52157	5.862788	0.678565
Methane	CH_4 (g)	1300 - 6000	85.81217	11.26467	-2.114146	0.13819	-26.42221
Nitrogen	N_2 (g)	298 - 6000	26.092	8.218801	-1.976141	0.159274	0.044434
Nitrogen Dioxide	NO_2 (g)	298 - 1200	16.10857	75.89525	-54.3874	14.30777	0.239423
Nitrogen Dioxide	NO_2 (g)	1200 - 6000	56.82541	0.738053	-0.144721	0.009777	-5.459911
Nitric Oxide	NO (g)	298 - 1200	23.83491	12.58878	-1.139011	-1.497459	0.214194
Nitric Oxide	NO (g)	1200 - 6000	35.99169	0.95717	-0.148032	0.009974	-3.004088
Nitrous Oxide	N_2O (g)	298 - 1400	27.67988	51.14898	-30.64454	6.847911	-0.157906
Nitrous Oxide	N_2O (g)	1400 - 6000	60.30274	1.034566	-0.192997	0.01254	-6.860254
Sulfuric Acid	H_2SO_4 (g)	298 - 1200	47.28924	190.3314	-148.1299	43.86631	-0.740016
Sulfuric Acid	H_2SO_4 (g)	1200 - 6000	139.2289	9.513663	-1.795577	0.118069	-15.61486
Hydrogen	H_2 (g)	298 - 1500	33.1078	-11.508	11.6093	-2.8444	-0.159665
Hydrogen	H_2 (g)	1500 - 6000	34.1434	0.503927	0.372036	-0.038599	-8.074761
Hydrogen	H (g)	298 - 6000	20.78603	0	0	0	0
Helium	He (g)	298 - 6000	20.78603	0	0	0	0
Water	H_2O(g)	500 - 1700	30.092	6.832514	6.793435	-2.53448	0.082139
Water	H_2O(g)	1700 - 6000	41.96426	8.622053	-1.49978	0.098119	-11.15764
Sulfur	S_2(g)	298 - 6000	33.51313	5.06536	-1.05967	0.089905	-0.211911
Sulfur Dioxide	SO_2(g)	298 - 1200	21.43049	74.35094	-57.75217	16.35534	0.086731
Sulfur Dioxide	SO_2(g)	1200 - 6000	57.48188	1.009328	-0.07629	0.005174	-4.045401

* Air is assumed to consist of 79 mole% N_2 and 21 mole% O_2.

For Metric Units

Unit Prefix	Multiple	Unit Prefix	Multiple
tetra (T)	10^{12}	deci (d)	10^{-1}
giga (G)	10^{9}	centi (c)	10^{-2}
mega (M)	10^{6}	milli (m)	10^{-3}
kilo (k)	10^{3}	micro (μ)	10^{-6}
hecto (h)	10^{2}	nano (n)	10^{-9}
deka (da)	10^{1}	pico (p)	10^{-12}

Conversion Factors

Length	m	ft	in	yd	mi
1 m	1	3.2808	39.3701	1.0936	6.2137E-04
1 ft	0.30480	1	12	0.33333	1.894E-04
1 in	0.02540	0.08333	1	0.027777	1.5782E-05
1 yd	0.91440	3	36	1	5.6812E-04
1 mi	1609.34	5280	63360	1760	1

Mass	kg	$\mathrm{lb_m}$	slug	ton	tonne
1 kg	1	2.20462	0.06852	0.001102	0.001
1 $\mathrm{lb_m}$	0.453592	1	0.031081	0.00050	4.5359E-04
1 slug	14.5939	32.174	1	0.01609	0.01459
1 ton	907.1847	2000	62.1619	1	0.90718
1 tonne	1000	2204.62	68.5218	1.10231	1

Area	1 $\mathrm{m^2}$	1 $\mathrm{ft^2}$	1 $\mathrm{in^2}$
1 $\mathrm{m^2}$	1	10.7639	1550.00
1 $\mathrm{ft^2}$	0.092903	1	144
1 $\mathrm{in^2}$	0.00064516	0.0069444	1

Example

1 $\mathrm{m^2}$ = 10.7639 $\mathrm{ft^2}$

Volume	$\mathrm{m^3}$	$\mathrm{ft^3}$	L	US gallon
1 $\mathrm{m^3}$	1	35.3147	1000	264.172
1 $\mathrm{ft^3}$	0.028317	1	28.317	7.48052
1 L	0.001	0.035315	1	0.264172
1 US gallon	0.0037854	0.133681	3.78541	1

Velocity	m/s	ft/s	mi/h
1 m/s	1	3.28084	2.23964
1 ft/s	0.30480	1	0.68182
1 mi/h	0.44704	1.46667	1

Temperature	K	R
1 K	1	1/1.8
1 R	1.8	1

Example

$T(^{o}\mathrm{C}) = T(\mathrm{K}) - 273.15 = [T(^{0}\mathrm{F}) - 32]/1.8$

$T(^{o}\mathrm{F}) = T(^{o}\mathrm{R}) - 459.67 = 1.8\,T(^{o}\mathrm{C}) + 32$

Conversion Factors

Density

	kg/m^3	lb_m/ft^3	lb_m/in^3
1 kg/m^3	1	0.06243	3.613E-05
1 lb_m/ft^3	16.0180	1	5.787E-04
1 lb_m/in^3	27679	1728	1

Force

	N	lb_f	dyne	kg_f
1 N	1	0.224809	100000	0.101972
1 lb_f	4.44822	1	444822	0.4536
1 dyne	0.00001	2.2481E-06	1	1.01972E-06
1 kg_f	9.80665	2.20462	980665	1

1 N = $kg\text{-}m/s^2$

Pressure

	Pa	bar	atm	torr	lb_f/in^2
1 Pa	1	1.0E-05	9.86923E-06	0.00750064	1.45037E-04
1 bar	1.0E+05	1	0.986923	750.064	14.5038
1 atm	101325	1.01325	1	760	14.6959
1 torr	133.322	1.33322E-03	1.31579E-03	1	0.0193367
1 lb_f/in^2	6894.76	0.068948	0.0680461	51.7151	1

1 Pa = 1 N/m^2 = 1 $kg/m\text{-}s^2$

Energy

	J	cal	Btu	lb_f ft	erg	eV
1 J	1	0.238845	9.47817E-04	0.73756	1.00E+07	6.242E+18
1 cal	4.1868	1	0.0039683	3.088025	4.1868E+07	2.613E+19
1 Btu	1055.055	251.996	1	778.169	1.055E+10	6.585E+21
1 lb_f ft	1.355818	0.323831	0.00128507	1	1.35582E+07	8.462E+18
1erg	1.00E-07	2.388E-08	9.478E-11	7.376E-08	1	6.242E+11
1 eV	1.6022E-19	3.827E-20	1.5185E-22	1.182E-19	1.6022E-12	1

1 J = N-m = $Pa\text{-}m^3$

Power

	W	hp	Btu/hr	lb_f ft/s
1 W	1	0.001341	3.413	0.73756
1 hp	745.7	1	2544.53	550.001
1 Btu/hr	0.293000	3.9003	1	0.216150
1 lb_f ft/s	1.35582	0.00181818	4.62641	1

mechanical hp

Examples

1 Btu = 1055.1 J

1 hP = 745.7 W

Physical Constants

Universal Gas Constant

8.31434621	J / mol K
8.314344621	Pa m^3 / mol K
1.985884	cal / mol K
0.08205746	atm L / mol K
1.985884	Btu / lbmol ^{o}R
1545.34896	lb_f ft / lbmol ^{o}R
10.73159	psi ft^3 / lbmol ^{o}R
0.7302413	atm ft^3 / lbmol ^{o}R

Standard Acceleration of Gravity

9.80665 m/s^2

32.174 ft/s^2